Logique & Analyse **194** (2006), 119–136

INFON LOGIC BASED ON CONSTRUCTIVE LOGIC

SEIKI AKAMA AND YASUNORI NAGATA

Abstract

Infon Logic was introduced by Devlin as a logic for situation theory. In this paper, we propose a version of infon logic, called constructive infon logic (CIL) based on Nelson's constructive logic with strong negation. CIL has constructible negation which is desirable to situated reasoning. We give Hilbert and tableau formulation of CIL and prove completeness with respect to Kripke semantics. Some issues related to situation theory are also discussed.

1. *Introduction*

Barwise and Perry [5] proposed *situation semantics* for natural language. Later, the main efforts were shifted to the foundation for situation semantics, called *situation theory*. However, the logic of situation theory was not entirely clear. Devlin [6] is the first attempt of this subject by introducing the so-called *infon logic*. A similar work can be also found in Barwise and Etchemendy [4] from a different perspective. They are very important since their logics can serve as a deductive system for situated reasoning. There are two crucial notions in infon logic. One is the concept of *infon* and the other is that of *situation*. Roughly speaking, infon is considered as a discrete item of information, and situation as some part of the activity of the world. These two notions are intimately connected. Because situations contain objects in some domain supporting *information*, the information *carried* by a situation has some logic. We can then understand that infon is a unit of information. These considerations lead us to work out infon logic.

There seem at least three requirements of infon logic. The first is to deal with *partiality* of information. In fact, situation may be partial and can support partial information. The second involves the *persistence* of information, which can be viewed as monotonicity by logicians. Obviously, persistence is one of the fundamental properties of information. Thirdly, infon logic should be used as the basis for situated reasoning. Unfortunately, the previous work concentrated on the first two things.

Infon logic has non-classical flavors due to the treatments of negation and quantifiers. This means that classical logic is not suited to outline a basis for infon logic. The point is in fact recognized by workers in situation theory. For instance, Barwise and Etchemendy used Heyting algebras and Devlin adopted a version of partial logic. However, there are other interesting possibilities.

In this paper, we develop *constructive infon logic* (CIL) based on Nelson's [7] *constructive logic with strong negation*. The rest of this paper is as follows. In section 2, we survey infon logic of Devlin. Section 3 gives a quick review of Nelson's constructive logics. Section 4 introduced constructive infon logic. We prove completeness of infon logic based on tableau calculus and Kripke semantics in section 5. Some theoretical issues in CIL are also discussed in section 6.

2. *Infon Logic*

In this section, we survey infon logic following Devlin [6]. Devlin identified the concept of *information* with the following:

objects $a_1, ..., a_n$ do/do not stand in the relation P.

Thus, information can be described by means of objects and relation holding these objects. Let P be n-place relation and $a_1, ..., a_n$ be appropriate objects for P. Then, $\langle\langle P, a_1, ..., a_n, 1\rangle\rangle$ is used to mean the informational item that $a_1, ..., a_n$ stand in the relation P, and $\langle\langle P, a_1, ..., a_n, 0\rangle\rangle$ is used to mean the informational item that $a_1, ..., a_n$ do not stand in the relation P.

An *infon* is an object of the form $\langle\langle P, a_1, ..., a_n, i\rangle\rangle$, where P is an n-place relation, $a_1, ..., a_n$ are appropriate objects for P, and i is the polarity equal to 1 or 0. Then, i is a value to denote the above two representations. If an infon corresponds to the way things actually are in our world, it is called a *fact*. From a traditional logical point of view, an infon correspond to an atomic sentence or its negation. Namely, it seems to be a basic representation of information. A *situation* is part of our world. Thus, a situation could be understood as partial possible worlds by modal logicians. Here, we do not go into the details of ontological natures of a situation. Let s be a situation and σ be an infon. We write $s \models \sigma$ to denote that σ is "made true by" s. If I is a set of infons and s is a situation, $s \models I$ to mean that $s \models \sigma$ for every infon σ in I.

Devlin's infon logic aimes at developing a logical calculus for complex infon. For doing so, logical connectives $\wedge$ (conjunction), $\vee$ (disjunction), and bounded quantifiers $\forall$ (for all), $\exists$ (for some) are introduced. Let σ and τ

be infons. Then, conjunction and disjunction are interpreted in the following way:

$$s \models \sigma \wedge \tau \text{ iff } s \models \sigma \text{ and } s \models \tau, \quad s \models \sigma \vee \tau \text{ iff } s \models \sigma \text{ or } s \models \tau$$

Let σ be an infon, x be a parameter and u be some set, a be an object given by an *anchor*. We simplify a situation theorist's notion of anchor by a suitable substitution. Then, existential and universal quantifier can be interpreted as follows:

$$s \models (\exists x \in u)\sigma(x) \text{ iff } s \models \sigma(a) \text{ for some } a \text{ in } u$$
$$s \models (\forall x \in u)\sigma(x) \text{ iff } s \models \sigma(a) \text{ for all } a \text{ in } u$$

Devlin did not introduce negation of an infon because the polarity of an infon can simulate negation.

One of the important properties of an infon is the property of *persistence*. This means that if $s \models \langle\langle P, a_1, ..., a_n, i\rangle\rangle$ for any situation s and appropriate objects $a_1, ..., a_n$ in s, then $s' \models \langle\langle P, a_1, ..., a_n, i\rangle\rangle$ for any situation s' which extends s.

Devlin gave a detailed treatment of his infon logic with other interesting notions. However, the presentation here is sufficient to our purposes in this paper. From the above exposition, infon logic is a version of partial predicate with two bounded quantifiers satisfying the property of persistence.

3. *Constructive Logic with Strong Negation*

To sketch another view of infon logic, we need a logic with appropriate negation and implication. One such candidate is *constructive logic with strong negation* originally proposed by Nelson [7]. Nelson's constructive logic with strong negation denoted by N is an extension of positive intuitionistic predicate logic with a new connective for *constructible falsity* or *strong negation* to overcome the non-constructive features of intuitionistic negation. Constructive logics with strong negation have been extensively studied by logicians for many years. Recently, these logics found several applications in computer science. N is an extension of the positive intuitionistic predicate logic with the following axioms for strong negation ($\sim$):

(N1) $A \wedge \sim A \rightarrow B$ (N2) $\sim\sim A \leftrightarrow A$
(N3) $\sim(A \rightarrow B) \leftrightarrow (A \wedge \sim B)$ (N4) $\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B)$

(N5) $\sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$ (N6) $\sim \forall x A(x) \leftrightarrow \exists x \sim A(x)$
(N7) $\sim \exists x A(x) \leftrightarrow \forall x \sim A(x)$.

The rules of inference are as follows:

(MP) $A, A \rightarrow B / B$ (EG) $A(c) \rightarrow B / \exists x A(x) \rightarrow B$
(UG) $B \rightarrow A(c) / B \rightarrow \forall x A(x)$

where c does not occur in B in (EG) and (UG).

