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BOHR'S MODELLING OF THE ATOM: A RECONSTRUCTION AND ASSESSMENT*

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Standard quantum mechanics notwithstanding, Bohr's celebrated model of the hydrogen atom is still taught as a paradigmatic example of successful modelling in physics. The seminal articles published by Bohr in 1913–1914 remain an inexhaustible source of inspiration for philosophers and historians of science alike who relish using them in a flexible way in order to support their cherished views on theory construction and heuristics. In deference to such a well- established tradition, I will have a close look at Bohr's original papers and propose a reconstruction of his *démarche* in the framework of the model-theoretic approach of theories. I will then argue that, contrary to widespread belief, Bohr's model is not inconsistent and that it can be interpreted to support the moderate and selective version of scientific realism that I favour.

1. *What is a model?*

In mathematics, a model is defined as a set-theoretical structure that satisfies some statements (Tarski 1953, Suppes 2002). A structure M is a set of individuals — which form a *domain* D — linked by a set of relations $r_1, r_2, \ldots, r_n$:

$$\mathcal{M} = \langle D, r_1, r_2, r_3, \ldots \rangle \text{ or } \mathcal{M} = \langle D, r_i \rangle \ (1 \leq i \leq n).$$

A structure becomes a model with respect to a set of propositions that it makes true or satisfies. This is the first face of the Janus-like (da Costa & French 2003), nature of models. But, in the empirical sciences — and this is the second face — scientists devote special interest to the *representational* function that models can perform. A preliminary condition for the success

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of scientific representation by means of mathematical structures is that we see entities (things, processes etc.) in the world as *systems, i.e.* sets of elements organized by some relations. This inaugural move may be called the *primary* or *original abstraction* (Ghins 2009). Then, as a further abstracting step, called *secondary abstraction*, we divide those entities into some parts which stand in specific relations among themselves. As scientists, we are interested only in some aspects of entities which we consider to be relevant. The relevant aspects bear on some division into parts, selected properties of those parts and selected relations among these properties. Thus, the domains involved in scientific structures are *properties*, and more specifically, numerical values of those properties.

Following Suppes (2002) and da Costa & French (2003), I wish to maintain that *homomorphism* is a necessary (*albeit* certainly not sufficient) ingredient for any successful representation in science. Let us first recall the definition of isomorphism, which is more restrictive than the notion of homomorphism. An isomorphism is a one-one function between two structures that preserve relations, *i.e.* the form. For example, if two elements of the domain D of a structure M stand in some relation r, their images in the targeted domain D^* of M^* stand in a corresponding relation r^*.

Mathematically, two structures $\mathcal{M} = \langle D, r_i \rangle$ and $\mathcal{M}^* = \langle D^*, r_i^* \rangle$ are said to be isomorphic if and only if there exists a one-one function f such that for all r_i and for all n-uple $(a_1, ..., a_n)$ of elements of D which stand in relation r_i, there exists a n-uple $(a_1^*, ..., a_n^*)$ of elements of D^* which stand in a r^*i such that $a_1^* = f(a_1), ..., a_n^* = f(a_n)$ (Suppes 2002, 54-57). We can then choose the structure $\mathcal{M}$ to represent the structure $\mathcal{M}^*$, and vice-versa. Now, a homomorphism is a function that, just as an isomorphism does, leaves relations invariant but is a many-one function. Two structures are homomorphic or *structurally similar* if some homomorphism has been constructed between them. Leaving the field of pure mathematics to enter the realm of empirical sciences, I claim that a mathematical structure becomes a model or a representation of a real (or imagined) system only if we have constructed a homomorphism or an isomorphism between these two structures.

For the reader who might be reticent with respect to what seems to be too restrictive a view of representation let me point out that the homomorphism condition is extremely *weak*. It is in fact possible to construct a homomorphism between any mathematical structure and any entity in the world provided we divide the latter into parts having properties standing in some relations. For an isomorphism to be constructed, we must deal with sets with the same cardinality. Isomorphism (*a fortiori* homomorphism) is cheap. For example, I could decide to construct an isomorphism between my office and the Milky Way by means of a suitable division of both into some parts (provided I make the two sets have the same cardinality), a selection of spatial relations between the elements of the respective domains and an appropriate mapping between them. Obviously, for a model to be useful it has to be informative[1], which necessitates additional requirements besides the construction of an isomorphism (or homomorphism). These requirements comprise the assumption that some propositions are true. We will come back to this all-important issue in due course.

2. *Modelling the observable: planetary motions*

Suppose we want to model the observable entity in the world we call the sky. First of all, we must look at the sky as a system and not as, say, the metaphorical dwelling of God. We then isolate in the nocturnal sky some bright spots seen to be moving with respect to other fixed bright spots. We call the first objects "planets" and the latter the "fixed stars". In a further abstracting step, these objects are considered to be points without extension whose colour and brightness are disregarded. We decide to pay attention to the changes in spatial relations of the five visible planets with respect to the fixed stars. In doing so, we have extracted from the sky a *system* whose elements are the observed positions and velocities of five planets. Notice that contrary to common practice, I take as the elements of the system not the planets themselves, but some of their *properties* namely their observable positions and velocities. Planets certainly are members of a system: the "solar system". But when it comes to science and measurements, number is what counts. It is not the planets themselves (even idealized as points) that make true or satisfy mathematical relations but the numerical values of their positions and velocities.

