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QUINE'S MASTER ARGUMENT

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Abstract

This paper examines what it suggests is essentially one argument, which Quine proposed in 1935 against the analyticity of logic, and in 1970 against taking Tarskian truth-definitions as giving the meanings of logical constants. I shall illustrate its wide relevance by arguing that it undermines some arguments of Dummett's, and that it generalizes the argument of Prior's "The Runabout Inference Ticket" and rebuts some replies to it.

Quine presented an argument I consider decisive on several issues, which I call his master argument. He presented one version of it in 1935, and another in 1970. I shall illustrate its wide relevance by using it to argue against some influential arguments of Dummett's, to generalize the argument of Prior's celebrated 'The Runabout Inference Ticket' and rebut some proposed circumventions of it.

1. *The Argument of 'Truth by Convention'*

If there is a notion that we can express perfectly clearly, but only at some length, we may well wish to use a briefer and more convenient technical term for it. The practice is common in mathematics; thus, rather than repeatedly saying 'domain of elements on which there is an associative binary operation, which contains an identity element under that operation, and every element of which has an inverse under that operation', we say 'group'. When such a technical term is first introduced, it is not appropriate to ask 'But is that really what a group is?', for one is proposing a convention for a new use of a term. So such stipulative definitions as 'a group is a domain of elements on which. . . ' are sometimes said to be "true by convention".

Logical empiricist philosophy did not hold that all truths were contingent factual matters to be learnt from experience; the truths of pure mathematics, in particular, were held to be necessary. It held that all *informative* truths were contingent and depended on experience (hence the 'empiricist'); but that e.g. pure mathematics was not informative at all, but *analytic*, expressing merely the meanings of words. Since it also held that the meanings of words was a matter of convention, pure mathematics could be explained as *true by convention*, like definitions. Typically, they held that pure mathematics was reducible by definitions to logic (hence the 'logical'), a belief largely inspired by Russell and Whitehead's *Principia Mathematica*.

This is relevant background to Quine's 1935 paper, 'Truth by Convention', which appeared in a volume in honour of Whitehead. In part I Quine presented an account of definitions as conventional, eliminable abbreviations. He made the point, as old as Aristotle, that any sequence of such definitions serves only to reduce, never entirely to eliminate, undefined notation. He made it clear what is involved in the task of reducing [some part of axiomatized] mathematics to logic. First there is a conceptual task, that of defining all the notions of [that part of] mathematics in the vocabulary of logic alone; then there is a doctrinal task, that of showing that its axioms, interpreted taking these definitions as abbreviations of logical claims, are truths of logic.

These are difficult enough tasks. But if their purpose is to show that [part of] mathematics is true by convention, or that it is analytic, more yet must be done:

> . . . in strictness we cannot regard mathematics as true purely by convention unless those logical principles to which mathematics is supposed to reduce are likewise true by convention. And the doctrine that mathematics is *analytic* accomplishes a less fundamental simplification for philosophy than would at first appear, if it asserts only that mathematics is a conventional transcription of logic, and not that logic is convention in turn: for if in the end we are to countenance any a priori principles at all which are independent of convention, we should not scruple to admit a few more, nor attribute crucial importance to conventions which serve only to diminish the number of such principles by reducing some to others.[1]

Quine then supposes that logic has been streamlined by having its primitive terms reduced by definition to a very small number that are not mutually definable. (This had already been done, in several ways.) He also supposes that these are not definable in other terms still available in our reduced language: in other words, that the process of reducing primitives has been carried as far as it can go. He also assumes that logic, in this "primitive" notation, has been adequately axiomatized — that a few logical truths ("the axioms")

[1] Quine 1935:80–81.

have been isolated, from which the others logically follow by a few logical rules that have also been articulated. (For most simple systems of logic, this too had been achieved in various ways.) Then he makes the point that if the *axioms* of logic are true by convention, it will not be by definition; for there are, by his assumptions, no expressions left in terms of which they can be defined. How then can we understand them as true by convention?