If $(N1)$ is deleted from N, we obtain paraconsistent constructive logic N^- of Almukdad and Nelson [2]. In N, we can define intuitionistic negation $(\neg)$ as follows: $\neg A \leftrightarrow (A \rightarrow \sim A)$. Clearly, strong negation is stronger than intuitionistic negation, namely $\sim A \rightarrow \neg A$, but the converse does not hold; see Akama [1].

A Kripke semantics for strong negation was developed by Thomason [8]; also see Akama [1]. A *strong negation model* for N is of the form $(W, \leq, w_0, val, D)$, where W is a set of worlds with the distinguished world w_0 such that $\forall w \in W(w_0 \leq w)$, $\leq$ is a reflexive and transitive relation on $W \times W$, val is a three-valued valuation assigning 1 (true), 0 (false), -1 (undefined) to the atomic formula $p(t)$ at $w \in W$ with parameter $t \in D(w)$ satisfying:

if $val(w, p(t)) = 1$ and $w \leq v$ then $val(v, p(t)) = 1$,
if $val(w, p(t)) = 0$ and $w \leq v$ then $val(v, p(t)) = 0$,

and D is a domain function from W to a set of variables such that if $w \leq v$ then $D(w) \subseteq D(v)$. Note that $V(w, A) = -1$ iff neither $V(w, A) = 1$ nor $V(w, A) = 0$. Then, we define the function $V(w, A)$ for any formula A.

$V(w, p(t)) = 1$ iff $val(w, p(t)) = 1$ for any atomic $p(t)$ with $t \in D(w)$,
$V(w, p(t)) = 0$ iff $val(w, p(t)) = 0$ for any atomic $p(t)$ with $t \in D(w)$,
$V(w, A \wedge B) = 1$ iff $V(w, A) = 1$ and $V(w, B) = 1$,
$V(w, A \wedge B) = 0$ iff $V(w, A) = 0$ or $V(w, B) = 0$,
$V(w, A \vee B) = 1$ iff $V(w, A) = 1$ or $V(w, B) = 1$,
$V(w, A \vee B) = 0$ iff $V(w, A) = 0$ and $V(w, B) = 0$,
$V(w, A \rightarrow B) = 1$ iff $\forall v(w \leq v$ and $V(v, A) = 1 \Rightarrow V(v, B) = 1)$,
$V(w, A \rightarrow B) = 0$ iff $V(w, A) = 1$ and $V(w, B) = 0$,
$V(w, \sim A) = 1$ iff $V(w, A) = 0$,
$V(w, \sim A) = 0$ iff $V(w, A) = 1$,

$V(w, \forall x A(x)) = 1$ iff $\forall v(w \leq v \Rightarrow V(v, A(t)) = 1$ for any $t \in D(v))$,
$V(w, \forall x A(x)) = 0$ iff $V(w, A(t)) = 0$ for some $t \in D(w)$,
$V(w, \exists x A(x)) = 1$ iff $V(w, A(t)) = 1$ for some $t \in D(w)$,
$V(w, \exists x A(x)) = 0$ iff $\forall v(w \leq v \Rightarrow V(v, A(t)) = 0$ for any $t \in D(v))$.

We say that A is true iff $V(w_0, A) = 1$. A is *valid*, written $\models_N A$, iff it is true in all strong negation models. A strong negation model for N^- needs the following extra clause.

$V(w, A \wedge \sim A) = 1$ for some w and for some formula A.

Alternatively, we can use a four-valued valuation in a strong negation model for N^-. A completeness proof for N may be found in Akama [1]. Thomason [8] proved that N with the constant domain axiom (CD): $\forall x(A(x) \vee B) \rightarrow (\forall x A(x) \vee B)$, where x is not free in B, has a Kripke semantics with constant domains. It is well known that (CD) is not acceptable for constructivists.

4. *Constructive Infon Logic*

We are now ready to develop constructive infon logic denoted by CIL. Our work is motivated by the idea that infons can be viewed as proofs (or disproofs) in a constructive setting. This leads us to produce something based on Nelson's constructive logic with strong negation. Although there seems to be an unwarranted presumption that the topics of constructive mathematics and situated reasoning are obviously related, variants of Nelson's logic can serve as a version of infon logic. This is a starting point of CIL. A similar idea can also be found in Wang and Mott [10] who proposed a first-order logic CF' with strong negation and bounded quantifiers which is a variant of CF of Thomason [9].

There are several points in the work of CIL to be addressed here. First, CIL provides natural negation. If negation is introduced into Devlin's infon logic, it obeys double negation law and de Morgan laws following Barwise and Perry [5]. This implies that Kleene's strong three-valued logic (or the negation-free fragment of Nelson's logic) can be used. We also note that intuitionistic negation is not appropriate in this context. It is then possible to express an infon as an atomic formula or a strong negation of an atomic formula. Second, CIL has *real* implication satisfying *modus ponens* and the deduction theorem, which is equivalent to intuitionistic implication. In general, situation theorists express the conditional as a conditional constraint, but existing formalizations of conditional constraints are far from adequate, largely because they represent attempts to come to grips with the slippery notion of situated reasoning. In contrast, the choice of intuitionistic implication can give an interesting idea of the representation of situated reasoning. Finally, a situation can be interpreted as a set of infons. And compounded infons are formed constructively. In addition, a situation is a piece of information with persistency. CIL can deal with these features using a variant of strong negation model given above.

The language L_{CIL} of CIL is that of N with a set of *bounders* $\mathcal{B}_L$ and the membership symbol $\in$. An *atomic formula* of CIL is an expression of the form $P(t_1, ..., t_n)$ or $c \in \beta$. Here P is n-place predicate symbol and $t_1, ..., t_n$ are terms, and c is a constant and $\beta \in \mathcal{B}_L$. Here, a term is defined as usual. Then, the formulation rule of quantified formulas reads: if $A(x)$ is a formula with a variable x and β is a bounder, then $\forall x \in \beta A(x)$ and $\exists x \in \beta A(x)$ are formulas. This can be extended for formulas with several variables.

A Hilbert style axiomatization of CIL is based on an axiomatization of positive intuitionistic propositional logic with the following axioms:

(C1) $A \wedge \sim A \rightarrow B$
(C2) $\sim\sim A \leftrightarrow A$
(C3) $\sim (A \rightarrow B) \leftrightarrow (A \wedge \sim B)$
(C4) $\sim (A \wedge B) \leftrightarrow (\sim A \vee \sim B)$
(C5) $\sim (A \vee B) \leftrightarrow (\sim A \wedge \sim B)$
(C6) $t \in \beta \wedge A(t) \rightarrow \exists x \in \beta A(x)$
(C7) $\forall x \in \beta A(x) \wedge t \in \beta \rightarrow A(t)$
(C8) $\sim \forall x \in \beta A(x) \leftrightarrow \exists x \in \beta \sim A(x)$
(C9) $\sim \exists x \in \beta A(x) \leftrightarrow \forall x \in \beta \sim A(x)$.