The system of positions and velocities is a perceived system since we can observe them for some point-like patches in the sky. We can then construct a rough structure whose domain is constituted by the positions and velocities of the planets seen from the earth. Such a structure, that we may call the *perceptual* or *phenomenal* structure, approximately organizes the planetary positions and velocities at various times according to their sizes as perceived through our unaided senses. (Notice again that the domain of the phenomenal model is not the set of planets, but a set of their properties — relational properties in fact since positions and velocities are observed in relation to the fixed stars.)

[1] An informative model can be highly impractical, but this is not the point at stake here.

Striving for precision, we further endeavour to measure the positions of the planets and their — angular — velocities by means of suitable instruments. The challenge at this stage of our *démarche* is to discover an adequate mathematical relation, *i.e.* a function which fits the data. This demanding task has kept the best minds busy from early antiquity up to the present[2] ... To make a long story short (keeping our eyes focussed on Bohr) and also to simplify the matter, we will suppose that we start with an observer located at the centre of the ellipses[3] that Kepler laboriously discovered by crunching the data patiently collected by his master Tycho-Brahe. We now make the simplifying assumption that planets move on circles. And we focus not on angular velocities but on *orbital frequencies* ω[4], that is the number of the complete revolutions of a planet during some unit of time.

We are presently in the process of constructing two structures related by a homomorphic function. These two structures are the *phenomenal structure* $\mathcal{O} = \langle O, o\rangle$ and the *data model* $\mathcal{D}_{\mathcal{K}} = \langle D_K, d_K\rangle$ which represents the phenomenal structure. The domain of the phenomenal model, obtained by *secondary* abstraction, contains the orbital frequencies and distances observed with unaided senses[5]; these are organized by an ordering relation o. The domain of the data model contains the *measured* orbital frequencies and distances to the centre. Kepler's data model is a structure $\mathcal{D}_{\mathcal{K}} = \langle D_K, d_K\rangle$ whose domain D_K contains the *measured* values of the planets orbital frequencies ω' and distances a' of the planets to the centre. (I will use $'$ to refer to variables that range over *measured* quantities, as opposed to *calculated* theoretical quantities). These data are approximately organized by the relation d_k which is a function $f(a') = \omega'$. Thus, $\mathcal{D}_{\mathcal{K}} = \langle \{\omega', a'\}, f(a') = \omega'\rangle$ with:

$$f(a') = \omega' = \frac{2\pi}{(a'^3 k)^{1/2}}$$

[2] Due to the perturbations by other planets and satellites, the exact form of the trajectory of a given planet for a long period of time (such as one hundred million years, which is a relatively short period with respect to the age of the earth) remains an open problem.

[3] The actual observations made by an observer on the earth are linked to the observations by an observer at the centre by a simple transformation.

[4] We use Bohr's notations: ω is not the angular velocity but the orbital frequency. On a circle with radius a the linear velocity u equals $2\pi\omega a$.

[5] Ancient astronomers had already observed that the orbital frequencies have different values for different planets and they also had hypothesized the order of distances for the different planets.

where k is Kepler's third law constant. ω' and a' are variables that range over the measured frequencies and distances of the five Keplerian planets: $\omega'_1, \ldots, \omega'_5; a'_1, \ldots, a'_5$. Propositions such as "The specific orbital frequency ω'_1 is equal to $f(a') = \omega' = 2\pi/(a'^3 k)^{1/2}$" are satisfied by this data model.[6] $\mathcal{D}_{\mathcal{K}}$ provides information on the motion of planets provided we know the meaning of the numerical symbols used (they denote or refer to the orbital frequencies and distances of the planets); in other words, we must know the *code* used in the representation.

This is of course not enough. Reliable information is conveyed only if the results of measurements are reasonably accurate: the informative content of the data model relies on the *truth* of some propositions, typically propositions that assign a measured orbital frequency to a given planet, such as "The orbital frequency *of planet 1* is ω'_1". Predicative propositions of this kind are not satisfied or made true by the data model; they are the conditions of possibility for the data model to possess some informative content. I will refer to predicative propositions of this kind as *ontic propositions* since they assert that some real entities possess some properties. So far, we have constructed the phenomenal structure and the data model. It is easy to construct a homomorphism between the two structures that associates to a measured orbital frequency and distance, an observable orbital frequency and distance, such that the structure is preserved. Then, we say that the data model represents the phenomenal structure, *i.e.* is a model in the representative sense.

One of Newton's momentous achievements was to merge Kepler's kinematical theory into a broader mathematical theoretical structure, namely a dynamical structure which introduces masses and forces. The Kepler-Newton model contains a specific force function which is none other than Newton's inverse square law of universal gravitation. Together with suitable initial conditions, we obtain a continuous array of *calculated* stationary orbits, associated with specific theoretical orbital frequencies ω and precise energy levels W.

