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A CONVERSE BARCAN FORMULA IN ARISTOTLE'S MODAL LOGIC

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Abstract

While critics correctly claim against Aristotle that universal affirmative and particular affirmative *de re* apodeictic propositions do not validly convert to a particular affirmative *de re* necessary counterpart, there is a type of valid convertibility for these propositions, a convertibility that at once reveals both an insight and a confusion on Aristotle's part, the confusion amounting at times to a fallacy of amphiboly. Recognition of this convertibility through the introduction of a "relational necessity" which is neither *de dicto* nor *de re* will resolve a longstanding anomaly generated by Aristotle's provocative claim that *being white* necessarily belongs to swans and to snow. By use of a modified Converse Barcan Formula, a distinctively Aristotelian ecthetic argument formally validating the apodeictic conversions is presented. Some implications of these results for the integrity and credibility of Aristotle's modal syllogistic are briefly mentioned.

> [A]n expository syllogism is evident in itself and does not require any further proof (Ockham [9]: ch. 26 (p. 174))

In *Prior Analytics* I.3, 25a32–34, Aristotle argued laconically for the convertibility of universal and particular affirmative apodeictic propositions: 'If a belongs to all or some b of necessity, it is necessary also that b belongs to some a; for if there were no necessity, neither would a belong to some b of necessity.'[1] The overall coherence of Aristotle's modal syllogistic, it

[1] All translations from *Prior Analytics* and other works in the *Organon* are from Aristotle [1]. So as to facilitate the presentation of Aristotle's reasoning in a current symbolism, here and elsewhere the translator's correct use of capitalized Latin letters for the general term variables has been replaced by their lower case counterparts.

is generally agreed, demands that such apodeictic propositions not be given a *de dicto* interpretation. Admittedly, such an interpretation of apodeictic convertibility at first blush seems entirely reasonable. For in the previous chapter (*An pr.* I.2, 25a14–22), Aristotle had argued that both the universal affirmative (A) propositional form 'a belongs to every b' $\{A[b](a)\}$ and the particular affirmative (I) 'a belongs to some b' $\{I[b](a)\}$ validly convert to the (I) 'b belongs to some a' $\{I[a](b)\}$.[2] As for Aristotle the particular quantifier $\{I\}$ does not connote existence ('exist(s)' functions as a subject or predicate term and not as a quantifier), these conversions are easily validated.[3] Now, as both '$A[b](a)$' and '$I[b](a)$' entail '$I[a](b)$', the positing of each as *de dicto* necessary $\{$'$L(A[b](a))$' and '$L(I[b](a))$'$\}$ implies by the laws of modal propositional logic the necessary *de dicto* particular '*Necessarily*$($*some*$[a](b))$'$\{$'$L(I[a](b))$'$\}$. There is, however, an insurmountable problem with this *de dicto* approach as an exclusive interpretation of Aristotle's meaning. For he later insists (*An pr.* I.9, 30a15–23) that a first figure syllogism with a universal apodeictic major, a universal affirmative assertoric minor, and a universal apodeictic conclusion is valid. But, Barbara-$^{L}XX^{L}X$, that is, the first figure inference '$LA[b](a), A[c](b) / \therefore LA[c](a)$' with a *de dicto* necessary major, an assertoric minor, and a *de dicto* necessary conclusion is obviously invalid.[4] In contrast, the inference pattern usually labeled Barbara-LXL $\{A[b](La), A[c](b) / \therefore A[c](La)\}$ is a perfect syllogism. Here a *de re* interpretation or a modalized predicate interpretation is given of the apodeictic major and the conclusion: '*Necessarily*-a' is posited as belonging to every b in the major and to every c in the conclusion. Such propositions we label A_L propositions while their particular counterparts are called I_L propositions.

[2] Subject terms are placed in brackets, while predicate terms are surrounded by parentheses. The small Latin letters a, b, c,... serve as variables representing general terms. A and I as well as E and O serve as both quantifiers and qualifiers in their work as proposition-forming functors for two-term arguments. As they quantify the subject term, these functors show whether the predicate is being attached to it or detached from it. A and I are attaching functors forming affirmative propositions, while E and O are detaching functors forming negative propositions; E and O thus show that the predicate is being denied of what the quantified subject denotes. N serves as a syntactical device or as part of a syntactical device basically designed to deny a predicate of a subject. Thus, in '$I[b]N(a)$' it works to show that a is being denied of some b. This propositional form is equivalent to '$NA[b](a)$' which shows a as being predicated of not every b, a form usually expressed as '$O[b](a)$'. '$N(A[b](a))$' in which the form '$A[b](a)$' is taken as a term being negated is also a possible construal.

[3] See section 1.2, below.

[4] For us there is a distinction without a difference between the necessary *de dicto* forms '$LA[b](a)$' and '$L(A[b](a))$'. But the *de dicto* form '$L(I[b](a))$' is weaker than the *de dicto* '$LI[b](a))$'. See footnote 8, below.

Unfortunately for the overall coherence of Aristotle's modal logic, counterexamples undermine the alleged validity of the convertibility of affirmative apodeictic propositions construed as *de re* necessary. I_L conversion, critics contend (cf. Patterson [10]: 154), is invalid. For instance, assume that the I_L proposition '*Something that*[is sitting on a mat](is *necessarily* a cat)' $\{I[m](Lc)\}$ is true. Its I_L converse '*Something that*[is a cat](is *necessarily* sitting on a mat)' $\{I[c](Lm)\}$, however, is false.[5] In addition, as shown in Patterson ([11]: 25), one can dispute by a counterexample the validity of the limited convertibility of the A_L form: it is true, given that rationality is necessary and essential to human beings, that being necessarily rational applies to everything that is a philosopher $\{A[p](Lr)\}$. But it is false that being necessarily a philosopher applies to something that is rational $\{I[r](Lp)\}$.