He suggests a device that might well do the trick:

> A word may, through historical or other accidents, evoke a train of ideas bearing no relevance to the truth or falsehood of its context; in point of *meaning*, however, as distinct from connotation, a word may be said to be determined to whatever extent the truth or falsehood of its contexts is determined. Such determination of truth or falsehood may be outright, and to that extent the meaning of the word is absolutely determined; or it may be relative to the truth or falsehood of statements containing other words, and to that extent the meaning of the word is determined relatively to those other words. A definition endows a word with complete determinacy of meaning relative to other words. But the alternative is open to us, on introducing a new word, of determining its meaning *absolutely* to whatever extent we like by specifying contexts which are to be true and contexts which are to be false.[2]

So the task Quine is now considering is a very specific one, required if the claim that logic is true by convention can be defended in this way. Not only must we be able to specify the truth-conditions of contexts of the primitives of logic in such a way that all of logic comes out as true, but we must be able to do this without using or supposing as already available to us either the notions whose contexts' truth-conditions we are thus specifying, or their equivalents. For if we could use these, either we would not, contrary to the assumption, have already reduced such notions by definitions as far as possible; or else we would not, contrary to the assumption, be introducing signs *new to our language* by explicit statement of truth-conditions. Quine presents his conclusion in two alternative formulations:

[2] Ibid.:82–83.

> In a word, the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions. Alternatively, the difficulty which appears thus as a self-presupposition of doctrine can be framed as turning upon a self-presupposition of primitives.[3]

But these are two difficulties, that need to be distinguished. Quine shows at length that to offer outright definitions for logical notions, and use them to determine what were the logical truths, we would have to understand, before and independently of the definitions, expressions equivalent to those being defined. We would need them in order to *state* such definitions — e.g. in defining 'x' we would have to say something like '*Every* expression of the form ... is to be counted true'. This is what he calls 'self-presupposition of primitives'. But then we would also need to *use* them in the required way; e.g. to *infer* from the definition and the observation 'This expression is of the form ... ' the truth of the expression in question, we must *already* understand 'every' well enough to recognize the soundness of such inferences. This is what he calls 'self-presupposition of doctrine'.

Quine's argument is presented in the context of the assumption that the process of reducing the number of logical notions through word-word definitions had already been carried as far as possible; that we are working within a language that lacks any equivalents for its basic logical constants. That, after all, is why it is "outright" rather than word-word definitions that are being considered. So his conclusion contradicts the assumptions of his argument. But there were two such assumptions. Since one, the claim that a chain of word-word definitions must terminate, is established, the force of the reductio falls on the other: that basic truths of logic can be founded on outright definitions. So if 'truth by convention' is understood, as it had been, in terms of such paradigms as the equivalences or identities presented as stipulative definitions, claims that logical truths are true by convention, and claims that the standard logical inferences are valid because of the meanings logical constants are given by convention, are *wrong*. Conventions of the kind of word-word definitions merely reduce the problem to that of inferences involving basic logical constants; and these constants *cannot* be given their meanings by outright definition.[4]

[3] Quine 1935:97.

[4] Quine did not put his conclusion so strongly or simply. He was irenically prepared in effect even to allow the *phrase* 'true by convention' to be used, as if itself by convention, of such stipulations, but then to argue that the customary *connotations* of the phrase did not apply, and that it was quite unclear what interesting and defensible claim was being made in so using it. This apparently harmless irenicism was unfortunate, for it enabled such defenders

The historic importance of this argument was directly to discredit the then widely accepted assumption that logic and arithmetic had been shown to be analytic. Indirectly, by raising doubts about this conclusion, it helped call into question what had been taken as self-evident, the very existence of a priori knowledge.