We then construct another model $\mathcal{H}_{\mathcal{K}} = \langle H_K, h_K \rangle$ the domain of which H_K contains the theoretical energy levels of the five planets W_n, their masses m_n, their orbital frequencies ω_n and their distances a_n to the centre:

[6] This appears to be redundant, even trivial. But see Suppes' very simple example of a structure $\mathcal{D}$ of two natural numbers 1, 2 organized by the order relation $\geq$, namely $\mathcal{D} = \langle \{1, 2\}, \geq \rangle$ which satisfies propositions such as $2 \geq 1$ (Suppes 2002, 26).

$H_K = \{W_n, m_n, \omega_n, a_n\}$ (the index n ranges over natural numbers that are assigned to the five planets). Such a set is organized by the function $f^*(= h_K)$:

$$f^*(m, \omega, a) = 2\pi^2 m\omega^2 a^2 = W \tag{1}$$

Now, we construct an isomorphism F between $\mathcal{H}_\mathcal{K}$ and $\mathcal{D}_\mathcal{K}$ that maps the theoretical orbital frequencies and distances in H_K into the measured orbital frequencies and distances in D_k : F is constructed such as $F(\omega_n) = \omega'_n$ and $F(a_n) = a'_n$. We must add the further condition that $\omega_n \simeq \omega'_n$ and $a_n \simeq a'_n$: the calculated values and the measured values have to be approximately equal. When $W_n = f^*(m_n, \omega_n, a_n)$ we also have $F(\omega_n) = f(F(a_n))$. $\mathcal{H}_\mathcal{K}$ is a substructure of the Newton-Kepler model $\mathcal{H}_{\mathcal{N}\mathcal{K}}$. We can say, using Bas van Fraassen's terminology, that the model $\mathcal{H}_{\mathcal{N}\mathcal{K}}$ is empirically adequate and that the phenomenal structure has been embedded in $\mathcal{H}_{\mathcal{N}\mathcal{K}}$.[7]

Our construction can be summarized in the following schema.

Sky (phenomenon)

$\downarrow$

Phenomenal structure of observed orbital frequencies and distances $\mathcal{O} = \langle O, o\rangle$

$\sim$ isomorphism

Data model of measured frequencies and distances $\mathcal{D}_\mathcal{K} = \langle D_K, d_K\rangle$

$\sim$ isomorphism F

Substructure $\mathcal{H}_\mathcal{K} = \langle H_K, h_K\rangle$ of the theoretical model

$\cap$

Newton-Kepler Theoretical model $\mathcal{H}_{\mathcal{N}\mathcal{K}} = \langle H_{NK}, h_{NK}\rangle$

[7] I introduced the additional notion of phenomenal model, which is not used by van Fraassen as an intermediary link between phenomena and data models (Ghins 2010).

The theoretical Newton-Kepler model here corresponds of course to a two-mass system, with one of the masses much bigger than the other, whose domain contains all the continuous possible energy levels for the smaller mass. Using this model, the energy *differences* between two possible states of the smaller mass can easily be calculated. This will be of some relevance when we get to the Bohr model.

3. *Modelling the unobservable: atomic energy levels*

The orbital frequencies of bright spots in the sky can be observed with the naked eye whereas the orbital frequencies of electrons revolving around a nucleus cannot. Yet few would dispute that modelling *un*observable systems plays a decisive role in physics. Typically, previously successful models are employed as guides for the construction of models of newly investigated systems. "Guide" is a vague term however. Happily, we can benefit from the work of the numerous scholars who in the past century have paid considerable attention to the *heuristic* role played by models. Mary Hesse (1966), among others, emphasized that in order to construct a model of a system (which need not be observable) that has not been hitherto modelled, we must assume that some kind of resemblance or *analogy* obtains between the newly investigated system and some system (which need not be observable either) for which a performing model is already at hand. As we shall presently see in our reconstruction of Bohr's successful attempt to model the hydrogen atom, the concept of analogy can be adequately and economically captured in this case by means of the sharing of some properties and the notion of homomorphism only, without having to resort to the notions of "quasi-structure" and "partial homomorphisms". On this issue, I disagree with what Newton Da Costa and Steven French (2003) contend.

In his celebrated 1913 papers "On the Constitution of Atoms and Molecules", Niels Bohr takes as his point of departure the scattering experiments performed with alpha particles by Geiger and Marsden. These experiments support Rutherford's *view* (*not* his *model*, in my sense: see below) of the atom as made of a massive and positively charged nucleus the radius of which is extremely small with respect to the dimensions of the atom and which is surrounded by electrons with very tiny mass. Those experiments clearly point to some "positive" (Hesse 1966) analogy between the solar system and the atomic system, as Jean Perrin had already suggested as early as 1901. In the atom, the nucleus and the electrons play the roles of the sun and the planets, respectively.

By pursuing this line of thought, we can construct a homomorphism between the *possible* orbital frequencies and distances of a mass gravitationally revolving around a much bigger mass and the possible orbital frequencies and distances to the nucleus of an electron in the hydrogen atom. As a first step we consider all possible orbits in conformity with what is suggested by the assumed positive analogy between the hydrogen atom and a single planet-sun system. The homomorphism maps the possible orbital frequencies and distances of a single planet into the possible electronic orbital frequencies and distances to the nucleus, in such a way that the structural relations between the orbital frequencies ω and distances a to the centre are preserved under the mapping. One supposes — as Rutherford and Bohr did — that ontic propositions asserting that electrons have orbital frequencies and distances to the nucleus (which is located at the centre of electronic circular orbits as Bohr assumes) are true, and therefore that electrons have energies corresponding to these orbital frequencies.

