



PRIOR’S TENSE-LOGICAL UNIVERSALISM

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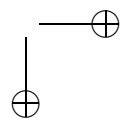
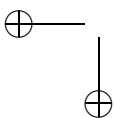
Abstract

Prior’s project of tense logic has both a formal-logical and a philosophical side. Both aspects were important for Prior. The paper suggests viewing Prior’s philosophical project as a continuation of the tradition of “logic as language”, or “universalism”, identified by van Heijenoort and Hintikka, respectively. The label “tense-logical universalism” is chosen in order to stress Prior’s emphasis on the foundational role of natural language for the three fields of logic, semantics, and philosophy of science. The paper draws on Prior’s works in all three mentioned areas. In particular, we comment on Prior’s universalist approach to model theory, which has recently given rise to hybrid logic, and on the interesting perspective on relativity theory offered through his writings.

1. *Introduction*

Prior conceived of himself as both a philosopher and a logician, and he held that “logic and general philosophy have more to bring to one another than is sometimes supposed” (Prior, 1957b, vii). Accordingly, his long-term project of tense logic has both a philosophical and a logical side. Much of that project took shape during Prior’s visit to Oxford, as the John Locke lecturer, in the academic year of 1955/56, with the subsequent publication of his seminal book, *Time and Modality*, from which the above quote is taken.

Prior’s project gained momentum during the thirteen years that he was able to continue his work afterwards. He pushed forward the technical development of tense logic while at the same time keeping a firm eye on the philosophical implications that he took his work to have. The main fields in which he believed such implications would reveal themselves were philosophy of logic and language and philosophy of science: Prior fought the rising Quinean orthodoxy of a modality-unfriendly understanding of logic and natural language semantics, and he was deeply concerned about the impact that relativity theory might have on his project.



After Prior's premature death, tense logic was developed further to provide for a great number of technically refined formal systems. Some of these systems have become everyday tools in computer science, linguistics, and philosophy of action. Prior's philosophical programme seems to have fared less well. I take this to be a contingent historical fact not to be explained by systematic shortcomings of that project, but rather by the fact that Prior's views on the foundational role of natural language do not go together well with the orthodoxy of the 1970s, nor of today. With respect to the distinction established by van Heijenoort (1967) and later generalised by Hintikka (1988), I will argue that Prior belongs to the universalist camp of "logic as language" rather than the by now orthodox model-theoretic point of view linked to "logic as calculus". Despite his few references to Frege, Prior is more of a Fregean than many would think. I will argue further that Prior's *logical* universalism is just one aspect of his threefold *tense-logical universalism*, the two other aspects being a *semantical* and a *scientific* universalism. In each of the three fields, Prior holds and defends strong views about the foundational character of our natural language: logic, semantics, and science are enterprises operating *within*, not from without our natural language.

Setting Prior in this light provides, or so I will claim, both an explanation of why some of his views have seemed so unorthodox and an incentive to pursue some aspects of his philosophical project further. While Prior's own technical contributions to logic have long been canonized and improved upon, Prior the philosopher can still be a source of valuable inspiration.

The paper is structured as follows: I will first lay out van Heijenoort's distinction between the views of "logic as calculus" and "logic as language" and show how both views can be generalised (section 2). I will then argue for my main claim that Prior should be viewed as a logical, semantical and scientific universalist in the Fregean tradition, drawing on Prior's work on the philosophy of logic, the semantics of natural languages, and philosophy of science (section 3). Finally, in section 4, I will show how Prior's tense-logical universalism leads to an interesting perspective on relativity theory, thus making good my second claim: 50 years after *Time and Modality*, there are still new insights to be gained from Prior's project of tense logic.¹

¹ Further points both about the historical embedding of Prior's project and about the interrelation of tense logic and relativity theory are argued for in detail in my German book, *Arthur Priors Zeitlogik* (Müller, 2002).

2. *Logic as calculus and logic as language*

2.1. *Historical stage-setting: Frege against the Booleans*

Formal logic, embodying the idea that logic could somehow benefit from the practices of mathematics, was started in the mid-19th century. Early on, two differing approaches were present. Boole wanted to treat logic as a branch of mathematics, not philosophy (cf. Boole, 1847, 13), describing his goal as follows: "I purpose to establish the Calculus of Logic, and [...] I claim for it a place among the acknowledged forms of mathematical analysis" (Boole, 1847, 3). Frege (1879), on the other hand, favoured a different understanding of the interrelation between mathematics and logic. According to Frege, logic was concerned with the most general laws of truth, and thus, was also a prerequisite for a clear understanding of mathematics. Thus the various branches of mathematics could at best provide hints for the proper development of logic, but it was out of the question to treat logic itself as a branch of mathematics.² Thus the subtitle of Frege's *Begriffsschrift* speaks of a formal language *modelled after* the language of arithmetics, not of the subsumption of logic under mathematics like Boole.

The widespread acceptance of the Boolean position, also in Germany (especially due to Ernst Schröder), was one of the main reasons why Frege's groundbreaking work was almost completely ignored until much later.³ With the conflict between Frege and the Boolean tradition, the two different conceptions of "logic as calculus" and "logic as language", as van Heijenoort (1967) later called them, were first present on the historical scene. Quite a number of debates in the history of logic can be fruitfully analysed in these or related terms. One of the key features of many of these debates is that both sides appear to be talking past each other, often failing to take the arguments of the other side seriously. This is, e.g., the case for the next debate in which Frege was involved, the one with Hilbert about the foundations of geometry.

Hilbert's *Grundlagen der Geometrie* (1899) marks a turning point towards the axiomatic practice of implicit definitions that is characteristic of much of modern mathematics. Hilbert's project can be coherently reconstructed in terms of the conception of "logic as calculus": Uninterpreted axioms are laid

²Rather, according to Frege's logicist thesis, mathematics should be viewed as a branch of logic. However, Frege's — ultimately unsuccessful — attempts at establishing logicism are independent from his views on the nature of logic.