2. *The Argument in ‘Philosophy of Logic’*

35 years later, in *Philosophy of Logic*, Quine made much the same point again, for a different purpose:

> A reasonable way of explaining an expression is by saying what conditions make its various contexts true. Hence one is perhaps tempted to see the above (Tarskian) satisfaction conditions as explaining negation, conjunction and existential quantification. However, this view is untenable; it involves a vicious circle. The given satisfaction conditions for negation, conjunction and quantification presuppose an understanding of the very signs they would explain, or of others to the same effect. A negation is explained as satisfied by a sequence when the constituent sentence is not satisfied by it; a conjunction is satisfied by a sequence when the one constituent sentence and the other are satisfied by it, and an existential quantification is satisfied by a sequence when the constituent sentence is satisfied by some suitably similar sequence. If we are prepared to avail ourselves thus of ‘not’, ‘and’ and ‘some’ in the course of explaining negation, conjunction and existential quantification, why not proceed more directly and just offer these words as direct translations?[5]

Though this time round the argument is pellucid, his target is made less clear. I will suggest some targets that it hits.

of orthodox analyticity theory as Carnap to reply in effect: yes, claims of analyticity are themselves analytic and so of course conventional; what’s the worry?

[5] Quine 1970:40.

3. *Dummett and the Redundancy Theory*

About what is meant by ‘explaining’ or ‘giving’ the meaning of an expression there are obscurities and disagreements. Fodor, I think, once complained that “the notion of ‘giving sense to a notion in a context’ is one I think has been given no sense in this context.” Much of how Dummett argues would make sense if for him, ‘gives the meaning’ implies at least: would successfully convey the sense to someone who does not already understand any synonymous expression.[6] Dummett took the thesis that giving truth-conditions of an expression was all that was required to give its meaning, in the sense of ‘sense’, as so central that were it to fail, the very notion of truth should be scrapped:

> Are we then to say that laying down the truth-conditions for a sentence is not sufficient to determine its sense, that something further will have to be stipulated as well? Rather than say this we should abandon the notions of truth and falsity altogether.[7]

He also assumed (as will become clear) that such giving of sense would be enough to justify those claims that were true in virtue of sense, and also that the theorems of logic would thus be true in virtue of their sense; in effect, a doctrine of the analyticity of logic.

In 1959 Dummett wrote a seminal paper, at the end of which he tentatively suggested accepting the redundancy theory, but in the course of which he changed his emphasis quite a bit; subsequently he changed his mind several times again. I say this not by way of reproach, but to explain how it is that much of the influence of his paper has been to promulgate a position it ended up rejecting. In 1978 he summed up “the fundamental contention” of the paper as

> that acceptance of the redundancy theory precluded the possibility of using the notion of truth in a general account of what it is to grasp the meaning of a sentence of the object-language, and in particular, of an account according to which an understanding of a sentence consisted in an apprehension of the condition for it to be true.[8]

[6] So the extent that my critical arguments below are found cogent, it is open to those who defend Dummett to spell out an alternative reading of ‘give the sense’ that vindicates his arguments, including the denial of circularity.

[7] Dummett 1959:11.

[8] Dummett 1973, Preface:xx–xxi.

This contention is absolutely sound. However, in this original paper, Dummett had presented this incompatibility largely by arguing against the redundancy theory on the basis of the contrary intuitions, and these arguments proved influential. Since he went on to show neither that nor how these intuitions failed, he left the impression that to reject them would be a costly and near-desperate measure to which we might be driven were we to insist on a redundancy theory. I shall show, to the contrary, that we are rationally driven to such rejection even if we find the redundancy theory absurd.

I shall label TT the thesis that the sense of logical particles is given by truth-tables in a way that justifies their use. The "redundancy theory" of truth, as Dummett uses the phrase, is that "p' is true [false]' is equivalent in meaning to '[not] p'. Here is how he sees TT as leading to rejecting the redundancy theory:

> ... we often make statements of the form 'P or Q' when the ultimate evidence for making them ... is neither evidence for the truth of P nor evidence for the truth of Q. The most striking instance of this is the fact that we are prepared to assert any statement of the form 'P or not P', even though we may have no evidence either for the truth of P or for the truth of 'not P'. In order to justify asserting 'P or not P', we appeal to the truth-table explanation of the meaning of 'or'. But if the whole explanation of the meanings of 'true' and 'false' is given by 'It is true that p if and only if p' and 'it is false that p if and only if not p', this appeal fails. The truth-table tells us, e.g., that from P we may infer 'P or Q' (in particular, 'P or not P'); ... The truth-table does not show us that we are entitled to assert 'P or not P' in every possible case, since this is to assume that every statement is either true or false; but, if our explanation of 'true' and 'false' is all the explanation that can be given, to say that every statement is either true or false is just to say that we are always justified in saying 'P or not P'.[9]

So if a redundancy theory were sound, appeal to truth-tables would fail to give the meaning of connectives in a way that justified their use.