Here, t is an arbitrary term. The rules of inference are as follows:

(MP) $A, A \rightarrow B/B$
(EG) $c \in \beta \wedge A(c) \rightarrow B/\exists x \in \beta A(x) \rightarrow B$
(UG) $B \rightarrow (c \in \beta \rightarrow A(c))/B \rightarrow \forall x \in \beta A(x)$.

Here, c does not occur in B. (C1)–(C5) are equal to (N1)–(N5). However, axioms for quantification need modifications due to the presence of bounded quantifiers. It is clear that the following equivalences are available in CIL.

$$A \vee B \leftrightarrow \sim (\sim A \wedge \sim B) \qquad A \wedge B \leftrightarrow \sim (\sim A \vee \sim B)$$
$$\exists x \in \beta A(x) \leftrightarrow \sim \forall x \in \beta \sim A(x) \quad \forall x \in \beta A(x) \leftrightarrow \sim \exists x \in \beta \sim A(x)$$

Next, we turn to a Kripke semantics for CIL. Let $\mathcal{V}_L$ be a set of variables, $\mathcal{C}_L$ be a set of constants, $\mathcal{P}_L^n$ be a set of n-place predicate symbols, and $\mathcal{B}_L$ be a set of bounders. A *constructive infon (CI) model* for CIL is of the form $(S, \leq, s_0, I, V, D)$, where S is a set of situations with the actual situation s_0

such that $\forall s \in S(s_0 \leq s)$, $\leq$ is a reflexive and transitive relation on $S \times S$, D is a domain function assigning sets of individuals to the elements of S satisfying that if $s \leq s'$ then $D(s) \subseteq D(s')$, and I is an *interpretation function* satisfying the following conditions:

(1) I_s is a partial function from $\mathcal{C}_L$ into $D(s)$ satisfying that for constant c, if $s \leq s'$ and $I_s(c)$ is defined, then $I_{s'}(c)$ is also defined and $I_s(c) = I_{s'}(c)$.
(2) for n-place predicate A, $I_s(A)$ is a partial function from $D(s)^n$ into $\{1, -1, 0\}$, and if $s \leq s'$, then $I_{s'}(A)$ is an extension of $I_s(A)$.
(3) for bounder $\in$, $I_s(\in)$ is a partial function from $\mathcal{B}_L$ into $pow(D(s))$ satisfying that if $s \leq s'$ then $I_s(\beta) \subseteq I_{s'}(\beta)$.

and V is a three-valued valuation function assigning 1 (true), 0 (false), -1 (undefined) to the atomic formula $A(c_1, ..., c_n)$ at $s \in S$ satisfying:

$V(s, A(c_1, ..., c_n)) = 1$ iff $I(c_1), ..., I(c_n)$ are defined and
$I_s(A)(I_s(c_1), ..., I_s(c_n)) = 1$,
$V(s, A(c_1, ..., c_n)) = 0$ iff $I(c_1), ..., I(c_n)$ are defined and
$I_s(A)(I_s(c_1), ..., I_s(c_n)) = 0$,
$V(s, A(c_1, ..., c_n)) = -1$ otherwise,
$V(s, c \in \beta) = 1$ iff $I(c)$ is defined and $I_s(c) \in I_s(\beta)$,
$V(s, c \in \beta) = 0$ iff $I(c)$ is defined and $I_s(c) \notin I_s(\beta)$,
$V(s, c \in \beta) = -1$ otherwise.

V can be extended for any formula as follows:

$V(s, A \wedge B) = 1$ iff $V(s, A) = 1$ and $V(s, B) = 1$,
$V(s, A \wedge B) = 0$ iff $V(s, A) = 0$ or $V(s, B) = 0$,
$V(s, A \vee B) = 1$ iff $V(s, A) = 1$ or $V(s, B) = 1$,
$V(s, A \vee B) = 0$ iff $V(s, A) = 0$ and $V(s, B) = 0$,
$V(s, A \to B) = 1$ iff $\forall s'(s \leq s'$ and $V(s', A) = 1 \Rightarrow V(s', B) = 1)$,
$V(s, A \to B) = 0$ iff $V(s, A) = 1$ and $V(s, B) = 0$,
$V(s, \sim A) = 1$ iff $V(s, A) = 0$,
$V(s, \sim A) = 0$ iff $V(s, A) = 1$,
$V(s, \forall x \in \beta A(x)) = 1$ iff $\forall s' \forall c \in D(s')(s \leq s'$ and $V(s', c \in \beta) = 1 \Rightarrow V(s', A(c)) = 1)$,
$V(s, \forall x \in \beta A(x)) = 0$ iff $\exists c \in D(s)(V(s, c \in \beta) = 1$ and $V(s, A(c)) = 0)$,
$V(s, \exists x \in \beta A(x)) = 1$ iff $\exists c \in D(s)(V(s, c \in \beta) = 1$ and

$V(s, A(c)) = 1)$,
$V(s, \exists x \in \beta A(x)) = 0$ iff $\forall s' \forall c \in D(s')(s \leq s'$ and $V(s', c \in \beta) = 1 \Rightarrow V(s', A(c)) = 0)$.

Here, we assume that every object has the same name. Note also that $V(s, A) = -1$ iff neither $V(s, A) = 1$ nor $V(s, A) = 0$. A is *true* iff $V(s_0, A) = 1$. A is *valid*, written $\models_{CIL} A$ iff it is true in all constructive infon models.

Theorem 4.1: *For any formula A and any situation s, s', we have:*

if $V(s, A) \neq -1$ and $s \leq s'$, then $V(s, A) = V(s', A)$.

Proof. By induction on the length of A. □

Theorem 4.2: *For any formula A and any s, we have:*

$V(s, A \wedge \sim A) \neq 1$.

Proof. Since V is a three-valued function, A and $\sim A$ cannot be simultaneously true in the same situation s. As a consequence, the statement of this theorem can be justified. □

Theorem 4.1 reveals that complex infons in CIL are persistent. By theorem 4.2, we mean that infons are consistent.

Now, it would be interesting to compare CIL with Wang and Mott's CF' in [10]. Although both systems are based on constructive logic with strong negation, there are some conceptual differences. First, Wang and Mott treat a universal quantifier *statically*, whereas we treat it *dynamically* as a constructivist usually assumes. Then, universally quantified sentences in CF' are not always persistent, and they must introduce the notion of a persistent bounder. The issue will be discussed later in connection with situation theory. Second, our bounders are partial but Wang and Mott's bounders are decidable. If the formula is of the form $c \in \beta$, then we think that it should be partial. On these differences, CF' needs an extra axiom $c \in \beta \vee \sim c \in \beta$ and the interpretations of the universal quantifier and negated existential quantifier are different from ours.

