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KNOWABILITY PRINCIPLE AND DISJUNCTION PROPERTY*

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Abstract

The so-called paradox of knowability is usually regarded as questioning the principle that all truth is knowable (knowability principle). In this paper we examine the connection of the principle of knowability with other principles (epistemic versions of the disjunction property), which concern the relationships between knowledge of a disjunction and knowledge of the disjuncts, and between knowledge of an existentially quantified sentence and knowledge of one of its instances. Some epistemic versions of the disjunction property are apparently weaker than the knowability principle. Still one of them seems to have paradoxical, or at least not easily acceptable, consequences as well. This puzzling result is diagnosed as depending on the association of a strongly intensional view of propositions with the impredicative way in which they are conceived and dealt with. If correct, the diagnosis directly applies to the paradox of knowability as well.

1

The relationships between truth and knowledge have always been a central issue in philosophy. In the last century, they have been examined in a very general and precise way and some puzzling results followed, among which the so-called *paradox of knowability*. It is reasonable to think that the generality of the approach is not, by itself, responsible for such results. Rather, they plausibly depend either on certain specific principles or on some underlying assumptions affecting the framework in which they are obtained. However, focusing on specific principles might not be helpful for the statement of a deep diagnosis. Instead, investigating their connections with other

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principles and with puzzles generated in a different way might suggest a different, perhaps better, way of understanding what went wrong.

The paradox of knowability is usually regarded as questioning the principle that all truth is knowable (knowability principle). Such a principle has interesting connections with the ideas that knowledge of a disjunction is related to knowledge of the disjuncts, and that knowledge of an existentially quantified sentence is related to knowledge of some of its instances (epistemic versions of the disjunction property).

This paper's first objective is to examine these connections by employing standard logical principles. Its second objective is to observe that if we have good reasons to hold that omniscience, or a weaker intuitionistic version of it, is not, and will not be, proved, then also an epistemic version of the disjunction property has paradoxical, or at least not easily acceptable, consequences. Since this epistemic version of the disjunction property is apparently weaker than the knowability principle, those puzzling consequences appear to be independent of the latter principle.

Arguably, they cannot depend on the reasonable assumptions that are made about what we factually (shall) know, or on the adoption of standard logical principles. According to a plausible diagnosis, they depend on both a strongly intensional view of propositions, and the impredicative way in which they are conceived and dealt with, which is implicit in the derivations customarily accepted in the literature.

2

In 1963 Frederic Fitch published a theorem, numbered as 1, which is now usually rewritten as:

(Fitch) $\forall p \neg \Diamond K(p \wedge \neg Kp)$

where "K" is taken to mean *it is known at some time (past, present or future) that*. (Fitch) asserts that no proposition can be known to be an unknown truth.

The proof of (Fitch) is well known. It is based on:

Dist $K(\alpha \wedge \beta) \vdash K\alpha \wedge K\beta$

Fact $\vdash K\alpha \rightarrow \alpha$

Nec If $\vdash \alpha$, then $\vdash \Box\alpha$

ER $\Box\neg\alpha \vdash \neg\Diamond\alpha$

and runs as follows:

1) $K(p \wedge \neg Kp)$	assumption
2) $Kp \wedge K\neg Kp$	from (1) by Dist
3) $Kp \wedge \neg Kp$	from (2) by Fact and standard logic
4) $\neg K(p \wedge \neg Kp)$	by (1)–(3), by denying (1) because of the contradiction (3)
5) $\Box\neg K(p \wedge \neg Kp)$	from (4) by Nec
6) $\neg\Diamond K(p \wedge \neg Kp)$	from (5) by ER

The principle (Fitch) follows from line (6) by universal generalization. It has some puzzling consequences.

