# LOOKING FOR NEW MATHEMATICAL CONCEPTS FOR THE MATERIAL WORLD
## WHITEHEAD'S INVESTIGATIONS INTO FORMAL ONTOLOGY

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*Abstract*

Read in December 1905 at the Royal Society of London, Alfred North Whitehead's paper "On mathematical concepts of the material world" is not only, according to Whitehead's own retrospective assessment, "the most original thing that he had done";[1] it also provides very interesting clues to understand Whitehead's contemporary collaboration with Bertrand Russell, as well as his later philosophical — epistemological and metaphysical — work.

## 1. *Russell and Whitehead's philosophical projects*

As is well known, Russell and Whitehead's *Principia mathematica* provided a major contribution to the logicist program which had first been designed in Gottlob Frege's *Grundlagen der Arithmetik* and partially carried out in both volumes of his *Grundgesetze der Arithmetik*. Though Frege himself only had planned to reduce *arithmetic* to logic, Bertrand Russell's own *Principles of mathematics* had extended this plan to *the whole of mathematics* — including calculus and geometry — as well as to the rational principles of the empirical sciences. Against Immanuel Kant's view according to which arithmetic, geometry and pure dynamics consisted of synthetic *a priori* judgements grounded on formal intuition and pure schematism, the logicist school intended to show that mathematical truths rely entirely on deductive logic and are therefore wholly analytic.

As far as arithmetic is concerned, the workability of this project had been thoroughly investigated by Frege. By using the notions of hereditary property and ancestral relation, the German logician had already given in 1879

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[1] Conversation with Victor Lowe on December the 2nd, 1936 (reported in Victor Lowe, *Alfred North Whitehead: the man and his work*, John Hopkins University Press, 1985, p. 296).

BRUNO LECLERCQ

the elements of a logical theory of series and of their inductive features.[2]
Five years later, he had defined natural numbers as sets of "equinumerous"
sets, i.e. as sets of sets which can be related together by a bijective rela-
tion — the number 0 is the set of empty sets, i.e. the set of the sets which
can be bijectively mapped with the extension of the concept "not identical
with itself".[3] In this field, Russell and Whitehead's *Principia mathematica*'s
specific relevance essentially consisted in improving Frege's system in order
to let it overcome the difficulties which had led to famous antinomies. As
everyone knows, a significant part of this job was done by Russell, who had
been struggling to solve this problem since 1901.

With regard to geometry, however, Whitehead's contribution to the logi-
cist program has notoriously been the most decisive. Before writing the
*Principia mathematica*, both thinkers had been working on the relations be-
tween geometrical entities as well as on the formal — i.e. structural and even
algebraic — properties of these interrelations.[4] But, after having finalized
axiomatic systems for projective and descriptive geometry,[5] Whitehead had
settled a subtle way of defining geometrical entities by logical construction:
the famous "extensive abstraction method".[6] Furthermore, his investigations
— to which we will return — of several Leibnizian models for Euclidean

[2] Gottlob Frege, *Begriffsschrift: eine der arithmetischen nachgebildete Formelsprache
des reinen Denkens*. Halle, 1879, engl. transl. *Conceptual notation and related articles*,
Oxford, Oxford University Press, 1972.

[3] Gottlob Frege, *Die Grundlagen der Arithmetik: eine logisch-mathematische Unter-
suchung über den Begriff der Zahl*, Breslau, 1884, engl. transl. *The Foundations of Arith-
metic: the logical-mathematical Investigation of the Concept of Number*, Oxford, Blackwell,
1960.

[4] Alfred North Whitehead, *A Treatise on Universal Algebra with Applications*, Cam-
bridge, Cambridge University Press, 1898, books III and IV; Bertrand Russell, *An Essay on
the Foundations of Geometry*, Cambridge, Cambridge University Press, 1897; Bertrand Rus-
sell, "Sur les axiomes de la géométrie", in *Revue de métaphysique et de morale*, 1899, vol. 7,
pp. 684–707. For a deep investigation of the relations between Whitehead's and Russell's
works on geometry, see Sébastien Gandon, "Russell et l'*Universal Algebra* de Whitehead. La
géométrie projective entre ordre et incidence (1898–1903)", in *Revue d'histoire des mathé-
matiques*, 2004, vol. 10, pp. 187–256.

