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THE PRIOR FUTURE

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Abstract
The paper presents an indexical semantics, based on times and worlds, for some of Prior's logics concerned with issues such as foreknowledge and determinism. In that sense Prior's logic of futurity can be presented as a completely standard modal/tense logic.

In both *Time and Modality* and *Past, Present and Future*, (Prior 1957, 1967) Prior concerns himself with what he calls *indeterminist time*. His response to this concern is to investigate logics in which indeterminist time can be formalised. My aim in this paper is to present a standard and classical indexical semantics for such logics, based on times and worlds. I shall then examine the connection between this indexical semantics, and the formal properties of Prior's logics, concentrating particularly on whether or not there is an incompatibility between the indexical view, and what Prior says about his logics and their connection with time and determinism. The paper will be principally devoted to chapter 7 of Prior 1967 (pp. 113–136). Much in that chapter is a more careful presentation and revision of chapters 9 and 10 of Prior 1957 (pp. 84–93, and 94–103.) In particular, the concern with a three valued logic, whereby indeterminate propositions have a 'neuter' truth value, which occupies much of chapter 9 and some of chapter 10 of the earlier book, has relatively little place in Prior 1967.[1]

The topic is made a little more difficult because of Prior's distrust of metalogic, and this is one feature of my discussion that I will try to be sensitive

[1] The three-valued logic that Prior 1957 principally has in mind seems to be the one described in a letter from Jan Łukasiewicz, which Prior includes on p. 53f of 1957. On p. 135 of Prior 1967, Prior describes it as "a little vexing that no one has yet been able to formalize satisfactorily the ancient and medieval view that predictions of future contingencies are 'neither true nor false'." What Prior notices is that many-valued logics quite often don't give good results for operators like conjunction, where the conjunction of two neuter propositions may be false rather than neuter. Prior seems to have become progressively more aware of the semantic problems posed by non-standard truth values. (An intermediate stage is Prior 1962.)

to. For simplicity I will only consider a propositional modal/tense logic with a denumerable set of propositional variables; p, q, r, ... etc., the material implication operator $\supset$, the standard false proposition (the 'falsum') $\perp$, the modal operator L and the tense operators P and F. I assume the following standard definitions of the other truth-functional operators:

$$\sim\alpha =_{\text{df}} \alpha \supset \perp$$
$$\alpha \vee \beta =_{\text{df}} \sim\alpha \supset \beta$$
$$\alpha \wedge \beta =_{\text{df}} \sim(\alpha \supset \sim\beta)$$

Intuitively, $P\alpha$ means that it was once true that α, and $F\alpha$ means that it will sometime in the future be true that α.[2] L is a modal operator, and $L\alpha$ means that it *must* be true that α. Dual to P is H, which may be defined as $\sim P\sim$ so that $H\alpha$ means that it has always been that α. G may be defined as $\sim F\sim$ and $G\alpha$ means that it is always going to be that α. The possibility operator M can be defined as $\sim L\sim$. (In all these cases the dual could be taken as primitive and the other defined in terms of it and negation.) It is well known that necessity and possibility have many senses. For each sense of L there is a corresponding M and *vice versa*. The particular sense of possibility and necessity that concerns this paper is what is sometimes called inevitability. The idea is that, while the past and present are fixed, the future is open. This sense of possibility is time-dependent — that is to say, whether something is or is not possible can change with the passage of time. Call this sense of necessity *temporal necessity*.

The model theory for such a language specifies that each wff is true or false in a possible world at a moment of time. It may be made precise as follows. A *frame* is an ordered quadruple $\langle$W,T,$<$,R$\rangle$, where W is a non-empty set of 'possible worlds', T is a set of 'times', $<$ is a strict linear ordering of T, and R is a family of dyadic equivalence relations between worlds, indexed by members of T. Intuitively $w\mathrm{R}_t w'$ means that a world w' is possible from the point of view of how things are in a world w at a time t. The semantics assumes that a linear temporal ordering is fixed across worlds. Probably this is not a plausible assumption, but it may be one Prior accepts. In any case, the assumption does not I think affect the points I am trying to make here. In order to convey the fact that the passage of time is the reduction of possibilities, R satisfies the following condition:

[2] Prior's tense-logical language is more expressive than the language introduced here. It is a language of what he calls 'metric tense logic' (Prior, 1967, chapter 6, pp. 95–112) in which you have wff like Pnp which means that p was true n units ago. I follow footnote 5 of Thomason 1970 in ignoring this complexity.