5. *Tableau and Completeness*

In this section, we describe a *tableau calculus* $TCIL$ for CIL for the purpose of a proof of completeness of CIL. A basic idea of tableau calculus is to employ *indirect proof* (cf. Smullyan [8]). Tableau calculus is regarded as a variant of a Gentzen system, and is convenient to prove completeness. It is also useful to represent proofs in CIL.

For the tableau calculus, we use the notion of a *signed formula*. If X is a formula, then TX and FX are signed formulas. TX reads "X is provable" and FX reads "X is not provable", respectively. If Γ is a set of signed formulas and A is a signed formula, then we simply write $\{\Gamma, A\}$ for $\Gamma \cup \{A\}$.

A tableau calculus $TCIL$ consists of *axioms* and *reduction rules*. A tableau is constructed by repeated applications of reduction rules until they cannot be applied. Let p be an atomic formula and A and B be formulas. Then, $TCIL$ is described as follows.

Tableau Calculus $TCIL$
Axioms
(AX1) $\{Tp, Fp\}$, (AX2) $\{T \sim p, F \sim p\}$, (AX3) $\{Tp, T \sim p\}$
Reduction Rules

$(T\wedge)\ \dfrac{\Gamma, T(A \wedge B)}{\Gamma, TA, TB}$ $\qquad (F\wedge)\ \dfrac{\Gamma, F(A \wedge B)}{\Gamma, FA;\ \Gamma, FB}$

$(T\vee)\ \dfrac{\Gamma, T(A \vee B)}{\Gamma, TA;\ \Gamma, TB}$ $\qquad (F\vee)\ \dfrac{\Gamma, F(A \vee B)}{\Gamma, FA, FB}$

$(T\rightarrow)\ \dfrac{\Gamma, T(A \rightarrow B)}{\Gamma, FA;\ \Gamma, TB}$ $\qquad (F\rightarrow)\ \dfrac{\Gamma, F(A \rightarrow B)}{\Gamma_T, TA, FB}$

$(T \sim \wedge)\ \dfrac{\Gamma, T(\sim (A \wedge B))}{\Gamma, T(\sim A);\ \Gamma, T(\sim B)}$ $\qquad (F \sim \wedge)\ \dfrac{\Gamma, F(\sim (A \wedge B))}{\Gamma, F(\sim A), F(\sim B)}$

$(T \sim \vee)\ \dfrac{\Gamma, T(\sim (A \vee B))}{\Gamma, T(\sim A), T(\sim B)}$ $\qquad (F \sim \vee)\ \dfrac{\Gamma, F(\sim (A \vee B))}{\Gamma, F(\sim A);\ \Gamma, F(\sim B)}$

$(T \sim\rightarrow)\ \dfrac{\Gamma, T(\sim (A \rightarrow B))}{\Gamma, TA, T(\sim B)}$ $\qquad (F \sim\rightarrow)\ \dfrac{\Gamma, F(\sim (A \rightarrow B))}{\Gamma, FA;\ \Gamma, F(\sim B)}$

$(T \sim\sim)\ \dfrac{\Gamma, T(\sim\sim A)}{\Gamma, TA}$ $\qquad (F \sim\sim)\ \dfrac{\Gamma, F(\sim\sim A)}{\Gamma, FA}$

$(T\forall)\ \dfrac{\Gamma, T(t \in \beta), T(\forall x \in \beta A(x))}{\Gamma, T(A(t))}$ $\qquad (F\forall)\ \dfrac{\Gamma, F(\forall x \in \beta A(x))}{\Gamma_T, T(c \in \beta), F(A(c))}$

$(T\exists)\ \dfrac{\Gamma, T(\exists x \in \beta A(x))}{\Gamma, T(c \in \beta), T(A(c))}$ $\qquad (F\exists)\ \dfrac{\Gamma, T(t \in \beta), F(\exists x \in \beta A(x))}{\Gamma, F(A(t))}$

$(T \sim \forall)\ \dfrac{\Gamma, T(\sim \forall x \in \beta A(x))}{\Gamma, T(c \in \beta), T(\sim A(c))}$ $\qquad (F \sim \forall)\ \dfrac{\Gamma, T(t \in \beta), F(\sim \forall x \in \beta A(x))}{\Gamma, F(\sim A(t))}$

$(T \sim \exists)\ \dfrac{\Gamma, T(t \in \beta), T(\sim \exists x \in \beta A(x))}{\Gamma, T(\sim A(t))}$ $\qquad (F \sim \exists)\ \dfrac{\Gamma, F(\sim \exists x \in \beta A(x))}{\Gamma_T, T(c \in \beta), F(\sim A(c))}$

Here, the constant t is arbitrary and the constant c satisfies the restriction that it must not occur in any formula of Γ or in the formula $A(x)$. Γ_T stands for $\{TX \mid TX \in \Gamma\}$. A *proof* of a sentence X is a closed tableau for FX. A tableau is a tree constructed by the above reduction rules. A tableau is *closed* if each branch is closed. A branch is closed if it contains the axioms above. We write $\vdash_{TCIL} A$ to mean that A is provable in $TCIL$.

Next, we turn to a completeness proof of the tableau calculus $TCIL$ with respect to constructive infon models. Let $S = \{TX_1, ..., TX_n,\ FY_1, ..., FY_m\}$ be a set of signed formulas, $(S, \leq, s_0, I, V, D)$ be a constructive infon model, and $s_0 \in S$. We say that w *refutes* S if

$$\begin{array}{ll} V(w, X_i) = 1 & \text{if } TX_i \in \Gamma, \\ V(w, Y_i) \neq 1 & \text{if } FY_i \in \Gamma. \end{array}$$

A set Γ is *refutable* if something refutes it. If Γ is not refutable, it is *valid*.

Theorem 5.1: (Soundness of $TCIL$) *If A is provable, then A is valid.*

Proof. If A is of the form of axioms, it is easy to show its validity. For reduction rules, it suffices to check that they preserve validity. For example, consider the rule $(T \sim \vee)$. We have to show that if $S, T(\sim (A \vee B))$ is refutable then $S, T(\sim A), T(\sim B)$ is also refutable. By the assumption, there is a constructive infon model $(S, \leq, s_0, I, V, D)$, in which s_0 refutes Γ and $V(s_0, \sim (A \vee B)) = 1$. This implies:

$$\begin{array}{lll} V(s_0, A \vee B) = 0 & \text{iff} & V(s_0, A) = 0 \text{ and } V(s_0, B) = 0 \\ & \text{iff} & V(s_0, \sim A) = 1 \text{ and } V(s_0, \sim B) = 1 \end{array}$$

Therefore, $S, T(\sim A), T(\sim B)$ is shown to be refutable.