[5] Alfred North Whitehead, *The Axioms of Projective Geometry*, Cambridge, Cambridge
University Press, 1906; *The Axioms of Descriptive Geometry*, Cambridge, Cambridge Uni-
versity Press, 1907. Oswald Veblen's influence on this work is well-known. See Oswald
Veblen, "A system of axioms for geometry", in *Transactions of the American Mathematical
Society*, 1904, vol. 5, n. 3, pp. 343–384.

[6] For a first explicit formulation of this method, which consists in defining an Euclidian
point with classes of converging volumes, see Alfred North Whitehead, "La théorie relation-
niste de l'espace", in *Revue de métaphysique et de morale*, 1916, vol. 23, p. 423. See also

geometry which could account for the current state of pure dynamics[7] had paved the way for the extension of the logicist claims to the whole realm of what Kant considered as synthetic *a priori* knowledge. And even though the fourth volume of the *Principia mathematica* — which was supposed to fulfil this reductive program — never came out, major tenants of logicism such as Russell and the early Carnap[8] have always seen Whitehead's work as a masterpiece of this plan.

By taking a closer look at Whitehead's 1905 text "On mathematical concepts of the material world", we would however like to reconsider what were Whitehead's own scientific tasks and aims when he started his collaboration with Russell. Though there were at that time lots of shared scientific interests and convictions between the later co-authors of the *Principia mathematica*, there were also significant differences in the ways Russell and Whitehead conceived the project of providing geometry and pure dynamics with logico-mathematical axiomatics as well as in the ways they conceived the epistemological plans this project was supposed to serve.

In this respect, the contrast between Russell's and Whitehead's concerns in 1905 is particularly striking. On one hand, Whitehead looks for adequate mathematical concepts for the vectorial forces, electromagnetic fields and even vortex rings which constitute the material world according to Lorentz, Maxwell or Kelvin, and he is thus led to question classical ontology together with the classical Newtonian model for physics. On the other hand, Russell enters into an ontological debate with Alexius Meinong as to the referential nature of definite descriptions;[9] and, from that moment onwards, the question of the ontological commitments of linguistic expressions will be a major concern in his work.

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Whitehead's *Enquiry concerning the Principles of Natural Knowledge*, Cambridge, Cambridge University Press, 1919. On this method and its development, see Adolf Grünbaum's "Whitehead's method of extensive abstraction", in *British Journal for the Philosophy of Science*, 1953, vol. 4, n. 15, pp. 215 et sq. On Russell's understanding and use of this method, see Guillaume Durand, "Whitehead et Russell: la discorde de 1917", in *Noesis*, 2008, vol. 13, pp. 237–250.

[7] Alfred North Whitehead, "On mathematical concepts of the material world", in *Philosophical transactions of the Royal Society of London*, Series A, 1906, vol. 205, pp. 465–525.

[8] See for example Bertrand Russell, *Analysis of Matter*, London, Kegan Paul, 1927, pp. 6–7, 22, 138, 290 et sq.; Rudolf Carnap, *Der logische Aufbau der Welt*, Leipzig, Felix Meiner Verlag, 1928, engl. transl. *The Logical Structure of the World*, University of California Press, 1967, §3, §73, §124.

[9] Bertrand Russell, "On denoting", in Mind, 1905, vol. XIV, n. 56, pp. 479–493. That this paper was aimed at Russell's own earlier stances as much as at Meinong's is well known.

BRUNO LECLERCQ

In 1905, Russell indeed became convinced that the fundamental distinction between concepts and objects — between propositional functions and their arguments — which Frege had drawn and used as the basis of his new *logical* analysis had some *ontological* significance. Frege was right in saying that concepts are characterized by definitory features — Frege's "*Merkmale*" — and have consequently an extension — Frege's "*Werthverlauf*" —, which can be empty. Frege's only mistake had been to treat proper nouns as definite descriptions with an intensional and an extensional meaning — "*Sinn*" and "*Bedeutung*" — rather than as immediately referential expressions. By treating definite descriptions and other conceptual expressions as proper nouns, Meinong committed the reverse — and much worse — mistake of endorsing a generalized referentialist conception of meaning, with an exuberant ontology as a result.[10]  On the contrary, Russell's project will be to stress the conceptual and *non-referential* nature of most linguistic expressions, including the constants for classes, which will soon be seen as incomplete symbols in the "no class theory".