We shall argue that, while critics correctly claim that A_L and I_L propositions do not validly convert to an I_L counterpart, there is a type of valid convertibility for these propositions, a convertibility that at once reveals both an insight and a confusion on Aristotle's part, the confusion amounting at times to a fallacy of amphiboly. Recognition of this convertibility through the introduction of a necessity which is neither *de dicto* nor *de re* will resolve a longstanding anomaly generated by Aristotle's provocative classification of some propositions that are *prima facie* only *de re* contingent as in some sense necessary. But, as we believe that Aristotle's argument for apodeictic conversion is implicitly and essentially an ecthetic one, it is first necessary to provide an explication of ecthesis in Aristotle's logic.

[5] Different uses of 'that' are at times operative. In the proposition '*Something that*[is an animal](is human)' $\{I[b](a)\}$, 'that' plays the role of a grammatical filler in formulating the particular (I) quantifier '*Something that*'. In '*Some*[thing](*that*$_k$ is an animal is human)' $\{I[c](b \cdot a)\}$, it functions as the conjunction '*and*' (*that*$_k$). This should be distinguished from its implicative role (*that*$_c$) in a claim such as '*Every*[thing](*that*$_c$ is human is mortal)' $\{A[c](b \supset a)\}$. The contradictory opposite here is '*Some*[thing](*that*$_k$ is human is not mortal)' $\{I[c](b \cdot N(a))\}$. The systematic ambiguity in the use of 'that' as a grammatical filler in contrast to its functioning as an implicative (*that*$_c$) and as a conjunction (*that*$_k$) demands a constant vigilance throughout any regimentation process. It must be stressed that '$A[b](a)$' whether understood as '*Whatever*[is human](is mortal)' or as its equivalent '*Everything that*[is human](is mortal)' entails but is not equivalent to '$A[c](b \supset a)$' interpreted as '*Every*[thing](*that*$_c$ is human is mortal)'.

These uses of 'that' all point to its function as a syntactical device or in traditional terminology as a syncategorematic term. In parsed expressions such as '*That*[lion](is tame)', '*That*[p](is true)', and '(Descartes thinks)*that*[p]', the syntactical role of 'that' is quite different: it functions as A or I do, that is, as a quantifier/qualifier attaching a parenthesized logical predicate to a bracketed logical subject. It is important to note that, as in modern analysis, traditional logic need not consider the grammatical subjects or predicates to be the logical ones. Moreover, the meaning of '*that*' in the lion example is quite different from its meaning in the Descartes example. For recognition of 'that' as a distinctive quantifier/qualifier, see Englebretsen ([2]: 91–93). For rejection of 'this' and 'that' as proper names, see Wittgenstein ([18]: 18–21).

1. *Ecthesis and the logic of the conversion of plain particular affirmative propositions*

Aristotle's ecthetic argument for the conversion of plain universal negative (E) propositions is presented in the second chapter of the first book of the *Prior Analytics*:

> First then take a universal negative with the terms a and b. Now if a belongs to no b, b will not belong to any a; for if it, b, does belong to some a (say to c), it will not be true that a belongs to no b — for c is one of the bs (*An pr.* I.2, 25a14–17).[6]

It is the cryptic second sentence that sketches the first formal proof actually given in the *Prior Analytics*. A sign of ecthetic reasoning is the presence of an exposed third term; in this case that term is c. Here the ecthetic proof, which establishes the validity of the direct convertibility of a plain particular affirmative (I) proposition, is embedded in an indirect argument designed to demonstrate the direct convertibility of the E proposition. Though recent commentators (see Mignucci [8]: 11) agree that a sign of ecthetic reasoning is the presence of an exposed term or ectheten in the explication of a particular proposition, controversy arises when it comes to specifying the nature of this term. Our own view is to regard the ectheten as an individually referring expression in a distinctively Aristotelian way, that is, to regard it as a proper name paradigmatically functioning as an abbreviation of an expression denoting an *individual-of-a-kind*.

1.1. *Ecthesis, and the nature of singular propositions*

If proper names are viewed in Aristotelian logic as abbreviations (paradigmatically as abbreviations for expressions denoting individuals-of-a-kind), it can follow predicate logic in regarding a particular proposition as a disjunction of singular propositions (cf. Quine [12]: 140). Thus, both the particular affirmative '$I[b](a)$' and the particular negative (O) propositional form '$O[b](a)$' can be understood as disjunctions of singular propositions being defined respectively as the expansions:

(Df.I) $I[b](a) =_{\text{df}} T_1[b](a) \vee T_2[b](a) \vee \ldots \vee T_n[b](a) \vee \ldots$

and

(Df.O) $O[b](a) =_{\text{df}} \perp_1[b](a) \vee \perp_2[b](a) \vee \ldots \vee \perp_n[b](a) \vee \ldots.$

[6] The translation here involves a slight alteration of that given for the passage by A.J. Jenkinson in Aristotle [1]. Cf. Mignucci ([8]: 11).