Next, consider the rule $(F \sim\rightarrow)$. By the assumption, there is a constructive infon model, in which s_0 refutes S and $V(s_0, \sim (A \rightarrow B)) \neq 1$. This implies:

$$V(s_0, A \rightarrow B) \neq 0 \quad \text{iff} \quad V(s_0, A) \neq 1 \text{ and } V(s_0, \sim B) \neq 1$$

Therefore, Γ, FA and $\Gamma, F(\sim B)$ are refutable.

For the rule $(T\forall)$, by the assumption, we have a constructive infon model, in which s_0 refutes Γ and $V(s_0, \forall x \in \beta A(x)) = 1$. This implies:

$$\begin{array}{lll} V(s_0, \forall x \in \beta A(x)) = 1 & \text{iff} & \forall s' \forall t \in D(s')(s_0 \leq s \text{ and } V(s', t \in \beta) \\ & & \quad \Rightarrow V(s', A(t)) = 1) \end{array}$$

Here, t is an arbitrary. By theorem 4.1, $\Gamma, T(t \in \beta), T(A(t))$ is also refutable. $(T \sim \exists)$ is similarly checked.

For the rule $(T\exists)$, from the assumption, there is a constructive infon model, in which s_0 refutes Γ and $V(s_0, \exists x \in \beta A(x)) = 1$. This implies:

$$V(s_0, \exists x \in \beta A(x)) = 1 \quad \text{iff} \quad \exists c \in D(s)(V(s_0, c \in \beta) \\ \text{and } V(s_0, A(c)) = 1)$$

Here, c is subject to the variable restriction. Then, $\Gamma, T(c \in \beta), T(A(c))$ is also refutable. $(T \sim \forall)$ is similarly checked. The verification of other rules presents no difficulty. □

A finite set of signed formulas Γ is *consistent* if no tableau for it is closed. An infinite set of signed formulas is consistent if every finite subset is consistent. If a set of signed formulas is not consistent, it is inconsistent.

Definition 5.2: *Let* P *be a set of parameters and* Γ *a set of signed formulas. We say that* Γ *is maximal consistent with respect to* P *if*
(1) every signed formula in Γ *uses only parameters of* P*,*
(2) Γ *is consistent,*
(3) for every formula A *with parameters in* P*, either* $TA \in \Gamma$ *or* $FA \in \Gamma$*.*

Here, we denote by $L(\mathcal{C})$ the new language extending the set of constants $\mathcal{C}_L$ of the original language L with a set of constants $\mathcal{C} = \{c_1, ..., c_n\}$.

Definition 5.3: *We say that a consistent set of signed formulas* Γ *is* $\mathcal{C}$*-saturated if*
(1) Γ *is maximal consistent with respect to* $\mathcal{C}_L \cup C$*,*
(2) if $T(\exists x \in \beta A(x)) \in \Gamma$*, then* $T(c \in \beta), T(A(c)) \in \Gamma$ *for some* $c \in \mathcal{C}_L \cup \mathcal{C}$*,*
(3) if $T(\sim \forall x \in \beta A(x)) \in \Gamma$*, then* $T(c \in \beta), T(\sim A(c)) \in \Gamma$ *for some* $c \in \mathcal{C}_L \cup \mathcal{C}$*.*

Lemma 5.4: *A consistent set of signed formulas* Γ_0 *can be extended to a maximal consistent set of signed formulas* Γ*.*

Proof. Since the language L has a countably infinite set of sentences, we can enumerate sentences $A_1, A_2, ...$ Now, we define for a consistent set of signed formulas Γ_0 a sequence of consistent sets of signed formulas $\Gamma_0, \Gamma_1, \Gamma_2, ...$in the following way:

$$\Gamma_{n+1} = \begin{cases} \Gamma_n \cup \{TA_{n+1}\} & \text{if} \quad \Gamma_n \cup \{TA_{n+1}\} \text{ is consistent,} \\ \Gamma_n \cup \{FA_{n+1}\} & \text{if} \quad \Gamma_n \cup \{FA_{n+1}\} \text{ is consistent,} \\ \Gamma_n & \qquad \text{otherwise.} \end{cases}$$

Then, we set:

$$\Gamma = \bigcup \Gamma_i$$

It is shown that Γ satisfies the desired properties of a maximal consistent set. $\square$

Lemma 5.5: *A consistent set Γ of signed formulas in L can be extended to a $\mathcal{C}$-saturated consistent set Δ of signed formulas in $L(\mathcal{C})$.*

Proof. Let $\Gamma_0 = \Gamma$. Extend Γ_0 to a set Γ_1 maximal consistent with respect to $\mathcal{C}_L$. Since $C = \{c_1, c_2, ...\}$ is a countable set of constants not in L. we can enumerate sentences of the form $\exists x \in \beta A(x)$ in $L(\mathcal{C})$ as $\exists x \in \beta A_1(x), \exists x \in \beta A_2(x), ...$ By definition $\forall x \in \beta A(x) \leftrightarrow \sim \exists x \in \beta \sim A(x)$, it suffices to check the case of $\exists x \in \beta A(x)$. Γ_i can be then defined for any $n \geq 1$ as follows:
$(i = 2n)$ Take the first formula of the form $\exists x \in \beta A_m(x)$. If $T(\exists x \in \beta A(x)) \in \Gamma_{n-1}$ but $T(c \in \beta), T(A(c)) \notin \Gamma_{n-1}$ for all $c \in \mathcal{C}_L \cup \{c_1, ..., c_n\}$, then set $\Gamma_{2n} = \Gamma_{2n-1} \cup \{T(c \in \beta), T(A(c))\}$.
$(i = 2n + 1)$ By lemma 5.4, we extend Γ_{2n} to Γ_{2n+1}, which is maximal consistent with respect to $\mathcal{C}_L \cup \mathcal{C}$.

Then, we define $\Delta = \bigcup \Gamma_i$. Here, we can easily check that Δ is C-saturated. Since each Γ_n is consistent, Δ is also consistent. Let $c \in \beta, A(c)$ be any sentences of $L(\mathcal{C})$ with $c \in \mathcal{C}$. From the maximality of Γ_{2n+1}, one of the conditions $TA \in \Gamma_{2n+1}, FA \in \Gamma_{2n+1}$, or $\Gamma_{2n+1} \cup \{TA\}$ and $\Gamma_{2n+1} \cup \{FA\}$ are provable, holds. Thus, Δ is shown to be maximal in $L(\mathcal{C})$. Finally, we check $T(\exists x \in \beta A(x)) \in \Delta$. We suppose $\exists x \in \beta A_n(x)$, i.e. the n-th enumeration. From the above construction, $T(c \in \beta), T(A(c)) \in \Delta$ for some $c \in \mathcal{C}_L \cup \{c_1, ..., c_n\}$ must hold. This implies that Δ is C-saturated. $\square$

We are now ready to define a *canonical* constructive infon model with respect to the tableau:

Definition 5.6: *Let $\mathcal{C}_0, \mathcal{C}_1, \mathcal{C}_2, ...$ be a countable sequence of disjoint countable sets of constants not occurring in L. We denote $\mathcal{C}_0 \cup \mathcal{C}_1 \cup ... \cup \mathcal{C}_n$ by $\mathcal{C}_n^*$. Then, we define a canonical constructive infon model $(S, \leq, s_0, I, V, D)$ as follows:*

(1) $S = \{\Gamma \mid \Gamma$ *is $\mathcal{C}_n^*$-saturated in $L(\mathcal{C}_n^*)$ for some* $n\}$,
(2) *If Γ is $\mathcal{C}_n^*$-saturated and $L(\Gamma) = L \cup \mathcal{C}_n^*$, then $D(\Gamma) = \mathcal{C}_n^*$,*
(3) *We define $\leq$ in the following way:*

$$\Gamma \leq \Delta \text{ iff } TA \in \Gamma \Rightarrow TA \in \Delta \text{ and } D(\Gamma) \subseteq D(\Delta),$$

(4) $I_\Gamma(c) = \begin{cases} c & \text{if } c \in \mathcal{C}_L \cup \mathcal{C}_n^*, \\ \text{undefined} & \text{otherwise.} \end{cases}$

(5) $I_\Gamma(\beta) = \{c \in \mathcal{C}_L \cup \mathcal{C}_n^* \mid (c \in \beta) \in \Gamma\}$.

(6) $V(\Gamma, A(c_1, ..., c_n) = \begin{cases} 1 & \text{iff } A(c_1, ..., c_n) \in \Gamma, \\ 0 & \text{iff } \sim A(c_1, ..., c_n) \in \Gamma, \\ -1 & \text{otherwise.} \end{cases}$

(7) $V(\Gamma, c \in \beta) = \begin{cases} 1 & \text{iff } I_\Gamma(c) \in I_\Gamma(\beta), \\ 0 & \text{iff } I_\Gamma(c) \notin I_\Gamma(\beta), \\ -1 & \text{otherwise.} \end{cases}$

It can be easily shown that a canonical constructive infon model is in fact a constructive infon model.

Lemma 5.7: *For any* $\Gamma \in S$ *in a canonical constructive infon model* $(S, \leq, s_0, I, V, D)$*, we have:*

(1) *if* $T(A \wedge B) \in \Gamma$*, then* $TA \in \Gamma$ *and* $TB \in \Gamma$*,*
(2) *if* $F(A \wedge B) \in \Gamma$*, then* $FA \in \Gamma$ *or* $FB \in \Gamma$*,*
(3) *if* $T(A \vee B) \in \Gamma$*, then* $TA \in \Gamma$ *or* $TB \in \Gamma$*,*
(4) *if* $F(A \vee B) \in \Gamma$*, then* $FA \in \Gamma$ *and* $FB \in \Gamma$*,*
(5) *if* $T(A \rightarrow B) \in \Gamma$*, then* $FA \in \Gamma$ *or* $TA \in \Gamma$*,*
(6) *if* $F(A \rightarrow B) \in \Gamma$*, then for some* Δ *such that* $\Gamma \leq \Delta$*,* $TA \in \Delta$ *and* $FB \in \Delta$*,*
(7) *if* $T(\sim (A \wedge B)) \in \Gamma$*, then* $T(\sim A) \in \Gamma$ *or* $T(\sim B) \in \Gamma$*,*
(8) *if* $F(\sim (A \wedge B)) \in \Gamma$*, then* $F(\sim A) \in \Gamma$ *and* $F(\sim B) \in \Gamma$*,*
(9) *if* $T(\sim (A \vee B)) \in \Gamma$*, then* $T(\sim A) \in \Gamma$ *and* $T(\sim B) \in \Gamma$*,*
(10) *if* $F(\sim (A \vee B)) \in \Gamma$*, then* $F(\sim A) \in \Gamma$ *or* $F(\sim B) \in \Gamma$*,*
(11) *if* $T \sim (A \rightarrow B) \in \Gamma$*, then* $TA \in \Gamma$ *and* $T(\sim B) \in \Gamma$*,*
(12) *if* $F \sim (A \rightarrow B) \in \Gamma$*, then* $FA \in \Gamma$ *or* $F(\sim B) \in \Gamma$*,*
(13) *if* $T(\sim\sim A) \in \Gamma$*, then* $TA \in \Gamma$*,*
(14) *if* $F(\sim\sim A) \in \Gamma$*, then* $FA \in \Gamma$*.*

Proof. (1): Suppose that $T(A \wedge B) \in \Gamma$ but $TA \notin \Gamma$ or $TB \notin \Gamma$. Since $\Gamma, T(A \wedge B)$ is maximal consistent, then $\Gamma, T(A \wedge B), TA$ is also consistent. But, it contradicts the maximality of Γ. Thus, $TA \in \Gamma$. Similarly, we can show that $TB \in \Gamma$. The proofs of (4), (8), (9), (11), (13) and (14) are similarly described.

(3): Suppose that $TA \notin \Gamma$ and $TB \notin \Gamma$. Since Γ is maximal, both Γ, TA and Γ, TB are inconsistent. This implies that they are provable. For a finite subset S of Γ, both S, TA and S, TB are inconsistent. By $(T\vee)$, $S, T(A \vee B)$

is also provable. This implies that $\Gamma, T(A \vee B)$ is provable and $T(A \vee B) \notin \Gamma$. We can justify (2), (5), (7), (10) and (12) in a similar way.

(6): From $F(A \to B) \in \Gamma$, we have Γ_T, TA, FB is not provable. Lemma 5.4 guarantees that Γ_T, TA, FB can be extended to a maximal consistent set Δ such that $TA \in \Delta$ and $FB \in \Delta$. □

Lemma 5.8: *For any* $\Gamma \in S$ *in a canonical constructive infon model* $(S, \leq, s_0, I, V, D)$*, we have:*