³The single more visible reaction to Frege's publication, a review by Schröder (1880), was quite negative: from a Boolean point of view, Frege's work appeared to be too complicated and ultimately superfluous. Frege tried, unsuccessfully, to point to the misunderstandings involved. For further historical discussion, cf. Sluga (1987).

down, and questions of consistency and independence can be tackled by giving various interpretations for the terms occurring in the axioms. Hilbert's own exposition is however not as clear as one might wish: He starts by claiming to analyse spatial intuition, but then conducts formal constructions using the real numbers. Frege initiated an exchange of letters in which he was quick to point out that Hilbert's use of the word "axiom" appeared shaky: On the one hand, Hilbert wanted the axioms to have geometrical content; on the other hand, the content he gave them was clearly not geometrical. Frege's critique led to a number of changes in Hilbert's terminology, but no full consensus was reached. Despite voices that try to put the blame for this on Frege's alleged incompetence,⁴ it appears to be more appropriate to view the Frege-Hilbert controversy as a fundamental clash between the "logic as language" conception of natural language as the starting point of all formal investigations, and the conception of "logic as calculus", which presupposes a reinterpretable or even an uninterpreted language to start with.⁵

2.2. Generalising the distinction

Van Heijenoort himself took the distinction between "logic as calculus" and "logic as language" to fade away after the first decades of the 20th century (cf. van Heijenoort, 1967, 328). However, subsequently it has been noted that van Heijenoort's classification can be generalised and then applied fruitfully to many later debates in the philosophy of logic and in other fields of philosophy. For this broader conception it is useful to separate two aspects of Frege's conception of logic. One is that contrary to some of his contemporaries, Frege takes a formal language to be a clarified fragment of our *interpreted* natural language,⁶ not an uninterpreted formal calculus. The second aspect of Frege's conception of logic is that our language is the basis of all argumentation, and thus there cannot really be a metalanguage point of view. This has two related consequences. One is that there remains little motivation for building up a formal metatheory, the other is that model-theoretic semantics cannot do any real foundational work. All of the mentioned aspects of Frege's view show up in Prior's writings, sometimes blended together as in Frege, sometimes more neatly separated. We will make this explicit below in commenting on Prior's attitude.

⁴ Cf., e.g., Hintikka (1988, 7).

⁵ For a similar assessment, and for more details about the Frege-Hilbert-debate, cf., e.g., Demopoulos (1994); Blanchette (1996); Rusnock (1995). Cf. also Hodges (1986, 141).

⁶ Drawing a parallel to the similar views of Peano's, Hodges (1986, 143) calls formal languages that are conceived of in this way, "Frege-Peano-languages".

Van Heijenoort's terminology is perhaps too suggestive of the divide between assuming an interpreted vs. an uninterpreted logical language. That once was an important issue, but it appears fair to say that the formalist doctrine of an uninterpreted formal language has little appeal today. A different terminology would therefore be useful. Hintikka (1988, 1) has suggested an alternative terminology for a generalisation of van Heijenoort's distinction: he speaks of the divide between "universalism" (language as the universal medium; logic as language) vs. the "model-theoretic conception" (logic as calculus).⁷ I will mostly follow Hintikka's suggestion with respect to the first camp, and I will alternate between the two names for the second camp. — The general picture is as follows:

2.2.1. *Logic as language (the universalist conception)*

According to that view, logic is primarily a tool for making finer distinctions *within* our natural language than is normally possible. Frege brings out this point when he compares logic with a microscope: Using a microscope presupposes using our eyes, and a microscope is not appropriate for all investigations. However, when it is appropriate, it can lead to astonishing insights (cf. Frege, 1879, XI). Logic is an extension of our native language, not something alien to it. When we conduct logic, we do not cease to speak our language, we just refine it. This means that the expressions used in logic already have a meaning. Of course, some clarifications are possible, but only within an hermeneutic process that starts from the way we talk and use the expressions in questions (cf. Frege, 1979, 207). There is no metalanguage point of view that could be used for foundational purposes.

2.2.2. *Logic as calculus (the model-theoretic conception)*

The conception of logic as calculus compares a logical language with an abstract structure such as those, e.g., studied in algebra. A formal language by itself does not have a meaning — or if it does, that meaning is not fixed, but can be changed in a global way. Before a formal logical language is applied, there is a step of interpretation assigning meaning to expressions. Asking for the meaning of an expression prior to interpretation is, according to this conception, like asking what is "the" neutral element of group theory: there is the neutral element of group G_1 , the neutral element of G_2 , and so forth, but it does not make sense to ask about "the" neutral element.⁸ A

⁷ Cf. also Hintikka and Hintikka (1986, Chap. 1).

⁸ Hodges (1986) draws a useful parallel to indexical expressions: Asking what "the" neutral element of group theory was outside the context of considering a specific group would be like asking where, of all places, "here" *really* was.

metalanguage point of view is available, and from this point of view, various languages can be studied formally.

2.2.3. *The crucial difference*

The real bone of contention between the two conceptions of logic is whether one can somehow "escape" natural language in describing a formal logical language. Proponents of "logic as calculus" answer in the affirmative, pointing to the practice of mathematics, which supplies meaning for formal languages from a point of view that apparently lies outside of natural language. Universalists, however, deny this, pointing out that *even describing* the practice of mathematics is itself an activity presupposing our natural language.

As far as ongoing debates are concerned, the distinction perhaps comes out best in different analyses of indexical expressions. From a universalist point of view, indexicals are an irreducible feature of our language; there appears to be little hope of providing more than, perhaps, translations among indexical expressions to elucidate the way they work. From a model-theoretic point of view, the situation is wholly different: given a metalanguage point of view, one can talk about "truth at an index" and thus give truth conditions for sentences containing indexicals. The issue is how much is gained by this move. There can be no doubt that there is a technical gain, but there may be a metaphysical loss since one has to posit a set of indexes (times, worlds) with perhaps unclear ontological status. Prior's position in this respect is especially interesting, since he apparently manages to get the best of both worlds (cf. section 3.1.3 below).

In the context of this paper, my aim is not to argue for or against any of the two positions that I have sketched. However, it is important to note that a failure to take seriously an author's reliance on the conception of "logic as language" does not only endanger that author's discussions with his contemporaries, but also the later historical appraisal of debates. As the conception of "logic as calculus", embodied in model-theoretic semantics for formal languages à la Tarski, has become orthodoxy, many writers have a-historically treated historical writings as presupposing that understanding of logic when in fact the authors were committed to the view of "logic as language". Frege has often been the victim of such misunderstanding, not just as concerns his debate with Hilbert.⁹

⁹Cf., e.g., Sluga (1980, 180f.) and Blanchette (1996) for a criticism of Dummett's reading of Frege.

In connection with Prior's project of tense logic it is crucial, both for an adequate historical assessment and for the prospects of deriving philosophical inspiration from his work, to view Prior as belonging to the universalist camp of "logic as language". In the following section I will substantiate this claim.

3. *Prior on logic, language, and science*

Prior is certainly not much of a Fregean. It is true that Prior's work does contain a number of references to Frege, including explicit praise for Frege's method of analysis (Prior, 1971, 52). Still, it appears fair to say that in most places, "Frege" is used merely as a label for a theory that takes sentences to refer to truth-values (e.g., Prior, 1957b, 71 and 1971, 50), and some of Prior's knowledge of Frege may have come from secondary sources (e.g., through Church). It would appear doubtful to name Frege as a major source of inspiration for Prior's views about logic and semantics. Despite this, it is striking how close Prior and Frege are with respect to a number of foundational issues: both subscribe to, and defend, the universalist conception of "logic as language".