(1) If $t_1 < t_2$ then for every w and w', if $w\mathrm{R}_{t_2}w'$ then $w\mathrm{R}_{t_1}w'$.[3]

A *model* is an ordered quintuple ⟨W,T,<,R,V⟩, where ⟨W,T,<,R⟩ is a frame and V is a value-assignment satisfying the following conditions for any $w \in$ W and $t \in$ T:

[Vp] For any propositional variable, p, either $\mathrm{V}(p, w, t) = 1$ or $\mathrm{V}(p, w, t) = 0$.

[V⊥] $\mathrm{V}(\perp, w, t) = 0$.

[V⊃] For any wff α and β, $\mathrm{V}(\alpha \supset \beta, w, t) = 1$ if either $\mathrm{V}(\alpha, w, t) = 0$ or $\mathrm{V}(\beta, w, t) = 1$; otherwise $\mathrm{V}(\alpha \supset \beta, w, t) = 0$.

Using the definitions above we have that $\mathrm{V}({\sim}\alpha, w, t) = 1$ iff $\mathrm{V}(\alpha, w, t) = 0$, $\mathrm{V}(\alpha \vee \beta, w, t) = 1$ iff either $\mathrm{V}(\alpha, w, t) = 1$ or $\mathrm{V}(\beta, w, t) = 1$, and $\mathrm{V}(\alpha \wedge \beta, w, t) = 1$ iff both $\mathrm{V}(\alpha, w, t) = 1$ and $\mathrm{V}(\beta, w, t) = 1$.

[VP] For any wff α and for any $w \in$ W and $t \in$ T, $\mathrm{V}(P\alpha, w, t) = 1$ if there is some $t' \in$ T, such that $t' < t$ and $\mathrm{V}(\alpha, w, t') = 1$; otherwise $\mathrm{V}(P\alpha, w, t) = 0$.

[VF] For any wff α and for any $w \in$ W and $t \in$ T, $\mathrm{V}(F\alpha, w, t) = 1$ if there is some $t' \in$ T such that $t < t'$ and $\mathrm{V}(\alpha, w, t') = 1$; otherwise $\mathrm{V}(F\alpha, w, t) = 0$.

[VL] For any wff α and for any $w \in$ W and $t \in$ T, $\mathrm{V}(L\alpha, w, t) = 1$ if for every $w' \in$ W such that $w\mathrm{R}_t w'$, $\mathrm{V}(\alpha, w', t) = 1$; otherwise $\mathrm{V}(L\alpha, w, t) = 0$.

In this paper I make a distinction between a *proposition* and a wff in the following way. For every model ⟨W,T,<,R,V⟩ and every sentence α the proposition expressed by α according to this model, may be defined as the set of pairs $\langle w, t\rangle$ such that $\mathrm{V}(\alpha, w, t) = 1$. Propositions are thus the language-independent entities which may be thought of as the meanings of wff, and a proposition p is *true* at t in w iff $\langle w, t\rangle \in p$. (I use p, q, ... etc. sometimes

[3] To get a closer link between the passage of time and the reduction of possibilities, one might think of adding to (1) the additional requirement that there is some w and w' such that $w\mathrm{R}_{t_1}w'$ but not $w\mathrm{R}_{t_2}w'$. Indeed, one could think of *defining* the passage of time in such a way that $t_1 < t_2$ is defined to hold iff these two conditions are satisfied. To get a linear time scale common to all worlds you would need to constrain R so that for any t_1 and $t_2 \in$ T, either, for every w and $w' \in$ W, if $w\mathrm{R}_{t_2}w'$ then $w\mathrm{R}_{t_1}w'$, or, for every w and $w' \in$ W, if $w\mathrm{R}_{t_1}w'$ then $w\mathrm{R}_{t_2}w'$. Another plausible condition is one which *identifies* a world with a set of possibilities over time: If $w_1\mathrm{R}_t w_2$ for every $t \in$ T, then $w_1 = w_2$. (If these conditions are imposed, the frames used in the discussion of (4) below may need to be more complex.)