If it is taken as true that there are unknown truths, i.e. $\exists p(p \wedge \neg Kp)$, then the following principle of knowability:

(KP) $\forall p(p \rightarrow \Diamond Kp)$

which claims that all truths are knowable, must be rejected, because its application to $p \wedge \neg Kp$ contradicts (Fitch). This is essentially theorem 2 by Fitch (1963). Notice that the rejection of (KP) amounts to the assertion of $\exists p(p \wedge \neg\Diamond Kp)$. So we have what is essentially theorem 5 by Fitch (1963):

$$\exists p(p \wedge \neg Kp) \vdash \exists p(p \wedge \neg\Diamond Kp)$$

which looks implausible, if not paradoxical, because it appears that the existence of something true which, as a matter of fact, is unknown, entails, on the basis of purely logical reasons, the existence of something true that is necessarily unknown.

If, on the other hand, the knowability principle is accepted, then it must be denied that there are unknown truths. Denying that there are unknown truths amounts to saying that all truths are known, i.e.:

(O) $\forall p(p \rightarrow Kp)$

Thus, if all truths are knowable, then all truths are known, i.e.:

$$\forall p(p \rightarrow \Diamond Kp) \vdash \forall p(p \rightarrow Kp)$$

That too appears, and appeared, implausible, because it turns out, on the basis of purely logical reasons, that the possible knowledge of all truths implies the actual knowledge of all truths.

Most people think that it can be taken as factually true that there are unknown truths, because, as far as we know, it appears that, as a matter of fact, there are propositions p such that $\neg Kp$ and $\neg K\neg p$. If the excluded middle $p \vee \neg p$ is accepted, it follows that:

$$\exists p(p \wedge \neg Kp)$$

whose negation (O) is implied by (KP). So it seems that we have a factual falsification of (KP), which is a general philosophical principle about knowledge.

3

Several diagnoses of this puzzling result have been provided. In order to choose among them, or possibly to try a new one, it might be useful to see:

1. whether there are significant inferential relations among (O), (KP) and some other principles ruling — intuitively — the extent of (possible) knowledge;
2. whether there are principles weaker than (KP) that generate puzzling results.

Some principles, which appear quite natural to consider, relate the knowledge of a disjunction to the knowledge of its disjuncts, or the knowledge of an existentially quantified sentence to the knowledge of one of its instances. These principles may be regarded as epistemic versions of the disjunction property for their analogy with the feature properly called 'disjunction property' in proof theory: a formal theory F has the disjunction property if, for any formula A and B:

$\vdash_F A \vee B$ implies $\vdash_F A$ or $\vdash_F B$

The following analogous principles of the distributivity of K will be taken into account

($K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (Kp \vee Kq))$

($\Diamond K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (\Diamond Kp \vee \Diamond Kq))$

($\Diamond K\exists$-Dist) $K\exists p\alpha(p) \rightarrow \exists p\Diamond K\alpha(p)$

as three (non equivalent) versions of the disjunction property for K. Their relations with (O) and (KP) can be analysed in a context in which, besides Dist, Fact, Nec, ER and universal generalisation, some other non-modal, modal and epistemic logical principles or rules are allowed. Of course, to use a larger amount of logic might increase the number of possible diagnoses when some contradiction or implausible result is derived. However, charging logic for it should be the last chance.

We shall start by considering what relation (O) and (KP) have to ($K\vee$-Dist) and ($\Diamond K\vee$-Dist). It is quite easy to see that:

($K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (Kp \vee Kq))$

and

(Known Ex Middle) $\forall pK(p \vee \neg p)$

are jointly equivalent to

(O) $\forall p(p \to Kp)$

Derivations for "(O) $\vdash$ ($K\vee$-Dist)" and "(O) $\vdash$ (Known Ex Middle)" are straightforward, but for the latter they are possible only in classical logic.

It is almost immediately proven (both classically and intuitionistically) that:

($K\vee$-Dist), (Known Ex Middle) $\vdash$ (O)

A possible derivation is the following:

1) p	assumption
2) $K(p \vee \neg p)$	from (Known Ex Middle)
3) $Kp \vee K\neg p$	from (2) by ($K\vee$-Dist)
4) $K\neg p \to \neg p$	from Fact
5) $\neg K\neg p$	from (1) and (4) by standard logic
6) Kp	from (3) and (5) by standard logic

Because "K" is taken to mean *it is known at some time (past, present or future) that*, obvious objections can be raised against ($K\vee$-Dist) from a classical viewpoint. Since ($K\vee$-Dist) is derivable from (O), it follows that they are also objections against (O).