Many of the basic intuitions and convictions that may lead one to that conception are shared by philosophers who would shy away from logic altogether: e.g., a sense for the richness of ordinary language and the many informal ways of argumentation that we are capable of. Prior certainly believed in the richness of ordinary discourse, but he also believed that that should not keep one from trying to formalise as much as possible; both aspects were important.¹⁰ These two aspects are however not easy to reconcile. Accordingly, Prior was often under attack from two sides. Slater remarks: "On the one hand Prior had to fight the natural language philosophers, who disliked his regimentation and rigour; but on the other hand he had to fight the pure formalists, who disliked the metaphysical import Prior took his work to have" (Slater, 1994, 2).

Despite all challenges, Prior held firmly on to his view of "logic as language". Prior's commitment to that view will now be made explicit by considering his work in logic (section 3.1), natural language semantics (section 3.2), and the philosophy of science (section 3.3).

¹⁰Cf., e.g., Prior's introduction to his "thank goodness" argument, where he says: "I'm a symbol-man rather than an ordinary-speech man myself, but I can see what the ordinary-speech men are worried about [...]" (Prior, 1976, 84).

3.1. *Logic*

As sketched above, Frege's logical universalism means that he sees formal logic as fundamentally of a piece with natural language. Prior holds the same view. This comes out very clearly through at least three different claims that Prior defends. The first two mostly pertain to the question of whether logic is a purely formal enterprise or an investigation of our interpreted language. The last issue is more directly about the model-theoretic viewpoint.

3.1.1. *What is logic?*

In his article "What is logic?", published posthumously, Prior considers various ways of delineating the field of logic.¹¹ First he considers the view that "logically true statements are ones which owe their truth entirely to the way in which we have chosen to speak" (Prior, 1976, 123). That view is closely tied to a formalist approach to logic, according to which logic is formal symbol manipulation operating with symbols that initially do not have a meaning. Prior calls that view "a muddle" (ibid.) and points out that meanings just won't stick to formal symbols arbitrarily. In substantiating his assessment, he reiterates a point of Frege's about the primacy of natural language: one can of course sharpen the meaning of certain words, such as "If" — but only if "there *is* a way of using the word 'If'" (124). Natural language comes first, and it is the basis for any agreement about the use of words. Clarification of meaning is an hermeneutic process operating within natural language, not an assignment of meaning "from nowhere". Prior thus clearly denies the possibility of a metalinguistic point of view that would allow an arbitrary assignment of meaning.

In his own conception, Prior distinguishes "a strict and a loose sense" of the term "logic" (128): In the strict sense, logic is only about "implication and universality; in a looser sense, it concerns itself with principles of inference generally, in all sorts of fields" (ibid.). In the end, it may be more or less fruitful to study "the logic" of some field — "some subjects do in fact have more order, more structure, more form, than others [...] and in these cases it is more proper than in others to speak of a 'logic' of the thing" (129). Tense logic is a worthwhile enterprise because the system of tenses exhibits a high degree of structure; a "logic of biology", on the other hand, seems to be a more doubtful project (129). Ultimately, one has to "try it out and see what happens" (ibid.). Just like natural language itself, Prior views logic as an historically developing human endeavour. This is the natural approach for

¹¹ In the article, Prior expands on material already contained in his logic textbook (Prior, 1962, 215–220).

the universalist, but it can hardly be made sense of by someone subscribing to "logic as calculus".

3.1.2. *Against formal rules: Tonk*

A debate in the early 1960s brings out further material that shows Prior's commitment to "logic as language" and interpreted formal languages. His article "The runabout inference-ticket" (Prior, 1976, 85–87), refined and defended in "Conjunction and contonktion revisited" (ibid., 159–164), gives a decisive, though hotly debated argument against a formalistic understanding of the logical operators.

The historical setting of Prior's argument is as follows:¹² With respect to operators like "and" and "or", Kneale (1956, 257) held that "[t]he special rules [...] determine the sense of the signs completely by fixing their roles in argument". Thus, once one knows how to employ the operators in inferences syntactically, one knows all there is to know about their semantics. Prior gives a precise definition of "and" according to this conception and then puts forth his challenge: In a completely parallel fashion, he is able to define an operator "tonk" as follows:

Its meaning is completely given by the rules that (i) from any statement P we can infer any statement formed by joining P to any statement Q by 'tonk' (which compound statement we hereafter describe as 'the statement P-tonk-Q'), and that (ii) from any 'contonktive' statement P-tonk-Q we can infer the contained statement Q. (Prior, 1976, 86)

The presence of that operator will of course trivialise any formal system, allowing one to infer Q from P in just two steps; Prior's example is to derive " $2 + 2 = 5$ " (Q) from " $2 + 2 = 4$ " (P) via (i) followed by (ii).

From the point of view of Prior's universalism, the diagnosis of this fatality is quite simple. Of course somebody can employ a definition like the one given above, in a legitimate case like "and", as a means of "putting people on the track of the meaning of a word" — Prior acknowledges that this means is often "the best we have" (Prior, 1976, 160). However, such definitions can only put us on the track of something if that something is already there, i.e., if our natural language already contains a word (such as "and") answering

¹²While Kneale is the primary target of "The runabout inference-ticket", Prior already developed the nucleus of his argument in his review of Tarski's collected writings (Prior, 1957a, 405f.), there directed against Popper's idea of a "logic without assumptions" (Popper, 1947).

to the definition. If there is no such word (as in the case of "tonk"), "we are being led up a tree" (Prior, 1976, 163). The attempt at *introducing* a significant sign by formal definitions is doomed: for the universalist, the signs already have a meaning; if they don't, no formal definition can supply one.

Prior draws a sharp distinction between a "symbolic game", in which one can lay down any rules one pleases, and significant language, in which sentences already have a meaning. The purported definition of "tonk" is perfectly legitimate for a symbolic game,¹³ but it is illegitimate for *logic*, which is after all concerned with meaningful sentences:

In a symbolic game, [...] we can just *decree* that a couple of wffs [well-formed formulae] with a sign between them is another wff, and lay down what rules we please for this newcomer; and after that there are no more questions to ask. But no one can *make* an expression consisting of two significant sentences joined by 'and' or 'or' or 'tonk' into a significant sentence just by (a) saying that it is one, and (b) giving us 'permission' to utter certain other sentences before or after it and insert a few 'therefores'. And if a man who does this says, further, that if we go through these permitted motions we shall never get into trouble (in particular, that we shall never pass from truth to falsehood), we have the right, and would be wise, [...] to try and check up on it. Sometimes checking up on it with the associated sentences whose truth-values we know (because we already understand them) will be enough to make it plain that we are being led up a tree. (This is the case with 'tonk'). (Prior, 1976, 163)

Logic is concerned with inference; it is not a symbolic game. Thus, meaningfulness is a prerequisite for logic: "only what already has a meaning can be inferred from anything, or have anything inferred from it" (Prior, 1976, 159).