For the sake of simplicity, I will skip the step of the construction of a model of orbital frequencies and distances for electrons and look at the construction of a model of the possible electronic *energy levels*, *orbital frequencies* and *distances* $\mathcal{H}_{\mathcal{R}} = \langle H_R, h_R \rangle$. h_R has the same mathematical form (1) as $f^*(= h_K)$ above. The domain H_R is continuous: in Rutherford's model the energies of the electrons can take continuous values. Obviously, $\mathcal{H}_{\mathcal{R}} = \langle H_R, h_R \rangle$ is *not* a data model since the orbital frequencies and distances are not measurable. Strictly speaking, $\mathcal{H}_{\mathcal{R}} = \langle H_R, h_R \rangle$ is the Rutherford *model*, even if Rutherford himself does not seem to have entertained hypotheses on the precise values of the electron's orbital frequencies and energies. Since the electron and the nucleus have opposite charges it is reasonable to suppose that they attract each other in compliance with Coulomb's law for the electrostatic force[8] which is an inverse square law just as Newton's law for the gravitational force is. We can thus construct an homomorphism between $\mathcal{H}_{\mathcal{R}} = \langle H_R, h_R \rangle$ and a Keplerian model (whose domain contains the properties of a single planet) $\mathcal{H}_{\mathcal{K}} = \langle H_K, h_K \rangle$ such that energy levels are sent to proportional energy levels. The Rutherford model $\mathcal{H}_{\mathcal{R}} = \langle H_R, h_R \rangle$ is a substructure of a Newton-Coulomb model $\mathcal{H}_{\mathcal{NC}} = \langle H_{NC}, h_{NC} \rangle$ in which the central force is not the gravitational force but the Coulomb force. This model is homomorphic, even isomorphic, to the Newton-Kepler model $\mathcal{H}_{\mathcal{NK}} = \langle H_{NK}, h_{NK} \rangle$ mentioned above given a function that sends energies to proportional energies etc. in such a way that the mathematical relations are preserved.

[8] The gravitational attraction nucleus-electron is much smaller than the electrostatic attraction and can be neglected.

So far the construction of the atomic model has been carried out on the basis of assumed *positive* analogies between the atomic model and the planetary model. We assumed the truth of ontic propositions which attribute to electrons specific properties that are *identical* to some properties of the planets. Electrons are supposed to have orbital frequencies and to occupy energy levels. Besides, the Coulomb force is mathematically similar to the Newtonian gravitational force. These ontic propositions permit the construction of structures which are not only structurally similar but also *resemble* one another in some particular respects: electrons are assumed to be like planets with respect to some selected properties. Then, the possible energy levels, masses, frequencies and distances of the orbiting body in the gravitational system are homomorphically sent to the possible energy levels, mass (which is constant in the present case), frequencies and energy levels of electrons.

The well-known quandary at this point is that Maxwell's laws of classical electrodynamics require that accelerated charges emit radiation. Accordingly, a revolving electron should not revolve on stationary orbits but spiral down to the nucleus while losing energy by emitting an electromagnetic radiation of continuous frequency. However, the experimental data first show that an atom does *not* emit any electromagnetic radiation except when it is excited, for example, when it is submitted to an electric discharge, and second that the emitted radiation spectrum is *not* continuous but discrete. Confronted with these experimental results, Bohr assumes first that electrons do *not* emit radiation while revolving on some *discrete* orbits which are consequently supposed to be stationary and second that an electron emits (or absorbs) a homogeneous (monochromatic) radiation only when jumping from one orbit to another.

Bohr provides a detailed calculation for the hydrogen atom, assuming — for simplicity — that a single electron moves on *circular* orbits with discrete *orbital frequencies* ω_n and therefore specific discrete energy levels W_n. Bohr imposes the following specific condition[9] for the quantization of energy as a function of ω_n, Planck's constant h and the quantum positive natural number n:

$$W_n = nh\frac{\omega_n}{2} \tag{2}$$

By using *classical* formulas only, Bohr deduces the mathematical expression for the energy levels as a function of the mass of the electron m, its

[9] Hendry (1993, p. 111) rightly stresses that Bohr uses Balmer's formula in order to get the precise mathematical formula for the quantification of orbits, whereas the principle of the quantification of the orbits is justified on other grounds. For Bohr's motivations behind assumption (2), see Norton (2000).

