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NATURAL LANGUAGE CONSISTENCY

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Abstract

Tarski’s assessment that natural language is inconsistent on account of the Liar Paradox is shown to be incorrect: what Tarski’s theorem in fact shows is that Truth is not a property of sentences but of propositions. By using propositions rather than sentences as the bearers of Truth, semantic closure within the same language is easily obtained. Tarski’s contrary assessment was partly based on confusions about propositions and their grammatical expression. But more centrally it arose through blindness to pragmatic factors in language — a blindness that was common in his time, and it has continued to the present day, in discussions of ‘Open Pairs’, and Yablo-type paradoxes, for instance. For completeness, it is also shown that the Fixed Point Theorem does not apply to propositions, because of categorical differences between sentences and propositions — also predicates and properties.

1

Natural language has resources that have not been copied into the formal languages of recent logic, and in at least two cases this has led to near intractable difficulties in that discipline. There are no reflexive pronouns in the languages of recent logic, which has produced misrepresentations of notorious predicates like ‘is not applicable to itself’ and ‘is not a member of itself’ (Slater 2004, 2005, 2006); and there are no nominalising devices in the languages of recent logic, which has engendered the abandonment not only of ‘that’-clauses, but also their referents: propositions. It is the latter difficulty that is the main focus of the present paper, although together the two difficulties have combined to generate what have been called ‘the paradoxes of self-reference’, one consequence of which has been the judgment that natural language is inconsistent, on account of such things as the Liar Paradox.

This view of natural language, of course, is a mirage brought about by the above removal of standard features of ordinary speech in the restricted languages that are instead considered within the formal tradition. But it needs to be shown in some detail how it can be that natural language, by retaining these features, does resolve the classic self-referential paradoxes. The first point to realize is why there is no direct analogue of the Fixed Point Theorem in connection with propositions. For it is that theorem which seemingly guarantees, in a very rigorous formal way, that there is no 'semantic closure', i.e. no consistent truth predicate both applicable to and definable within the same language. But only a glance at it is sufficient to show that it has no direct analogue with propositions. For it has the form:

$$q \equiv A(gn'q'),$$

and equates certain propositions with propositions about the Gödel numbers of sentences that express them. So the only point that needs to be made with respect to natural language is the simple one that the propositions expressible in a language, unlike the sentences in that language, cannot be numbered.

The point is obvious, given that natural language contains explicit indexicals. But we shall see that it holds much more widely. Thus while there is one sentence 'this is not true', innumerable propositions may be made with it, depending on the chosen referent of 'this'. And that multiplicity of propositions not only defeats any direct analogue of the Fixed Point Theorem applying, it also immediately resolves the supposed self-referential paradox that might be formulated in this case. For if the referent of 'this' is taken to be the sentence 'this is not true' itself, then the proposition then made by the sentence is simply true. It is true that the sentence 'this sentence is not true' is not true, because truth attaches to the proposition it makes in the given circumstances, instead. Only a failure to separate the sentence itself from the proposition it is chosen to make could result in the belief that the sentence both was not true, and was true, so the contradiction is immediately avoided once the proposition and the sentence are separated. Of course that requires distinguishing a mentioned, i.e. quoted sentence from a used sentence preceded by the nominaliser 'that', and more will be said about that in due course.

Before that it is important to see that the 'indexicality' of language extends much further than in such explicit cases where a demonstrative like 'this' is involved.

To see that wider presence of the required features we must see, preeminently, the contextuality of what Quine called an 'eternal sentence' like 'the sentence at the top of page n of book B is not true'. For it would appear that the contextuality of such 'eternal sentences' is what has primarily been missed, since the location of (some token of) such a sentence (at the top of page n of book B) might be taken to be a plain fact about the actual world, and so seemingly not one in a special, limited context. Hence, it would seem, the sentence alone could be taken to carry the appropriate truth-value, though of course, then, a very puzzling one. But in this case the variability of truth-value of the associated propositions is with respect to *different possible worlds*. Here is where a historical point gains some significance: it is not an historical accident that the self-referential paradoxes arose forcefully in logicians minds before context dependence and indexicality were studied fully, for what was also characteristic of that period was that *possible worlds* were not studied either. Indeed, if one keeps to that early twentieth century mind-set one will still find the self-referential paradoxes puzzling, particularly followers of Quine on Modality, and eternal sentences, of course, since one needs to understand fictional contexts in order to realize that the actual world is just another context. Contrariwise, moving out of that mind-set, and in particular moving away from the influence of Tarski, the resolutions of such paradoxes become extraordinarily easy, as above. For what is true in the given 'eternal sentence' case is simply that the sentence at the top of page n of book B is not true, while the key point to realize, to avoid contradiction, is that to say that, i.e. to say that *that proposition* is true is not to say that the sentence itself is true. What is true is not the sentence, but what the sentence says on a self-referential interpretation.