for propositions, and sometimes for propositional variables. Obviously the difference must be kept clear.) I suspect that Prior would not have been sympathetic to this way of looking at the matter, and part of my aim is to see whether the *logics* he uses are compatible with this kind of 'Platonism'.[4]

I shall refer to all this as the *indexical semantics*.[5] A wff is *valid* in the indexical semantics iff it is true at all worlds and times in all models on all frames. Among the principles valid in the indexical semantics are the S5-like principles

(2) $Lp \supset p$

and

(3) $\sim Lp \supset L\sim Lp.$

(These are Prior's L1 and L3, 1967, p. 125. His L2 is the standard K axiom $L(p \supset q) \supset (Lp \supset Lq)$.) (3) claims that if a given proposition is not determinately true then this fact, that it is not determinate, is itself a determinate fact. (3) might seem at variance with the interpretation of L in terms of time-dependent inevitability, but we need to recall that, according to the indexical semantics, L is interpreted at a time t in a world w, in terms of what is happening *at* t in w. We can think of R_t in two ways. One is that, from the point of view of w_1 at t, w_2 is a way the world could become. The other is that the worlds accessible from w at t are the worlds which, in some intuitive sense, are just like w up to and including t, and differ only in what happens after t. Why should these two ways be equivalent? Suppose that w_1 and w_2 differ before or at t — that an event e say occurs in w_1 but not in w_2. From the point of view of worlds like w_1 up to t, nothing you do after t could change the fact that e occurs in all of them. So nothing that you do in any of them after t can make w_2 a way the world could be. And this applies

[4] In chapter 3 of Prior 1971 ('Platonism and quantification', pp. 31–47) Platonism is described on p. 32 as a programme "which eliminates parts of speech by multiplying entities", and the burden of the chapter seems to be to express a preference for parts of speech. As far as metalogic goes one might look at the reservations expressed in Prior 1967, pp. 41–45, and the somewhat dismissive comments, in Prior 2003, p. 211, about understanding a relational frame in terms of a more-expensive-than relation between teacups.

[5] Although the modal/tense language investigated here contains as primitive tense and modal operators only P, F and L, others can easily be added. Thus for instance a logical necessity operator $\Box$ can be introduced so that $\Box\alpha$ is true at t in w iff α is true at t in every w' in w, whether or not w' is accessible from w. And if the frame were to include a 'nearness' relation, a dyadic counterfactual operator $\Box\!\!\rightarrow$ could be interpreted in the manner of Lewis 1973 and others. Operators to do with choice and agency in such frameworks, are discussed in Belnap 1996.

to *any* difference between w_1 and w_2 before or at t. So not $w_1\mathrm{R}_t w_2$. Conversely, since R_t represents a quasi-logical possibility, *any* world like w_1 up to t counts as possible in this sense.[6] Other notions of temporal possibility arise by restricting this one.

Prior's worry about the future is based on the claim that while the past (and perhaps the present) is necessary, in the sense of being unchangeable, yet the future is not. Certainly there does seem a sense in which this is so, but, as Prior goes on to shew (Prior 1967, p. 117) this sense may be difficult to pin down. Prior considers the wff

(4) $Pp \supset LPp.$

(4) is not valid according to the indexical semantics. Consider a frame with two worlds w_1 and w_2, and two times t_1 and t_2 with $t_1 < t_2$. R_t is universal in W for all $t \in \mathrm{T}$. Put $\mathrm{V}(p, w_1, t_1) = 1$ and $\mathrm{V}(p, w_2, t_1) = 0$. Then $\mathrm{V}(Pp, w_1, t_2) = 1$. But $\mathrm{V}(Pp, w_2, t_2) = 0$; and so $\mathrm{V}(LPp, w_1, t_2) = 0$. So $\mathrm{V}((4), w_1, t_2) = 0$. To make (4) valid one must require that $\mathrm{V}(p, w_1, t_1) = \mathrm{V}(p, w_2, t_1)$, or, more generally:

(5) for all w and $w' \in \mathrm{W}$, all $t \in \mathrm{T}$ and every variable p, if $w\mathrm{R}_t w'$ then $\mathrm{V}(p, w, t) = \mathrm{V}(p, w', t)$.