charge e, Planck's constant h and the quantum number n:

$$W_n = \frac{2\pi^2 me^4}{n^2 h^2} \tag{3}$$

It is important to notice that the kinematical orbital frequency ω_n and the distance to the nucleus a_n do not occur in this formula. At this point, we construct a model $\mathcal{H}_{\mathcal{B}} = \langle H_B, h_B \rangle$ the domain H_B of which contains energies and natural numbers. These are structured by equation (3). Thus $\mathcal{H}_{\mathcal{B}} = \langle \{W_n, n\}, W_n = f_B(n) \rangle$ with $f_B(n) = 2\pi^2 me^4/n^2h^2$. Then, we establish a homomorphic function B from H_B to H_R that sends energies and natural numbers to equal energies and orbital frequencies in such a way that if $W_n = 2\pi^2 me^4/n^2h^2$ then $B(W_n) = 2\pi^2 mB(n)^2 a^2$. So far, our reconstruction of Bohr's *démarche* can be summarized thus:

Two-mass system		Hydrogen atom
$\downarrow$ abstraction		$\downarrow$ abstraction
Keplerian model		Rutherford's model
$\mathcal{H}_K =< H_K, h_K >$	$\sim$	$\mathcal{H}_R =< H_R, h_R >$
	Homomorphism (positive analogy)	
		$\sim B$
		$H_B =< H_B, h_B >$

Except for the quantization condition (2) for the energy levels, Bohr proceeded by relying on formulas of classical mechanics and the Coulomb force law only. He was of course fully aware that his assumption that the electrons do not emit radiation while circulating on some orbit flatly contradicts classical electromagnetism. Yet, such a contradiction does *not* imply that his model is *internally* inconsistent, as we will show below. Now, the energy levels of the electrons are not experimentally knowable unless some hypothesis is made about the link that connects them with some measurable properties, namely the emitted electromagnetic frequencies ν. Bohr assumes that when an electron transits from one orbit n' to an orbit n closer to the nucleus (with n smaller than n'), it emits a monochromatic quantum of radiation $h\nu$ whose energy is equal to the *difference* of the mechanical energies associated with these orbits:

$$W_n - W_{n'} = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) = h\nu$$

And thus:

$$\nu = \frac{2\pi^2 me^4}{h^3}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \tag{4}$$

For $n = 2$, we recover Balmer's formula. Although the values of the emitted frequencies are not directly observable, colours surely are and can be analyzed by means of a spectrometer. We thus get a data model of frequencies and natural numbers $\mathcal{D}_\nu = \langle D_\nu, d_\nu\rangle$ whose domain $D_\nu = \{\nu, n, n'\}$ is structured by formula (4) with the condition $n < n'$. Such a data model is homomorphic to a model of *differences* of energies which can be easily constructed from $\mathcal{H}_\mathcal{B}$. If we take n' to be infinite, we get a model $\mathcal{D}^*_\nu$ which is homomorphic to the model $\mathcal{H}_\mathcal{B}$. In this particular case of the binding of a free electron, energy levels are sent to electromagnetic frequencies in such a way that pairs (W_n, n) satisfying (3) are mapped into pairs (ν, n) satisfying (4). $\mathcal{D}^*_\nu$ is the model of electromagnetic frequencies emitted by a free electron when falling to an orbit associated with the quantum number n. Empirical adequacy is achieved if the calculated electromagnetic frequencies are approximately equal to the observed frequencies for given natural numbers. The kinematic characteristics of the classical orbits of the electrons, namely the orbital frequency ω and the distance a to the nucleus — *albeit* not observable — are readily calculable from the measured values of the emitted frequencies.

A sceptical reader may rightly question at this stage the faithfulness of our reconstruction of Bohr's *démarche*. Well, in his original paper, Bohr claims that his theory is based on two main assumptions and two special ones. The two main assumptions are:

> "1. That the dynamical equilibrium of the systems in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the system between two stationary states cannot be treated on that basis.
> 2. That the latter process is followed by the emission of a *homogeneous* radiation, for which the relation between the frequency and

the amount of energy emitted is the one given by Planck's theory." (Bohr 1913, p. 7)

And the two special assumptions are:

"1. That the different stationary states correspond to the emission of a different number of Planck's energy quanta.[10]
2. That the frequency of the radiation emitted during the passing of the system from a state in which no energy is yet radiated out to one of the stationary states, is equal to half the frequency of revolution of the electron in the latter state." (Bohr 1913, p. 8)

The cash value of these assumptions is the mathematical theory that Bohr formulates on their basis and which we presented above. As we saw, in order to account for the spectral lines for the hydrogen atom (and also for the ionized helium atom with one electron) there is no need to consider the mechanical trajectory that the electron would perhaps follow when passing from an orbit to another, nor the stationary trajectories: Bohr's model remains silent on those hypothetical trajectories. The only thing that matters is the differences in mechanical energies of the initial and final states of the electron, which correspond to the energy $h\nu$ of the "homogeneous" (monochromatic) emitted quantum. At the end of the day, what counts in physics are not the assumptions formulated in common language, but the equations and the structures that satisfy them, *i.e.* the theories that account for the data. When tackling philosophical problems it seems advisable to focus on the latter rather than on the former.

4. *Is Bohr's theory inconsistent?*

As of today, the view that the Bohr model is inconsistent still prevails among philosophers and historians of physics[11]. Several authors (Priest 2002, da Costa and French 2003, Bueno 2006) even contend that Bohr's atomic theory is a prime example for displaying the fecundity of some inconsistent models in physics and they use it as a springboard to promote some paraconsistent logical systems as tools for understanding model construction in

[10] This refers to the special case when a free electron falls on an energy level W_n.