With Russell's 1905 text, the epistemological fight of logicism against Kant's synthetic *a priori* knowledge comes to coincide with the ontological fight against Meinong's semantical entities since, as Rudolf Carnap's *Aufbau* will show, the logical construction of the abstract "objects" used in mathematics and empirical science can also be seen as the demonstration of the purely conceptual nature of these so-called "objects". Now, this second fight *only partially* meets Whitehead's own ontological fight against substantialism. As we shall see, what the logical construction of points of space and matter particles exhibits is, for Whitehead, the *relational* rather than *substantial* nature of these entities; relations precede their terms, so that the status of the system's entities entirely depends on the way the axiomatic system is built up. From his *Universal Algebra* on, Whitehead had always been interested in formal structures. And, in this respect, logicism, which was to be developed on Russell's brilliant logic of relations, was only one way among others to exhibit the formal relations between scientific entities.[11]  Though "the project of deducing mathematics from logic appealed to Whitehead", as

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[10] On Brentano's school's and Frege's school's diverging views on logic and ontology, see Bruno Leclercq, "Les présupposés d'existence de l'école de Brentano à l'école de Frege", in *Philosophie*, 2008, vol. 97, pp. 26–41.

[11] On this see Luca Gaeta, "Order and change", in M. Weber, W. Desmond, (eds), *Handbook of Whiteheadian Process Thought*, vol. 2, Frankfurt, Ontos Verlag, 2008, pp. 105–117, especially pp. 106–107. See also Luca Gaeta, *Segni del cosmo: logica e geometria in Whitehead*, Milan, LED Edizioni, 2002.

Russell says in his famous obituary text,[12] Ivor Grattan-Guinness is therefore right to maintain that "for Whitehead, logicism was only a part, albeit a central one, of a broader philosophical picture of mathematics".[13]

Now, this throws some new light on Whitehead's later work. Even though, as Luca Gaeta rightly claims, the "anachronistic attitude of tracing Whitehead's metaphysics back to 1905"[14] should better be avoided, it is in reverse quite obvious that the careful working out of Whitehead's late philosophy of process should not be considered separately from his early rigorous elaboration of new concepts for geometry and pure dynamics. There is indeed strong continuity between Whitehead's deep and thorough criticism of classical substantialist ontology while attempting to find suitable mathematical formal systems for the new developments of physics, and his later resolute and detailed defence of an ontology based on events and processes from a cosmological and even metaphysical point of view. Of course, it would be an overstatement to claim that Whitehead's late metaphysical work is "nothing but" the philosophical expression of the fundamental mathematical structures of pure dynamics he had previously exhibited. But it is certainly true that many theses of *Process and reality* would seem unwarranted and arbitrary were they not grounded on the logico-mathematical framework which is put forward in such texts as "On mathematical concepts of the material world".[15] And this means that epistemological goals prevailed over ontological goals in Whitehead's development of thought, while the reverse was probably true in Russell's case. A good example of this is provided by the use of Occam's razor in "On mathematical concepts of the material world".

[12] Bertrand Russell, "Whitehead and Principia mathematica", in *Mind*, 1948, vol. LVII, n. 226, pp. 137–138.

[13] Ivor Grattan-Guinness, "Foundations of mathematics and logicism", in M. Weber, W. Desmond, (eds), *Handbook of Whiteheadian Process Thought*, vol. 2, Frankfurt, Ontos Verlag, 2008, pp. 97–103. On the topic of the relations of logicism with the more general project of the foundations of mathematics, see also Ivor Grattan-Guinness, *The Search for Mathematical Roots 1870–1940: Logics, Set Theories, and the Foundations of Mathematics from Cantor through Russell to Gödel*, Princeton University Press, 2000.

[14] Luca Gaeta, "Order and change", art. cit., p. 114.

[15] Alfred North Whitehead, *Process and reality. An essay in cosmology*, New York, MacMillan, 1967. While Newtonian cosmology "emphasized the 'receptacle' theory of space-time" (pp. 108–109), Whitehead's cosmology claims that, being constructed within the theory, actual entities have to *become* in the space-time continuum (pp. 104–105) and are nothing but particular concrescences of the whole universe so that every actual entity is present in every other actual entity (pp. 79–80). Furthermore, as actual entities are not substances but processes, motion cannot be significantly attributed to them (p. 119).