The idea behind (5) is this. $w\mathrm{R}_t w'$ is supposed to entail that w and w' are alike up to t. So that if p is a proposition only about what is the case at t, its truth at t should not depend on what happens after t, and so should not depend on R_t. The problem with a restriction like (5) is that it is not a property which holds for complex wff even if imposed on the variables. For consider a frame with three times $t_1 < t_2 < t_3$, where R_{t_1} and R_{t_2} are universal in W, but $w\mathrm{R}_{t_3} w'$ iff $w = w'$. Let $\mathrm{V}(p, w_1, t_3) = 1$ and $\mathrm{V}(p, w, t) = 0$ for every other w and t. Since neither $w_1\mathrm{R}_{t_3} w_2$ nor $w_2\mathrm{R}_{t_3} w_1$ we have satisfied (5) for p. But $\mathrm{V}(PFp, w_1, t_2) = 1$, while $\mathrm{V}(PFp, w_2, t_2) = 0$; and so $\mathrm{V}(LPFp, w_1, t_2) = 0$. As Prior notes on p. 122f, the problem appears in the context of a logical system in the rule of uniform substitution for propositional variables. Prior suggests using two classes of variable, where only one kind of variable satisfies (4). Alternatively, he suggests that (4) be retained for all variables, but that the axioms which are to hold for all wff be stated as

[6] Lewis 1976 has argued that there is a sense in which you can no more change the future than you can change the past. Nothing you do in a world w can make it the case that w is different from the way it is. But to say that you *can* do something in w is to say that you *do* do it in a w' open to you, and, in the present situation, $w\mathrm{R}_t w'$ marks off the w's which are the worlds open to you in w at t. All these worlds coincide with w up to t and the different possibilities open to you in w at t are represented by these different worlds. The asymmetry between past and future is built into R via condition (1).

schemata, so that the logic does not contain any rule of uniform substitution for variables. Then, while (4) remains a theorem its schematic version

(6) $P\alpha \supset LP\alpha$

is not valid for all α, and, in particular, the instance of (6),

(7) $PFp \supset LPFp$

is not a valid wff. Prior even suggests, in the case of variables, adding

(8) $p \supset Lp$

and indeed (8) is valid under the restriction (5). Obviously the schematic version of (8)

(9) $\alpha \supset L\alpha$

should not be not valid for all α, since, as Prior notes (1967, p. 123), this makes L vacuous. The point of view of the present paper is semantic, and while the indexical semantics (with or without (5)) may indeed specify an axiomatisable class of wff, this paper is not in the business of studying it. Indeed (9) may be considered to *define* what it is for α to be solely about the past and present, and there is no reason to suppose that there is any syntactical way of marking out this class of sentences. At any rate (5) is not part of the indexical semantics of this paper. Prior, I suspect, may have supposed that his logical languages reflect, in some way, metaphysical truth. The idea would be that all truths are made up of simple truths, which are truths about a particular time.[7]

Prior offers two ways of understanding the future. One is what he calls the *Ockhamist* way, and one is what he calls the *Peircean* way. Prior 1967, p. 126, produces a semantics for Ockhamist tense logic. Although he does not use the word, Prior's 'frames' (which I shall call 'Priorian frames') are

[7] The desire to discover the logically perfect language, whose atomic sentences 'picture' the atomic facts, is a view which drove much influential metaphysics of the early 20th century; the *locus classicus* perhaps being the *Tractatus Logico-Philosophicus* (Wittgenstein 1921).