[11] The prevalence of this opinion is probably due to the lingering influences of Lakatos (1970) and Feyerabend (1978) who saw in Bohr's theory an exemplary case of inconsistent physical theory.

science. My aim here is not to dispute the utility of paraconsistent logics for understanding scientific modelling in some specific instances, but to briefly show, following Peter Vickers[12], that the particular case of Bohr's model does not provide solid grounds for supporting the agenda of the friends of paraconsistency.

Before we show this, some clarifying remarks on my construal of theories are in order. A physical theory contains two ingredients: first, a class of models and second, a set of propositions (namely interpreted statements or sentences) that are made true or satisfied by these models. As we saw above, models perform a double role: they are possible representations of other structures and they also satisfy some propositions. In our reconstruction of Bohr's model, we mainly concentrated on the representative role of models. It is time we focus on propositions and pay attention to the sets of relations that are supposed to hold between the elements of a domain. For example, the Newton-Coulomb model $\mathcal{H}_{\mathcal{NC}} = \langle H_{NC}, h_{NC} \rangle$ is supposed to make true the formulas for the energy levels of a system of two charged bodies linked by a central electrostatic force. Without entering into technical subtleties and thus not pretending to achieve complete rigor[13] we suppose that the domain H_{NC} not only contains theoretical values for energy levels, but also for masses, positions, charges and velocities and that the h_{NC} are the mathematical relations that hold between the values of those physical variables. Thus H_{NC} makes true the Coulomb force law and the other classical laws for point mechanics.

The connection with a real system is achieved through propositions that assert that some entities in the world, such as electrons, have such and such properties that take the values that belong to the domain of the model. Such propositions, when true, permit our models to escape from the realm of pure mathematics and reach out as representations of real systems in the world. These propositions — which I called *ontic* propositions[14] do *not* belong to

[12] For a detailed discussion of the inconsistency charges that have been levelled against Bohr's model, see Vickers (forthcoming, chapter 3).

[13] For a rigorous treatment the reader is advised to consult the classical works by Patrick Suppes summarized in his *Representation and Invariance of Scientific Structures* (2002). However in our presentation of models, the elements of the domains are the values of some physical properties and not things such as points, electrons etc. Strictly speaking, it is these values that satisfy the mathematical formulas.

[14] Ontic propositions are to be carefully distinguished from "coordinative" or "bridging" principles. Such principles have been introduced in the framework of the syntactic view of theories in order to semantically interpret sentences or statements and were supposed to be part and parcel of theories. Such principles, in the context of the model-theoretic approach,

the theory but are the conditions of possibility of its successful application to systems in the physical world.

If we take the set of mathematical formulas or laws that are satisfied by the models described above, we would have a hard time to deduce from them a proposition of the form "p and not-p" by means of the truth-preserving rules of standard logic. As Vickers aptly remarks, no such deduction is to be found in the rather extended literature on the issue of the consistency of Bohr's model. Granted, the absence of a derivation of a contradictory proposition from Bohr's formulas does not prove the consistency of his theory. In the present case however, there is no doubt that the burden of proof rests on the shoulders of those who wish to saddle Bohr with an inconsistency charge.

As Vickers and others (Bartelborth 1989a, 1989b, Hendry 1993, Hettema 1995) have maintained, we must carefully distinguish between *internal* and *external* consistency. If we confine ourselves to Bohr's mathematical theory of the hydrogen atom its consistency is not in jeopardy. In fact, the major charge of inconsistency rests on the contradiction between the assumption that the electron does not emit electromagnetic radiation while orbiting on an orbit n (especially on the lowest orbit for which n equals 1, *e.g.* the ground state) on the one hand and the law of emission of radiation by accelerated charges in classical electrodynamics on the other hand.

> "(...) the assertion that the ground state was stable, so that an electron in such a state would not radiate energy (...). *This* is the central inconsistency." (Da Costa and French 2003, p. 91)

This contradiction certainly is an annoying feature of Bohr's theory in the context of the accepted background physics of the time, but such inconsistency is merely *external*. It surely is possible, according to the construal of scientific theories advocated here, to avoid embracing holism and to separate Coulomb's formula from the rest of Maxwell's theory of electromagnetism[15], as Bartelborth argues:

keep us confined to the world of representations and do not allow the latter to reach out to external realities (Ghins 2010). When we assert ontic propositions, we know the meanings of the terms that occur in them; they thus do not function as interpretative principles. Let me stress that ontic propositions such as "Planet 1 has mass m_1", "An electron n has energy W_n" do not commit us to any particular metaphysics or ontology. I am not against all forms of metaphysics, but I wish to stress that specific metaphysical views must be defended by means of elaborate arguments connected with observation (Ghins 2007).

[15] Such a separation is advocated by Hendry (1993) and Norton (2000).

