LOGICAL CONSEQUENCE AS TRUTH-PRESERVATION

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Abstract
It is often suggested that truth-preservation is insufficient for logical consequence, and that consequence needs to satisfy a further condition of relevance. Premises and conclusion in a valid consequence must be relevant to one another, and truth-preservation is too coarse-grained a notion to guarantee that. Thus logical consequence is the intersection of truth-preservation and relevance.

This situation has the absurd consequence that one might concede that the conclusion of an argument was true (since the argument had true premises and was truth-preserving); yet should refuse to infer the conclusion from the premises, in the absence of demonstration of the relevance of the premises to the conclusion.

The error lies in giving insufficient attention to the notion of truth-preservation. Relevance is no separable ingredient in the analysis of logical consequence, but a necessary condition of it. If an argument really is truth-preserving, then that in itself is enough to show that the premises are (logically) relevant to the conclusion.

1. *Implication and Relevance*

1.1. *The Paradoxes of Material Implication*

It is well-known that C.I. Lewis[1] criticized Russell and others for representing implication by a truth-functional connective. In fact, he went so far as to claim that, for this reason, Russell and Whitehead had not properly established the theorems of *Principia Mathematica* [22]. For their claim that the postulates implied the theorems, if all they meant by 'imply' was material implication, '⊃', said no more than that either the postulates were false, or the theorems true. This was not adequate for their proof, he said.

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[1] [5], p. 242; cf. [6], p. 432; [7], p. 247; see also p. 241 n. 1.

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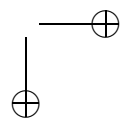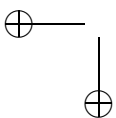
Norbert Wiener [23] (p. 656) replied that Lewis had here committed the fallacy of denying the antecedent. Just because Russell had the wrong sense of 'imply' did not mean that he had not shown that the postulates implied the theorems. In other words, Wiener interpreted Lewis as arguing that if Russell had understood 'imply' properly, he would have made sure that the postulates properly implied the theorems; but he misunderstood 'imply' and so *Principia Mathematica* was not what it seemed, the postulates did not imply the theorems, and the proofs were wanting.

Lewis was suitably provoked to restate his case [8]. Material implication, as used in *Principia Mathematica* [22], is not our usual notion of implication. It is responsible for what he calls the "peculiar" theorems, including those commonly known as the paradoxes of material implication:

(M1) $$-p \supset (p \supset q)$$

(M2) $$q \supset (p \supset q)$$

(with '$-$' for 'not' and '$\supset$' for material implication). But we never use this part of the theory, he claims, so the theory contains a redundant part:

> "Proof" requires that a connection of content or meaning or logical connection be established. And this is not done for the postulates and theorems in material implication ... For a relation which does not indicate relevance of content is merely a connection of "truth-values", not what we mean by a "logical" relation or "inference". ([8], p. 355)

He described his task in setting up his calculus of strict implication as developing a theory without the "peculiar" theorems and without the redundancy.

Lewis' inspiration in this task was an idea of Hugh MacColl's, that implication must show not only that it is not the case that the postulates are true and the theorems false, but that this is impossible. Implication must give a guarantee that truth is preserved, that the consequent of a true implication is true whenever the antecedent is true. MacColl and Lewis interpret this condition in the same way, in symbols:[2]

$$(pq')^{\eta}$$

$$\sim(p \, -q),$$

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[2] [13], §74, p. 78; [9], p. 293.

in each case read: it is impossible (η: MacColl; $\sim$: Lewis) that p and (concatenation) not- ($'$: MacColl; $-$: Lewis) q. In each case, concatenation represents extensional conjunction, 'p and q', true if and only if each of p and q is true.

1.2. *The Paradoxes of Strict Implication*

Suppose p is impossible: then so is 'p and not-q'. The first paradox of strict implication results immediately from the definition of strict implication. If p is impossible, then any proposition whatever is strictly implied by p. Similarly, if 'not-q' is impossible, that is, if q is necessary, so too is 'p and not-q'. So if q is necessary, any proposition whatever strictly implies it.