## 2. *New mathematical axiomatics for physics...*

Whitehead's 1905 text starts with the exposition of the sort of ontology which classical physics implies:[16]

> In geometry, as derived from the Greeks, the simple elements of space are *points*, and the science is the study of the relations between points. Points occur as members of the fields of these relations. Then matter (the ultimate "stuff" which occupies space) in its final analysis, even if it is continuous, consists of entities, called *particles*, associated with the points by relations which are expressed by saying that a particle *occupies* (or *is at*) a point. [...] Thus "occupation" is a triadic relation holding in each specific instance between a particle of matter, a point of space, and an instant of time. According to this concept of the material world, which we will call the *Classical concept*, the class of ultimate existents is composed of three mutually exclusive classes of entities, namely *points of space*, *particles of matter* and *instants of time*.

The rest of the text then consists in challenging this classical framework. In this regard Leibniz's theory of the relativity of space provides the starting point. Some philosophical arguments plead for a new investigation of several forms of this theory. While the classical "concept" requires two separate classes of objective reals to be related to the instants of time, namely the points of space and the particles of matter, Leibnizian concepts intrinsically link matter and spatial extension. Now, as Whitehead says, "Occam's razor — Entia non *multiplicanda praeter necessitatem* — formulates an instinctive preference for a monistic against a dualistic concept".[17]

This should however not be interpreted as an ontological but as an epistemological stance. Indeed, the main disadvantage of the classical — dualistic — concept is that it requires that the positions of all particles in space at each moment be specified on top of the "essential relations" which are given by the laws of the axiomatic system. Since classical geometry, including non-euclidean geometry, only grasps an unchanging world of space, there is no link between the essential relations (which are order relations between points as in Veblen's system) and the time relation (which is a serial relation

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[16] Alfred North Whitehead, "On mathematical concepts of the material world", art. cit., p. 467.

[17] Ibid., p. 468.

ordinally similar to the serial relation that generates the series of real numbers). And this is why particles, i.e. entities moving in space, are needed.[18] But this means that the classical concept demands an indefinite, if not infinite, number of relations between particles of matter, points of space and instants of time, relations which are "extraneous" to the system itself. And, for Whitehead, this is what goes against Occam's razor:[19]

> Judged by Occam's principle, this class of extraneous relations forms a defect.

A first step in order to overcome these defect has been suggested by Russell in §441 of his *Principles of mathematics*:[20]

> We may replace a material point by a many-one relation whose domain is a certain one-dimensional series, and whose converse domain is contained in a certain three-dimensional series. To obtain a material universe, so far as kinematical considerations go, we have only to consider a class of such relations subject to the condition that the logical product of any two relations of the class is to be null. This condition insures impenetrability. If we had that the one-dimensional and the three-dimensional series are to be both continuous, and that each many-one relation is to define a continuous function, we have all the kinematic conditions for a system of material particles, generalized and expresses in terms of logical constants.

Now, if, as Leibniz suggested, geometry and dynamics are tackled together, i.e. if time is involved into the "geometrical" theses, lines and planes can be treated as instantaneous geometrical relations between moving points

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[18] "It is plain that the only relevant function of a material point is to establish a correlation between all moments of time and some points of space" (Bertrand Russell, *Principles of mathematics*, London, Allen and Unwin, 2nd ed., 1937, p. 468).

[19] Alfred North Whitehead, "On mathematical concepts of the material world", art. cit., p. 469.

[20] Bertrand Russell, *Principles of mathematics*, London, Allen and Unwin, 2nd ed., 1937, p. 468.

BRUNO LECLERCQ

or particles of Descartes' ether[21] and therefore get on with one single extraneous relation, namely a tetraedric relation which involves the four dimensions and determines the reference "kinetic axes" for the measurement of velocity.[22] According to this axiomatic system, which is Whitehead's concept number III, velocity (including restfullness) indeed becomes the main stake of dynamics, since points of space and geometrical figures disappear as time goes on.