branching structures in which any point can have more than one possible future.[8] The simplest non-trivial Priorian frame is

$$\begin{array}{ccc} & & y \\ & \nearrow & \\ x & & \\ & \searrow & \\ & & z \end{array}$$

in which the points y and z are in the future of x, but neither is in the future of the other. Prior does not postulate instants here, although he has briefly introduced dates on p. 120. (He was always bothered about the metaphysical status of instants; see p. 188f.) Prior then introduces what he calls *prima facie* assignments. A prima facie assignment gives a truth value to each propositional variable at each point in a Priorian frame. Clause (1) of Prior's truth definition, (10) in this paper, says

(10) Each propositional variable is arbitrarily assigned a single truth-value at each point.

In (10) prima facie assignments are playing the role of possible worlds. At the time Prior was writing, many logicians supposed that using assignments instead of worlds avoided unwelcome ontological commitments, and some such motivation may be in play here. But there are risks. The principal risk is connected with the view of necessity as truth in all assignments, since without restrictions this can validate unwanted formulae. Prior's clause (4) on p. 126, here (11), states

(11) The assignment to $L\alpha$ at x gives it truth if α is given truth in all its prima-facie assignments at x, otherwise falsehood.

Unrestricted, (11) has the consequence that $\sim Lp$ is valid, since at any element in any frame there is a prima facie assignment at which p is false there, and so $\sim Lp$ is always true there. The standard answer here, which we must take Prior to be assuming, is that the model contains a set of designated prima facie assignments. Once this answer has been given, the way is open to letting the frame contain a set of entities which can index these assignments, and which we can call 'possible worlds' — though, from the

[8] If other senses of possibility are obtained by restricting R_t-possibility, it makes plausible the supposition that many senses of 'counterfactual' possibility might involve going back to a time at which something not now possible *was* possible. Locutions like English's 'might have been' then become very natural.

point of view of logic, the indices can be anything at all. We can then have a single model which gives a value to every wff with respect to each of these indices, and whatever other indices the frame may specify. A second reason for disquiet concerns the fact that for many wff there is no unique truth value at an element in a Priorian frame. Prior's clause for F, here (12), states

(12) The prima-facie assignment to $Fn\alpha$ at a given point x for a given route to the right of x, gives it the value assigned to α at the distance n along that route from x. (If the line branches within this distance, there may be different prima-facie assignments to $Fn\alpha$ at x.)

Consider the Priorian frame mentioned above, with the following assignment to the variables:

$$\begin{array}{cccc} & & y & p \\ & \nearrow & & 1 \\ x & & & \\ & \searrow & & \\ & & z & p \\ & & & 0 \end{array}$$

(The value of p at x is arbitrary.) It should be clear that there is only one assignment of values here; and it makes LFp false at x. Now consider the truth value at x of Fp in this model. Prior is correct that this is a relative matter, but the relativity is not relativity to an *assignment*; rather the relativity seems to be to what Thomason 1970 p. 267, in his study of Prior's Ockhamist logic, calls a 'history'.[9] Relative to the path xy, Fp is true at x, while relative to the path xz, Fp is false at x. This is in line with the way Prior tells it on p. 127, though it does not mention assignments. The relativity does not apply to the propositional variables, since Prior assumes them to have a truth value at each element which is unaffected by the truth values to wff at other elements. But if prima facie assignments are in effect possible worlds, it seems best to follow Thomason's section 7 (*op cit*, pp. 276–279) and define the truth of a wff at a time and a history. Like Prior, Thomason imposes the requirement that variables have truth values at each point absolutely, which

[9] Belnap 1996, p. 241, claims to distinguish between a world, and a possible history in the actual world. I am not sure what to make of this distinction, and I am treating the 'worlds' in the indexical semantics as corresponding to Belnap's and Thomason's 'histories', except that they are taken as primitive and not analysed as linear strings of 'times'.

is of course (5) above; but, given a definition of truth which is relative to a history, there is no need to impose (5) on the variables.[10]