The crucial point is that it is not possible for a referential sentence to specify which world it is used in, since that is a matter of pragmatics. Even if it was possible that a code might be provided giving the various referents, in different possible worlds, of phrases like 'the sentence at the top of page n in book B', etc., still what world the phrase is used in is a matter of pragmatics. So the code itself shows that such phrases are trans-world, if not 'contextually' indexical, and so ambiguous, along with standard referential indexicals like demonstratives, and pronouns. In the general case of the basic type of puzzling syntactic identity

$t =$ 't is not true',

the grammatical point is that 't' is quoted on the right hand side, and so is just mentioned without reference to a world. So no specific referent can be involved there. But the referent of 't' *in this world* is involved on the left hand side, because it is not mentioned but used, and we are speaking in this world. But if one tries to improve on this by trying to insert the context of utterance into the sentence itself, constructing something like

$t =$ 't when uttered in this world/in the actual world is not true' then one either has brought in an explicit indexical ('this world'), or an equivalent to one — remembering David Lewis on 'the actual world' (Lewis 1986).

Of course, to see how one can use referential phrases 'in other worlds' one has to remember also that other possible worlds are not entirely abstract objects, since we can imagine entering them, which is a process that takes place in this world. So there is no difficulty in transferring oneself, in one's imagination if not in reality, to another possible world, or situation. In the case of linguistic fictions this commonly involves certain context markers, like 'Once upon a time', for instance. Maybe once upon a time an old man was reading page n of a book, B, the first sentence on that page [i.e. page n of book B] being 'The first sentence on page n of book B is not true'. In this case, at its unquoted place the subsequently quoted referential phrase refers to the page the old man was reading in the fiction, i.e. the possible world. Without the 'Once upon a time', the story starting 'a man was reading…' might be fiction or non-fiction, although the same kind of linguistic cross-reference would still occur, from the referential 'the first sentence…' back to the previous introductory description of the old man. In non-anaphoric uses of referential phrases, i.e. when they are 'deictic', there is no context marker like 'Once upon a time', or even explicit introductory description. But, now, *the absence of such a context marker* is not part of the sentence(s) that follow, so, even when there is direct reference to the actual world, that is a matter of the pragmatics, not the semantics of the utterance, and therefore not something in the sentence alone, in itself.

Wouldn't there still be a paradox if we considered whether the sentence token

> This sentence token is not true

was true or false? Here we, in this world, are referring to a specific set of words also in this world, and there is therefore no possibility of ambiguity of reference. But, still, what is true is *that that sentence token is not true*, not the sentence token itself. It is the proposition expressible by the sentence token which is true, while the sentence token itself is not true.

2

The above points about the categorical differences between sentences and propositions show up in a variety of individual ways in connection with further particular discrete cases in the area. Indeed, the paradoxes where indexicality is showable by direct inspection of the sentences, along with elementary cases involving less plain and evident indexicals, like 'the first sentence on page n of book B is not true', clearly give good inductive grounds for the belief that there is indexicality in all related cases people have found paradoxical. Note that supposed examples of necessarily self-referential sentences, such as 'this very sentence is false' do not escape from the indexical category, since in fact the referent of the 'this very sentence' still has to be determined with a gesture, and might be to some other sentence, so the 'self-reference' is not properly necessary. If one gives names to sentences there is the same problem as we shall see arises with numbering systems, since it is not in 'sentence A is not true' itself that it is sentence A in some list, if it is so, and that very same sentence might make a different proposition using a different naming system. It is the resulting difference between sentences and propositions that resolves a number of further paradoxes.

Recently, for instance, there has been much discussion about what are now called 'open pairs' (e.g. Sorensen 2003, Armour-Garb and Woodbridge 2006). Here are some sentence-proposition facts regarding what are often presented as

1: 2 is not true,
2: 1 is true.