> "(...) the only necessary theory-element from classical electrodynamics for Bohr's theory is quasi-electrostatics for point-particles, because what Bohr really needed from classical electrodynamics was the concept of electric charge and Coulomb's law." (Bartelborth 1989a, p. 221)

If Bohr's theory is consistent, there is no need to resort to quasi-structures as Da Costa and French propose (2003, p. 91). In a quasi-structure, three kinds of relations are considered. R_1 are the relations that we assume to hold between elements of the domain; R_2 are the relations that we suppose not to hold between these elements; R_3 are the relations about which we are non-committal with respect to their holding or not between the elements of the domain (Da Costa and French 2003, p. 19). It is then not difficult to show that a partial structure can satisfy a set of propositions in which some propositions contradict each other[16]. However, in my reconstruction of Bohr's theory, some parts of electrodynamics are simply ignored since they are not necessary to deduce the values of the emitted frequencies. This, I suggest, is just a consequence of the abstraction that is involved in any modelling *démarche*. In itself, Bohr's model does not suppose that some laws of electrodynamics may or may not obtain. His model does not take into account the classical mathematical relation between the cinematic acceleration and the emission of radiation when the electron is in some discrete stationary energy state; simply because *ex hypothesi* the electron does not emit radiation in such a state.

As Vickers stresses, the physical community of the time perceived Bohr's theory exactly in this way, namely as using only electrostatics and not electrodynamics. Typical of the attitude of contemporary physicists is the following quotation by Millikan.

> "Bohr's first assumption (...) when mathematically stated takes the form $e^2/R^2 = (2\pi\omega)^2 mR$ in which e is the charge of the electron, ω the orbital frequency, and m the mass of the electron. This is merely the assumption that the electron rotates in a circular orbit... The radical element in it is that it permits the negative electron to maintain this orbit or to persist in this so-called "stationary state" without radiating energy even though this appears to conflict with ordinary electromagnetic theory." (Millikan 1917, quoted by Vickers 2009, p. 247)

[16] See Vickers (2009) for some graphic examples.

However, such contemporary reactions to Bohr's theory show that the — coherent — unification of physics was a value actively pursued by physicists. Such a value has been praised, and rightly so, up to the present day. When a successful theory which appears to contradict some accepted laws is successful, scientists are confronted with what Larry Laudan (1977) called an external conceptual problem. Scientists then embark on the challenging task of revising significant parts of accepted physics in order to build a coherent broader theory. As history teaches us, those efforts are often remarkably fruitful. In the episode that concerns us here, such efforts gave rise to standard quantum mechanics and led to the view that classical electromagnetism remains a good approximation for a defined class of systems. Stressing the virtues of inconsistency (if there are any) shouldn't hinder the pursuit of consistency and the benefits it has yielded to scientific progress.

5. *Bohr's theory of the atom and scientific realism*

We will leave aside here the metaphysical issues connected with scientific realism and address the epistemological questions that hinge upon the legitimacy of our beliefs about the existence of some entities and the truth of propositions about them. Given the empirical success of Bohr's theory, what are the beliefs that we are reasonably authorized to entertain about atoms? In what follows the issue at stake is not so much the identification of the beliefs actually embraced by Bohr and the physical community of his time, but to ponder the arguments that the scientific realist is entitled to adduce in favour of some belief commitments.

As seen above, a scientific theory is a class of models together with the set of propositions that satisfy them. By itself a scientific theory is independent from the existence of any entity in the world. Classical mechanics can be presented in textbooks without being concerned with the actual existence of mechanical systems to which it may be applied, just as Euclidean geometry can be presented independently of the actual existence of rigid rods. If we disregard how Kepler was led to formulate his laws, his theory can be considered in itself without paying attention to planets. Frederick Suppe (1974) has defended a very plausible counterfactual account of theories according to which scientific laws are not committed to the existence of entities but solely assert that if entities of a specific kind existed, then their behaviour would be described by specific mathematical formulas. A theory must then leave open the possibility of the existence of some systems in the world. For this to happen, the theory must be logically consistent. Thus, since there is no cogent reason to question the internal consistency of Bohr's theory, this removes a serious obstacle in the way of a selective realist interpretation of it.

With respect to a given theory, do we have reasons to believe that some existing entities display a behaviour that can be — at least partially — correctly described by the theory? Kepler's theory can be applied to planets, the latter being identified with some bright spots moving in the sky. To apply a theory to real entities is to assume the truth of some ontic predicative propositions about those entities. When applying Kepler's theory to planets, we assume the truth of the proposition "Planets have positions and velocities". Since planets as well as their positions and velocities are directly observable, there is every reason to accept the truth of this ontic proposition. Then, the empirical adequacy of Kepler's theory provides good grounds to believe that the planets have definite trajectories. We stressed that ontic propositions are not to be included as components of theories. This is a pre-condition for the possible application of a theory to a wide variety of entities. Kepler's theory can evidently be applied to systems other than the solar system, such as planet-satellite systems, and in general to any physical system composed of two bodies which interact according to an inverse square force law. It is hardly necessary to mention that Newtonian point mechanics can be applied to an even broader class of entities.

