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CUT-ELIMINATION AND COMPLETENESS IN DYNAMIC TOPOLOGICAL AND LINEAR-TIME TEMPORAL LOGICS

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Abstract
Two Gentzen-type sequent calculi L_ω and L_ω^- are introduced. Some dynamic topological and linear-time temporal logics are subsumed in L_ω and L_ω^-. The cut-elimination theorems for L_ω and L_ω^- and the completeness theorem for L_ω^- are uniformly proved based on an embedding-based method.

1. *Introduction*

Dynamic topological logic (DTL), which is a combination of S4 and temporal logic, has recently been studied by several researchers (see e.g., [1, 8, 9, 10, 12]). DTL provides a context for studying the confluence of the topological semantics for S4, topological dynamics, and temporal logic [9]. Two bimodal (next-interior) fragments of DTL, which are called S4F (functional) and S4C (continuous), were first introduced by Artemov et al. [1]. In [1], some Gentzen-type cut-free sequent calculi and Hilbert-type axiom schemes were introduced for S4F and S4C, and the complete topological and Kripke-type semantics were obtained for S4F and S4C. An alternative formulation of cut-free sequent calculus for S4C was studied by Mints [12].

Trimodal DTLs were formalized semantically by Kremer and Mints [9], combining the S4 modal operator $\Box$ (interior) and the linear-time temporal operators X (next), and G (globally or henceforth). Although sequent calculi for S4F and S4C have been studied, sequent calculi for trimodal DTLs have not been studied yet. The reasons may be that the trimodal DTL of homeomorphism was shown to be not recursively axiomatizable by Konev et al. [10], and that this logic requires the following infinitary axiom scheme [9]:

$$\mathrm{G}\alpha \leftrightarrow \alpha \wedge \mathrm{X}\alpha \wedge \mathrm{XX}\alpha \wedge \mathrm{XXX}\alpha \wedge \cdots \infty.$$

In this paper, two Gentzen-type cut-free sequent calculi L_ω and L_ω^- that can derive this infinitary axiom scheme are introduced by combining linear-time temporal logic and infinitary logic.

Linear-time temporal logic (LTL), which has the temporal operators X, G and F (eventually), has been studied by many researchers (see e.g., [3, 11, 13] and the references therein). A Gentzen-type cut-free sequent calculus LT_ω for LTL was introduced by Kawai [7]. *Infinitary logic* (IL), which has the infinitary conjunction $\bigwedge$ and the infinitary disjunction $\bigvee$, has been studied by many logicians (see e.g., [6, 16] and the references therein). Gentzen-type cut-free sequent calculi for IL and its modal extensions have also been studied. A sequent calculus, called here LK_ω, for IL was introduced and studied in the 1950s. A sequent calculus, called here $\mathrm{S4}_\omega$, for a modal extension of IL was used as a base system for game theory [6].

The results of this paper are then summarized as follows. A new sequent calculus L_ω for a logic that includes S4F, S4C and the S4H of homeomorphism is introduced by combining LT_ω and $\mathrm{S4}_\omega$. A sequent calculus L_ω^-, which is an integration of LT_ω and LK_ω, is also introduced as the $\Box$-less subsystem of L_ω. The cut-elimination theorem for L_ω is proved using a theorem for syntactically embedding L_ω into $\mathrm{S4}_\omega$. The completeness theorem (w.r.t. Kripke semantics) for L_ω cannot be shown since $\mathrm{S4}_\omega$ is known to be Kripke-incomplete. The cut-elimination and completeness theorems for L_ω^- are proved uniformly by combining two theorems for syntactically and semantically embedding L_ω^- into LK_ω. The proposed embedding theorems are regarded as modified extensions of the embedding theorem [5] of LT_ω into LK_ω. An embedding-based cut-elimination proof for LT_ω was obtained in [5]. However, an embedding-based completeness proof for LT_ω has not yet been obtained. The proposed embedding-based completeness proof for L_ω^- is thus a new technical contribution of this paper. This technique can also be applied to the completeness theorem for LT_ω.

The merits of the results of this paper are summarized as follows:

1. L_ω and L_ω^- give a natural sequent-style formalization for Kremer-Mints' infinitary axiom scheme for trimodal DTLs. L_ω is a natural extension of S4F, S4C, S4H, $\mathrm{S4}_\omega$, LT_ω and LK_ω. L_ω^- is a natural extension of LT_ω and LK_ω.
2. A simple, easy and uniform proof of the cut-elimination and completeness theorems for L_ω^- is obtained based on the syntactical and semantical embedding theorems of L_ω^- into LK_ω. This proof method is also applicable to LT_ω.
3. A Baratella-Masini style cut-free 2-sequent calculi [2] for L_ω and L_ω^- can easily be obtained by using the cut-elimination-preserving translations proposed in [4], although this result is omitted in this paper.

2. *Sequent calculus*

The following list of symbols is adopted for the language of the underlying logic: (countable) propositional variables $p_0, p_1, \ldots,$ $\rightarrow$ (implication), $\neg$ (negation), $\bigwedge$ (infinitary conjunction), $\bigvee$ (infinitary disjunction), $\Box$ (interior), X (next), G (globally) and F (eventually). Remark that the standard binary connectives $\wedge$ (conjunction) and $\vee$ (disjunction) are regarded as special cases of $\bigwedge$ and $\bigvee$, respectively.

Definition 2.1: *Let F_0 be the set of all formulas generated by the standard finitely inductive definition with respect to $\{\rightarrow, \neg, \Box, \mathrm{X}, \mathrm{G}, \mathrm{F}\}$ from the set of propositional variables. Suppose that F_t is already defined with respect to $t = 0, 1, 2, \ldots$. A non-empty countable subset Θ of F_t is called an* allowable set. *The expressions $\bigwedge \Theta$ and $\bigvee \Theta$ for an allowable set Θ are considered below. We define F_{t+1} from $F_t \cup \{\bigwedge \Theta_t, \bigvee \Theta_t \mid \Theta$ is an allowable set in $F_t\}$ by the standard finitely inductive definition with respect to $\{\rightarrow, \neg, \Box, \mathrm{X}, \mathrm{G}, \mathrm{F}\}$. The set F_ω, which is called the set of formulas, is defined by $\bigcup_{t<\omega} F_t$, and an expression in F_ω is called a* formula. *An expression of the form $\Gamma \Rightarrow \Delta$ where Γ and Δ are finite (possibly empty) sets of formulas is called a* sequent.