A further step is taken when one uses linear objective reals as elements of the system. If straight lines come at first, points of space become complex entities which are defined as classes of intersecting straight lines.[23] Now, if this idea is combined with the previous one, one ends up with a system grounded on instantaneous geometrical relations between straight lines: "The proposition $R; (abcdt)$ can be read as the statement that the objective real $a$ intersects the objective reals $b$, $c$, $d$ (in the order $bcd$) at the instant $t$".[24] Then, one can either — and this is concept IVa — conceive dynamics as distinct from geometry, i.e. conceive particles of matter as ontologically separate from these instantaneous geometrical relations between straight lines, and this will thus draw back to the necessity of specifying an indefinite number of extraneous relations between particles and their "lines of force"; or again — and this is concept IVb — one can tackle geometry and physics together and treat particles or corpuscles as complex entities which are defined as instantaneous classes of intersecting linear real objectives. Though very different from the classical conception of particles in physics, this last conception matches with Lorentz'model of electrons as nodes of electromagnetic forces — and thus of vectorial quantities — inside a field.[25]

By adding the central notion of "dimension" to the theory of interpoints, concept V then defines points as *incessantly passing away classes of objective reals*, which are capable of the "various and complicated structures"[26] which they are assigned to by contemporary physics.

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[21] In *The concept of nature* (Cambridge University Press, 1920, p. 78), Whitehead will criticize the notion of «ether» as a vain and desperate attempt to save some remainders of a materialist and substantial theory of nature.

[22] Alfred North Whitehead, "On mathematical concepts of the material world", art. cit., pp. 480–482.

[23] Ibid., pp. 482–484.

[24] Ibid., p. 484.

[25] Ibid., pp. 488–492.

[26] Ibid., p. 505.

### 3.  ... *resulting in a new formal ontology for the material world*

Aside the question of the intrinsic merits and limits of these several proposals for new physical interpretations of geometry — merits and limits which could, for example, be compared with concurrent proposals such as Einstein's special theory of relativity or Minkowski space time[27] —, there are obvious ontological stakes in this work. And the whole text even seems to be structured round them. Should the system be monistic or dualistic? Should it be absolutistic or relativistic regarding on space? Should the basic elements be ponctual or linear? Each "concept" — i.e. each model for Euclidean geometry — gives specific answers to these different and partially independent questions. And this really is the way the mathematician enters into the philosophical ground ; the investigation of the relations between geometry and pure dynamics leads him to discuss several mathematical concepts that are related to as many ontological options.

Whitehead, as we have seen, explicitly opts for monistic — as opposed to "dualistic" — concepts. The above-mentioned references to Occam's razor however show that what matters most to Whitehead is theoretical rather than ontological parsimony. While Russell explicitly puts his philosophical work under the "nominalistic" maxim which Rudolf Carnap will later quote as the foreword of his *Aufbau* — "Wherever possible, logical constructions are to be substituted for inferred entities"[28] —, Whitehead looks for the simplest axiomatic system which could account for the empirical data and contemporary theories of physics. "Consonance with some modern physical ideas" as well as "superior simplicity"[29] are the explicit leitmotives of Whitehead's formal ontology; and they are what demands to break with the

[27] Albert Einstein, "Zur Elektrodynamik bewegter Körper", in *Annalen der Physik*, 1905, vol. 17, pp. 891–921; Hermann Minkowski, "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern", in *Nachrichten von Königliche Gesellschaft der Wissenschaften zu Göttingen, mathematisch-physikalische Klasse*, 1908, pp. 53–111. For a discussion of Whitehead's relations with Minkowski's and Einstein's theories, see, in the present volume, Ronny Desmet, "The Minkowskian Background of Whitehead's theory of gravitation".

[28] Bertrand Russell, "The Relation of Sense-Data to Physics", in *Scientia*, 1914; *Our knowledge of the external world*, London, Open Court, 1914; Rudolf Carnap, *Der logische Aufbau der Welt*, op. cit. Following a distinction drawn by Russell in his intellectual autobiography ("My mental development", in P.A. Schilpp ed., *The philosophy of Bertrand Russell*, Library of living philosophers, 1946, pp. 14–15), we could say that Whitehead sees Occam's razor as a principle of "minimum vocabulary" while at that time Russell saw it as a principle of "parsimony as regards entities".

[29] Alfred North Whitehead, "On mathematical concepts of the material world", art. cit., p. 524.

BRUNO LECLERCQ

old substantialist schemes. The main reasons for replacing some alledged entities by logical constructions are thus epistemological rather than onto-logical: particles of matter were the elementary objects that suited the ob-servations and laws of newtonian physics, but they have now become what Bachelard would call "epistemological impediments" and they should be re-placed by vectorial events, movements or processes, which are the immedi-ate observable phenomena as well as the fundamental objects of contempo-rary physics. Though vectorial events, movements and processes used to be defined in terms of particles of matter, the latter ones should now be defined in terms of the former ones:[30]

> The reason for the original introduction of "matter" was, without doubt, to give the senses something to perceive. If a relation can be perceived, this concept II [which abolishes the particles and trans-form the triadic extraneous relations into dyadic relations between points of space and instants of time] has every advantage over the classical concept.