Accordingly, Prior holds that the use of "inference" by subscribers to a formalistic understanding of logic rests on equivocation and wishful thinking:

'Inference' in the sense of a permitted transformation of wffs in a purely symbolic game neither presupposes meaning nor gives it. For here the permitted moves are framed entirely in terms of the design of the symbols, independently of their interpretation. [...] But to

¹³ Even though, as Prior notes, "such a game would be rather less interesting to play than noughts-and-crosses", allowing one to infer anything from anything (Prior, 1976, 160).

believe that anything of this sort can take us beyond the symbols to their meaning, is to believe in magic. (Prior, 1976, 159f.)

The lesson that Prior would like to draw is clear: Logic operates within the already meaningful realm of ordinary discourse, just formalising what is already there. This is exactly Frege's idea of "logic as language". By providing his "tonk" argument, Prior poses a challenge to the alternative view of "logic as calculus": subscribers to that idea must find a means of ruling out the addition of "tonk" to their formal language. Prior suggests that this cannot be done independently of (universalist) considerations of meaning. As far as I can see, this argument of Prior's in favour of universalism seems to stand unrefuted.¹⁴

3.1.3. *Model-theoretic semantics from a universalist perspective*

Perhaps the most impressive sign of Prior's commitment to logical universalism is his reaction to, and reinterpretation of, the methods of model-theoretic semantics for temporal and modal logics. In this line of argumentation, Prior does not just oppose the now perhaps obsolete assumption of uninterpreted formal languages, but takes a stance that pertains to the current discussion about the fundamentals of temporal and modal semantics.

Prior was among the first discoverers of the so-called Kripke semantics (relational semantics) for temporal and modal logics.¹⁵ He thus knew about the power of these methods. The idea of relational semantics is to interpret modal or temporal operators as quantifiers over so-called possible worlds in a relational structure called a frame. Thus, "necessarily ϕ " is true at a world w if and only if ϕ is true at all worlds related to w , and "possibly ϕ " is similarly interpreted by means of existential quantification. There are well-known correspondences between properties of the relation in a frame and modal formulae valid in models based on that frame; e.g., reflexive frames support the axiom schema "if necessarily ϕ , then ϕ ".

From the model-theoretic point of view, relational semantics is just what semantics should be like. Initially, there is a meaningless modal calculus containing, apart from classical logic, two new operators. Through a step of interpretation that is not part of the formal system, these operators, along

¹⁴ Early praise for "tonk" came, e.g., from Belnap (1962), who opposed his views but still called Prior's paper "a gem"; this assessment is reinforced by Sundholm (1986, 486). Cf. also Martin-Löf (1987).

¹⁵ Cf. Goldblatt (2005) and Copeland (2002) for historical details. Cf. also Copeland (1996, 10–15).

with everything else, are assigned a meaning via the inductive clauses of relational semantics. From the same point of view that one takes on for the step of interpretation, one can then also ask metasystematic questions about the relation between the syntactic side of the formal language (with its proof theory) and the semantical side (with its notion of validity and logical consequence). That point of view is also a natural starting place for investigating the syntactic/semantic correspondence between axioms and properties of frames pointed to above. It all falls into place quite naturally.

As a logical universalist, Prior cannot make sense of the point of view from which we purportedly assign a meaning to our logical language. For him, modal logic is just a refined way of normal talk. A coherent universalist picture would thus require one to choose between either discarding the model-theoretic results and techniques, or finding a means of making sense of these results and techniques in terms of "logic as language". Given Prior's insight into the power of model theoretic semantics, simply discarding the technique is out of the question for him.¹⁶ Thus he has to bite the bullet. And this is exactly what he does — in fact, one of his most far-reaching technical projects, which occupied him during much of his last years, is precisely the project of making sense of model-theoretic techniques from a universalist point of view. That work, most of which was published posthumously as the book *Worlds, Times and Selves*, co-authored by Kit Fine (Prior and Fine, 1977), was all but neglected for many years. It has however recently been rediscovered under the name of "hybrid logic", an approach to modal logic that has found many fruitful applications, e.g., in computer science.¹⁷

While hybrid logic can be, and has been, employed by universalists and model-theoreticians alike, Prior's own view of that enterprise is distinctly universalist. He starts from his assumption of intensional nominalism (cf. Prior, 1976, 190): It is important to take modalities logically seriously (intensionalism), but possible worlds are not respectable entities (nominalism). Thus, it must be possible to replace model-theoretic talk of times and possible worlds and relations between them by metaphysically more respectable talk that takes the tenses or modalities themselves to be primary: "Tense-logic is for me, if I may use the phrase, *metaphysically fundamental* [...]" (Prior and Fine, 1977, 37; Prior's emphasis). Apart from this metaphysical argument, there is also a (universalist) semantical claim that favours Prior's tense-logical reconstruction of model-theoretic semantics: "We understand 'truth in states of affairs' because we understand 'necessarily'; not

¹⁶ Prior acknowledges that "in our technical work, [...] we always turn to 'U-calculi' in which the terminology is decidedly more abstract [...]" (Prior, 1967, 75) — 'U-calculus' is Prior's name for the theory of relational structures.

¹⁷ Cf. Blackburn (2000) for an introduction to the subject and to its applications.

vice versa" (Prior and Fine, 1977, 54). The parallel to Frege is striking: Just as Frege held that mathematics cannot be the basis of logic since mathematics presupposes logical thinking, so Prior holds that model-theoretic semantics cannot be the basis of modalities since the modalities are primary for our understanding of natural language.

The technical results supporting Prior's reduction of relational semantics to tense logic are somewhat involved and will not be given in any detail here. It must be said that Prior did not achieve all the technical results that would have been required to see his project through. Important first steps are taken in Chapter V and Appendix B.3 of *Past, Present and Future* (Prior, 1967); later material is contained in Prior and Fine (1977). A first full reconstruction of Prior's project, showing that it is in fact feasible, is given by Øhrstrøm (1997); cf. also Øhrstrøm and Hasle (1995, Chap. 2.9).