Greek lower-case letters $\alpha, \beta, \ldots$ are used to denote formulas, and Greek capital letters $\Gamma, \Delta, \ldots$ are used to represent finite (possibly empty) sets of formulas. For any $\sharp \in \{\Box, \mathrm{X}, \mathrm{G}, \mathrm{F}\}$, an expression $\sharp\Gamma$ is used to denote the set $\{\sharp\gamma \mid \gamma \in \Gamma\}$. The symbol ω is used to represent the set of natural numbers. Lower-case letters i, j and k are used to denote any natural numbers. An expression $\mathrm{X}^i\alpha$ for any $i \in \omega$ is defined inductively by $(\mathrm{X}^0\alpha := \alpha)$ and $(\mathrm{X}^{n+1}\alpha := \mathrm{X}\mathrm{X}^n\alpha)$.

Sequent calculi L_ω and L_ω^- are then introduced below.

Definition 2.2: (L_ω and L_ω^-) *The initial sequents of* L_ω *are of the form: for any propositional variable p,*

$$\mathrm{X}^i p \Rightarrow \mathrm{X}^i p.$$

The inference rules of L_ω *are of the form:*

$$\frac{\Gamma \Rightarrow \Delta, \alpha \quad \alpha, \Sigma \Rightarrow \Pi}{\Gamma, \Sigma \Rightarrow \Delta, \Pi}\ (\mathrm{cut}) \qquad \frac{\Gamma \Rightarrow \Delta}{\Sigma, \Gamma \Rightarrow \Delta, \Lambda}\ (\mathrm{we})$$

$$\frac{\Gamma \Rightarrow \Sigma, \mathrm{X}^i\alpha \quad \mathrm{X}^i\beta, \Delta \Rightarrow \Pi}{\mathrm{X}^i(\alpha{\rightarrow}\beta), \Gamma, \Delta \Rightarrow \Sigma, \Pi}\ ({\rightarrow}\mathrm{left}) \qquad \frac{\mathrm{X}^i\alpha, \Gamma \Rightarrow \Delta, \mathrm{X}^i\beta}{\Gamma \Rightarrow \Delta, \mathrm{X}^i(\alpha{\rightarrow}\beta)}\ ({\rightarrow}\mathrm{right})$$

$$\frac{\Gamma \Rightarrow \Delta, \mathrm{X}^i\alpha}{\mathrm{X}^i\neg\alpha, \Gamma \Rightarrow \Delta}\ (\neg\text{left}) \qquad \frac{\mathrm{X}^i\alpha, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \mathrm{X}^i\neg\alpha}\ (\neg\text{right})$$

$$\frac{\mathrm{X}^i\alpha, \Gamma \Rightarrow \Delta\ (\alpha \in \Theta)}{\mathrm{X}^i(\bigwedge\Theta), \Gamma \Rightarrow \Delta}\ (\bigwedge\text{left}) \qquad \frac{\{\ \Gamma \Rightarrow \Delta, \mathrm{X}^i\alpha\ \}_{\alpha\in\Theta}}{\Gamma \Rightarrow \Delta, \mathrm{X}^i(\bigwedge\Theta)}\ (\bigwedge\text{right})$$

$$\frac{\{\ \mathrm{X}^i\alpha, \Gamma \Rightarrow \Delta\ \}_{\alpha\in\Theta}}{\mathrm{X}^i(\bigvee\Theta), \Gamma \Rightarrow \Delta}\ (\bigvee\text{left}) \qquad \frac{\Gamma \Rightarrow \Delta, \mathrm{X}^i\alpha\ (\alpha \in \Theta)}{\Gamma \Rightarrow \Delta, \mathrm{X}^i(\bigvee\Theta)}\ (\bigvee\text{right})$$

where Θ *is an allowable set,*

$$\frac{\mathrm{X}^i\alpha, \Gamma \Rightarrow \Delta}{\mathrm{X}^i\Box\alpha, \Gamma \Rightarrow \Delta}\ (\Box\text{left}) \qquad \frac{\mathrm{X}^i\Box\Gamma \Rightarrow \mathrm{X}^k\alpha}{\mathrm{X}^i\Box\Gamma \Rightarrow \mathrm{X}^k\Box\alpha}\ (\Box\text{right})$$

$$\frac{\mathrm{X}^{i+k}\alpha, \Gamma \Rightarrow \Delta}{\mathrm{X}^i\mathrm{G}\alpha, \Gamma \Rightarrow \Delta}\ (\text{Gleft}) \qquad \frac{\{\ \Gamma \Rightarrow \Delta, \mathrm{X}^{i+j}\alpha\ \}_{j\in\omega}}{\Gamma \Rightarrow \Delta, \mathrm{X}^i\mathrm{G}\alpha}\ (\text{Gright})$$

$$\frac{\{\ \mathrm{X}^{i+j}\alpha, \Gamma \Rightarrow \Delta\ \}_{j\in\omega}}{\mathrm{X}^i\mathrm{F}\alpha, \Gamma \Rightarrow \Delta}\ (\text{Fleft}) \qquad \frac{\Gamma \Rightarrow \Delta, \mathrm{X}^{i+k}\alpha}{\Gamma \Rightarrow \Delta, \mathrm{X}^i\mathrm{F}\alpha}\ (\text{Fright}).$$

L_ω^- *is obtained from* L_ω *by deleting* $\{$(□left), (□right)$\}$.

It is remarked that the contraction rules

$$\frac{\alpha, \alpha, \Gamma \Rightarrow \Delta}{\alpha, \Gamma \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow \Delta, \alpha, \alpha}{\Gamma \Rightarrow \Delta, \alpha}$$

are not used in L_ω^- and L_ω since the antecedents and conclusions of sequents are sets of formulas.