[9] Dummett 1959:6–7. Notice that Dummett interprets those who offer B as an explication of A as holding that 'saying A' is the same as 'saying *we are justified in saying that* B'; this is of course in general mistaken. It is not relevant to the argument here, but he can legitimately attribute this equating only to those who espouse a "verificationist" account of 'true' and offer B as an explication of '*A is true*'. It is especially mistaken in this case, for he is explicitly considering those who offer a redundancy rather than a verificationist account of 'true'.

However, such an appeal would fail in any case. It falls foul of the following dilemma: it is either unnecessary or ineffective.

(1) For an appeal to the truth-table account of the meaning of the connectives to justify asserting anything, e.g. 'P or not P', an understanding of the meaning (as thus explicated) must suffice for the justification.
(2) For such an appeal to be needed, we must lack such understanding until the truth-table explication is given to us.
(3) But if we do lack such understanding, we will fail to understand the explication, and so it will not be effective.

Dummett is right in remarking that the truth-table does not show us that we are entitled to assert 'P or not P' in every possible case, and that the extra assumption required is that every statement is either true or false. He is also right that since according to the redundancy theory, asserting that every statement is either true or false is equivalent to asserting 'P or not P' in all cases, and so that if the redundancy theory is right the justification would be the sheerest question-begging. But for this conclusion, the redundancy theory is redundant. Let us set it aside. After all, for the extra information 'every statement is either true or false' to help us, we must already understand the idiom 'either ... or ... '. So if we do not understand it already, the truth-table account is useless. But if we do already understand it, the account is not needed for us to understand it. So the claim that the truth-tables give the meaning of the connectives, and so provide the justification for (e.g.) the law 'P or not P', is indefensible. Nothing in this argument depends on accepting a redundancy theory. It is simply an adaptation of Quine's master argument.

Understanding is no all-or-nothing matter. Does this dilemma fail by wrongly treating it as such? Can we defend TT by building in the idea that understanding develops in stages? Let us assume that prior to the truth-table explication, we have a more basic understanding of logical connectives than it expounds: say, the understanding provided by their introduction and elimination rules. This might well have been in Dummett's mind. For this interpretation fits with his partiality towards intuitionistic logic, whose elimination rules licence concluding from A to no more than would, by the introduction rules, have warranted A in the first place; a feature nice enough to be a major independent motivation for such partiality. Given this feature, reminiscent of definitions, it is tempting to say that the entire meaning is provided by the introduction rules alone, and that this being the entire meaning justifies the elimination rules. It fits with his saying "we can give an account of the meaning of 'and' by saying that we are in a position to assert 'P and Q' when and only when we are in a position to assert P and in a position to assert Q."[10] It fits with his finding it particularly problematic that "we often assert 'P or Q' when we are not either in a position to assert P or in a position to assert Q. ... we are prepared to assert any statement of the form 'P or not P', even though we may have no evidence either for the truth of P or for the truth of 'Not P'."[11] It fits, too, with his remark that "we learn the sense of the logical operators by being trained to *use* statements containing them" and with his illustrating this entirely in terms of introduction and elimination rules for the operators.[12]

The assumption we are considering is that such rules provide our basic understanding, not our complete understanding, of the connectives. The stronger assumption would lead to no logic stronger than intuitionistic logic, and so would not lead to acceptance of excluded middle, i.e. of the universal truth of 'p or not p'. The weaker leaves room for a classical "topping up" of the basic meaning of 'or'; and adherence to excluded middle demands it. The suggestion is that truth-tables would then provide just the required topping-up. And if a basic understanding of 'or' is already provided in the rules 'We are in a position to assert 'P or Q' when we are in a position to assert P' and 'We are in a position to assert 'P or Q' when we are in a position to assert Q', there need be no circularity in the appearance of 'or' in the truth-table explication.