($K\vee$-Dist) might be objectionable even from an intuitionistic viewpoint. First of all let us observe that to assert ($K\vee$-Dist) is to say that a method is known which transforms any proof that it is at some time known that $p \vee q$ into a proof that it is at some time known that p or that it is at some time known that q. Then consider that "it is at some time known that" is an empirical operator in the sense that a proof of "it is at some time known that s" appears to depend on empirical information concerning the proof activity displayed by an empirical subject or a community of empirical subjects and different empirical information is usually required to prove different sentences of the form "it is at some time known that s". As a consequence, if the proof that it is at some time known that $p \vee q$ does not directly involve a proof of one of the disjuncts, and new empirical information is needed to transform it into a proof of a specific disjunct, then it is not evident either that such new information will be available or that a method can provide it. So, because of the empirical nature of K, there seem to be reasons to refrain from asserting the intuitionistic validity of ($K\vee$-Dist) and since ($K\vee$-Dist) is intuitionistically derivable from (O), (O) turns out to be not assertable.

Let us formally remark that if the principle $\forall p(p \to \Diamond p)$ is added to the original framework, then (KP) derives from (O) in a straightforward way.

So, by virtue of Fitch’s derivation of (O) from (KP) and the proof of the equivalence between (O) and the conjunction of ($K\vee$-Dist) and (Known Ex Middle), (KP) too — when $\forall p(p \rightarrow \Diamond p)$ is assumed — turns out to be classically equivalent to the conjunction of ($K\vee$-Dist) and (Known Ex Middle).

It is, however, more interesting to wonder whether

($\Diamond K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (\Diamond Kp \vee \Diamond Kq))$

and

(Known Ex Middle) $\forall p K(p \vee \neg p)$

are jointly equivalent to

(KP) $\forall p(p \rightarrow \Diamond Kp)$

Clearly

1a) (KP) $\vdash$ ($\Diamond K\vee$-Dist)

1b) (KP) $\vdash$ (Known Ex Middle)

using classical reasoning for (1b). On the other hand, it is also clear that

2) ($\Diamond K\vee$-Dist), (Known Ex Middle) $\nvdash$ (KP)

Informally, let us suppose there is a world w where ($\Diamond K\vee$-Dist) and (Known Ex Middle) hold, a certain proposition p is true in w, p is never known in any world accessible from w, in w', accessible from w, p is false and it is known that $\neg p$. Then $\Diamond Kp$ is false in w.

$$\begin{array}{ccc} w & \longrightarrow & w' \\ \circlearrowleft & & \\ p, \neg Kp & & \neg p, \neg Kp, K\neg p \end{array}$$

Let us notice that this counterexample depends on the existence of contingent propositions. If we restrict (KP) to the propositions p such that $\Box p \vee \Box \neg p$, we get that $\forall p((\Box p \vee \Box \neg p) \rightarrow (p \rightarrow \Diamond Kp))$, which is equivalent to

(KP*) $\forall p(\Box p \rightarrow \Diamond Kp)$.

It can be shown that:

3) ($\Diamond K\vee$-Dist), (Known Ex Middle) $\vdash$ (KP*)

For:

1) $\Box p$	assumption
2) $K(p \vee \neg p)$	(Known Ex Middle)
3) $\Diamond Kp \vee \Diamond K\neg p$	by ($\Diamond K\vee$-Dist)
4) $\Box(K\neg p \rightarrow \neg p)$	by Fact and Nec
5) $\Diamond K\neg p \rightarrow \Diamond\neg p$	from (4) by T modal logic
6) $\neg\Diamond K\neg p$	from (1) and (5) by T modal logic
7) $\Diamond Kp$	from (3) and (6) by standard logic