And the other "ontological" choices of "On mathematical concepts of the material world", notably the relativistic and linearistic options, are com-manded by similar reasons.

In 1905, Whitehead already is in complete support of Leibniz' arguments for the relativistic view of space upon which his later work will elaborate. Now, it is noteworthy — and very significant — that, even though Whitehead owes probably much of his knowledge on Leibniz to Russell's book,[31] Rus-sell himself did not support this relativistic view at that time and that he only got converted to it later through the influence of Whitehead's developments. The early Russell indeed considered internal relations as incompatible with the atomistic world on which he wanted to build logicism. Whitehead, how-ever, could easily accept these internal relations because he attached more

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[30] Ibid., p. 480. In *The Concept of nature* (op. cit., pp. 56–57), Whitehead will simi-larly say that simultaneity is immediately perceived while instants have to be built as logical concepts.

[31] In his intellectuel autobiography, Whitehead will however minimize the importance of this reading: "My knowledge of Leibniz's investigations was entirely based on L. Couturat's book, *La Logique de Leibniz*, published in 1901" ("Autobiographical notes", in P.A. Schilpp ed., *Alfred North Whitehead. The Library of Living Philosophers*, vol. III, 1941).
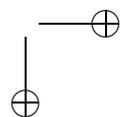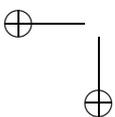
importance to relations than to individual entities.[32] And this, as a few commentators have already argued, makes Whitehead on many points closer to Hilbert's formalist than to Frege's logicist view of mathematics.[33]

In the same way, Whitehead also firmly decides upon a "linearistic" rather than "ponctualistic" conception of the basic elements of ontology. From a formalist viewpoint, Oswald Veblen had already shown that you can define points as classes of lines rather then the other way round.[34] Because it matches with an electrodynamic interpretation of particles, Whitehead undertakes this way of developping geometry. Besides, Whitehead favours a "dynamic" approach of his relativistic and linearistic view of space; geometrical relations hold between moving points, so that there are no more steady figures. As it is entangled with dynamics, geometry essentially relates to time and change. Of course, lots of this text's readers will see there obvious foreshadows of Whitehead's later vectorial and then processual metaphysics. But, once more, these philosophical positions are commanded by epistemological requirements such as theoretical simplicity and agreement with the new developments of physics rather than by ontological stances, such as nominalism.

## 4. *Conclusion*

Undoubtedly is formal ontology the main concern of "On mathematical concepts of the material world". Unlike Russell, however, Whitehead does not start with ontological arguments but rather draws ontological conclusions from epistemological arguments: "The general problem is here discussed purely for the sake of its logical (i.e. mathematical) interest. It has an indirect bearing on philosophy by disentangling the essentials of the idea of material world from the accidents of one particular concept".[35]

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[32] On Russell and Whitehead's standpoints towards the relativistic theory of space, see Jean-Pascal Alcantara, "Le rôle des mathématiques dans la genèse du système de Whitehead", in F. Beets et al. eds., *De l'algèbre universelle à la théorie naturelle*, Ontos Verlag, «Chromatiques whiteheadiennes», 2004 ; see also in the present volume Jean-Pascal Alcantara, "On relations in Leibniz, British Neo-realism and Whitehead".

[33] In the present volume, see in particular Rosen Lutskanov's text «Whitehead's early philosophy of mathematics and the development of Hilbert's formalism».

[34] Oswald Veblen, "A system of axioms for geometry", art. cit..

[35] Alfred North Whitehead, "On mathematical concepts of the material world", art. cit., p. 465.

In Wittgenstein's words, it could perhaps be said that the early Whitehead's formal systems intended to exhibit — to "show" — rather than to describe — to "tell" — the formal structure of the world. Only later on will Whitehead venture onto the metaphysical field and try to speak thereof he had previously prefered to be silent. But still at that time, he will warn against the temptation to let one be led only by philosophical stances and maintain that "the chief error in philosophy is overstatement".[36]

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[36] Alfred North Whitehead, *Process and reality. An essay in cosmology*, op. cit., p. 11.

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