3.1.4. *The absence of metasystematic questions*

A further sign of Prior's universalism is the almost total absence of metasystematic questions from his work. One case in point is his (rather limited) presentation of completeness results in his detailed book on *Formal Logic* — quite unlike the material given to students nowadays. At one place where he points to general metatheoretic work by Tarski, he even puts the word "interpretation" in scare quotes (Prior, 1962, 234n.). In a similar vein, he plays down any metaphysical implications of models consisting of instants used in formal completeness proofs for tense logic: "for formal completeness proofs we can [...] think of teacups, and for metaphysics we can paraphrase the instants away altogether [...]" (Prior and Fine, 1977, 83). Unlike the mainstream modal logicians of the time, Prior appears to be not much interested in questions of soundness and completeness for modal systems — as far as I can see, the only mainstream completeness proof in Prior's published writings occurs in the appendix to *Worlds, Times and Selves*, which was written not by Prior, but by Fine (Prior and Fine, 1977, 164–168).¹⁸

¹⁸ Again, the parallel with Frege is striking. From a contemporary (i.e., model-theoretic) point of view, it appears odd that almost four decades elapsed between the formal specification of a proof-theoretic system for propositional and predicate logic (Frege, 1879) and the first completeness proof for the propositional part (the result of Bernays (1926) dates from 1918; Post (1921) is the first published proof). Frege did not consider the question. This is certainly not due to the technical challenges posed by the completeness proof; the best explanation seems to be that within the (at that time) mainstream view of "logic as language", metasystematic questions did not arise (cf. Dreben and van Heijenoort (1986) for a similar assessment).

3.1.5. *Lessons from universalist model theory*

There are two lessons to be drawn from Prior's examination of model-theoretic semantics. Firstly, that aspect of his work shows clearly Prior's deep commitment to the view of "logic as language". He invests a great deal of difficult technical and conceptual work into his project of making sense of universalism, all the time holding firmly on to his conviction that a real clarification of temporal or modal logic can only come from within our natural language, not from some abstract mathematics. Secondly, the episode and its historical continuation show that universalism may be difficult to defend, but in the end, the technical difficulties can be mastered. We will draw a similar lesson from Prior's semantical universalism.

3.2. *Language*

Logical universalism is a historically respectable doctrine which, in the hands of Russell, once had been established as the mainstream point of view.¹⁹ In comparison, *semantical* universalism seems to many to be not just less respectable, but even provably wrong.

According to semantical universalism, natural language is the universal medium of inquiry — even when it comes to investigations of the semantics of natural language itself. This view appears to be quite commonsensical — after all, *The Journal of Semantics*, which, among other things, deals with the semantics of the English language, is itself written in English, and it is difficult to imagine which other means for dealing with semantics could be available. However, there is an obvious threat to the doctrine of semantical universalism in the form of the liar paradox: One of the things that Tarski showed in his famous paper on "The concept of truth in formalized languages" (Tarski, 1935) is that natural language appears to be inconsistent because it allows for sentences of the type "This sentence is false".²⁰ Tarski later gave a more thorough analysis of the problem of languages that are able to talk about their own semantics. He claimed that no consistent language can be semantically closed, i.e., no consistent language can contain its own semantics (cf. Tarski, 1944, 672f.). Semantics must therefore

¹⁹ Prior even alludes to Russell's universalist slogan, "Logic is about the real world like zoology, only at a more abstract level" (Russell, 1919, 169), when he claims that "Philosophy, including Logic, is not primarily about language, but about the real world" (Prior in Copeland, 1996, 45).

²⁰ Tarski's actual demonstration is more subtle than that, and there are good reasons for such subtleties. For a lucid account that also does justice to the historical setting, cf. Kühne (2003, Chap. 4).

be a metalinguistical enterprise. While Tarski held that natural language was generally too fuzzy to allow for precise claims, his guess was "that a language whose structure has been exactly specified and which resembles our everyday language as closely as possible would be inconsistent" (Tarski, 1944, 673). A number of loopholes may remain: the structure of an exact variant of natural language could be different from the structure of a formal language envisaged by Tarski by, e.g., allowing for ineliminable ambiguity or a non-uniform truth predicate. Still, Tarski's verdict appears to rule out semantical universalism once and for all.

Prior's universalist enterprise thus appears to be in a losing position. But just as Prior found within universalism the resources to deal with the challenges of model theory, so he also found a way to deal with Tarski's challenge. Again, Prior wasn't quite able to see his work through to completion — the best he wrote on the subject is contained in the draft of his book *Objects of Thought*, which was published only posthumously (Prior, 1971).²¹

Maybe it is best to quote at length from Prior to give his view of the problem, which already points to his solution to be described below. Under the chapter heading "Tarskian and non-Tarskian semantics", Prior describes the difference between his and Tarski's analysis of the semantic notion of truth as follows:

The truth and falsehood with which Tarski is concerned are genuine properties of genuine objects, namely sentences. The truth and falsehood with which we have been concerned here might be described as properties not of sentences but of propositions; but this means that they are only quasi-properties of quasi-objects, and it might be less misleading to say that we have not been concerned with the adjectives 'true' and 'false' at all but rather with the adverbs 'truly' and 'falsely'. The basic form which Tarski defines is 'The sentence *S* is a true one'; the form which we define is not this, but rather '*x* says truly (thinks correctly, fears with justification) that *p*'. And we define this quite simply as [...] '*x* says (thinks, fears) that *p*; and *p*'. [...] From these definitions and ordinary logic we may deduce all such statements as that

- (A) If anyone says that snow is white, then he says so truly if and only if snow is white.

From Tarski's definitions and ordinary logic, we may deduce the

²¹ There are parallels to his paper on Buridan on self-reference (Prior, 1976, 130–146). Cf. also Prior (1957a).

truth of all such statements as

- (B) The sentence 'Snow is white' is true if and only if snow is white,

and indeed for him the deducibility of all such statements is a criterion of satisfactoriness for a definition of truth. There may not seem to be that much difference between (A) and (B), but in fact the differences are considerable.

In the first place, there are quotation-marks in (B) but not in (A). These in fact belong to Tarski's informal exposition rather than his rigorous theory; but it is essential to his theory that in sentences of his type (B) the sentence which is *used* in the second clause should be *mentioned* (by name — however the name be formed) in the first. In (A), on the other hand, the sentence 'Snow is white', which is *used* more than once, is not *mentioned* at all (it nowhere goes into quotation-marks, or is spelt, or given a 'Gödel number', or named or designated in any way). (B) is *about* the sentence 'Snow is white', (A) is from beginning to end not about this but about snow. (Prior, 1971, 98f.)

The main difference is thus that Prior views the adverbial forms "truly" and "falsely" as primary, whereas the Tarskian tradition is interested in the *predicates* "true" and "false". Since Tarski's formal results only pertain to the truth-*predicate*, it may well be possible to give a satisfactory semantical analysis of truth that takes the adverbial form "truly" to be primary and that does not rule out languages containing their own semantics.

Prior's definition of the adverbial use of the notion of truth, which he proclaims to be primary in the above quote, is as follows:

x says truly that p if and only if x says that p , and p . (cf. Prior, 1971, 98)

This definition is to be read generally, i.e., with x and p quantified universally. Here a first problem may be thought to loom: What kind of quantification is involved? Quantification over persons (in the x position) is generally held to be unproblematic, but what about the propositional quantifier binding the variable p ? Is that to be read objectually, leading to unwanted ontological commitment to propositions as particulars, or substitutionally, leading to conflict with our natural language in which there simply aren't enough names for all that there is (cf. Quine, 1969, 95)? In fact that is a false alternative.