The indexed T, abbreviating 'This' or 'That' or 'The individual that', is a proposition-forming quantifier/qualifier limiting the reference of the bracketed subject term b, which is a general term, to a specific individual as it shows that a is being attached to the uniquely quantified subject. Thus, if $b =$ human and $a =$ runs, '$I[b](a)$' represents '*Some*[human](runs)'. '$T_1[b](a) \vee T_2[b](a)$' shows that one individual human {this human} runs *or* another individual human {that human} different from the first is running. $\perp$ is a proposition-forming quantifier/qualifier limiting the reference range of the bracketed subject and general term b to a specific individual as it shows that the parenthetical predicate term a is being denied of it. Thus, if '$O[b](a)$' represents '*Some*[human]*not*(runs)', '$\perp_1[b](a) \vee \perp_2[b](a)$' can be taken to show that one individual human {this human} is not running *or* another individual human {that human} different from the first is not running. '$\perp[b](a)$', which is equivalent to '$T[b]N(a)$', is the contradictory opposite of '$T[b](a)$'.[7] Accordingly, the expansions

(Df.A) $A[b](a) =_{\text{df}} T_1[b](a) \cdot T_2[b](a) \cdot \ldots \cdot T_n[b](a) \cdot \ldots$

and

(Df.E) $E[b](a) =_{\text{df}} \perp_1[b](a) \cdot \perp_2[b](a) \cdot \ldots \cdot \perp_n[b](a) \cdot \ldots$

can be deduced respectively from (Df.O) and (Df.I) to define universal affirmative and universal negative plain propositions.[8]

Two ecthetic rules can now be laid down, one for particular affirmative propositions, the other for particular negative propositions. Analogous to the non-implicative rule of Existential Instantiation in predicate logic, they are:

(ECI) $I[b](a) / \therefore T_i[b](a)$

and

(ECO) $O[b](a) / \therefore T_i[b]N(a)$ $\{= \perp_i[b](a)\}$,

[7] '$T[b]N(a)$', which denies a of *that* b, is equivalent to '$NT[b](a)$' which denies the proposition expressed by '$T[b](a)$' by showing that a belongs to not *that* b. '$N(T[b](a))$' in which the proposition itself is negated can also be allowed as an equivalent expression.

[8] In Aristotle's logic 'b' and 'a' can represent either substantial (essential) terms or ones merely expressing accidents. In a sempiternal universe such as Aristotle's the possibility of individuals of a kind coming to be and passing away *ad infinitum* must be recognized. The individual designated through 'T_i' need not be a substance now actually existing. Additionally, each individual substance possesses an infinity of accidents, each of which theoretically can be designated.

In this context, it is easy to understand why the *de dicto* necessary '$L(A[b](a))$', unpacked as '$L(T_1[b](a) \cdot T_2[b](a) \cdot \ldots \cdot T_n[b](a) \cdot \ldots)$', is equivalent to '$LA[b](a)$', understood as '$LT_1[b](a) \cdot LT_2[b](a) \cdot \ldots \cdot LT_n[b](a) \cdot \ldots$'. But, '$L(I[b](a))$', unpacked as '$L(T_1[b](a) \vee T_2[b](a) \vee \ldots \vee T_n[b](a) \vee \ldots)$', is obviously weaker than '$LI[b](a)$', construed as '$LT_1[b](a) \vee LT_2[b](a) \vee \ldots \vee LT_n[b](a) \vee \ldots$'. See footnote 4, above.

where T_i serves to designate one of the individual bs (having no previous occurrence in a deductive context) which a is being affirmed (or denied) of. The corresponding ecthetic rules modeled on Universal Instantiation are accordingly

(ECA) $A[b](a)/\therefore T_i[b](a)$

and

(ECE) $E[b](a)/\therefore T_i[b]N(a)\ \{= \perp_i[b](a)\}$,

where T_i designates an arbitrarily selected individual b to which a does (or does not) belong. We shall assume that each of these ecthetic rules can be extended to their various modal counterparts. For example, (ECI$_L$) would permit the instantiation of ‘$T_i[b](La)$’ from ‘$I[b](La)$’ according to the same restrictions analogously applied and understood.

In his ecthetic proof of I convertibility Aristotle uses a proper name (he calls it c, let us call it α) to designate *the* a denoted by the phrase ‘$T[a]$’ in ‘$T[a](b)$’. Just as *the*$_1$ *human* or *the*$_1$ *individual that is human* might be dubbed by the paradigmatic proper name and rigid designator ‘Socrates’, so *the*$_i$ b denoted by ‘$T_i[b]$’ might be dubbed by the proper name ‘β_i’. Let individual variables be tied to specific general term variables. Thus, ‘$[\alpha_i](b)$’ is shorthand for ‘$T_i[a](b)$’, while ‘$[\beta_i](b)$’ is shorthand for ‘$T_i[b](b)$’. We follow Thom ([13]: 303 and [14]: 170–74) in letting the small Greek letter or individual variable be tied to its small Latin letter counterpart representing a general term.