Unfortunately, this route too is of no use for its purpose. Much the same dilemma blocks the way. The crucial premise in the supposed justification of 'P or not P' by appeal to truth-tables is 'every statement is either true or false'. But how is the 'either ... or' of *this* premise to be understood? If the understanding of 'either ... or' in this premiss is *merely* that provided by the two introduction rules, the premiss is *unwarranted*, and the justification fails. The premiss is only justified on the *already topped up* reading; indeed, the need for topping up was *because* of the need to justify it. However, if we here *already* understand 'or' in the topped up sense, there is no need for appeal to truth-tables to provide such understanding. The dilemma is as before: either the job can't be done, or it doesn't need doing. So even with a picture of meaning being laid on in layers, first by introduction rules and second by truth-tables, there is still circularity in the upper layer.

Dummett is aware that the presence of 'and' in the text quoted to note 10 suggests a circularity. In its immediate sequel he tries to scotch this idea:

[10] Dummett 1959:6.

[11] Ibid.

[12] Dummett 1959:17.

> ... We can give an account of the meaning of 'and' by saying that we are in a position to assert $\ulcorner P$ and $Q\urcorner$ when and only when we are in a position to assert P and in a position to assert Q. (This is not circular: one could train a dog to bark only when a bell rang *and* a light shone without presupposing that it possessed the concept of conjunction.) But, if we accept a two-valued logic, we cannot give a similar explanation of the meaning of 'or'. We often assert $\ulcorner P$ or $Q\urcorner$ when we are not either in a position to assert P or in a position to assert Q.[13]

This rebuttal is gloriously irrelevant. For what he is arguing is not circular is: giving an account of the *meaning* of 'P and Q' by *saying* that we may assert it just when we may assert P *and* may assert Q. No-one who does not already understand 'and' will learn its meaning *from this account*, even if they are patted on the head, given some meat and told 'Good dog'.

Quine's master argument shows that neither the proof-theory nor the semantic theory of logical constants can be elucidated except on the basis of prior understanding of such constants or their functional equivalents. In this clear sense at least, it is false that truth-tables give or explain the meaning of logical constants. Dummett says

> We naturally think of truth-tables as giving the explanation of the sense which we attach to the sign of negation and to the connectives, an explanation which will show that we are justified in regarding certain forms of statement as logically true. It now appears that if we accept the redundancy theory of 'true' and 'false' — the theory that our explanation gives the whole meaning of these words — the truth-table explanation is quite unsatisfactory. More generally, we must abandon the idea which we naturally have that the notions of truth and falsity play an essential role in any account either of the meaning of statements in general or of the meaning of a particular statement.[14]

But that truth-tables fulfil this role is refutable, independently of the redundancy theory.

Dummett saw an analogous difficulty for negation:

[13] Dummett 1959:6.

[14] Dummett 1959:7.

> A popular account of the meaning of the word 'true' ... is that 'It is true that P' has the same sense as the sentence P.[15] ... If we are to give an explanation of the word 'false' parallel to our explanation of the word 'true' we shall have to say that 'It is false that P' has the same sense as the negation of P. In logical symbolism there exists a sign which, put in front of a sentence, forms the negation of that sentence; but in natural languages we do not have such a sign. We have to think to realise that the negation of 'No one is here' is not 'No one is not here' but 'Someone is here'; there is no one rule for forming the negation of a given sentence. Now according to what principle do we recognize one sentence as the negation of another? It is natural to answer: the negation of a sentence P is that sentence which is true if and only if P is false and false if and only if P is true. But this explanation is ruled out if we want to use the notion of the negation of a sentence in order to explain the sense of the word 'false'.[16]

So far, this seems to me (but for the idea that it is "natural" to offer this answer) lucid and correct. The infixed 'not' in English sometimes forms a negation but sometimes does not: 'John does not like bran' negates 'John does like bran', but 'Somebody does not like bran' does not negate 'Somebody likes bran'. So in judging what is the negation of what, we go not just by (surface) syntactical features, but more by our understanding of sense. (I use the word without prejudice; that is, setting aside problems about whether the usual connotations of 'sense' suggest a right theory about precisely what kind of 'more' is required.) According to what principle, Dummett asks, do we recognize one sentence as the negation of another? He in effect demands that a reasonable answer must deal with sameness of sense; just as with the question of how we recognize meaninglessness, (surface) syntax does not provide the whole story. And this seems right.