From a universalist point of view, the use of propositional quantifiers is as unproblematic as the use of other quantifiers. After all, e.g., the existential quantifier stands for "something", and Prior does "not think that any formal definition of 'something' is either necessary or possible" (Prior, 1971, 35). In the end, any explanation of quantification will have to revert to our means of quantification in natural language.²² Formally, propositional quantification can thus be modelled after our natural language device for propositional quantification, viz., the use of prosentences like "things are so".²³

A definition of the adverbial use of "truly" may be useful, but it appears not to cover all cases of our use of the notion of truth. Sometimes we *do* predicate truth of sentences. Prior concedes that point, and he suggests an extension of his adverbial definition given above. The key ingredient in his definition of truth for sentences, over and above propositional quantification, is the operator "... means that ...", which belongs to a useful class of formal devices which are "predicates at one end and connectives at the other" (Prior, 1971, 19), forming a sentence out of a sentence and a name. Prior gives abundant examples of the employment of such devices in natural language (Prior, 1971, 16ff.), thus justifying their use from his universalist perspective.

With the help of these formal means, Prior suggests to

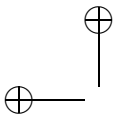
[d]efine '*x* is a true sentence' as '*x* is a sentence, and for all *p*, if *x* means that *p*, then *p*'. (Prior, 1971, 104)²⁴

When compared with Tarski's rather involved definition of truth for sentences in terms of satisfaction (Tarski, 1935), this truth definition appears both concise and easy to understand: rendering the propositional quantifier by prosentences, one can rephrase it as "*x* is a true sentence iff *x* is a sentence, and however *x* says that things are, thus they are". — But isn't

²² Cf. Williams (1995, 152): "*Some* use of "some" must be taken as primitive."

²³ In discussing this issue, Prior mentions Ramsey and points to examples given by Wittgenstein (cf. Prior, 1971, 37ff.). Some aspects of Prior's suggestion have been taken up by proponents of a "prosentential theory of truth" (Grover et al., 1975; Grover, 1992); cf. also Brandom (1994, 301–306). Terminologically, the difference between the prosentential theory of Grover, Camp and Belnap and Prior's own theory is sometimes blurred. Both Brandom (ibid.) and Williams (1995, 150) use the label "prosentential theory" for what is in fact closer to Prior's original proposal.

²⁴ Prior's definition is a close relative of the "modest conception of truth" championed by Künne in his book on *Conceptions of Truth* (Künne, 2003). He claims that "Prior gets things exactly right" (347) — even though his own reconstruction has an existential quantifier in place of Prior's universal one. Cf. Künne (2003, Chap. 6) for a detailed discussion, which also tracks down the original Polish sources of many of the ideas involved.



this a Pyrrhic victory for the universalist? After all, Tarski’s formal result about the inconsistency of semantically closed formal languages threatens: in virtue of the truth definition given above, our language appears to contain its own truth-*predicate* after all.

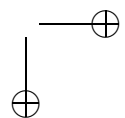
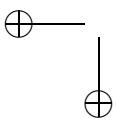
In fact, if one goes through Tarski’s derivation of a liar-type contradiction in detail, one ends up with the following diagnosis:²⁵ For the liar-sentence *s*, there is a *p* such that *s* means that *p*, and not-*p*, and there is a *q* such that *s* means that *q*, and *q*. A bit more colloquially, the liar sentence means that things are one way, which they aren’t, and it means that things are another way, and things are that other way. Is this contradictory? It certainly is if the “means that” operator is functional, i.e., if any sentence can mean only one thing. That assumption is operative in Tarski’s proof of the inconsistency of semantically closed formal languages. However, Tarski does not claim that our natural language *is* a formal language fulfilling this assumption. Rather, as was pointed out above, he remains cautious, claiming only that a formal reconstruction of our natural language would in all likelihood lead to inconsistency.

Our natural language is not a formal language (cf. Prior, 1957a, 408). By denying that sentences in our natural language mean just one thing, the universalist can easily fend off the threat of inconsistency. Prior considers this option carefully, weighing it against another way of restoring consistency by denying the liar sentence a meaning altogether:

In a well-organized language each correctly formed sentence would have precisely one meaning; or even if different inscriptions or utterances of ‘the same sentence’ ([. . .] *type*) might have slightly different meanings [. . .], still at least a particular single inscription or utterance ought to have precisely one meaning. Either, therefore, we must admit that our language is *not* as well organized as this; or we must deduce [. . .] that *nothing* can mean that something that it means is false. (Prior, 1971, 105)

Whatever the ultimate verdict on this question turns out to be: one possibility of saving semantical universalism from the threat of inconsistency would already be enough to show that Prior holds a systematically tenable universalist position with respect to natural language semantics.

²⁵ Cf. Slater (1986), Slater (1994, 49–55), and Müller (2003).



3.3. *Science*

Logical universalism is a respectable doctrine that Prior held on to firmly. Semantical universalism, though apparently less respectable as a doctrine, was defended by Prior, with good arguments. In this section, we will point out a third dimension of Prior's tense-logical universalism, which may be called "scientific universalism": Prior also tried to defend the view that scientific terms depend for their meaning on our natural language practice. In this field the stakes are even higher than in the case of semantical universalism: Firstly, scientific universalism isn't much of a doctrine really, and secondly, Prior's opponent isn't 'just' Tarski, but the whole of modern relativistic physics. Or so it might seem.

It would be inappropriate to call Prior a philosopher of science, but he knew that his project of tense logic would at one point or other have to deal with the challenges that our intuitive conception of time allegedly has to face due to the discoveries of modern physics. In fact, Prior's whole project was attacked as "ill-advised because grounded in bad physics" (Massey, 1969, 31): the project was accused of presupposing a Newtonian conception of time in which the notion of absolute simultaneity made sense. As that assumption had been found to be empirically inadequate by relativistic physics, the project appeared untenable. Prior devoted a lot of effort to countering that challenge; in fact, the last published talk he gave in his life was exactly about that issue.²⁶

One can identify two lines of discussion in Prior's investigation of relativistic physics. The first is formal in character, and amounts to asking which formal properties a temporal or modal-logical system based on the relativistic relation of causal accessibility should have. Prior is able to differentiate logically between the appropriate Diodorean modal systems for general relativity (S4) and for special relativity (Prior, 1967, 204f.).²⁷ His conjecture that the latter should be exactly S4.2 was later vindicated by Goldblatt (1980). Further research along these lines includes Phillips (1998, 2001) and Shapirovsky and Shehtman (2003). This is respectable and respected technical work on the axiomatic foundations of physical theories.