As term variables in Aristotle’s logic can range over either essential or accidental characteristics,[9] it necessary to differentiate two types of proper names which can serve as the ectheten: those which do and those that do not function as rigid designators. If b is a genus or species or any other Aristotelian essential term such as a difference or a *proprium*, ‘β_i’ is a rigid designator, a paradigmatic proper name; if b is an Aristotelian accident, then ‘β_i’ merely fixes its referent to an individual such as *the one that philosophizes* or *the individual that is white*, that is, to *the*$_i$ *philosopher* or to *the*$_i$ *white*. Proper names so used in effect abbreviate Russellian definite descriptions such as ‘the one that is presently king of France’.[10] An indexed ‘the’

[9] Aristotle illustrates the validity of E, A, and I convertibility by using ‘pleasure’ and ‘good’ as instances of the variables, while the counterexample devised to invalidate O convertibility contains the essential terms ‘human’ and ‘animal’ (*An pr.* I.2, 25a6–13). Thus, ‘*The*$_i$[pleasure]’ and ‘*The*$_i$[good]’, expressions denoting accidental characteristics, can be abbreviated by different proper name ectheten which fix their denotations without any suggestion that these phrases refer to Platonic subsistent forms or that their proper name proxies denote Aristotelian primary substances.

[10] It might be helpful to sketch a possible context for an Aristotelian employment for such an expression. Like Russell, the Aristotelian term logician confronted with the sentence ‘The

can thus function like 'This' or 'That' or 'The one that' in indicating that the general term being quantified is being designated as unique.

1.2. *The ecthetic proof of* I *convertibility*

Aristotle's ecthetic argument for the convertibility of the particular affirmative proposition can accordingly be sketched as:

[A]:	(1)	Iab	[As.][11]
	(2)	$LA[a](a)$	[Pr.][12]
	(3)	$T_i[a](b)$	[1,ECI]
	(4)	$A[a](a)$	[2,Nec Elim]
	(5)	$T_i[a](a)$	[4,ECA][13]
	(6)	Iba	[5,3]

The inference from (5) and (3) to (6) is crucial. We take Aristotle in the *Prior Analytics* as using c as a proper name for the arbitrarily selected individual-of-a-type denoted by '$T_i[a]$' at steps (3) and (5).[14] With α_i used

present king of France is bald' would rewrite it so as to displace the grammatical subject. Part of the rewrite might be

(i) *The*$_1$[man](*that*$_k$ is presently king of France is bald)

of the form '$T_1[c](b \cdot a)$'. As '*No*[man](is presently king of France)' is true, (i) is false. Its contradictory opposite

(ii) *The*$_1$[man](*that*$_c$ is presently king of France is not bald) $\{T_1[c](b \supset N(a))\}$

is true. But (i) entails the particular affirmative

(iii) *Something that*[is presently king of France](is bald) $\{Iba\}$.

By ECI, the inference to '$T_i[b](a)$' could occasion the use of the proper name 'β_i' as a referent fixer abbreviating '*the*$_i$ *one that*[is presently king of France]' in the context of arguing for the conversion of (iii). See section 1.2.

[11] To avoid clutter parentheses and brackets will often not be used in the presentation of the simple assertoric forms.

[12] Łukasiewicz ([6]: 104–05)) says that the forms 'Aaa' and 'Iaa' do not explicitly appear in Aristotle's works, but were recognized by his disciples. In [5]: 88, he cites them as axioms expressing laws of identity in his construction of syllogistic as a deductive system.

[13] It has been objected (by a referee) that the Axiom of Identity only allows an inference to '$T_i[a] = T_i[a]$' $\{T_i[a]T_i(a)\}$. However, for us this conclusion is only an additional one easily justified through the use of Df.A′, developed in section 2, and the thesis '$T_i[a] \neq T_j[a]$' $\{T_i[a]N(T_j(a)\}$.

[14] The derivation of (6) from (5) and (3) is accepted as evident via the form TTI-3, that is, from two affirmative propositions with the same individuating quantifier as premisses to a particular affirmative conclusion. The reasoning is in the third figure as a is the subject of both

as a proper name shorthand for '$T_i[a]$', in place of Aristotle's c, (3) is logically equivalent to

(3.1) $[\alpha_i](b)$,

while (5), instantiating (4), is equivalent to

(5.1) $[\alpha_i](a)$.

Aristotle's c is b, but also it is necessarily the case that c, that is, α_i, is a $\{L([\alpha_i](a))\}$. With (3.1) as the minor premiss and (5.1) as the major premiss, the conclusion to (6) is via an expository syllogism, that is, via a third or last figure inference with a proper name subject as the middle term. The form is SSI-3.[15] Such an expository syllogism, if Ockham is correct, does not itself need to be validated. It should be noted that the conjunction of (3.1) with (5.1) is logically equivalent to '$[\alpha_i](b \cdot a)$', again reflecting Aristotle's insistence that both b and a belong to c. Since it is equivalent to (3.1), (3) can be regarded as entailing (6) with (5.1) having the status of a suppressed necessarily true major premiss.

2. *Predicate expansion in Aristotelian logic*

It is possible and somewhat natural on the bases of the definitions already given of the four propositional forms constituting the traditional Aristotelian square of opposition to supplement them with definitions containing expanded predicates. Relevant for our considerations are two such definitions:

(Df.A′) $A[b](a) =_{\text{df}} A[b](T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$

and

(Df.I′) $I[b](a) =_{\text{df}} I[b](T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$.

Recognizing the possibility of defining all plain singular and general propositions in terms of expanded individuating predicates, we can also define the various *de re* apodeictic propositions on this model. Two definitions of immediate interest to us are:

premisses. Aristotle's own use of c here creates the illusion that the inference contains three distinct term variables when in fact only two are involved. The use of a proper name as an abbreviation of an individually quantified denoting expression is not essential to Aristotelian ecthetic proofs.