They can be extended quite easily to other cases, such as Yablo's (Yablo 1993, see also, for instance, Goldstein 2006). Maybe the idea is that the examples given can be expunged of contextual elements, and turned into 'eternal sentences' (the matter dealt with before), but as they stand these cases even more clearly do not give rise to any paradox, once the very evident contextuality is spelt out. For, if '1' and '2' are supposed to be names of sentences, then, for a start the proper, full expression is quotational:

Sentence 1 = 'Sentence 2 is not true',
Sentence 2 = 'Sentence 1 is true'.

The puzzle then seems to be that if sentence 1 is true then (because of what sentence 1 says) sentence 2 is not true, but that requires, doesn't it (because of what sentence 2 says), that sentence 1 is not true? Hence, seemingly, sentence 1 has to be not true. But that makes sentence 2 true, surely (because of what sentence 1 says), and therefore we seem to be able to deduce that sentence 1 is true (because of what sentence 2 says)? But once we remember the sentence-proposition distinction it becomes clear that the sentences on the right are neither true nor false in themselves, indeed, in themselves they are not the sort of thing that can have truth-values. To gain a truth-value they need to be fully interpreted, which in this case means they need to be used in connection with a list of sentences in a numerical order, as on the left. Then what proposition sentence 1 can be used to make in the given context, namely that sentence 2 (here) is not true, is true, showing one must distinguish very carefully, again, the sentence from what it says in the context of the given list, i.e. what proposition it makes there. For sentence 1 is not true, while the proposition made by it, in this context, is true. And likewise, in reverse, with sentence 2, since the proposition that it makes is that sentence 1 (here) is true, which is not true. So not only sentence 2, but also the proposition it makes in the context is not true.

As before, this kind of point must also be made in the central, 'self-referential' case, where, for instance,

Sentence 3 = 'Sentence 3 is not true'.

For here what is true is not (contradictorily) sentence 3 but the proposition made by it in this context, namely that sentence 3 (here) is not true. Hence there is no Liar paradox in this case, and it is the functioning of the pragmatic context that has been primarily overlooked by theorists who find further paradoxes in this area, i.e. those who find Open Pairs and Yablo-type cases puzzling.

Notice, in connection with 'that'-clauses, that one cannot just nominate a self-referential propositional identity, in the same way as a 'self-referential' sentential one. There is no barrier to naming the sentence 'sentence 3 is not true' as 'sentence 3', since the identity of the sentence is independent of what the phrase 'sentence 3', in it, refers to. Not so with any attempt to name, for instance, the proposition that proposition 3 is not true, as 'proposition 3' (c.f. Kripke 1975, note 5). Thus what people have in mind with

3: 3 is not true

might be not a sentential identity but a propositional one:

Proposition 3 is that proposition 3 is not true.

But to know the identity of the proposition on the right hand side of this identity one needs to already know the referent of the 'proposition 3' used in making that proposition. So its referent is settled, and one is not free to nominate the referent of the expression subsequently. Of course, no referent of the expression is given by the identity as stated, since that is circular and so leads to an infinite regress, as Kneale pointed out (Kneale 1972, 242). Yet another way of realising this point is by seeing that substitution into the right hand side of the previous sentential identity is impossible, because of the quotation, whereas substitution into the right hand side of the propositional identity is possible, because of the lack of quotation. The same point holds with other phrases referring to propositions, e.g.

What sentence 3 says is that what sentence 3 says is not true,

although care must be taken to distinguish this from the unproblematic:

What sentence 3 says is that sentence 3 is not true.

The general, overriding point is that there is no syntactic self-reference, since any reference is only given through an interpretation. Thus one must primarily remember, for instance, that a sentential identity like

$t =$ 't is not true',

does not itself show that some sentence is about itself, since it does not entail that

$(\exists x)(x =$ 'x is not true'$)$,

by existential generalisation, and neither can 't' be replaced throughout by a quotation name for the supposedly self-referential sentence, since nothing of the form

'p' $=$ "'p' is not true"

is possible, because nothing can be a proper part of itself. By contrast, if one talks not about the identity of a sentence but about *the content* of it, in a sentence where there is not *direct quotation*, but *indirect speech*, such as

t says that t is not true,

there follows unproblematically that

$(\exists x)(x$ says that x is not true$)$,

and so that something is self-referential. And there is an equal possibility of providing a quotation name for a sentence of the required kind, since there is nothing against cases like the following being true (on a given interpretation):

'p' says that 'p' is not true.