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The following existential version of the epistemic disjunction property is easily derivable from (KP):

($\Diamond K\exists$-Dist) $K\exists p\alpha(p) \rightarrow \exists p\Diamond K\alpha(p)$

It is worth noting that the existential quantifier does not occur within the scope of a modal operator in the antecedent of ($\Diamond K\exists$-Dist). So, no model similar to the ones involved in classical counterexamples to the Barcan schema (i.e. models in which some entities do not exist in some world) can be employed to argue against ($\Diamond K\exists$-Dist). Indeed, by itself, ($\Diamond K\exists$-Dist) does not require propositions to exist *in* every world (at least not in the sense in which something physical is said to exist in a world if it is an entity of the world). However, we are assuming that any proposition can be evaluated relative to every world.

Reasonably, (KP) does not follow from any number of instances of ($\Diamond K\exists$-Dist), since ($\Diamond K\exists$-Dist) seems to provide no means to move from a proposition's truth, whatever it is, to the knowledge of its truth. In any case, ($\Diamond K\exists$-Dist) has some puzzling consequences, because it is equivalent to:

$$\forall p\neg\Diamond K\alpha(p) \rightarrow \neg K\exists p\alpha(p)$$

An instance of this schema is:

$$\forall p\neg\Diamond K(p \wedge \neg Kp) \rightarrow \neg K\exists p(p \wedge \neg Kp)$$

The antecedent is just (Fitch). So:

($\neg$K unknown) $\neg K\exists p(p \wedge \neg Kp)$

i.e., it is never known that there is a true proposition that is unknown. This might already be puzzling for some people who think that science provides reasons to be sure that there are truths never known at any time, such as, for example, the proposition stating the number of planets in a very far galaxy.

However, a supporter of an epistemic notion of truth, or an optimist classical epistemologist, could try to argue against such reasons and for the correctness of ($\neg K$ unknown). We need not take into account the kind of arguments he could provide. Let us just wonder whether he can assume as conjecture that $\exists p(p \wedge \neg Kp)$ is false; that is, from a classical viewpoint:

(O) $\forall p(p \rightarrow Kp)$

Such an assumption would commit him to

$$K\forall p(p \rightarrow Kp)$$

because an instance of (O) is

$$\forall p(p \rightarrow Kp) \rightarrow K\forall p(p \rightarrow Kp)$$

However, it is very hard to see how it might be possible that there is a time when we know that all truths, including those not already taken into account, will be known, if they are not known already. Motivations for doubting that such possibility is open might be so strong to lead us to conclude that $\exists p(p \wedge \neg Kp)$ is true.

May these considerations indicate that we avail of a proof of $\exists p(p \wedge \neg Kp)$? If so, we can assert $K\exists p(p \wedge \neg Kp)$ against (¬K unknown).

The consequence of (◊K∃-Dist):

(¬K unknown) $\neg K\exists p(p \wedge \neg Kp)$

is problematic even when the intuitionistic point of view is adopted. Its proper intuitionistic meaning is the following: it cannot be proved, at any time, that a proposition is true, i.e. provable, and such that it is absurd that it is at some time proved. That is intuitionistically plausible. However, intuitionistic negation appears too strong when applied to an empirical operator or predicate, since it cannot be used to say that the operator or the predicate does not apply without implying that its application is absurd in a way that cannot be affected by any future empirical information.

Given the strong meaning of intuitionistic negation, if $\neg K\exists p(p \wedge \neg Kp)$ is intuitionistically justified, then also $K\neg\exists p(p \wedge \neg Kp)$ seems to be justified. The reason is that, from an intuitionistic viewpoint, to rightly assert that it is absurd that a certain proposition q is proved at some time we should possess a proof that it is impossible that q. Let us remark that having a proof that

> It is absurd that [q is true, i.e. provable, and it is absurd that it is at some time proved that q]

does not amount to having a proof that

> It is absurd that [q is true, i.e. provable, and, as mere matter of fact, it is at no time proved that q]

The latter statement denies that provability of q and no actual knowledge of q at any time are compatible, but this possibility is admitted from the point of view according to which intuitionistic proofs can be potentially available but not necessarily known at some time.