Definition 2.3: ($\mathrm{S4}_\omega$ and LK_ω) *A sequent calculus* $\mathrm{S4}_\omega$ *for an infinitary version of the modal logic* S4 *is obtained from* L_ω *by deleting* (Gleft), (Gright), (Fleft), (Fright) *and replacing* i, k *by* 0 *(i.e., deleting every occurrence of* X*). The modified inference rules for* $\mathrm{S4}_\omega$ *by replacing* i, k *by* 0 *are denoted by labeling "*0*" in superscript, e.g.,* ($\rightarrow$left^0). *A sequent calculus* LK_ω *for classical infinitary logic is obtained from* $\mathrm{S4}_\omega$ *by deleting* (□left^0) *and* (□right^0).

The following cut-elimination theorem is well-known (see e.g., [16, 6] and the references therein).

Proposition 2.4: *Let* L *be* $\mathrm{S4}_\omega$ *or* LK_ω*. The rule* (cut) *is admissible in cut-free* L*.*

An expression $L \vdash S$ or $\vdash S$ is used to denote the fact that a sequent S is provable in a sequent calculus L.

Note that the rules ($\bigwedge$right), ($\bigvee$left), (Gright) and (Fleft) have infinite premises. The sequents of the form: $\mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\alpha$ for any formula α are provable in cut-free L_ω and cut-free L_ω^-. This fact is proved by induction on the complexity of α. Hence, in the following discussion, these sequents are sometimes regarded as the initial sequents of L_ω and L_ω^-.

Proposition 2.5: *Let L be L_ω or L_ω^-. The rule*

$$\frac{\Gamma \Rightarrow \Delta}{\mathrm{X}\Gamma \Rightarrow \mathrm{X}\Delta}\ (\mathrm{Xregu})$$

is admissible in cut-free L.

Proof. By induction on the proofs P of $\Gamma \Rightarrow \Delta$ in cut-free L. □

An expression $\alpha \Leftrightarrow \beta$ is used as an abbreviation of two sequents $\alpha \Rightarrow \beta$ and $\beta \Rightarrow \alpha$.

Proposition 2.6: *The following sequents are provable in cut-free L_ω and cut-free L_ω^-: for any formulas α, β, any allowable set Θ and any $i \in \omega$,*

1. $\mathrm{X}^i(\alpha{\rightarrow}\beta) \Leftrightarrow \mathrm{X}^i\alpha{\rightarrow}\mathrm{X}^i\beta$,
2. $\mathrm{X}^i\neg\alpha \Leftrightarrow \neg\mathrm{X}^i\alpha$,
3. $\mathrm{X}^i(\sharp\Theta) \Leftrightarrow \sharp(\mathrm{X}^i\Theta)$ *where* $\sharp \in \{\bigwedge, \bigvee\}$,
4. $\mathrm{G}\alpha \Leftrightarrow \bigwedge\{\mathrm{X}^i\alpha \mid i \in \omega\}$,
5. $\mathrm{F}\alpha \Leftrightarrow \bigvee\{\mathrm{X}^i\alpha \mid i \in \omega\}$,
6. $\mathrm{G}\alpha \Rightarrow \mathrm{X}\alpha$,
7. $\mathrm{G}\alpha \Rightarrow \mathrm{XG}\alpha$,
8. $\mathrm{G}\alpha \Rightarrow \mathrm{GG}\alpha$,
9. $\alpha, \mathrm{G}(\alpha{\rightarrow}\mathrm{X}\alpha) \Rightarrow \mathrm{G}\alpha$ *(time induction).*

The following sequents are provable in cut-free L_ω: for any formula α and any $i \in \omega$,

10. $\mathrm{X}^i\Box\alpha \Leftrightarrow \Box\mathrm{X}^i\alpha$.

Proof. We show some cases.

(4):

$$\dfrac{\dfrac{\{\ \mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\alpha\ \}_{\mathrm{X}^i\alpha\in\{\mathrm{X}^i\alpha\ |\ i\in\omega\}}}{\{\ \mathrm{G}\alpha \Rightarrow \mathrm{X}^i\alpha\ \}_{\mathrm{X}^i\alpha\in\{\mathrm{X}^i\alpha\ |\ i\in\omega\}}}\ (\mathrm{Gleft})}{\mathrm{G}\alpha \Rightarrow \bigwedge\{\mathrm{X}^i\alpha \mid i\in\omega\}}\ (\textstyle\bigwedge\text{right})\quad \dfrac{\dfrac{\{\ \mathrm{X}^j\alpha \Rightarrow \mathrm{X}^j\alpha\ \}_{j\in\omega}}{\{\ \bigwedge\{\mathrm{X}^i\alpha \mid i\in\omega\} \Rightarrow \mathrm{X}^j\alpha\ \}_{j\in\omega}}\ (\textstyle\bigwedge\text{left})}{\bigwedge\{\mathrm{X}^i\alpha \mid i\in\omega\} \Rightarrow \mathrm{G}\alpha}\ (\mathrm{Gright}).$$

(9):

$$\dfrac{\begin{matrix}\vdots\\ \{\alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^k\alpha\}_{k\in\omega}\end{matrix}}{\alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{G}\alpha}\ (\mathrm{Gright})$$

where $\mathrm{L}_\omega \vdash \alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^k\alpha$ for any $k \in \omega$ is shown by mathematical induction on k as follows. The base step is obvious using (we). The induction step can be shown as follows.

$$\dfrac{\dfrac{\begin{matrix}\vdots\ \text{ind.hyp.}\\ \alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^k\alpha\end{matrix} \quad \mathrm{X}^{k+1}\alpha \Rightarrow \mathrm{X}^{k+1}\alpha}{\alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha), \mathrm{X}^k(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^{k+1}\alpha}\ (\to\text{left})}{\alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha), \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^{k+1}\alpha}\ (\mathrm{Gleft})$$

where the last sequent of this proof is equivalent to the sequent $\alpha, \mathrm{G}(\alpha\to\mathrm{X}\alpha) \Rightarrow \mathrm{X}^{k+1}\alpha$.