The basic confusion, therefore, is a use-mention confusion and it is primarily separating clearly use from mention that shows there is no problem with any version of the Liar Paradox — or Open Pair, or Yablo type case, as above. For it is only on a certain interpretation that paradoxes arise, and it is now clear that that is an additional, intensional matter beyond any extensional, direct speech identity. The fact that the interpretation is an intensional matter is what brings in the need for indirect speech, and 'that'-clauses. But that then shows that what is relevantly true in paradoxical cases is simply that some sentence is not true, while also showing that to say that some sentence is not true is not to say that what is true is some sentence. What is true is what some sentence says on a given interpretation, i.e. the proposition it makes in those circumstances.

3

But the pragmatic dimension involved in 'that'-clause use extends well beyond indexical sentences, eternal sentences, and listed sentences. It reaches even the area of mathematical sentences, which at one time were thought to be immune to truth-value variability. For, while it was maybe quite plausible to believe that one can have a truth predicate of elementary mathematical sentences such that, e.g.

T'$2 + 3 = 5$' if and only if $2 + 3 = 5$,

the realisation that the cases where this kind of equivalence has a chance of holding are very special cases has now begun to dawn — given Tarski's own theorem, showing that there cannot be such a 'T' in general, even in Arithmetic. If there are alternative models for arithmetic sentences, as Gödel demonstrated, then no formal equivalence like the one displayed is available, in general, and only *the use* of the sentence on the left in connection with the standard model for Arithmetic would produce a necessary *propositional* equivalence,

It is true that $2 + 3 = 5$ if and only if $2 + 3 = 5$.

But that is an instance of Horwich's Equivalence Scheme (Horwich 1998), not of Tarski's Truth Scheme (Tarski 1956).

Quite a lot hangs on this. What is true, in Gödel's First Incompleteness Theorem, for instance, is not the sentence '$(x)Fx$', for a certain predicate 'F', but the proposition that all natural numbers are F, i.e. the proposition expressed by the sentence when it is used with respect to the standard model.

The inability of the logical tradition to represent such a propositional referring phrase as 'that all natural numbers are F' has, by contrast, made it seem that what is true or false on the standard interpretation is still the (mentioned) sentence '$(x)Fx$'; but only that sentence in a certain use, preceded by 'that', refers to the item that has the truth-value. For the formula '$(x)Fx$' is indexical, because the universe of discourse of the quantification is variable, and that allows different propositions to be made with this same sentence, while only one such proposition is claimed to be true.

The major consequence of this, not always drawn, is a very large one indeed: that humans are categorically different from Turing machines. For while, like Turing Machines, humans can utter sentences such as '$(x)Fx$', they can also do something Turing Machines cannot, namely use sentences like '$(x)Fx$' to state things about different models. In particular, a Turing Machine would have to not only utter '$(x)Fx$', but also use it pragmatically in connection with one model rather than another, and specifically in connection with the standard, intended model, if it was to state that all natural numbers are F. But a Turing Machine lacks the required power of choice to select the standard model, and thereby any capacity to prove that Fn for any natural number n, even if it can generate the sentence 'Fn' for every numeral 'n'.

The central point is that Tarski, although he continuously expressed propositions, and made statements, was not conscious in a theoretical way either of the existence of propositions and statements, or their distinctive grammar. But, of course, the idea that sentences are the bearers of Truth was not just Tarski's opinion. There were many reasons advanced by theorists, in the early decades of the twentieth century, for the abandonment of statements and propositions, and concentration instead on sentences as the bearers of semantic assessments. More important in the present context was the main practical measure to the same effect: the preference for an unnatural language in which such abstract objects could not even be referred to, or talked about. The principal kinds of expression that do that, in natural language, are the 'that'-clauses focussed on above. 'That'-clauses are substantival phrases such as occur in subject-predicate sentences like 'That the Kneales showed how to refer to properties and propositions is true, but not well known' (see, for instance, the O.E.D. under 'that' as a conjunction). Intensional Logic, as currently developed, deals with related expressions. It deals with the 'cleft' form of such sentences, i.e., in the case illustrated, 'It is true, but not well known, that the Kneales showed how to refer to properties and propositions'. But that would be symbolised as involving the operators 'it is true that', and 'it is not well known that', and so it would be expressed in a language in which 'that'-clauses have no distinct, substantival place.