[15] S and $ thus respectively represent affirmative and negative singular propositions with proper name subjects and general term predicates.

(Df.A_L) $A[b](La) =_{\text{df}} A[b](LT_1(a) \vee LT_2(a) \vee \ldots \vee LT_n(a) \vee \ldots)$

and

(Df.I_L) $I[b](La) =_{\text{df}} I[b](LT_1(a) \vee LT_2(a) \vee \ldots \vee LT_n(a) \vee \ldots)$.

Now, as the counterexamples preclude any validation of A_L and I_L conversion, one might despair at the prospect of capturing any underlying logic for Aristotle’s position. However, we can begin anew and stress with Richard Patterson ([11]: 8) that Aristotle himself sometimes ‘speaks in a third way’ about the modal operator, a way which determines it as attaching neither to the dictum nor to the predicate ‘but rather to *the manner of the predicate’s applying to the subject*.’[16] This for us, though not for Patterson, means that the two necessary *de re* propositional forms of interest might be differentiated from the “relational” forms A_{L*} $\{A[b]L(a)\}$ and I_{L*} $\{I[b]L(a)\}$ defined with expanded individuating predicates respectively as:

(Df.A_{L*}) $A[b]L(a) =_{\text{df}} A[b]L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$

and

(Df.I_{L*}) $I[b]L(a) =_{\text{df}} I[b]L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$.

2.1. *The relation between A_L (I_L) and A_{L*} (I_{L*}) propositions*

A_L and A_{L*} propositions are obviously not logically equivalent. To conflate them constitutes a fallacy of amphiboly. It can be established that the A_L form entails its A_{L*} counterpart:

[B]:	(1)	$A[b](La)$	[As.]
	(2)	$A[b](LT_1(a) \vee LT_2(a) \vee \ldots \vee LT_n(a) \vee \ldots)$	[1,Df.A_L]
	(3)	$A[b]L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$	[2]
	(4)	$A[b]L(a)$	[3,Df.A_{L*}][17]

[16] Unless, otherwise noted, emphases are always in the text being quoted. The paradigm text grounding this interpretation is *An. pr.* I.8, 29b29–35.

[17] A referee for this journal wonders whether Argument [B] should be translated into modal predicate logic as the easily invalidated inference from

(i) $(\forall x)(Bx \supset \Box Ax)$

to

(ii) $(\forall x)\Box(Bx \supset Ax)$.

The short answer is that it should not. As mentioned in footnote 5, we construe ‘$A[b](a)$’ as entailing but not equivalent to ‘$A[c](b \supset a)$’. The entailment is easily validated by an indirect argument using ecthesis. By way of counterexample to the right to left inference: ‘*Every*[member of the Lyceum](*that*$_c$ is talking is walking)’ can be true without it being true that ‘*Whatever* [is talking](is walking)’. Thus, in our Aristotelian term logic ‘$A[b](La)$’ entails but is not equivalent to

(i′) $A[c](b \supset La)$,

As in modal propositional logic '$\Box\phi \vee \Box\chi$' $\{L\phi \vee L\chi\}$ entails but is not equivalent to '$\Box(\phi \vee \chi)$'$\{L(\phi \vee \chi)\}$, the analogous inference here from (2) to (3) is evident as the predicate of (2) entails but is not equivalent to the predicate of (3).[18] An indirect argument would show that the negation of (3), that is,

(3′) $O[b]L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$

entails

(3*) $I[b](NLT_1(a) \cdot NLT_2(a) \cdot \ldots \cdot NLT_n(a) \cdot \ldots)$,

the contradictory opposite of (2). A_{L*} propositions are thus not equivalent to their A_L counterparts. Analogously, I_L propositions are stronger than but not equivalent to their I_{L*} counterparts.

In this context Aristotle's perplexing recognition of certain characteristics as predicated accidentally of some things and with necessity of others can be explicated. Take *being white*, for example. According to Aristotle, it accidentally belongs to humans, but necessarily belongs to swans and snow.[19] We thus propose that the A_{L*} claim 'Every swan necessarily is white', parsed as '*Every*[swan]*necessarily*(is white)' $\{A[s]L(T_1(w) \vee$

the counterpart to (i) in modal predicate logic. Now, '$A[b]L(a)$' entails but is not equivalent to

(ii′) $A[c](b \supset L(a))$,

a claim quite different from

(ii*) $A[c]L(b \supset a)$,

the term logic counterpart to (ii). As 'La' $\{LT_1(a) \vee LT_2(a) \vee \ldots \vee LT_n(a) \vee \ldots\}$ entails '$L(a)$' $\{L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)\}$, it follows that (i′) entails (ii′), but not (ii*).

It is helpful to note that the *de dicto* necessary '$LA[b](a)$' can be posited as entailing '$A[c]L(b \supset a)$', that is, (ii*). Intuitively, '*Necessarily every*[bachelor](is unmarried)' entails '*Every* [thing]*necessarily*(*that*$_c$ is a bachelor is unmarried)'. But, '$LA[b](a)$' and '$A[b](La)$' are logically independent forms.

[18] Where $\chi = \sim\phi$ Aristotle himself recognizes that '$\Box(\phi\vee\chi)$' does not entail '$\Box\phi\vee\Box\chi$': '…it is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for a sea-battle to take place tomorrow, nor for one not to take place — though it is necessary for one to take place or not to take place' (*De Int.* ch. 9, 19a30–33, translated by J.L. Ackrill, in Aristotle [1]).