(10):

$$\dfrac{\dfrac{\mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\alpha}{\mathrm{X}^i\Box\alpha \Rightarrow \mathrm{X}^i\alpha}\ (\Box\text{left})}{\mathrm{X}^i\Box\alpha \Rightarrow \Box\mathrm{X}^i\alpha}\ (\Box\text{right}) \qquad \dfrac{\dfrac{\mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\alpha}{\Box\mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\alpha}\ (\Box\text{left})}{\Box\mathrm{X}^i\alpha \Rightarrow \mathrm{X}^i\Box\alpha}\ (\Box\text{right}).$$

□

In the following, we give some remarks on Proposition 2.6.

1. The sequents listed in (1)–(3) and (10) correspond to the characteristic axioms for some next-interior fragments of DTL. In fact, a Hilbert-style axiomatization of S4C is obtained from that of S4 by adding the following axiom schemes and inference rule:
   (a) $\mathrm{X}(\alpha \circ \beta) \leftrightarrow \mathrm{X}\alpha \circ \mathrm{X}\beta$ where $\circ \in \{\to, \wedge, \vee\}$,
   (b) $\mathrm{X}\neg\alpha \leftrightarrow \neg\mathrm{X}\alpha$,
   (c) $\mathrm{X}\Box\alpha\to\Box\mathrm{X}\alpha$,

(d)

$$\frac{\alpha}{\mathrm{X}\alpha}\ .$$

2. In particular, the sequents of the forms $\mathrm{X}\Box\alpha \Rightarrow \Box\mathrm{X}\alpha$ and $\Box\mathrm{X}\alpha \Rightarrow \mathrm{X}\Box\alpha$ listed in (10) respectively correspond to the *continuous axiom*, which characterizes the continuity property of the function f on the topological space X of the underlying dynamic topological system (X, f), and the *homeomorphism axiom*, which characterizes the open mapping property of f in (X, f). If a function is a continuous open bijection, then the function is called a *homeomorphism*. For details, see [8] and the references therein.
3. In order to prove the sequents listed in (10), we need the fact that the parameters i and k in (□right) and (□left) can be different. Indeed, the applications of (□right) in the left and right proof figures in the proof of (10) correspond to the cases $k = 0$ and $k = i$, respectively.
4. The sequents listed in (4) and (5) correspond to the characteristic axioms for a full DTL with a homeomorphism f on a topological space X. Intuitively, (4) and (5) are respectively interpreted as follows [10]: for a given subset V of X,
   (a) $\mathrm{G}V := \bigcap\{f^{-i}(V) \mid i \in \omega\}$
   (b) $\mathrm{F}V := \bigcup\{f^{-i}(V) \mid i \in \omega\}$

   where f^{-i} means the i-times iteration of the inverse mapping of f.
5. The sequents listed in (6)–(9) correspond to the characteristic axioms for LTL.

In the following, we present a comparison of other sequent systems.

1. A sequent calculus $\mathrm{S4F}_G$ [1] for S4F is obtained from a standard sequent system for S4 by adding (Xregu). The rules (→right), (→left) and (□left) were shown to be admissible in cut-free $\mathrm{S4F}_G$ [1].
2. A sequent calculus $\mathrm{S4C}_G$ [1] for S4C is obtained from $\mathrm{S4F}_G$ by adding the rule of the form:

$$\frac{\Box\mathrm{X}\Box\alpha, \Gamma \Rightarrow \Delta}{\mathrm{X}\Box\alpha, \Gamma \Rightarrow \Delta}\ .$$

3. Mints' sequent calculus for S4C is similar to $\mathrm{S4C}_G$, and uses the rule of the form
$$\frac{\mathsf{B} \Rightarrow \alpha}{\mathsf{B} \Rightarrow \Box\alpha}$$
where B is a set of formulas of the form $\mathrm{X}^i\Box\alpha$. It is noted that this rule dose not allow to derive the sequent $\Box\mathrm{X}\alpha \Rightarrow \mathrm{X}\Box\alpha$ of homeomorphisms. In order to derive such a sequent, the rule ($\Box$right) proposed in this paper is needed.

4. A sequent calculus for a bimodal version of DTL with a homeomorphism, called S4H in [8], is then regarded as the $\{\rightarrow, \wedge, \vee, \Box, \mathrm{X}\}$ fragment of L_ω.

5. Kawai's LT_ω [7] for LTL is obtained from L_ω by deleting ($\Box$left), ($\Box$right) and replacing the $\{\bigwedge, \bigvee\}$ rules by the standard binary $\{\wedge, \vee\}$ rules. The cut-elimination and completeness theorems for LT_ω was shown in [7] using Schütte's method.

6. A *2-sequent calculus* $2\mathrm{S}\omega$ for LTL, which is a natural extension of the usual sequent calculus, was introduced by Baratella and Masini, and the cut-elimination and completeness theorems for this calculus were proved based on an analogy between LTL and Peano arithmetic with ω rule [2]. A direct syntactical equivalence between LT_ω and $2\mathrm{S}\omega$ was shown by introducing the translation functions that preserve cut-free proofs of these calculi [4]. Baratella-Masini-style cut-free 2-sequent calculi can also be obtained for L_ω and L_ω^-.

7. The rule (Xregu) was first introduced by Artemov et al. [1] for formalizing S4F and S4C. This rule is more expressive than the following standard inference rule for the modal logic K:
$$\frac{\Gamma \Rightarrow \alpha}{\Box\Gamma \Rightarrow \Box\alpha}$$
in which X is represented by $\Box$. The corresponding logic for the propositional LK with (Xregu), which was called KF, is identified as characteristic for total (serial) and functional (deterministic) binary relations in the Kripke semantics. Axioms for KF were introduced by Prior in the 1950s as the axioms for the modality that can represent "tomorrow it will be the case that." For more information on KF, see e.g., [1, 14, 15] and the references therein. The system L_ω is thus also regarded as an extension of the sequent calculus for KF.