[15] For this remark to make grammatical sense, it must be a sentence that substitutes for the first occurrence of 'P' and a *name* of a sentence that substitutes for its second. This infelicity can be patched up, e.g. by replacing 'It is true that P' with 'P is true'. Is it mere pedantry to draw attention to such easily mended infelicities? I suggest not, because they hide matters that can prove crucial. I show that we cannot understand semantically lowered expressions only in virtue of prior understanding of semantically raised ones, and that this undermines arguments Dummett took to be powerful; a notation that hid which was which may have helped him fail to realize this.

[16] Dummett 1959:4–6.

He then suggests that we can only understand the general concept of a sentence's negation in terms of the concept of falsity, as that sentence which is true exactly when the original is false and vice versa.

But it *could not be* that we only understood our idioms in virtue of such semantic ascent. Ascent would not enable us to recognize (e.g.) that 'Everybody likes bran' is a negation of 'There is somebody who does not like bran' and that 'John likes bran' is a negation of 'John doesn't like bran' *unless we already understood these four sentences*; in particular, unless we already understand such idioms as 'doesn't' and 'not'. And in understanding the signs which serve to negate, we understand enough to be in a position to notice that they function in the same way. This is a judgment about sense. In fact, it seems exactly the judgment required to satisfy Dummett's reasonable demand. So the rebuttal of Dummett is essentially the same dilemma as before: if we do not already understand the logical particles we use, appeal to the notions of truth and falsehood will fail to provide such understanding; if we already have such understanding, such appeal is not needed to provide it. This is again applying Quine's master argument.

4. *Russell on the Meaning of Truth-Functional Conjunctions*

TT can perhaps be fathered on Bertrand Russell; for having explained truth-tables, he added "The meaning of disjunction will be entirely explained by the above schema."[17] The cited schema in effect says: 'p or q' is true if both 'p' and 'q' are true, and is true if 'p' is true and 'q' is not true, and is true if 'p' is not true and 'q' is true, and is not true if both 'p' and 'q' are not true. So Quine's query has force: if we are thus prepared to make free use of 'if' and 'not' in explaining the meaning of 'or', why not do it by a direct definition: p or $q =_{\mathrm{df}}$ if not p, q?

Russell may have thought he had explained the truth-conditions of the contexts of such terms "outright". But he had not; he had explained them only by use of synonymous expressions. I doubt if he even thought he had; his main concern in "explaining the meaning" was to explain the *reference*, and in the case of these conjunctions, to argue that they did not have any: 'you must not look about the real world for an object which you can call 'or''.[18]

Russell took himself to be elaborating Wittgenstein's ideas;[19] but Wittgenstein of course thought that one could show what one could not state, and

[17] Russell 1919:65.

[18] Loc. cit.

[19] Ibid.:31.

may well have thought that the truth-table picture showed how truth-functional connectives worked. Still, without a prose explication (of just the kind deemed unstatable) of what the picture is meant to do, it would generally fail to do its showing; and once such an explication is understood the picture is superfluous, except perhaps as a pictorial mnemonic.

5. *Prior's Runabout Inference Ticket*

Prior's brief 'The Runabout Inference Ticket', aptly described by Belnap as "a gem, reminding one of Lewis Carroll's 'What the Tortoise said to Achilles'", is like that paper in drawing no explicit moral. Prior introduces by definition a connective 'tonk', with a perfectly good introduction rule (half of the introduction rules for 'or'):

$$\frac{A}{A \text{ tonk } B}$$

and with a perfectly good elimination rule (half of the elimination rules for 'and'):

$$\frac{A \text{ tonk } B}{B}$$

whose use would remarkably simplify both proofs and the task of finding them: any B could be derived from any A in two steps.