So, on the one hand, it appears to be impossible to assert, in an intuitionistic framework, that a proposition is always unknown as a mere matter of fact. On the other hand, the existence of such a heavy expressive limitation might be contested, since it is not fully clear what is expressed by sentences involving empirical notions from an intuitionistic viewpoint. Thus, for the sake of the argument, let us suppose that $\exists p(p \wedge \neg Kp)$ can be understood as stating the existence of a provable proposition never in fact proved. If we assume that it should be always open if some provable propositions not yet acknowledged are never proved, we might assert

(¬K no unknown) $\neg K \neg \exists p(p \wedge \neg Kp)$

However,

(weak O) $\neg \exists p(p \wedge \neg Kp)$

is intuitionistically equivalent to

$$\forall p(\neg Kp \rightarrow \neg p)$$

which, together with (¬K unknown) and (¬K no unknown) implies a contradiction (Percival (1990)). Thus

(¬weak O) $\neg\neg \exists p(p \wedge \neg Kp)$

would be assertable as a consequence of (¬K unknown) and (¬K no unknown).

Then, since (weak O) is an intuitionistically valid consequence of (KP), we can conclude that, as far as (¬weak O) is justified, the denial of (KP) is also justified. But it seems that the denial of (KP) cannot be asserted from any intuitionistic point of view. Even if proofs are conceived as constructions potentially accessible, it cannot be taken as false that they can be actually recognised.

In the end, can we say that we have an argument leading to the intuitionistically unacceptable denial of (KP), independent of the implausibility of the classical consequence (O) of (KP)? Indeed, there are at least three possible conclusions. The argument might be taken as a refutation of the assumption that $\exists p(p \wedge \neg Kp)$ can be understood as stating the existence of a provable proposition never in fact proved, or, more radically, as a refutation of the possibility of embedding the intuitionistic notions of truth and knowledge in Fitch's framework (see Cozzo (1994), Martino and Usberti (1994)), or even as a refutation of the conception of proofs as constructions which can be only potentially accessible. In what follows we shall propose a more general and basic analysis, which, however, is not incompatible with any of the possible reactions just listed.

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Let us sum up, starting from the easiest observations:

1.

($K\vee$-Dist) $\forall p K(p \vee q) \rightarrow \forall p(Kp \vee Kq)$

(Known Ex Middle) $\forall p K(p \vee \neg p)$

are jointly equivalent to

(O) $\forall p(p \rightarrow Kp)$

2.

(KP) $\forall p(p \rightarrow \Diamond Kp)$

implies

($\Diamond K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (\Diamond Kp \vee \Diamond Kq))$

(Known Ex Middle) $\forall p K(p \vee \neg p)$

3.

($\Diamond K\vee$-Dist) $\forall p \forall q(K(p \vee q) \rightarrow (\Diamond Kp \vee \Diamond Kq))$

(Known Ex Middle) $\forall p K(p \vee \neg p)$

imply

(KP*) $\forall p(\Box p \rightarrow \Diamond Kp)$.

Point 3 might suggest that (KP) should be restricted to non-contingent propositions. Such a restriction does not allow (KP) to be applied to the assumption "$p \wedge \neg Kp$", which is contingent because Kp is contingent; so the derivation of (O) is blocked. Could this restriction be taken as a solution to the paradox? Some philosophers, such as N. Tennant (1997) and M. Dummett (2001), tried to avoid the derivation of (O) by limiting the generality of (KP), so that (KP) does not hold for propositions of the form $p \wedge \neg Kp$. Even if some of these proposals have an intrinsic interest, they, in the end, are more or less ad hoc restrictions (see Hand and Kvanvig (1999), Brogaard and Salerno (2002), Rosenkranz (2004), Douven (2005)) and, for this reason, should not be accepted as a solution to the paradox. Something similar can be said of other kinds of restrictions, such as those proposed by D. Edgington (1985) and others. Thus our answer to the above question is that restricting the validity of (KP) to non-contingent propositions should not be taken as a satisfactory solution to the paradox.