8. L_ω^- is regarded as an integration of both LT_ω and LK_ω. Thus, L_ω^- is a sequent calculus for a natural extension of both LTL and IL.

3. *Cut-elimination*

Definition 3.1: *We fix a countable non-empty set Φ of propositional variables, and define the sets $\Phi_i := \{p_i \mid p \in \Phi\}$ $(1 \leq i \in \omega)$ and $\Phi_0 := \Phi$ of propositional variables where $p_0 = p$. The language $\mathcal{L}_{\mathrm{L}_\omega}$ (or the set of formulas) of L_ω is defined using Φ, $\rightarrow$, $\neg$, $\bigwedge$, $\bigvee$, $\Box$, X, G and F in the same way as in Definition 2.1. The language $\mathcal{L}_{\mathrm{S4}_\omega}$ of $\mathrm{S4}_\omega$ is defined using $\bigcup_{i\in\omega}\Phi_i$, $\rightarrow$, $\neg$, $\bigwedge$, $\bigvee$, $\Box$ in a similar way as in Definition 2.1.*

A mapping f from $\mathcal{L}_{\mathrm{L}_\omega}$ to $\mathcal{L}_{\mathrm{S4}_\omega}$ is defined as follows.

1. $f(\mathrm{X}^i p) := p_i \in \Phi_i$ $(i \in \omega)$ *for any* $p \in \Phi$ *(especially,* $f(p) := p \in \Phi$*),*
2. $f(\mathrm{X}^i(\alpha{\rightarrow}\beta)) := f(\mathrm{X}^i\alpha){\rightarrow}f(\mathrm{X}^i\beta)$,
3. $f(\mathrm{X}^i\neg\alpha) := \neg f(\mathrm{X}^i\alpha)$,
4. $f(\mathrm{X}^i(\sharp\Theta)) := \sharp f(\mathrm{X}^i\Theta)$ *where* Θ *is an allowable set,* $\sharp \in \{\bigwedge, \bigvee\}$, *and* $f(\mathrm{X}^i\Theta)$ *is the result of replacing every occurrence of a formula* α *in* $\mathrm{X}^i\Theta$ *by an occurrence of* $f(\alpha)$,
5. $f(\mathrm{X}^i\Box\alpha) := \Box f(\mathrm{X}^i\alpha)$,
6. $f(\mathrm{X}^i\mathrm{G}\alpha) := \bigwedge\{f(\mathrm{X}^{i+j}\alpha) \mid j \in \omega\}$,
7. $f(\mathrm{X}^i\mathrm{F}\alpha) := \bigvee\{f(\mathrm{X}^{i+j}\alpha) \mid j \in \omega\}$.

We also define the languages $\mathcal{L}_{\mathrm{L}^-_\omega}$ (for L^-_ω) and $\mathcal{L}_{\mathrm{LK}_\omega}$ (for LK_ω) as the $\Box$-less sublanguages of $\mathcal{L}_{\mathrm{L}_\omega}$ and $\mathcal{L}_{\mathrm{S4}_\omega}$, respectively. A mapping f from $\mathcal{L}_{\mathrm{L}^-_\omega}$ to $\mathcal{L}_{\mathrm{LK}_\omega}$ is obtained from the above defined mapping by deleting the condition 5. We also use the same name f for this mapping.

An expression $f(\Gamma)$ denotes the result of replacing every occurrence of a formula α in Γ by an occurrence of $f(\alpha)$, e.g., if $\Gamma \equiv \{\alpha, \beta, \gamma\}$, then $f(\Gamma) \equiv \{f(\alpha), f(\beta), f(\gamma)\}$.

Theorem 3.2: (Syntactical embedding) *Let Γ and Δ be sets of formulas in $\mathcal{L}_{\mathrm{L}_\omega}$, and f be the mapping defined firstly in Definition 3.1. Then:*

1. $\mathrm{L}_\omega \vdash \Gamma \Rightarrow \Delta$ *iff* $\mathrm{S4}_\omega \vdash f(\Gamma) \Rightarrow f(\Delta)$.
2. $\mathrm{L}_\omega - (\mathrm{cut}) \vdash \Gamma \Rightarrow \Delta$ *iff* $\mathrm{S4}_\omega - (\mathrm{cut}) \vdash f(\Gamma) \Rightarrow f(\Delta)$.

Let Γ and Δ be sets of formulas in $\mathcal{L}_{\mathrm{L}^-_\omega}$, and f be the mapping defined secondly in Definition 3.1. Then:

1. $\mathrm{L}^-_\omega \vdash \Gamma \Rightarrow \Delta$ *iff* $\mathrm{LK}_\omega \vdash f(\Gamma) \Rightarrow f(\Delta)$.

2. $\mathrm{L}_\omega^- - (\mathrm{cut}) \vdash \Gamma \Rightarrow \Delta$ *iff* $\mathrm{LK}_\omega - (\mathrm{cut}) \vdash f(\Gamma) \Rightarrow f(\Delta)$.

Proof. We show only the case for L_ω. Since the case (2) can be obtained as the subproof of the case (1), we show only (1) in the following.

($\Longrightarrow$): By induction on the proof P of $\Gamma \Rightarrow \Delta$ in L_ω. We distinguish the cases according to the last inference of P, and show some cases.

Case ($\mathrm{X}^i p \Rightarrow \mathrm{X}^i p$): The last inference of P is of the form: $\mathrm{X}^i p \Rightarrow \mathrm{X}^i p$. In this case, we obtain $\mathrm{S4}_\omega \vdash f(\mathrm{X}^i p) \Rightarrow f(\mathrm{X}^i p)$, i.e., $\mathrm{S4}_\omega \vdash p_i \Rightarrow p_i$ ($p_i \in \Phi_i$).