Difficulties with the recognition of propositional referring phrases of the form 'that p' are therefore a large part of what have made Liar sentences seem paradoxical. Frege’s content stroke, i.e. the horizontal line that he sometimes used to indicate the thought expressed by a sentence, has not been incorporated into the generality of logic texts that have followed his formal work, and that has caused many of the difficulties (Slater 2001, 2008). Using a quotational form “‘p’” as an alternative to ‘that p’, as is commonly done, confuses syntactic expressions with their semantic and pragmatic readings, and leads to misunderstandings about the differences between Tarski’s Truth Scheme and Horwich’s Equivalence Scheme.

One difference between Tarski’s Truth Scheme and the Equivalence Scheme of Horwich, for example, is that only the latter applies to indexical cases. Horwich’s is a propositional schema, viz

> the proposition that p is true if and only if p,

whereas Tarski’s is a sentential schema:

> the sentence x is true if and only if p,

where what replaces ‘x’ is a name of a sentence whose translation into the metalanguage replaces ‘p’. The difference is most pointed in the homophonic sentential case, which parallels very closely the propositional one. For what replaces ‘x’ then is a quotation-name of the sentence that replaces ‘p’, not that sentence itself. So one could have, for instance,

> that he is happy is true if and only if he is happy,

while one cannot have

> ‘he is happy’ is true if and only if he is happy.

Certainly there would be less need to make the distinction if all sentences were unambiguous and non-indexical, i.e. had just one interpretation, since then facts about propositions could be mapped 1–1 onto facts about sentences. But the central question, as we have seen, is whether sentences are non-indexical in the required way.

It has to be said that the difference between ‘that p is true’, and “‘p’ is true” may not be completely appreciated even in Horwich’s informal work, since he thinks there are still paradoxical cases of his propositional schema. But ‘it is true that’ is the null modality in the modal system KT, i.e. an ‘L’ for which it is necessary that $Lp \equiv p$, and so one cannot have $p \equiv \neg Lp$, since the modal system KT is consistent. Horwich’s thought against this is in terms of ‘THE PROPOSITION FORMULATED IN CAPITAL LETTERS IS NOT TRUE’, which he abbreviates to ‘#’ (Horwich 1998, 40–41). He also quite generally abbreviates ‘the proposition that p’ to ‘$< p >$’. Naturally he gets ‘#’ from ‘<#> is true’, and ‘¬#’ from ‘<#> is not true’. But he also wants to derive ‘<#> is not true’ from ‘#’ (‘whose subject, said to be not true, turns out to be the proposition <#>’) and ‘<#> is true’ from ‘¬#’ (‘which says of <#> that it is *not* not true’). So he ends up saying that <#> is true if and only if <#> is not true.

There clearly must be something wrong with this argument if ‘It is true that p’ is equivalent to ‘That p is true’ and the former is quite consistent. But where does Horwich go wrong? He goes wrong through thinking that a specific proposition is expressed in the case in question. Specifically, what the contradiction shows is that the referring phrase ‘THE PROPOSITION FORMULATED IN CAPITAL LETTERS’ must be non-attributive, i.e. Millian, allowing ‘#’ not to express a proposition any more than the indexical sentence ‘This is not true’ does, on its own.

A more commonly presented example, requiring much the same kind of solution, arises with, for example,

> The proposition expressed by this sentence token is not true,

when the referent of ‘this sentence token’ here is supposed to be the sentence token just indented (which has to be added, since a token of the very same sentence, of course, could make reference to a quite different sentence token). But there is no paradox here, since that would arise only if a specific, single proposition was expressed by the sentence token in question. It was Prior, in recent times, who first came to suspect, in connection with such cases, that there was the possibility of ambiguity, preventing a single proposition being expressed. Thus he said (Prior 1971, 106): ‘We could then say that if x means that x is false it will have two contradictory meanings — that it is false and that it is true’. But the exploration of such possibilities arose even earlier, in the work of Thomas Bradwardine. See for complete details Read 2010.

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