Besides the need to avoid ad hocness, there are other, more general, reasons to look for another diagnosis. The derivation of (O) from (KP) does not exploit the empirical interpretation attributed to "K". The empirical interpretation of "K" makes (O) look paradoxical, but paradoxicality cannot depend on just such an interpretation. Neither does only (KP) have puzzling or unacceptable consequences. In fact, we showed that:

4.

($\Diamond K\exists$-Dist) $K\exists p\alpha(p) \rightarrow \exists p\Diamond K\alpha(p)$

implies

($\neg$K unknown) $\neg K\exists p(p \wedge \neg Kp)$

And ($\neg K$ unknown) is not easily acceptable too, since it seems to conflict with ($\neg K$ no unknown), that is with the rejection of an intuitionistically weaker form of (O) (classically equivalent to (O)). Thus, interestingly enough, ($\Diamond K\exists$-Dist), which is apparently weaker than (KP), is enough to generate puzzling, somewhat paradoxical, results.

6

However, we do not think that the above arguments should be taken as refutations of anything — in particular, not as refutations of ($\Diamond K\exists$-Dist), and so, a fortiori, of (KP) — but just as symptoms of some very deep difficulties affecting the conceptual framework in which these elementary pieces of reasoning are produced. Surely the empirical meaning of "K" has a role, but it also seems that a major role is played by some general features of the notion of knowledge represented by "K", and of the way of taking propositions and reasoning on them.

Let us first observe that when K is applied to complex propositions in passages such as those

from $\forall p(p \rightarrow \Diamond Kp)$
to $p \wedge \neg Kp \rightarrow \Diamond K(p \wedge \neg Kp)$

or

from $\forall p(p \rightarrow Kp)$
to $\forall p(p \rightarrow Kp) \rightarrow K\forall p(p \rightarrow Kp)$

an implicit use is made of an impredicative comprehension principle for propositions, since $p \wedge \neg Kp$, for any given value of the variable p, and $\forall p(p \rightarrow Kp)$ can be taken as instances of the variable p only if such formulas are assumed to express values of p, i.e. propositions. Moreover, if we adopt the point of view of *Principia Mathematica*, “K” stands for a propositional function whose range of significance turns out to involve propositions expressed by formulas where “K” occurs. It appears that some entities belonging to the range of significance of K presuppose K, so that the principle of vicious circle is violated. Such a violation is quite explicitly described by Russell and Whitehead:

> a function is not a well-defined function unless all its values are already well-defined. It follows from this that no function can have among its values anything which presupposes the function, for if it had, we could not regard the objects ambiguously denoted by the function as definite until the function was definite, while conversely [...] the function cannot be definite until its values are definite. This is a particular case, but perhaps the most fundamental case, of the vicious-circle principle. (p. 39)

The violation of the vicious circle principle is mostly clear if propositions are conceived as constituted by what the significant parts of the sentences by which they are expressed stand for. It might be objected that there is no reason to adopt such a Russellian notion of proposition in an analysis of the paradox of knowability. Indeed, the reasons brought up by Paseau (2008) and Linsky (2009) do not strictly depend on the adoption of the outlined Russellian notion of proposition. However, we need a notion of proposition which has a similar degree of intensionality, for (O) can appear paradoxical only if propositions are intensionally understood. If all true propositions were identified, then nothing paradoxical would affect the claim that all true propositions are known since a true proposition is known (apart from sceptical objections). Neither necessarily equivalent propositions can be identified if their knowledge is taken into account in a non idealised way, for the truth-value of a knowledge attribution Kp (as usually understood) strongly depends on the propositional structure of p and the identity of p’s constituents. This implies that we need a highly intensional notion of proposition which should look very similar to the Russellian one as concerns the degree of fine-grainedness.