Unlike the structures used by Prior and Thomason, the frames of the present paper are not branching structures, but it is easy to generate such structures in the indexical semantics. An element in a Priorian frame can be understood in an indexical frame $\langle$W,T,<,R$\rangle$ as an equivalence class of worlds. Where $w \in$ W, let $|w|_t = \{w' : w'\mathrm{R}_t w\}$. Then define a relation $\prec$ as follows:

(13) $|w|_{t_1} \prec |w'|_{t_2} =_{\mathrm{df}} (|w'|_{t_2} \subseteq |w|_{t_1} \wedge |w'|_{t_2} \neq |w|_{t_1})$

With definition (13) it is clear that $\prec$ is transitive and asymmetrical. To ensure that the structure is a tree we need to have that if two elements x and y are both in the past of some element z then either $x = y$ or $x \prec y$ or $y \prec x$. So suppose that $|w_1|_{t_1} \prec |w_3|_{t_3}$ and $|w_2|_{t_2} \prec |w_3|_{t_3}$. Now $w_3 \in |w_3|_{t_3}$ and so $w_3 \in |w_1|_{t_1}$ and $w_3 \in |w_2|_{t_2}$, and so $|w_1|_{t_1} = |w_3|_{t_1}$ and $|w_2|_{t_2} = |w_3|_{t_2}$. But either $t_1 = t_2$ or $t_1 < t_2$ or $t_2 < t_1$. So either $|w_3|_{t_1} = |w_3|_{t_2}$, or, $|w_3|_{t_1} \neq |w_3|_{t_2}$ and, by (1), $|w_3|_{t_1} \subseteq |w_3|_{t_2}$ or $|w_3|_{t_2} \subseteq |w_3|_{t_1}$. One can also make an indexical frame out of a Priorian frame, with the worlds being chains of elements.[11]

What then is the connection with determinism? Chapter 7 of Prior 1967 begins by raising the theological question of God's omniscience. Following Jonathan Edwards, Thomas Aquinas and others, Prior seems to have felt that if God knows the future, then the future must be determined, and therefore that a God who is omniscient can only be so in a deterministic universe. Why should this be so? Presumably it is this: that a deterministic world is a world in which there is no distinction between truth and temporal necessity. That is, a world w is deterministic iff for every t, if $w\mathrm{R}_t w'$ then $w = w'$. For such a w (9) is true for every $t \in$ T. (This of course makes determinism a contingent matter.) One account of knowledge requires that one knows a

[10] Thomason invokes van Fraassen's notion of a 'supervaluation'. A supervaluation is a family of valuations, which, in a rough and ready way, corresponds to the use of worlds in the present paper. 'Rough and ready' because the 'supertruth' of a supervaluation plays no role in the semantic values of the component expressions in the final sentences, and is introduced principally to define various notions of logical consequence between sentences. Thomason distinguishes on p. 273 between the validity of the implication $\alpha \supset \beta$, and the validity of the passage from α to β. The difference arises because the latter case is defined to hold where β is true for every way the future could develop provided α is. Thus, although $\alpha \supset L\alpha$ is not valid, the passage from α to $L\alpha$ *is*. In terms of the indexical semantics, the difference is between the validity of $\alpha \supset \beta$ and the validity of $L\alpha \supset L\beta$, and the legitimacy of the move from α to $L\alpha$ is reflected by the validity of $L\alpha \supset LL\alpha$.

[11] A formal proof of the equivalence between the indexical semantics and the Prior/Thomason semantics would require another paper; and, for such a result, the indexical frames might need to satisfy some additional conditions like those mentioned in footnote 3.

proposition when one believes it *because it is true*. On this account, the truth of p is what *causes* one to know it; and for this reason it might seem that p must be temporally necessary. That is to say, for all w, t and α, if $K_a\alpha$ means that a knows that α

(14) If $\mathrm{V}(K_a\alpha, w, t) = 1$ then $\mathrm{V}(L\alpha, w, t) = 1$.