Case ($\Box$right): The last inference of P is of the form:

$$\frac{\mathrm{X}^i \Box\Gamma \Rightarrow \mathrm{X}^k \alpha}{\mathrm{X}^i \Box\Gamma \Rightarrow \mathrm{X}^k \Box\alpha}\ (\Box\mathrm{right}).$$

By induction hypothesis, we have $\mathrm{S4}_\omega \vdash f(\mathrm{X}^i \Box\Gamma) \Rightarrow f(\mathrm{X}^k \alpha)$, i.e., $\mathrm{S4}_\omega \vdash \Box f(\mathrm{X}^i \Gamma) \Rightarrow f(\mathrm{X}^k \alpha)$. Then, we obtain

$$\frac{\begin{matrix}\vdots\\ \Box f(\mathrm{X}^i \Gamma) \Rightarrow f(\mathrm{X}^k \alpha)\end{matrix}}{\Box f(\mathrm{X}^i \Gamma) \Rightarrow \Box f(\mathrm{X}^k \alpha)}\ (\Box\mathrm{left}^0)$$

where $\Box f(\mathrm{X}^i \Gamma) \Rightarrow \Box f(\mathrm{X}^k \alpha)$ coincides with $f(\mathrm{X}^i \Box\Gamma) \Rightarrow f(\mathrm{X}^k \Box\alpha)$ by the definition of f.

Case (Gleft): The last inference of P is of the form:

$$\frac{\mathrm{X}^{i+k} \alpha, \Gamma \Rightarrow \Delta}{\mathrm{X}^i \mathrm{G}\alpha, \Gamma \Rightarrow \Delta}\ (\mathrm{Gleft}).$$

By induction hypothesis, we have $\mathrm{S4}_\omega \vdash f(\mathrm{X}^{i+k} \alpha), f(\Gamma) \Rightarrow f(\Delta)$, and hence obtain:

$$\frac{\begin{matrix}\vdots\\ f(\mathrm{X}^{i+k} \alpha), f(\Gamma) \Rightarrow f(\Delta)\end{matrix} \quad (f(\mathrm{X}^{i+k} \alpha) \in \{f(\mathrm{X}^{i+j} \alpha) \mid j \in \omega\})}{\bigwedge\{f(\mathrm{X}^{i+j} \alpha) \mid j \in \omega\}, f(\Gamma) \Rightarrow f(\Delta)}\ (\textstyle\bigwedge \mathrm{left}^0)$$

where $\bigwedge\{f(\mathrm{X}^{i+j} \alpha) \mid j \in \omega\}$ coincides with $f(\mathrm{X}^i \mathrm{G}\alpha)$ by the definition of f.

Case (Gright): The last inference of P is of the form:

$$\frac{\{\ \Gamma \Rightarrow \Delta, \mathrm{X}^{i+j} \alpha\ \}_{j \in \omega}}{\Gamma \Rightarrow \Delta, \mathrm{X}^i \mathrm{G}\alpha}\ (\mathrm{Gright}).$$

By induction hypothesis, we have $\mathrm{S4}_\omega \vdash f(\Gamma) \Rightarrow f(\Delta), f(\mathrm{X}^{i+j}\alpha)$ for all $j \in \omega$. Let Θ be $\{f(\mathrm{X}^{i+j}\alpha) \mid j \in \omega\}$. We obtain

$$\dfrac{\begin{array}{c}\vdots\\ \{\ f(\Gamma) \Rightarrow f(\Delta), f(\mathrm{X}^{i+j}\alpha)\ \}_{f(\mathrm{X}^{i+j}\alpha)\in\Theta}\end{array}}{f(\Gamma) \Rightarrow f(\Delta), \bigwedge\Theta}\ (\bigwedge\mathrm{right}^0)$$

where $\bigwedge\Theta$ coincides with $f(\mathrm{X}^i\mathrm{G}\alpha)$ by the definition of f.

($\Longleftarrow$): By induction on the proof Q of $f(\Gamma) \Rightarrow f(\Delta)$ in $\mathrm{S4}_\omega$. We distinguish the cases according to the last inference of Q. We show only some cases.

Case (cut): The last inference of Q is of the form:

$$\dfrac{f(\Gamma_1) \Rightarrow f(\Delta_1), \beta \quad \beta, f(\Gamma_2) \Rightarrow f(\Delta_2)}{f(\Gamma_1), f(\Gamma_2) \Rightarrow f(\Delta_1), f(\Delta_2)}\ (\mathrm{cut}).$$

Since β is in $\mathcal{L}_{\mathrm{S4}_\omega}$, we have the fact $\beta = f(\beta)$. This fact can be shown by induction on β. Then, by induction hypothesis, we have: $\mathrm{L}_\omega \vdash \Gamma_1 \Rightarrow \Delta_1, \beta$ and $\mathrm{L}_\omega \vdash \beta, \Gamma_2 \Rightarrow \Delta_2$. We then obtain the required fact: $\mathrm{L}_\omega \vdash \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2$ by using (cut) in L_ω.

Case ($\bigwedge$right^0):

Subcase (1): The last inference of Q is of the form:

$$\dfrac{\{\ f(\Gamma) \Rightarrow f(\Delta), f(\mathrm{X}^i\alpha)\ \}_{f(\mathrm{X}^i\alpha)\in f(\mathrm{X}^i\Theta)}}{f(\Gamma) \Rightarrow f(\Delta), \bigwedge f(\mathrm{X}^i\Theta)}\ (\bigwedge\mathrm{right}^0)$$

where $\bigwedge f(\mathrm{X}^i\Theta)$ coincides with $f(\mathrm{X}^i(\bigwedge\Theta))$ by the definition of f. By induction hypothesis, we have $\mathrm{L}_\omega \vdash \Gamma \Rightarrow \Delta, \mathrm{X}^i\alpha$ for all $\mathrm{X}^i\alpha \in \mathrm{X}^i\Theta$, i.e., for all $\alpha \in \Theta$. Then, we obtain

$$\dfrac{\begin{array}{c}\vdots\\ \{\ \Gamma \Rightarrow \Delta, \mathrm{X}^i\alpha\ \}_{\alpha\in\Theta}\end{array}}{\Gamma \Rightarrow \Delta, \mathrm{X}^i(\bigwedge\Theta)}\ (\bigwedge\mathrm{right}).$$