It should be clear that if a Russellian notion of proposition, or a similar one, is adopted, both the involvement of the notion of knowledge and the involvement of quantification on all propositions in the proposition whose knowledge is attributed allow the generation of possible vicious circles, and it does not seem relevant that knowledge, understood as the real cognitive activity, is represented by means of an operator or by a predicate.

These considerations appear to be enough to motivate an account for the paradoxicality of the obtained outcomes in accordance with a suggestion by Alonzo Church. In his referee’s report, referring to the former title of Fitch’s paper “A definition of value”, Church says:

> Of course the foregoing refutation of Fitch’s definition of value is strongly suggestive of the paradox of the liar and other epistemological paradoxes.

and goes on to hint at the possibility of applying “the standard devices for avoiding the epistemological paradoxes”.

Church’s suggestion was followed by Linsky (2009), who provided a uniform account of various paradoxical arguments (Fitch, Hintikka, Fitch-B, Preface) by using the idea of logical types of propositions. A Russellian analysis of the paradox of knowability is further developed in Giaretta (2009). The idea of logical types of propositions can be found in Russell (1908) and Whitehead and Russell (1910), is formulated in a precise technical version in Church (1976), is applied in an intensional context in Church (1984), and this application is discussed in Anderson (1989).

A Russellian distinction of logical types of propositions requires the introduction of different knowledge predicates (or operators) for propositions of different levels of complexity, so that the proper knowledge predicate to be applied to a proposition expressed by a sentence involving a knowledge predicate of level n or quantification over all propositions of level n should have at least level $n+1$.

In particular, Linsky’s introduction of type distinctions in the proof of (Fitch) leads to:

1) $K^{(2)}(p^1 \wedge \neg K^{(1)}p^1)$	assumption
2) $K^{(2)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1$	from (1) by the suitable typed instance of Dist
3) $K^{(2)}p^1 \wedge \neg K^{(1)}p^1$	from (2) by the suitable typed instance of Fact
4) $\neg K^{(2)}(p^1 \wedge \neg K^{(1)}p^1)$	?

where it turns out that step 3 does not involve any formal contradiction, since $K^{(2)}p^1$ does not imply $K^{(1)}p^1$, and so $\neg K^{(1)}p^1$ does not imply $\neg K^{(2)}p^1$ either. So step (4) cannot be deduced by reductio. Moreover, other passages from the above arguments are not legitimate anymore, when types are introduced:

from $\forall p^1(p^1 \rightarrow \Diamond K^{(1)}p^1)$
to $p^1 \wedge \neg K^{(1)}p^1 \rightarrow \Diamond K^{(1)}(p^1 \wedge \neg K^{(1)}p^1)$

from $\forall p^1(p^1 \rightarrow K^{(1)}p^1)$
to $\forall p^1(p^1 \rightarrow K^{(1)}p^1) \rightarrow K^{(1)}\forall p^1(p^1 \rightarrow K^{(1)}p^1)$

It is clear that a Russellian analysis and therapy can be applied also to the puzzling consequences of ($\Diamond K\exists$-Dist). The analysis appears to be motivated independently of (KP)'s acceptability. Moreover the therapy has a principled motivation: the effect of blocking the paradox is the consequence of a conceptual wide scope analysis based on the principle of vicious circle, which has also other applications. From this point of view such approach provides a solution quite different from Dummett's restriction of (KP) to basic, non logically complex, statements and also from Tennant's restriction of (KP) to Cartesian statements, i.e. to statements whose knowledge (at some time) is not provably inconsistent. We do no claim that these different solutions are not motivated, but only that it is less clear that they are motivated in principled way.

If we are right, then the paradox of knowability on the one hand and, on the other, the classical antinomies (liar paradox, Russell paradox, and so on) are but examples of the same kind of difficulty. Alternatives to the broadly Russellian analysis and therapy we sketched above should not give up this insight.

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