Prior 1962 (see p. 49 of Prior 2003) distinguishes between knowing and guessing in a manner which suggests that he thinks one can only *know* what is already determined. Say that a being a is *omniscient* at t in a world w iff a knows every proposition which is true at t in w. Thus for every α, we would have

(15) $\mathrm{V}(K_a\alpha, w, t) = \mathrm{V}(\alpha, w, t)$.

Putting (14) and (15) together, we have that if any being is omniscient in w at t then

(16) If $\mathrm{V}(\alpha, w, t) = 1$ then $\mathrm{V}(L\alpha, w, t) = 1$

and if a being is omniscient in w at every t then w is a deterministic world. The causal account of knowledge supposes that God knows something *because it is true*, and perhaps Prior saw an inconsistency here between that and the supposition that God is responsible for whatever is the case. (See Prior 1976, p. 108.) On p. 114 of 1967, Prior notes Edwards' insistence that he is *not* saying that God's foreknowledge *causes* things to happen. None of this however goes any way to supposing that we need give up the indexical semantics introduced above, whereby every proposition has a definite truth value at every time in every world.

The semantics offered in the Ockhamist account is classical about truth, and Prior seems to sense this. Whether or not it troubled him, he goes on to present a more radical response to the problem of foreknowledge, which gets closer to his conviction that the reason why God cannot know the future may be because there is *nothing to know*. The problem with the Ockhamist solution from Prior's point of view might well arise from the fact that it *can* be given a classical construal, and that, if it can, then it is not a sufficiently accurate articulation of the view that, in some important sense, the future doesn't exist. In the indexical semantics, provided that there is no 'end-of-time' moment, i.e., provided that for every t there is a t' such that $t < t'$, then the following wff is valid:

(17) $\sim Fp \supset F\sim p$.

For what Prior calls the 'Peircean' answer (1967, p. 128), (17) is not valid. If p is a proposition about the future then it may not now exist, and so Fp might fail and $\sim Fp$ be true. But if p does not exist neither does $\sim p$, and so $F\sim p$ is not true either, and (17) would be false. What should be said about this? Prior (*op cit* p. 130) attributes to Michael Shorter the claim that, for the Peircean, F really means LF, so that (17) is really

(18) $\sim LFp \supset LF\sim p.$

In contrast to (17), (18) does indeed fail in the indexical semantics. In the frame presented in the discussion of (4) we need only make $\mathrm{V}(p, w_1, t_2) = 1$ and $\mathrm{V}(p, w_2, t_2) = 0$ to have (18) fail at t_1 in either world. This seems to mean that the Peircean is simply an Ockhamist with a restricted language. That may have been what Shorter was suggesting, but matters like this can be tricky. One way of interpreting intuitionistic logic is by means of Kripke frames, according to which the intuitionist's truth is the classicist's necessary truth, and intuitionistic logic emerges as a sub-system of classical S4 modal logic with restrictions on expressibility. An intuitionist will object to this construal, and Prior seems to have similar reservations about the Ockhamist understanding of the Peircean response. Just as the intuitionistic propositional calculus can be interpreted directly in a Kripke frame without translation into S4, so, in Prior's semantics on p. 132 for Peircean logic, the clause for F reads:

(19) The actual assignment to $Fn\alpha$ at x gives it truth if all its prima-facie assignments do; otherwise falsehood.

In the indexical semantics the corresponding assignment would be

(20) For any wff α and for any $w \in \mathrm{W}$ and $t \in \mathrm{T}$, $\mathrm{V}(F\alpha, w, t) = 1$ if for every w' such that $w\mathrm{R}_t w'$ there is some $t' \in \mathrm{T}$ such that $t < t'$ and $\mathrm{V}(\alpha, w', t') = 1$; otherwise $\mathrm{V}(F\alpha, w, t) = 0$.

How then might the Peircean explain the possibility of true predictions, and how might an Ockhamist understand the explanation? Prior (1967, p. 131) offers the following example:

(21) It was the case an hour ago that you were saying 'Eclipse will win', and now he is winning.