²⁶ Prior gave his talk on "The Notion of the Present" at the Oberwolfach conference of the International Society for the Study of Time, 31 August–6 September 1969 (Prior, 1970). Prior died on 6 October 1969 in Trondheim.

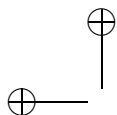
²⁷ The Diodorean definition of "necessarily ϕ " is "it is now and always will be the case that ϕ ".

Prior's second line in the discussion of relativity theory is both more philosophical and more problematic. Philosophically, it amounts to facing Massey's challenge and showing tense logic to be tenable in a relativistic universe. The problem comes about as follows: Common lore propounded by Einstein and others has it that the empirical discoveries of relativistic physics force us to give up many of our commonsensical conceptions about space and time. For example, we intuitively assume that for any two events it makes sense to ask which one occurred first, and that this question must have a definite answer. Even when really short times are involved and we are epistemically not in a position to find out the answer (such as might happen in the finish of a race), we are certain that there must *be* an answer. However, according to relativistic physics, for many pairs of events (*viz.*, for those that are space-like related, meaning they cannot influence each other causally) that questions does *not* have a definite answer, because it is itself underdetermined — it only becomes determinate once we specify the frame of reference with respect to which the temporal order of the two events is to be evaluated. The answer may then differ, depending on the frame of reference.

Prior lucidly expresses the problem in his article "The notion of the present" (Prior, 1970). He considers the case of a distant pulsating body. Suppose we have just observed a pulsation; we know that it happened a long time ago, because light from the distant body takes a long time to reach us. If we consider the pulsation after the one we have just observed and "ask whether this next pulsation, although we won't of course observe it for a while, is in fact going on right now, or is really still to come, or has occurred already", we appear to be in a fix: On the one hand, "[t]he difference between pulsating — really and actually pulsating — and merely having pulsated or being about to pulsate, is as clear and comprehensible a difference as any we can think of, being but one facet of the great gulf that separates the real from the unreal, what is from what is not." On the other hand, "the special theory of relativity appears to deny" exactly this difference, because the required notion of "absolute" simultaneity is in many cases just not to be had" (Prior, 1970, 247f.).

How should we react to this? Is the "clear and comprehensible" ontological difference we know so well, just an illusion? Can we really just change our notions of space and time at will, as the physics community proposes? Or is it possible to save our intuitions?

Prior seems not to have reached a definite conclusion, but he leans towards the universalist position that our natural language conception of time is primary. In his posthumously published article "Some free thinking about time", he strongly proclaims that our intuitive ontological view of time should be preserved:



So far, then, as I have anything that you could call a philosophical creed, its first article is this: I believe in the reality of the distinction between past, present, and future. I believe that what we see as a progress of events *is* a progress of events, a *coming to pass* of one thing after another, and not just a timeless tapestry with everything stuck there for good and all. (Prior in Copeland, 1996, 47).

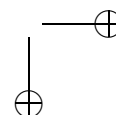
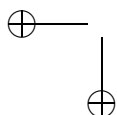
And he continues with the following statement of scientific universalism:

When an event *X* is happening, another event *Y* either *has* happened or *has not* happened — ‘having happened’ is not the kind of property that can attach from one point of view but not from another. On the contrary, it’s something like *existing*; in fact to ask what has happened *is* a way of asking what exists, and you can’t have a thing existing from one point of view but not from another, although of course its existence may be *known* to one person or in one region, without being known to or in another.

So it seems to me that there’s a strong case for just digging our heels in here and saying that, relativity or no relativity, if I say I saw a certain flash before you, and you say you saw it first, one of us is just wrong — or misled it may be, by the effect of speed on his instruments — even if there is just no physical means whatever of deciding which of us it is. To put the same point another way, we may say that the theory of relativity isn’t about *real* space and time, in which the earlier-later relation is defined in terms of pastness, presentness, and futurity; the ‘time’ which enters into the so-called space-time of relativity theory isn’t this, but is just part of an artificial framework which the scientists have constructed to link together observed facts in the simplest way possible, and from which those things which are systematically concealed from us are quite reasonably left out. (Prior in Copeland, 1996, 50f.)²⁸

The argumentation is as follows: *we know* what it means for two things to be simultaneous to each other, long before physicists ever thought of the matter. Indeed their whole physical theorising presupposes our everyday notions,

²⁸ Cf. Prior (1968, 43): “I simply cannot see how the presentness, pastness or futurity of any state of affairs can be in any way relative to the *persons to whom* this state of affairs is known.” Cf. also Prior (1970, 248).



including the notions of time and simultaneity.²⁹ Simultaneity is already a meaningful concept, and it is not open to physicists to change that meaning — they also have to operate within our natural language, and a point from which that language could be treated as an object is just not available.³⁰ A similar point holds for our conception of time: we have a fixed conception of time, and if physicists create a theory about “local time”, they may of course do this; the critical point is that we shouldn’t confuse the two notions.³¹ To draw a parallel to the beginning of section 2: Frege could criticise the Booleans for trying to base logic on mathematics, when in fact mathematics already presupposed logic and logical thinking. Similarly, Prior criticises those (like Quine, 1960, 172) who would urge us to revise our conceptions of space and time, when in fact scientists have to presuppose those very notions in order to formulate their theories.³²

So apart from logical and semantical universalism, it appears that Prior subscribes to scientific universalism as well. But is that a good thing?

4. *A relativistic logic of points of view*

In the previous section it has been argued that Prior’s tense-logical programme amounts to subscribing to a threefold universalism:

- (1) *Logical universalism.* Logic is a refinement of our natural language, not something external to it. The logician, operating with his or her formal calculus, is much like the scientist who operates with refined tools to broaden the range of sense perception. Frege’s image of the microscope neatly summarises this view. Prior’s tense-logical programme leads him to defend logical universalism by giving a universalist account of model theoretic techniques.
- (2) *Semantical universalism.* Investigations into the semantics of natural languages are conducted within the very medium that they are about.

²⁹ Interestingly, Einstein conceded that point already in his celebrated 1905 article on special relativity (Einstein, 1905, 28).

³⁰ Cf. Prior’s remarks about believing in magic, quoted above in connection with “tonk”.

³¹ For Lorentz (1904, 15), “local time” was a metaphor; he puts the term in scare quotes. In Einstein (1905), the scare quotes are gone. Prior puts himself on Lorentz’s side; cf. the scare quotes around “time” in the quotation from Copeland (1996, 51) above.

³² At this point Prior almost seems to lean towards a Kantian position.

This leads to an apparent threat of circularity, but that challenge can be met in a formally perspicuous manner.

- (3) *Scientific universalism*. Science is bound to employ our natural language. A large-scale revision of our concepts, such as the shift in meaning of "time" and "simultaneous" allegedly made necessary by relativity theory, is neither possible nor necessary.