(1) *if* $T(\forall x \in \beta A(x)) \in \Gamma$*, then* $\forall c \in D(\Gamma)(T(c \in \beta) \in \Gamma \Rightarrow T(A(c)) \in \Gamma)$*,*
(2) *if* $F(\forall x \in \beta A(x)) \in \Gamma$*, then* $\exists \Delta \geq \Gamma \exists c \in D(\Delta)(T(c \in \beta) \in \Delta$ and $F(A(c)) \in \Delta)$*,*
(3) *if* $T(\exists x \in \beta A(x)) \in \Gamma$*, then* $\exists c \in D(\Gamma)(T(c \in \beta) \in \Gamma$ and $T(A(c)) \in \Gamma)$*,*
(4) *if* $F(\exists x \in \beta A(x)) \in \Gamma$*, then* $\forall c \in D(\Gamma)(T(c \in \beta) \in \Gamma \Rightarrow F(A(c)) \in \Gamma)$*,*
(5) *if* $T(\sim \forall x \in \beta A(x)) \in \Gamma$*, then* $\exists c \in D(\Gamma)(T(c \in \beta) \in \Gamma$ and $T(\sim A(a)) \in \Gamma)$*,*
(6) *if* $F(\sim \forall x \in \beta A(x)) \in \Gamma$*, then* $\forall c \in D(\Gamma)(T(c \in \beta) \in \Gamma \Rightarrow F(\sim A(a)) \in \Gamma)$*,*
(7) *if* $T(\sim \exists x \in \beta A(x)) \in \Gamma$*, then* $\forall c \in D(\Gamma)(T(c \in \beta) \in \Gamma \Rightarrow T(\sim A(c)) \in \Gamma)$*,*
(8) *if* $F(\sim \exists x \in \beta A(x)) \in \Gamma$*, then* $\exists \Delta \geq \Gamma \exists c \in D(\Delta)(T(c \in \beta) \in \Delta$ and $F(\sim A(a)) \in \Delta)$.

Proof. (1): Assume $T(\forall x \in \beta A(x)) \in \Gamma$ but $T(c \in \beta) \in \Gamma$ and $T(A(a)) \notin \Gamma$ for all $c \in D(\Gamma)$. Since $\Gamma, T(\forall x \in \beta A(x))$ is consistent, so is $\Gamma, T(\forall x \in \beta A(x)), T(c \in \beta), T(A(c))$. So Γ is not maximal. Thus, $T(A(c)) \in \Gamma$. We can similarly deal with (3), (5) and (7).

(2): assume $F(\forall x \in \beta A(x)) \in \Gamma$. If c does not occur in Γ, then $\Gamma_T, T(c \in \beta), F(A(c))$ is consistent. By lemma 5.5, we can extend $\Gamma_T, T(c \in \beta), F(A(c))$ to Δ such that $T(c \in \beta), F(A(c)) \in \Delta$ for some $c \in D(\Delta)$. We can handle (4), (6) and (8) in a similar way. □

Theorem 5.9: *For any* $\Gamma \in S$ *in a canonical constructive infon model and any formula* A*,*

$TA \in \Gamma$ *iff* $V(\Gamma, A) = 1$,
$FA \in \Gamma$ *iff* $V(\Gamma, A) \neq 1$.

Proof. By induction on A. The case A is an atomic formula is immediate.

(1) $A = B \wedge C$:

$$\begin{array}{lll}
T(B \wedge C) \in \Gamma & \text{iff} & TB \in \Gamma \text{ and } TC \in \Gamma \\
 & \text{iff} & V(\Gamma, B) = 1 \text{ and } V(\Gamma, C) = 1 \\
 & \text{iff} & V(\Gamma, B \wedge C) = 1 \\
F(B \wedge C) \in \Gamma & \text{iff} & FB \in \Gamma \text{ or } FC \in \Gamma \\
 & \text{iff} & V(\Gamma, B) \neq 1 \text{ or } V(\Gamma, C) \neq 1 \\
 & \text{iff} & V(\Gamma, B \wedge C) \neq 1
\end{array}$$

The case $A = \sim (B \vee C)$ is treated similarly.

(2) $A = B \vee C$:

$$\begin{array}{lll}
T(B \vee C) \in \Gamma & \text{iff} & TB \in \Gamma \text{ or } TC \in \Gamma \\
 & \text{iff} & V(\Gamma, B) = 1 \text{ or } V(\Gamma, C) = 1 \\
 & \text{iff} & V(\Gamma, B \vee C) = 1 \\
F(B \vee C) \in \Gamma & \text{iff} & FB \in \Gamma \text{ and } FC \in \Gamma \\
 & \text{iff} & V(\Gamma, B) \neq 1 \text{ and } V(\Gamma, C) \neq 1 \\
 & \text{iff} & V(\Gamma, B \vee C) \neq 1
\end{array}$$

The case $A = \sim (B \wedge C))$ is treated similarly.

(3) $A = B \rightarrow C$:

$$\begin{array}{lll}
T(B \rightarrow C) \in \Gamma & \text{iff} & \forall \Delta \geq \Gamma (T(B \rightarrow C) \in \Delta) \\
 & \text{iff} & \forall \Delta \geq \Gamma (FB \in \Delta \text{ or } TC \in \Delta) \\
 & \text{iff} & \forall \Delta \geq \Gamma (V(\Delta, B) \neq 1 \text{ or } V(\Delta, C) = 1) \\
 & \text{iff} & V(\Gamma, B \rightarrow C) = 1 \\
F(B \rightarrow C) \in \Gamma & \text{iff} & \exists \Delta \geq \Gamma (TB \in \Delta \text{ and } TC \in \Delta) \\
 & \text{iff} & \exists \Delta \geq \Gamma (V(\Delta, B) = 1 \text{ and } V(\Delta, C) \neq 1) \\
 & \text{iff} & V(\Gamma, B \rightarrow C) \neq 1
\end{array}$$

(4) $A = \sim\sim B$:

$$\begin{array}{lll}
T(\sim\sim B) \in \Gamma & \text{iff} & TB \in \Gamma \\
 & \text{iff} & V(\Gamma, B) = 1 \\
 & \text{iff} & V(\Gamma, \sim B) = 0 \\
 & \text{iff} & V(\Gamma, \sim\sim B) = 1 \\
F(\sim\sim B) \in \Gamma & \text{iff} & FB \in \Gamma \\
 & \text{iff} & V(\Gamma, B) \neq 1 \\
 & \text{iff} & V(\Gamma, \sim B) \neq 0 \\
 & \text{iff} & V(\Gamma, \sim\sim B) \neq 1
\end{array}$$

(5) $A = \exists x \in \beta B(x)$:

$$\begin{array}{lll}
T(\exists x \in \beta B(x)) \in \Gamma & \text{iff} & \exists c \in D(\Gamma)(T(c \in \beta), T(B(c)) \in \Gamma) \\
 & \text{iff} & \exists c \in D(\Gamma)(V(\Gamma, c \in \beta) = 1 \\
 & & \quad\quad \text{and } V(\Gamma, B(c)) = 1) \\
 & \text{iff} & V(\Gamma, \exists x \in \beta B(x)) = 1
\end{array}$$

$$
\begin{array}{lll}
F(\exists x \in \beta B(x)) \in \Gamma & \text{iff} & \forall c \in D(\Gamma)(F(c \in \beta) \in \Gamma \text{ or } F(B(c)) \in \Gamma \\
& \text{iff} & \forall c \in D(\Gamma)(V(\Gamma, c \in \beta) \neq 1 \\
& & \quad \text{or } V(\Gamma, B(a)) \neq 1) \\
& \text{iff} & V(\Gamma, \exists x \in \beta B(x)) \neq 1
\end{array}
$$

The case $A = \sim \forall x \in \beta B(x)$ is treated similarly.