He does not characterize in detail the doctrines of "analytic validity" he intends this connective to discredit. In context, the doctrine seems to be that 1) connectives, e.g. 'and', can be introduced by definition as having just the meaning specified by stipulating introduction and elimination rules for them, 2) this guarantees that the claim that inferences in accord with such rules are valid is analytic, and 3) it is not necessary to have an antecedent idea of the independent meaning of the connectives.

In the best of the replies, Belnap undertook to defend the first and third of these claims. He did not explicitly discuss the second; though clearly, a major philosophical question Prior was raising is whether it is defensible. What Belnap saw as required was a motivated way of discriminating among proposed definitions, so that those of connectives like 'and' proved acceptable while those of connectives like 'tonk' proved unacceptable. His key ideas were that there were uncontroversial logical requirements for soundness of definitions, which, though they had not before been explicitly adapted to the question of defining connectives through intelim rules, could straightforwardly be so adapted; and that so adapted, they would discriminate acceptable from unacceptable connectives. The two requirements he elaborates are those of conservativeness and of uniqueness.

Conservativeness Belnap understood standardly: a definition is conservative iff no formula in notation available in the system before it is introduced is provable with its help that was not already provable without it. The system relative to which one's very first definitions should be judged for conservativeness is, Belnap suggests, the pure theory of deducibility as expounded by Gentzen. This contains an axiom, $A \vdash A$, and rules of weakening, permutation, contraction and transitivity. Conservativeness guarantees relative consistency. Belnap collapses the demand for existence into that for such consistency: "One bears the onus of proving at least consistency (existence)."[20]

His definition of uniqueness is this. A binary connective plonk is unique iff, if any connective plink obeys the defining intelim rules of plonk,

(1) $A_1, \ldots, B$-plonk-$C, \ldots, A_n \vdash D$ if and only if $A_1, \ldots, B$-plink-C, $\ldots, A_n \vdash D$,

and

(2) $A_1, \ldots, A_n \vdash B$-plonk-$C$ if and only if $A_1, \ldots, A_n \vdash B$-plink-$C$.

The analogous account for unary connectives would be that a connective squidge is unique iff, if any connective squodge obeys the defining intelim rules of squidge,

(1) $A_1, \ldots,$ squidge-$B, \ldots, A_n \vdash C$ if and only if $A_1, \ldots,$ squodge-B, $\ldots, A_n \vdash C$,

and

(2) $A_1, \ldots, A_n \vdash$ squidge-B if and only if $A_1, \ldots, A_n \vdash$ squodge-B.

6. *Technical Digression*

However, there are some technical problems with defending the connectives of any logic in these terms. The basic relevant facts are these.

1) Adjoined to the pure theory of deducibility, the customary rules for conditionals do not define a unique connective. 2) Assuming the conditional thus

[20] Belnap 1962.

standardly introduced, the negation rules for classical logic are not conservative. 3) The negation rules for any logic weaker than classical logic do not define a unique connective.

The customary intelim rules for conditionals are

$$(A_1, \ldots, A_n, A \vdash B) > (A_1, \ldots, A_n \vdash (A \to B))$$

and

$$(A \to B), A \vdash B$$

If these are adjoined to the pure theory of deducibility, we do not have

$$((A \to B) \to A) \vdash A.$$

After all, these rules give the intuitionistic fragment of the logic of pure conditionals, of which "$(((A \to B) \to A) \to A)$" is not a theorem.

However, let us define '⇒' by the introduction rule for '→', and *two* elimination rules, the rule for '→' and the rule

$$((A \Rightarrow B) \Rightarrow A) \vdash A.$$

This gives us the classical material conditional. Since this last rule does not hold for the connective '→' but does hold for '⇒', they are distinct connectives; so since all the rules that hold for '→' hold for '⇒', '→' is not unique.

Customary classical intelim rules for 'not' are

$$(A_1, \ldots, A_n, A \vdash B), (C_1, \ldots, C_k, A \vdash \text{not } B) > (A_1, \ldots, A_n, C_1, \ldots, C_k \vdash \text{not } A)$$

and

$$\text{not not } A \vdash A.$$

If to the system consisting of the pure theory of deducibility plus the rules for '→' we adjoin these rules as the definition of 'not', we find that it is now demonstrable that

$$\vdash (((A \to B) \to A) \to A)$$

So the definition is not conservative. We can adjoin a definition — of 'nohow', say — which *is* conservative; the introduction rule is as for 'not', but the elimination rule is instead

A, nohow $A \vdash B$

'Not' is classical and 'nohow' intuitionistic negation. 'Nohow', however, is not unique. For 'not' also obeys all its rules, but we do not have

nohow nohow $A \vdash A$.