In the case of (1) and (2), our argument to the effect that Prior held these views also amounted to an argument for holding these views themselves to be consistent and of systematic interest. That step is still lacking for (3), and it appears uncertain whether it can be provided. Our aim in this section is to give a short sketch showing that Prior's scientific universalism also marks a systematically tenable position.³³

As the varying tone of the quotes given in the previous section shows, Prior resists changes in our conception of time, but appears not to be quite sure what else one can say. Prior would certainly not downplay scientifically established results. We believe that it is best to read Prior as offering *two* different answers to the challenges of relativity theory. This accords well with his view of the role of the logician as the one to point out the available options:

The logician must be rather like a lawyer [...] in the sense that he is there to give the metaphysician, perhaps even the physicist, the tense-logic that he wants, provided that it be consistent. He must tell his client what the consequences of a given choice will be [...], and what alternatives are open to him; but I doubt whether he can, *qua* logician, do more. (Prior, 1967, 59; cf. also 1968, 134)

The two options that Prior discusses are the following:

- a. Metaphysically, there *is* an absolute notion of simultaneity (a preferred rest frame) — even if epistemically, we can never find out anything about it. Thus, our talk about simultaneity, past, present, and future makes literal sense, even if we may be at a loss to judge about the truth or falsity of such talk in some cases. Formally, we should just stick to tense logic in its original formulation, which is metaphysically fully justified.

³³ Apart from Müller (2002, Chap. 4), the articles Müller (2004, 2006) provide additional details.

- b. Metaphysically, one should remain agnostic as to the existence of a preferred rest frame (perhaps leaving it to scientists to provide hints). Our talk about simultaneity, past, present, and future, when conceived of in the original manner of tense logic, may or may not make literal sense. On the formal side, one should explore options for alternative tense logics that answer the relativistic challenge directly.

Option a. Option (a) amounts to a neo-Lorentzian view that posits a preferred rest frame. Such a view had been thought untenable following Einstein, but recent discussion has shown that strong, implausible verificationist assumptions are needed to argue against it (cf. Smith, 1998, 146). In fact, a formal result by Rakić (1997) shows precisely that from a formal point of view, there is nothing wrong with assuming a preferred rest frame: the axiomatic theory of special relativity plus a referred rest frame can be shown to be a conservative extension of the original theory. Apart from these formal results, there may even be physical reasons for assuming a notion of absolute simultaneity. While arguments based on quantum-mechanical distant correlations (cf., e.g., Popper, 1984, 54) might be viewed with suspicion, a notion of “cosmic time” that would supply the sought-for relation of absolute simultaneity is available in many models of general relativity, and it is clear that special relativity is not the whole truth about our universe.³⁴ Thus, option (a) is clearly a tenable view. Prior gives the following assessment:

One possible reaction to this situation, which to my mind is perfectly respectable though it isn't very fashionable, is to insist that all that physics has shown to be true or likely is that in some cases we can never *know*, we can never *physically find out*, whether something is actually happening or merely has happened or will happen. I'm sure there *are* questions which are perfectly genuine and intelligible questions but which seem to be incapable of being answered. (Prior, 1970, 248)

Prior is certainly right about the unfashionableness, even if in view of the possible physical justifications just mentioned, his overall assessment may be somewhat too pessimistic. But whatever the merits of subscribing to option (a), it seems clear that for the tense-logical project, that option marks a dead end. It either amounts to an “ignorabimus”, or it passes the buck to the

³⁴ Nor, apparently, is general relativity (due to quantum effects) — so that it is unclear how one can accord much metaphysical weight to any current physical theory anyway. — Further physical motivations for a notion of absolute simultaneity are discussed by Øhrstrøm (1988).

physicists; no new formal results are forthcoming.

Option b. The tense-logically more interesting option is option (b), because it urges one to take the relativistic challenge seriously as a logician, leading to the project of a "logic of points of view". Prior does not develop that option in great detail, even though he provides a clear sketch of what one has to do. In fact, Prior sketches two possible "logics of points of view". The first one is still tied to the assumption that there *is* a preferred rest frame, but it provides the means for a language in which that rest frame is not, or only partially, known (cf. Prior, 1968, 133f.). It thus takes a middle position between options (a) and (b). We will not consider that first "logic of points of view" here. The second system is more in line with option (b). Contrasting that system with the first, Prior describes it as follows:

We can, all the same, develop the logic of 'points of view' on the basis of a syntax which does *not* thus suggest that there is a 'real' (though only partly knowable) course of events which represents these various systematically related appearances. We might describe this alternative syntax in a very general way as follows: instead of using the plain *p* for a quite impersonal 'It is (really) the case that *p*', we use it for 'It appears (or is the case) from *this* point of view that *p*', or 'It is the case with *this* person or particle that *p*'. That is, the prefix 'It appears from *this* point of view that — ' or 'It is the case with *this* person or particle that — ' is one which has the same sort of vacuity in this language as 'It is *now* the case that — ' has in ordinary tense logic; it does not need to be expressed, but is understood in all that we say. We then describe what appears to be the case from other points of view, or what is the case with other persons or particles, by using quasi-modal operators which take us from 'this' point of view or particle to the other ones, very much like operators like 'It will be the case that — ' take us to other 'nows' from 'this' now. They would, I think, be operators corresponding to the 'signal relation' of relativistic physics. Associated with these other points of view or particles are other time-series.

In this passage, Prior sketches a way of bridging the gap between tense logic and relativity theory that leads to interesting insights for both sides. With respect to logic, the project of a "logic of points of view" leads to a broadened understanding of the role of parameters of truth for modal languages, and to treating all parameters of truth alike. If in a relativistic setting, the truth value of assertions about simultaneity depends on the reference frame, then

that shows the reference frame to be a parameter of truth, and an appropriate universalist formalisation needs to do justice to that fact.³⁵

There is also a lesson for relativity theory, which is mostly one of meta-physical and semantical modesty. Despite the fact that Prior himself was not overly optimistic (cf. Prior, 1968, 134), his project of a "logic of points of view" leads to a formalisation of our natural temporal discourse that is immune against the demands of scientific language reform. The way we talk about time is scientifically respectable; it is just that a little formalisation may be required to convince the opponent. Prior's three-fold tense logical universalism thus amounts to formalisation at the service of philosophy of logic, of language, and of science, and it gives fresh impulses for all three disciplines.

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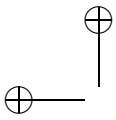
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³⁵ Cf. Müller (2004) for some of the formal details, and Belnap (2005) for the general programme, which Belnap calls "flat pre-semantics". A universalist understanding of model theoretic techniques, sketched in section 3.1.3 above, is essential for the tenability of that programme from the point of view of tense-logical universalism.

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