Prior's aim in discussing (21) is to argue that, despite what the Ockhamist might think, the Peircean can make good sense of what it is for a prediction to become true — even if when uttered it was not (definitely) going to be true.[12] For the Ockhamist there are two claims about the future that a predictor could make. A predictor who simply says

(22) Eclipse will win

is making a statement of the form $F\alpha$, which may or may not be true depending on what the future holds. A predictor who says

(23) It is now definite that Eclipse will win

is making a statement of the form $LF\alpha$, and (23) might still have been false even if Eclipse turns out to win. The Peircean, it seems, cannot allow a predictor to distinguish between (22) and (23). The most that (21) can do is allow us to assert, in an hour's time, a connection between a previous prediction, and what is now happening. In a posthumously published paper (Prior 1976) Prior returns to the problem of prediction. He notes on p. 100 that he would be aggrieved if he bet someone that Phar Lap would win, and his companion declined to pay up, on the ground that when the prediction was made it was *not* unpreventably true, even though Phar Lap *did* turn out to win.

The aim is to shew how a Peircean can understand an *Ockhamist's* future tense operator, which I will refer to as f.[13] Prior points out on p. 103 that the Peircean can *mention* the Ockhamist's operator, but cannot use it. His response is ingenious. In the first place it is couched in terms of 'assertions' or 'statements', and what he seems to have in mind is a semantics of *utterances*. Unlike sentences, utterances are temporally located. u takes place at a time t_u, and once t_u is past u no longer takes place. Suppose then that someone produces an utterance u of a sentence $f\alpha$, as it might be

(24) Phar Lap will win in two minutes.

Prior notices that, even if it was not definite that Phar Lap would win, what *is* definite is the connection between the truth of the prediction, and Phar Lap's winning. Prior's discussion, on pp. 103–108 of 1976, proceeds mostly by

[12] Prior's earlier version of his solution, on p. 94 of Prior 1957, attributes it to Ryle 1954. (The example of Eclipse's winning is in fact Ryle's, 1954 p. 19f.)

[13] In Prior 1976, Prior no longer uses the terms 'Ockhamist' or 'Peircean', but prefers 'small-letter language' and 'big-letter language'. I have retained 'Ockhamist' and 'Peircean', even in describing this later work, in order to conform with the earlier usage. Prior (p. 103) uses 'will' for the Ockhamist's future tense and WILL for the strong Peircean sense of 'definitely will', or 'unpreventably will'. So I have used a lower case version of F, since, in this Peircean logic, F is the *Peircean* future operator.

way of the Phar Lap example, which involves a fixed future time; but what he says is capable of generalisation. For a start, we have that, if α is any wff, and u is an utterance of $f\alpha$, then, at t_u, the following is true:

(25) $G(\alpha \supset u \text{ is true})$

where the Peircean G is the Ockhamist's LG, and cannot be defined as $\sim F \sim$. (25) should be straightforward. Since u is an utterance it takes place at a time, t_u, and if, at any time later than t_u, α should turn out to be true, we may say that u is a true prediction. (25) has a partial converse. Recall that, for the Peircean, u only becomes *true* if what it says becomes definitely so, and this means that, at t_u, we have

(26) $G(u \text{ is true } \supset F\alpha)$.

This solution comes at a cost:

> Basically what must be abandoned, if we are to use betting language correctly yet indeterministically, is the assumption that the present truth of something must consist in its accordance with something in the non-linguistic world around one at the time the assertion is made. (1976, p. 101)

One form that the cost will take is that both (25) and (26) appear to involve a language which contains its own truth predicate, and so precautions will need to be taken to avoid paradox. On the Ockhamist account, a metalinguistic treatment of the future is unnecessary, and one might feel that the Peircean price is too high. Prior, however, may have felt it worth paying.

I have tried in this paper to make the case that Prior's concern with issues such as foreknowledge and determinism can all be addressed in a classical indexical semantics. In that sense the logic of futurity is a completely standard modal/tense logic, whose semantics proceeds by giving a recursive indexical truth definition. Prior's work on tense logic is of course among the precursors of this development, and it is because he tried to articulate his views on time in a formal way that we have a precise semantic framework in which to address it — even if it is not perhaps quite a framework of which he would have approved.

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