[19] It is Paul Thom ([15]: 137, n. 14 and 141, n. 30) who highlights the possibility that for Aristotle a term may be predicated necessarily of some individuals and contingently of others. His key reference is to *An. pr.* I.19, 38a32–33, where Aristotle in the context of developing another example says that white belongs of necessity to swan. (Cf. *An. pr.* I.19, 38b20, and I.15, 35a20–24. In *Categories* 10, 12b35–41, whiteness is ascribed necessarily to snow.) In *Topics* IV.1, 120b15–121a1, Aristotle claims that white is a qualitative accident and not a genus either of snow or swan, both of which are considered to be [secondary] substances.

$T_2(w) \vee \ldots)\}$, be distinguished from the A_L claim 'Every swan is necessarily white' $\{A[s](LT_1(w) \vee LT_2(w) \vee \ldots)\}$.[20] We shall argue that Aristotle would accept the former as also *de dicto* contingently true. This would be based both on an enumerative induction in light of his (limited) experience of swans as always possessing the attribute of *being white* and on a suppressed premiss to be introduced below.[21] *Whiteness* could then be recognized as what later would be called a necessary accident of snow and swans.[22] Acceptance of the relational A_{L*} claim as true would allow Aristotle to reject its A_L *de re* counterpart. Such a proposition would be taken as expressing attributes that are both necessary and essential. Thus, while the A_L '*Every*[swan](is *necessarily* white)' is false, the A_L '*Every*[swan]((is)*necessarily an*[animal])' is true and, for Aristotle, necessarily true *de dicto* as well.

2.2. I_{L*}/I_{L*} *convertibility and a suppressed premiss*

A counterexample successfully used against the conversion of I_L propositions — the presumably true I_L

(i) *Something that*[is a philosopher](is *necessarily* human)

as entailing by conversion the false I_L

(ii) *Something that*[is human](is *necessarily* a philosopher)

— cannot be used against the entailment by conversion of I_{L*} propositions. If (i) is true, so also by I_L/I_{L*} entailment is the I_{L*}

(iii) *Something that*[is a philosopher]*necessarily*(is human).

But, if the false (ii) entails the I_{L*}

(iv) *Something that*[is human]*necessarily*(is a philosopher),

[20] If, following modern syntactical definitions, we define a modal formula containing quantifiers as *de re* if and only if *all* its quantifiers appear outside the scope of any modal connective, then both the A_L form '$A[b](La)$' and the A_{L*} form '$A[b]L(a)$' can be classified *prima facie* as *de re* forms. But, with the former explicated as equivalent to '$A[b](LT_1(a) \vee LT_2(a) \vee \ldots \vee LT_n(a) \vee \ldots)$' and the latter as '$A[b]L(T_1(a) \vee T_2(a) \vee \ldots \vee T_n(a) \vee \ldots)$', it is clear that neither formula shows the quantifiers T_1, T_2,... as outside the scope of the necessity operator. Our introduction of explicit quantifiers into the predicate of Aristotelian propositions thus requires recognition of stronger and weaker affirmative necessary propositions in what can be called the divided sense. The former we have dubbed as exhibiting a *de re* necessity in the primary sense, the latter exhibits what we have stipulated as showing a "relational" necessity.

[21] See section 2.2.

[22] For an historical overview on the notion of a necessary accident, see van Rijen [17]: 132–53.

this cannot be the ground for dismissing (iv) as false. Perhaps (iv) follows from (iii) and is true. The truth of (iv) might appear anomalous. But, predicating '*necessarily*(is this philosopher or is that philosopher or…)' of a human still leaves open the possibility that each human is possibly not a philosopher. '*That* human necessarily is a philosopher and *that same* human is not necessarily a philosopher' $\{T_1[h]L(p) \cdot T_1[h](NLp)\}$ is a consistent claim. The first conjunct does not entail '$T_1[h](Lp)$', the contradictory of the second, and the second conjunct does not entail '$T_1[h]NL(p)$', the contradictory of the first. In contrast both '$T_1[h]L(p) \cdot T_1[h]NL(p)$' and '$T_1[h](Lp) \cdot T_1[h]N(Lp)$' are explicitly inconsistent. Here sameness of the referent of the quantified subject is shown not said. With 'Socrates' as a rigid designator and Aristotelian proper name abbreviating '*That*[human]', one can also consistently remark '[Socrates]*necessarily*(is a philosopher) and [Socrates]((is)*not necessarily a*[philosopher])'.[23]

To establish but not demonstrate $\mathsf{I}_{L*}/\mathsf{I}_{L*}$ convertibility by ecthesis it is necessary to accept what shall be labeled a term logic *Converse Aristotle Formula*, an intermodal theorem which we cast as

(CAF) $L(LAba \supset A[b]L(a))$

or as the rule '$LAba/ \therefore A[b]L(a)$'. This formula must be distinguished from the term logic *Converse Barcan Formula*

(CBF) $L(LAba \supset A[b](La))$

or as the rule '$LAba/ \therefore A[b](La)$'. CBF must be rejected: by it the true *de dicto* necessary '*Necessarily whatever*[is a bachelor](is unmarried)' entails the false *de re* necessary '*Whatever*[is a bachelor](is *necessarily* unmarried)'.[24] By CAF it only entails the weaker relationally necessary '*Whatever*[is a bachelor]*necessarily*(is unmarried)', a claim compatible with the

[23] If in '[Tully](is Cicero and is a philosopher)' 'is' functions as shorthand for 'is identical to', '[Tully](is the same philosopher as Cicero)' follows through a necessarily true premiss without using indexed quantifiers within the predicate. The same conclusion, of course, follows from '[Tully](is Cicero and is this philosopher)'. Expressions of relative identity such as 'is the same philosopher as' are reducible to formulas expressing absolute identity. '[Tully] *necessarily*(is the same philosopher as Cicero)' can be truly asserted, but not '[Tully](is necessarily the same philosopher as Cicero)'. In contrast, '[Tully](is necessarily the same human as Cicero)' is true.