(6) $A = \forall x B(x)$:

$$
\begin{array}{lll}
T(\forall x \in \beta B(x)) \in \Gamma & \text{iff} & \forall \Delta \geq \Gamma \forall c \in D(\Delta)(T(c \in \beta) \in \Delta \\
& & \quad \Rightarrow\ T(B(c)) \in \Delta \\
& \text{iff} & \forall \Delta \geq \Gamma \forall c \in D(\Delta)(V(\Delta, c \in \beta) \\
& & \quad \Rightarrow\ V(\Delta, B(c)) = 1) \\
& \text{iff} & V(\Gamma, \forall x \in \beta B(x)) = 1 \\
F(\forall x \in \beta B(x)) \in \Gamma & \text{iff} & \exists \Delta \geq \Gamma \exists c \in D(\Delta)(T(c \in \beta) \in \Delta \\
& & \quad \text{and } F(B(c)) \in \Delta \\
& \text{iff} & \exists \Delta \geq \Gamma \exists c \in D(\Delta)(V(\Delta, B(c)) \neq 1 \\
& \text{iff} & V(\Gamma, \forall x \in \beta B(x)) \neq 1
\end{array}
$$

The case $A = \sim \exists x \in \beta B(x))$ is similarly treated. □

Theorem 5.10: (Completeness Theorem) $\vdash_{TCIL} A$ *iff* $\models_{CIL} A$.

Proof. The soundness (⇒)was already stated as theorem 5.1. For the completeness (⇐), it suffices to show that an open tableau is refutable by a counter constructive infon model. This can be done by theorem 5.9. Then, the completeness theorem follows by contraposition. □

Our construction can also state the strong completeness theorem, i.e. $\Gamma \vdash_{TCIL}$ iff $\Gamma \models_{CIL} A$. Here, $\Gamma \models_{CIL} A$ is a semantic consequence relation.

6. *Some Theoretical Issues*

In this section, we discuss some theoretical issues in CIL in relation to situation theory. From a logical perspective, we focus on the logical connectives, i.e. negation, implication and universal quantifier. We start with the problem of negation. As stated above, in Devlin's [6] infon logic there is no negation. However, polarity in an infon can play a role of negation. Because infon logic is formalized in a partial setting, the negation is not classical negation.

Barwise and Etchemendy's [4] *Heyting infon algebra* assumes that infon logic is intuitionistic logic. Unfortunately, intuitionistic negation is too weak to be used for infon logic. Strong negation is thus appropriate to be used for infon logic. But, there is a further extension. If we allow a contradiction in a situation, the resulting logic should be *paraconsistent*. Situation theory assumes an incomplete situation, but does not assume an inconsistent situation. We have no reason to reject the existence of inconsistent situation. However, an alternative view on situations in constructive infon logic is possible, if we replace the underlying logic N by N^- of Almukdad and Nelson [2]. It is not difficult to modify the semantics and tableau given above to accommodate to inconsistent situations.

Second, the conditional is of special interest from a logical viewpoint. In CIL, the implication is intuitionistic implication. As is well known, the interpretation of the intuitionistic implication $A \to B$ is that there is a *construction* which transforms a proof of A into a proof of B. This could be paraphrased in a situation theoretic setting as "there is an information flow from an infon A to an infon B".

There are, however, other possibilities of information flow different from the one given by the intuitionistic implication. In view of constructive logics with strong negation, Wansing [11] studied substructural constructive logics by means of Kripke models. A more elaborated treatment of implication in connection with information may be found in the tradition of *relevance logic*; see Anderson, Belnap and Dunn [3]. In particular, the so-called *Routley-Meyer semantics* for relevant implication uses a three-place relation suitable to give an intuitive meaning of information flow.

Third, we consider the issue of quantification again. Although existential quantifier presents no difficulty, universal quantifier gives rise to several interpretations. We think that there are at least two intriguing interpretations, namely static and dynamic interpretations. The static interpretation, which is usually assumed by situation theorists, reads:

$$V(s, \forall x A(x)) = 1 \text{ if } \forall c \in D(s)(s, V(A(c)) = 1)$$

The dynamic interpretation adopted by intuitionists reads:

$$V(s, \forall x A(x)) = 1 \text{ iff } \forall s' \geq s \forall c \in D(s')(s', V(A(c)) = 1)$$

Here, we neglect bounders because they do not affect the discussion below.

The static interpretation is simpler than the dynamic one. But, the price is to give up the persistency. The static interpretation was also defended by Wang and Mott [10], but they need some restriction on formulas. We adopt the dynamic interpretation since our aim is to develop an infon logic as a version of constructive logic with strong negation. However, one may support the static interpretation if one is not an intuitionist. These considerations reveal that constructive infon logic can be regarded as a starting point of the logic of situations. Some of its extensions should be worked out to address the issues discussed in this section.

ACKNOWLEDGEMENTS

We are grateful to a referee for his comments.

Seiki Akama
Department of Information Sciences, Teikyo Heisei University,
2289 Uruido, Ichihara, Chiba 290-0193, Japan
akama@thu.ac.jp
and
Yasunori Nagata
Department of Electrical and Electronics Engineering, Faculty of Engineering,
University of the Ryukyus, 1 Senbaru Nishihara, Okinawa 903-0213, Japan
ngt@eee.u-ryukyu.ac.jp

REFERENCES

[1] Akama, S. (1988): Constructive predicate logic with strong negation and model theory, *Notre Dame Journal of Formal Logic*, 29, 18–27.

[2] Almukdad, A. and Nelson, D. (1984): Constructible falsity and inexact predicates, *Journal of Symbolic Logic*, 49, 231–33.

[3] Anderson, R., Belnap, N. and Dunn, J. (1992): *Entailment vol. 2*, Princeton University Press, Princeton.

[4] Barwise, J. and Etchemendy, J. (1990): Information, infons, and inference, K. Cooper, K. Mukai, and J. Perry (eds.), *Situation Theory and its Applications, vol. 1*, 33–78, CSLI Lecture Notes 22, Stanford.

[5] Barwise, J. and Perry, J. (1983): *Situations and Attitudes*, MIT Press, Cambridge, Mass.

[6] Devlin, K. (1991): *Logic and Information*, Cambridge University Press, Cambridge.

[7] Nelson, D. (1949): Constructible falsity, *Journal of Symbolic Logic*, 14, 16–26.

[8] Smullyan, R. (1968): *First-Order Logic*, Springer, Berlin.

[9] Thomason, R.H. (1969): Semantical study of constructible falsity, *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 15, 247–57.

[10] Wang, X. and Mott, P. (1998): A variant of Thomason's first-order logic CF based on situations, *Notre Dame Journal of Formal Logic*. 39, 74–93.

[11] Wansing, H. (1993): *The Logic of Information Structures*, Springer, Berlin.