Subcase (2): The last inference of Q is of the form:

$$\dfrac{\{\ f(\Gamma) \Rightarrow f(\Delta), f(\mathrm{X}^{i+j}\alpha)\ \}_{f(\mathrm{X}^{i+j}\alpha)\in\{f(\mathrm{X}^{i+j}\alpha)\ \mid\ j\in\omega\}}}{f(\Gamma) \Rightarrow f(\Delta), \bigwedge\{f(\mathrm{X}^{i+j}\alpha) \mid j \in \omega\}}\ (\bigwedge\mathrm{right}^0)$$

where $\bigwedge\{f(\mathrm{X}^{i+j}\alpha) \mid j \in \omega\}$ coincides with $f(\mathrm{X}^i\mathrm{G}\alpha)$ by the definition of f. By induction hypothesis, we have $\mathrm{L}_\omega \vdash \Gamma \Rightarrow \Delta, \mathrm{X}^{i+j}\alpha$ for all $\mathrm{X}^{i+j}\alpha \in \{\mathrm{X}^{i+j}\alpha \mid j \in \omega\}$, i.e., for all $j \in \omega$. Then, we obtain

$$\dfrac{\begin{matrix}\vdots \\ \{\ \Gamma \Rightarrow \Delta, \mathrm{X}^{i+j}\alpha\ \}_{j\in\omega}\end{matrix}}{\Gamma \Rightarrow \Delta, \mathrm{X}^i\mathrm{G}\alpha}\ (\mathrm{Gright}).$$

□

Theorem 3.3: (Cut-elimination) *Let L be* L_ω *or* L_ω^-. *The rule* (cut) *is admissible in cut-free L.*

Proof. We show only the case for L_ω. Suppose $\mathrm{L}_\omega \vdash \Gamma \Rightarrow \Delta$. Then, we have $\mathrm{S4}_\omega \vdash f(\Gamma) \Rightarrow f(\Delta)$ by Theorem 3.2 (1), and hence $\mathrm{S4}_\omega -$ (cut) $\vdash f(\Gamma) \Rightarrow f(\Delta)$ by Proposition 2.4. By Theorem 3.2 (2), we obtain $\mathrm{L}_\omega -$ (cut) $\vdash \Gamma \Rightarrow \Delta$. □

4. *Completeness*

Let Γ be a set $\{\alpha_1, ..., \alpha_m\}$ $(m \geq 0)$ of formulas. Then, Γ^* represents $\alpha_1 \vee \cdots \vee \alpha_m$ if $m \geq 1$, and otherwise $\neg(p{\rightarrow}p)$ where p is a fixed propositional variables. Also Γ_* represents $\alpha_1 \wedge \cdots \wedge \alpha_m$ if $m \geq 1$, and otherwise $p{\rightarrow}p$ where p is a fixed propositional variables. The symbol $\geq$ or $\leq$ is used to represent a linear order on ω.

A semantics for L_ω^- is defined below.

Definition 4.1: *Let Θ be an allowable set. Timed valuations I^i $(i \in \omega)$ are mappings from the set of all propositional variables to the set $\{t, f\}$ of truth values. Then, timed satisfaction relations $\models_i \alpha$ $(i \in \omega)$ for any formula α are defined inductively by:*

1. *$\models_i p$ iff $I^i(p) = t$ for any propositional variable p,*
2. *$\models_i \bigwedge\Theta$ iff $\models_i \alpha$ for any $\alpha \in \Theta$,*
3. *$\models_i \bigvee\Theta$ iff $\models_i \alpha$ for some $\alpha \in \Theta$,*
4. *$\models_i \alpha{\rightarrow}\beta$ iff not-($\models_i \alpha$) or $\models_i \beta$,*
5. *$\models_i \neg\alpha$ iff not-($\models_i \alpha$),*
6. *$\models_i \mathrm{X}\alpha$ iff $\models_{i+1} \alpha$,*

7. $\models_i \mathrm{G}\alpha$ *iff* $\models_j \alpha$ *for any* $j \geq i$,
8. $\models_i \mathrm{F}\alpha$ *iff* $\models_j \alpha$ *for some* $j \geq i$.

A formula α *is called* L_ω^--valid *if* $\models_0 \alpha$ *holds for any timed satisfaction relations* $\models_i$ $(i \in \omega)$. *A sequent* $\Gamma \Rightarrow \Delta$ *is called* L_ω^-*-valid if so is the formula* $\Gamma_* \rightarrow \Delta^*$.

A semantics for LK_ω is defined below.

Definition 4.2: *Let* Θ *be an allowable set. A valuation* I *is a mapping from the set of all propositional variables to the set* $\{t, f\}$ *of truth values. A satisfaction relation* $\models \alpha$ *for any formula* α *is defined inductively by:*

1. $\models p$ *iff* $I(p) = t$ *for any propositional variable* p,
2. $\models \bigwedge \Theta$ *iff* $\models \alpha$ *for any* $\alpha \in \Theta$,
3. $\models \bigvee \Theta$ *iff* $\models \alpha$ *for some* $\alpha \in \Theta$,
4. $\models \alpha \rightarrow \beta$ *iff not-*$(\models \alpha)$ *or* $\models \beta$,
5. $\models \neg\alpha$ *iff not-*$(\models \alpha)$.

A formula α *is called* LK_ω-valid *if* $\models \alpha$ *holds for any satisfaction relation* $\models$. *A sequent* $\Gamma \Rightarrow \Delta$ *is called* LK_ω*-valid if so is the formula* $\Gamma_* \rightarrow \Delta^*$.

As well known, the following completeness theorem holds for LK_ω.