[24] The textbook version of the Converse Barcan Formula is sometimes presented as '$\Box(\forall x)Ax \rightarrow (\forall x)\Box Ax$'. Our term logic transformation of this finds it ambiguous between '$L(LAba \supset A[b]L(a))$' {CAF} and '$L(LAba \supset A[b](La))$' {CBF}. There is a third formula which the term logician can endorse, one which perhaps best conveys from the Aristotelian standpoint the basic intuition underlying the Converse Barcan Formula: $L(LAba \supset A[c]L(b \supset a))$. Cf. the second paragraph in footnote 17, above. As the textbook Converse Barcan Formula correctly authorizes the inference '$\Box(\forall x)(Bx \supset Ax)/ \therefore (\forall x)\Box(Bx \supset Ax)$', Aristotelian term logic can also posit in tandem: $LA[c](b \supset a)/ \therefore A[c]L(b \supset a)$.

de re universal negative possibility (E_M) proposition '*Whatever*[is a bachelor](is *possibly not* unmarried)', though not with the relational E_{M*} '*Whatever*[is a bachelor]*possibly not*(is unmarried)' {*Nothing that*[is a bachelor] *necessarily*(is unmarried)}. By CAF the true $^L\mathsf{A}$ '*Necessarily whatever*is white' entails the A_{L*} '*Whatever*[is white]*necessarily*(is white)'. Thus, if '*Every*[swan](is white)' is true, it follows that '*Every*[swan] *necessarily*(is white)' is true also.[25] Of course, this A_{L*} conclusion is compatible with asserting the E_M '*No*[swan](is *necessarily* white)', the contradictory of the false I_L '*Some*[swan](is *necessarily* white)'. The contingency of the whiteness of swans is not being denied as through CAF the once wild Aristotelian claim asserting that *being white* necessarily belongs to swans has been tamed: whiteness still retains its status as an accidental characteristic. A discovery of black swans would not undermine Aristotle's basic position.

It is necessary to differentiate CAF from another acceptable formula or rule of inference due to Thom ([15]: 149):

(TF) $L(A[b](La) \supset A[a](La))\ \{A[b](La)/\therefore A[a](La)\}$.

This formula allows that if an attribute is essentially predicated of any characteristic whatsoever, then it is essentially predicated of itself. Thus, '*Whatever*[is white](is *necessarily* an animal)' entails '*Whatever*[is an animal](is *necessarily* an animal)'. Now, if $b =$ is human and $a =$ is an animal, '$LA[b](La)$' can be posited. This implies '$LA[a](La)$'. As '$A[a](La)$' entails '$A[a]L(a)$', '$LA[a]L(a)$' holds only for essential characteristics, whereas by CAF the weaker '$A[a]L(a)$' holds for all characteristics, essential or accidental.

Through CAF an ecthetic argument for $\mathsf{I}_{L*}/\mathsf{I}_{L*}$ conversion on the model of Argument [A] for I conversion can now be developed:

[C]:	(1)	$I[b]L(a)/\therefore I[a]L(b)$	
	(2)	$LA[b](b)$	[Pr.]
	(3)	$A[b]L(b)$	[2,CAF]
	(4)	$T_i[b]L(a)$	[1,ECI_{L*}]
	(5)	$T_i[b]L(b)$	[3,ECA_{L*}]
	(6)	$I[a]L(b)$	[5,4,$\mathsf{T}_{L*}\mathsf{T}_{L*}\mathsf{I}_{L*}$-3]

As A_{L*} entails I_{L*}, $\mathsf{A}_{L*}/\mathsf{I}_{L*}$ conversion is also easily established. More significantly, since *de re* necessary propositions entail their relationally necessary counterparts, both $\mathsf{A}_L/\mathsf{I}_{L*}$ conversion and $\mathsf{I}_L/\mathsf{I}_{L*}$ conversion are *a fortiori* established.

[25] The essence of the argument is a two-term "syllogism" in Barbara-L_*XL_*: *Whatever*[is white]*necessarily*(is white)$\{A[w]L(w)\}$, *Every*[swan](is white)$\{A[s](w)\}/\therefore$ *Every*[swan]*necessarily*(is white) $\{A[s]L(w)\}$. The validation is via a *reductio* using ecthesis and an expository syllogism.