Arguably, we cannot directly observe the energies of the planets. Surely, their energies can be readily calculated by means of simple mathematical equations. Yet from an epistemic point of view "calculable" is not to be conflated with "directly observable", that is, observable with the unaided senses or without instruments (van Fraassen 1980, p. 15). Taken in isolation, the Newton-Kepler theory does not provide reasons to believe in the truth of (ontic) propositions such as "This planet has the energy level value W". Arguments in favour of the truth of these propositions have to be sought elsewhere and are thus *external* to the Newton-Kepler model. What are these arguments? It is reasonable to suppose that a property that can be measured by a variety of independent and concordant experimental methods belongs to a real entity, just as we feel entitled to believe that an observable entity is rectangular when we have performed a variety of empirical checks (Ghins 1992 and 2009). Clearly, various independent methods are available for measuring the masses and energies of ordinary observable bodies such as rocks and billiard balls and these methods give concordant numerical results. This situation pleads in favour of our belief that terrestrial observable bodies do possess some mass and energy. Even though in the history of astronomy the planets have been thought to have a nature very different from terrestrial objects (think of Aristotle's ether), Galileo's observations and more recently the landing of artificial satellites and humans on some celestial bodies give us sufficient reasons to believe in the similarity of planets with rocks and billiard balls in some relevant respects. As a consequence, the realist is entitled to conclude that planets also have a mass and a (kinetic and potential) energy.

Granted, the successes of Bohr's model, namely its ability to account for the frequencies of spectral lines[17] but also the prediction of the correct value for the Rydberg constant (which strongly impressed Einstein), do not lend credence to the existence of electrons. First of all, electrons — unlike planets — are unobservable entities. The reasons to believe in their existence must then come from some evidence which is external to Bohr's theory. Electrons are entities which are supposed to have an electric charge and a mass the values of which can be measured by a variety of experimental methods — independent of spectrometry — that deliver concordant quantitative results. (Some of these methods were already known in Bohr's time). From such evidence, one is justified to believe in the existence of electrons, namely particles that have a mass m and a charge e equal to $9.11\text{x}10^{-31}kg$ and $1.6\text{x}10^{-19}coul.$ respectively.

Even though Bohr was quite aware that electrons behave in a significantly different way from ordinary macroscopic bodies in some respects, it was quite natural for him to suppose, pending evidence to the contrary, that electrons move on trajectories around the nucleus. In Bohr's theory, the orbital periods ω and radiuses a of their trajectories are easily calculable on the basis of the values of the emitted frequencies. However, those kinematical magnitudes cannot be measured by means of different independent experimental methods. Surely, the position of an electron can be measured in certain instances, but not while it is allegedly orbiting around a nucleus. It is not possible to perform measurements of the successive positions supposedly occupied by an electron circling a nucleus, whereas such measurements are feasible for the successive positions occupied by a given planet.

Anjan Chakravartty (2007, chapter I) proposes an illuminating distinction of unobservable properties into two disjoint classes: those which are *detectable* by means of instruments and those which are not detectable but play an *auxiliary* role in the explanation of phenomena. Detectable properties are those, in the words of Chakravartty, "with which we have managed to forge significant causal contact" (2007, p. 60). According to Chakravartty the scientific realist should be committed to the existence of detectable properties

[17] This holds true not only for the hydrogen atom but also for the ionized helium (Pickering's lines).

only and suspend his/her judgement on the existence of auxiliary properties. I contend that even more stringent conditions should be required in order to justify the legitimacy of existence beliefs: only properties that are measurable by means of distinct and independent experimental procedures that lead to concordant quantitative values (up to an acceptable degree of approximation) can legitimately be attributed to some entities[18].

Thus, it is reasonable to be committed to the truth of propositions that assert that "Electrons have mass, charge and energy". On the other hand, it would be better for the realist to refrain from claiming that bound electrons have a trajectory, simply because we have not managed to forge strong causal contact with their positions and velocities. Besides, unlike what occurred in the case of planets, we lack sufficient evidence that electrons are similar to ordinary bodies with respect to positions and velocities. Therefore, we are in a position to conclude that the positions and velocities of bound electrons are not detectable. These are auxiliary properties only and there is no convincing reason for committing ourselves to the belief that electrons possess them. An agnostic attitude *à la van Fraassen* (1980, p. 72) is to be recommended with respect to the existence of these properties. Such austere selective realism does *not* exclude the possibility that further evidence might be provided in the future in favour of electron trajectories. As of today such evidence is still wanting.

6. *Conclusion*

Our proposed reconstruction of Bohr's modelling of the hydrogen atom is meant to contain the minimal amount of ingredients to understand first the supposed analogies between the solar system and the atomic system and second the capacity of his theory to account for the hydrogen spectral lines. This rather economical, even hygienic, reconstruction shows that Bohr's theory is not inconsistent and that in order to understand the heuristic *démarche* that led to Bohr's theory, it is not necessary to resort to Da Costa's and French's "quasi-structures" and "quasi-isomorphisms". In light of this, it has been argued that the scientific realist is reasonably entitled to believe that electrons exist and occupy discrete energy levels in the hydrogen atom, whereas he or

[18] Unlike Chakravartty, I think that properties need some anchoring in entities identified by means of observable properties. Thus, an electron is what is causally responsible for some specific phenomena identified independently of theory change and which is endowed with different properties in various theories (Ghins 2012).

she should adopt an agnostic attitude with respect to the existence of electron trajectories.

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