Proposition 4.3: *For any sequent* S, $\mathrm{LK}_\omega \vdash S$ *iff* S *is* LK_ω*-valid.*

In order to apply the mapping f in Definition 3.1, we assume the languages which are based on $\mathcal{L}_{\mathrm{L}_\omega^-}$ and $\mathcal{L}_{\mathrm{LK}_\omega}$ by constructing Φ and $\bigcup_{i \in \omega} \Phi_i$, respectively.

Lemma 4.4: *Let* f *be the mapping defined in Definition 3.1. For any timed satisfaction relation* $\models_i$ $(i \in \omega)$*, there exists a satisfaction relation* $\models$ *such that for any formula* α *in* $\mathcal{L}_{\mathrm{L}_\omega^-}$,

$$\models_i \alpha \text{ iff } \models f(\mathrm{X}^i\alpha).$$

Proof. Let Φ be a set of propositional variables and Φ_i be the set $\{p_i \mid p \in \Phi\}$ of propositional variables with $p_0 := p$.

Suppose that

I^i $(i \in \omega)$ are mappings from Φ to $\{t, f\}$.

Suppose that

I is a mapping from $\bigcup_{i\in\omega}\Phi_i$ to $\{t, f\}$.

Suppose moreover that

$I^i(p) = t$ iff $I(p_i) = t$.

Then, the claim is proved by induction on the complexity of α.

• Base step:

Case ($\alpha \equiv p \in \Phi$): $\models_i p$ iff $I^i(p) = t$ iff $I(p_i) = t$ iff $\models p_i$ iff $\models f(\mathrm{X}^i p)$ (by the definition of f).

• Induction step:

Case ($\alpha \equiv \bigwedge\Theta$ where Θ is an allowable set) $\models_i \bigwedge\Theta$ iff $\models_i \beta$ for any $\beta \in \Theta$ iff $\models f(\mathrm{X}^i\beta)$ for any $\beta \in \Theta$ (by induction hypothesis) iff $\models f(\mathrm{X}^i\beta)$ for any $f(\mathrm{X}^i\beta) \in f(\mathrm{X}^i\Theta)$ iff $\models \bigwedge f(\mathrm{X}^i\Theta)$ iff $\models f(\mathrm{X}^i \bigwedge\Theta)$ (by the definition of f).

Case ($\alpha \equiv \bigvee\Theta$ where Θ is an allowable set): Similar to Case ($\alpha \equiv \bigwedge\Theta$ where Θ is an allowable set).

Case ($\alpha \equiv \alpha_1{\rightarrow}\alpha_2$): $\models_i \alpha_1{\rightarrow}\alpha_2$ iff not-($\models_i \alpha_1$) or $\models_i \alpha_2$ iff not-($\models f(\mathrm{X}^i\alpha_1)$) or $\models f(\mathrm{X}^i\alpha_2)$ (by induction hypothesis) iff $\models f(\mathrm{X}^i\alpha_1){\rightarrow}f(\mathrm{X}^i\alpha_2)$ iff $\models f(\mathrm{X}^i(\alpha_1{\rightarrow}\alpha_2))$ (by the definition of f).

Case ($\alpha \equiv \neg\beta$): $\models_i \neg\beta$ iff not-($\models_i \beta$) iff not-($\models f(\mathrm{X}^i\beta)$) (by induction hypothesis) iff $\models \neg f(\mathrm{X}^i\beta)$ iff $\models f(\mathrm{X}^i\neg\beta)$ (by the definition of f).

Case ($\alpha \equiv \mathrm{X}\beta$): $\models_i \mathrm{X}\beta$ iff $\models_{i+1} \beta$ iff $\models f(\mathrm{X}^{i+1}\beta)$ (by induction hypothesis) iff $\models f(\mathrm{X}^i\mathrm{X}\beta)$.

Case ($\alpha \equiv \mathrm{G}\beta$): $\models_i \mathrm{G}\beta$ iff $\models_j \beta$ for any $j \geq i$ iff $\models f(\mathrm{X}^j\beta)$ for any $j \geq i$ (by induction hypothesis) iff $\forall k \in \omega$ [$\models f(\mathrm{X}^{i+k}\beta)$] iff $\models \gamma$ for any $\gamma \in \{f(\mathrm{X}^{i+k}\beta) \mid k \in \omega\}$ iff $\models \bigwedge\{f(\mathrm{X}^{i+k}\beta) \mid k \in \omega\}$ iff $\models f(\mathrm{X}^i\mathrm{G}\beta)$ (by the definition of f).

Case ($\alpha \equiv \mathrm{F}\beta$): Similar to Case ($\alpha \equiv \mathrm{G}\beta$). □

Lemma 4.5: *Let f be the mapping defined in Definition 3.1. For any satisfaction relation $\models$ and any $i \in \omega$, there exists a timed satisfaction relation $\models_i$ such that for any formula α in $\mathcal{L}_{\mathrm{L}_\omega^-}$,*

$\models f(\mathrm{X}^i\alpha)$ *iff* $\models_i \alpha$.

Proof. Similar to the proof of Lemma 4.4. □

Theorem 4.6: (Semantical embedding) *Let f be the mapping defined in Definition 3.1. For any formula α in $\mathcal{L}_{\mathrm{L}_\omega^-}$, α is L_ω^--valid iff $f(\alpha)$ is LK_ω-valid.*

Proof. By Lemmas 4.4 and 4.5. We take 0 for i. □

Combining Theorems 3.2 and 4.6, we can derive the following completeness theorem for L_ω^-.

Theorem 4.7: (Completeness) *For any sequent S,* $L_\omega^- \vdash S$ *iff S is* L_ω^-*-valid.*

Proof. Let $\Gamma \Rightarrow \Delta$ be S and α be $\Gamma_* \rightarrow \Delta^*$. It is sufficient to show that $L_\omega^- \vdash \Rightarrow \alpha$ iff α is L_ω^--valid. We show this as follows. $L_\omega^- \vdash \Rightarrow \alpha$ iff $LK_\omega \vdash \Rightarrow f(\alpha)$ (by Theorem 3.2) iff $f(\alpha)$ is LK_ω-valid (by Proposition 4.3) iff α is L_ω^--valid (by Theorem 4.6). □

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