'$A[b]L(a)$' does entail 'Aba'.[26] Now, through CAF '$LAaa$' entails '$A[a]L(a)$'; thus, if Aba, then $A[b]L(a)$. 'Aba', however, remains a weaker claim than '$A[b]L(a)$'.[27] 'Aba' implies but does not entail '$A[b]L(a)$' as the premiss '$A[a]L(a)$' is not to be posited as a necessary dictum. CAF does entail '$LLA[a](a) \supset LA[a]L(a)$'. Though Aristotle can accept '$LA[a](a)$', we cannot ascribe an acceptance of the characteristic axiom of S4 to him and then infer an acceptance on his part of '$LLA[a](a)$' and a consequent acceptance of '$LA[a]L(a)$', a formula which through TF can only hold for essential characteristics. If, as we are suggesting, Aristotle claims that '*Every*[swan]*necessarily*(is white)' is true on the basis of the inductively established truth of '*Every*[swan](is white)', his appeal to the suppressed premiss '*Whatever*[is white]*necessarily*(is white)' to establish it is not an appeal to a premiss that can be explicated as a necessarily true dictum without an illicit reliance on S4. In this way the respective collapses of A_{L*} and I_{L*} propositions into plain A and I propositions are avoided.

3. *Retrospect and prospect*

We have argued that the defect in Aristotle's treatment of the convertibility of universal and particular affirmative apodeictic propositions lies in its failure to distinguish between those propositions that contain a necessary modalized predicate and those that express a "relational" necessity. In short, Aristotle conflates A_L $\{A[b](La)\}$ and A_{L*} $\{A[b]L(a)\}$ propositional forms, a conflation he himself would likely dub a fallacy of amphiboly. The distinction between these two forms has the added benefit of enabling an Aristotelian to differentiate in a syntactical way the predication of essential characteristics from the predication of necessary accidental characteristics. Furthermore, we can now be reasonably confident that in contrast to the invalid $\mathsf{A}_L/\mathsf{I}_L$ and

[26] Following Thom ([16]: 34), we attribute to Aristotle such rules of *L-subordination* as '$A[b](La)/ \therefore Aba$' and '$I[b](La)/ \therefore Iba$'. Here we add the rule of *L_*-subordination* '$A[b]L(a)/ \therefore Aba$'. For a critique of the radical thesis stating Aristotle's rejection of this rule's claim that necessarily belonging entails plain belonging, see Patterson [11]: 106–15.

[27] Relational necessity itself, we have seen, is a weaker type of necessity than the *de re* necessity expressed as a modalized predicate. Though '$LA[b](a)$' entails '$A[b]L(a)$', the latter does not entail the former. For, while it is possible that '*Whatever*[is white]*necessarily* (is an animal)' be true, '*Necessarily whatever*[is white](is an animal)' is false. Both the "Aristotelian" Formula

(AF) $L(A[b]L(a) \supset LA[b](a))$ $\{A[b]L(a)/ \therefore LA[b](a)\}$

and the Barcan Formula

(BF) $L(A[b](La) \supset LA[b](a))$ $\{A[b](La)/ \therefore LA[b](a)\}$

must be rejected.

$\mathsf{I}_L/\mathsf{I}_L$ conversions, $\mathsf{A}_L/\mathsf{I}_{L*}$ and $\mathsf{I}_L/\mathsf{I}_{L*}$ conversions are valid. Perhaps most significantly, the distinction provides a means for dealing with the classical problem of the two Barbaras. In treating first figure syllogisms with one necessary premiss and one plain premiss, Aristotle claimed that Barbara-LXL $\{A[b](La), A[c](b)/ \therefore A[c](La)\}$ is valid and that Barbara-XLL $\{Aba, A[c](Lb)/ \therefore A[c](La)\}$ is invalid.[28] It has been shown that Aristotle's position is inconsistent, if he maintains that $\mathsf{I}_L/\mathsf{I}_L$ convertibility is valid.[29] But, we suggest, Aristotle need accept only the weaker $\mathsf{I}_L/\mathsf{I}_{L*}$ convertibility as valid. The integrity of his position can be maintained, though he must also now recognize that both Barbara-L_*XL_* $\{A[b]L(a), A[c](b)/ \therefore A[c]L(a)\}$ and Barbara-XL_*L_* $\{Aba, A[c]L(b)/ \therefore A[c]L(a)\}$ are valid forms.

We have relied in Argument [C] on CAF as a means to validate $\mathsf{I}_L/\mathsf{I}_{L*}$ convertibility through $\mathsf{I}_{L*}/\mathsf{I}_{L*}$ convertibility. But, this argument is not demonstrative: '$A[b]L(b)$' is not a necessary dictum. Furthermore, acceptance of its truth would license an inference from '$I[b](a)$' to '$I[b]L(a)$' for any b and a. It seems odd to say "'*Some* [human](is a philosopher)' implies '*Some*[human]*necessarily*(is a philosopher)'". Admittedly, once it is recognized that this is not an assertion of an entailment and that there is no implication here of predicating *being a philosopher* as an essential characteristic of a human, the oddity is somewhat reduced, though not completely removed. The task ahead is at least threefold: (a) to develop a *demonstration* of $\mathsf{I}_L/\mathsf{I}_{L*}$ convertibility which does not rely upon CAF; (b) to justify apart from CAF Aristotelian conclusions to such claims as '*Every* [raven]*necessarily*(is black)'; and (c) to test the validity of CAF by among other projects developing convincing counterexamples to the alleged validity of $\mathsf{I}_{L*}/\mathsf{I}_{L*}$ convertibility.[30]

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[28] See *An. pr.* I.9, 30a15–29.

[29] See Hintikka ([4]: 138–44) as well as McCall ([7]: 13–14) and Geach ([3]: 204).

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