Any negation rules strong enough to yield this yield classical logic, while any not so strong fail to be unique just because this rule could consistently be added. Does this show that uniqueness and conservativeness cannot be satisfied together? No, because our assumptions have included those about the *order* in which rules have been introduced. If we *first* adjoin 'not' to the pure theory of deducibility and *then* introduce '$\rightarrow$' as above, conservativeness and uniqueness will be satisfied by both definitions, and '$\rightarrow$' will serve as the classical conditional.

Does this make Belnap's suggested repair adequate? Here is a reason for doubt. Since it is natural to introduce '$\rightarrow$' before 'not', it is clear that a natural and easy way of introducing the connectives violates his requirements. We only realize that an alternative order meets them in virtue of much subsequent metatheory. But if we are to adapt general logical accounts of definition to the task of defining connectives, we surely should demand what such accounts generally demand: not only that our definitions have certain desirable features (such as relative consistency), but that they *decidably* satisfy certain requirements which are independently motivated and which give a general guarantee that they have such features.

The analogue for connectives of one such general requirement is, informally stated, this. What is by definition derivable from an expression should coincide with what by definition licences its derivation. This, along with the idea that the intelim rules provide definitions, harks back to Gentzen's original exposition of intelim rules in 1934: 'The introductions represent, as it were, the "definitions" of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. . . In eliminating a symbol, we may use the formula with whose terminal symbol we are dealing only 'in the sense afforded by the introduction of the symbol.'[21]

[21] Gentzen 1934:80.

This Prawitz called the principle of inversion. Now though intuitionistic logic can be presented in a way that satisfies this requirement, no logic with stronger negation-elimination rules can; and while classical logic can be presented in a way that satisfies uniqueness, no logic with weaker negation-elimination rules can. So if we have negation, we cannot have both inversion and uniqueness.

Because I think inversion a plausible enough demand and a beautiful enough result, I would not make Belnap's uniqueness a requirement on intelim rules for connectives, ruling out as it would intuitionistic '→' as well as 'nohow'. Indeed, Belnap says that uniqueness is not as essential as conservativeness. But I would not want to rule out classical 'not' either, or even the myriad connectives definable in modal and relevance logics. There is room, I think, for a variety of standards of stringency of introduction of connectives through intelim rules, as there is room for a plurality of logics.

7. *Back to the Master Argument*

The substantive question is whether any of these standards guarantee the analyticity of the claims that inferences in accordance with such rules are valid. It was primarily to discredit this claim that Prior concocted 'tonk'. So even if 'tonk' can be barred as a monster by finding relevant features distinguishing it from 'and', the substantive question had in general been settled Prior's way in 1935, by Quine's master argument. For this shows that the strategy of defending the analyticity of the claim that inferences involving basic logical constants are valid by giving their meanings by outright definition cannot work. This tells also against Stevenson's reply to Prior, that "the meaning of connectives ... should be stated in terms of truth-function statements in a meta-language".[22] It clearly holds also if the proposed definitions are in terms of intelim rules rather than truth-conditions.

There are merits to taking Gentzen's account of deducibility as a basis for the proof-theory of different connectives. But it is important not to be hoodwinked into thinking that they enable the introduction of all our logical constants de novo. For they implicitly assume and use some basic 'if-then' connective and some implicit notion of generality, 'for all'. To the extent that logical connectives can be characterized using the theory of deducibility, it is only relative to such implicit primitives that their bedeutung is determined.

Though there is not space here to elaborate the argument, it is here left as an exercise for the reader to show how Quine's 1970 argument also bears

[22] Stevenson 1962:127.

on the early versions of Davidson's programme, in which he argued that the meanings of languages were given by their Tarskian truth-definitions.

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