These results are paradoxical because, as in the case of material implication, an implication has arisen between propositions p and q which appear to have no "connection of content". To be sure, strict implication does arguably amount to more than a "connection of truth-values", for if p strictly implies q, we know not only that either p is false or q true, but that this is necessarily so. Nonetheless, what is paradoxical is that it appears that an implication can hold between p and q in virtue of a property of only one of p or q, independent of the other. Lewis indeed admits that they are analogues of the "peculiar" propositions of material implication:[3]

(S1) $$\sim p \dashv 3 (p \dashv 3 q)$$

(S2) $$\sim - q \dashv 3 (p \dashv 3 q)$$

(where $\dashv 3$ is strict implication). However, he accepts these as "sound logical principles" (p. 339), even though they at first appear "not in accord with the 'proper' sense of implies" (*ibid.*).

Lewis' derivation of the paradoxes of strict implication in his later volume (co-authored with Langford), *Symbolic Logic* [12], is well known—or at least the derivation of the first, a derivation already known in the twelfth century [14]. His argument in the *Survey* [9] (pp. 336-8) is somewhat different, depending on the principle of Antilogism, rather than Disjunctive Syllogism (or *Modus Tollendo Ponens*). First, note that any impossible proposition $p - p$ strictly implies its own negation. For $p - p \dashv 3 p + -p$ by Simplification and Addition (where '$+$' denotes extensional disjunction), and $p + -p \dashv 3 - (p - p)$ by a De Morgan transformation.

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[3] [9], p. 335.

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Suppose now that $p \prec -p$. Then $p - q \prec -p$ by Strengthening the Antecedent (or Simplification and Transitivity), and so

$$p -- p \prec q$$

by Antilogism, i.e.,
$$p \prec q.$$
Then, since we have shown that

$$p -p \prec -(p -p)$$

we can conclude that
$$(p -p)(p -p) \prec q$$
or, contracting,
$$(p -p) \prec q.$$
Thus if p is impossible, it entails anything, and so

(S1) $$\sim p \prec (p \prec q).$$

(S2) follows by contraposition. Accordingly, Lewis convinces himself that (S1) and (S2) are not peculiar, but sound principles of implication.

1.3. *Relevance*

But what has Lewis' derivation of (S1) shown? He claims ([9], p. 339) that it shows that impossible propositions imply anything, and defies "anyone [to] think that this validates the doubtful theorems of Material Implication [(M1) and (M2)]" or "to show that some proposition that is *false but not impossible* implies anything and everything" (*ibid.*). Yet (M1) and (M2) are as much theorems of the calculus of Material Implication as (S1) and (S2) are theorems of the calculus of Strict Implication. Their proof is straightforward. Defining $p \supset q$ as $-(p -q)$, we have $p -q \supset p$ by Simplification, and so $-p \supset -(p -q)$ by Contraposition, that is,

(M1) $$-p \supset (p \supset q);$$

and again by Simplification we have $qp \supset q$, and so by "Peano's Principle of Exportation" ([9], p. 231),

(M2) $$q \supset (p \supset q).$$

What these proofs show is that, if we suppose that $\dashv3$ represents implication (real implication), then an impossible proposition implies any proposition, and if $\supset$ represents implication, even a false proposition implies any other. Neither proof shows that these are sound principles about implication. They do show that if they are not sound principles of implication then in the one case either Simplification, Antilogism or Transitivity must fail for it, or in the other, either Simplification, Contraposition, Exportation or Transitivity. What they do not show is that proof does not after all require a connection of content, or that impossible propositions imply anything and everything, or that (S1) and (S2) are not after all peculiar.

We are left then with Lewis' claim that relevance is a necessary condition on inference and proof. Lewis, following MacColl, thought that relevance could be reduced to modality: rejecting the truth-functional analysis as permitting irrelevance, they both sought to capture the essence of implication by insisting on a necessary connection of truth-values. But that cannot be done. The paradoxes of strict implication show that Lewis' and MacColl's versions don't succeed. Bob Meyer gave a proof that no comparable formula can do it.[4] He showed that

- if $p \rightarrow q$ expresses implication in any system S extending the calculus of truth-functions, M, but weaker than or equal to that defined by the 3-point Sugihara matrix $(-1, 0, 1)$; and
- if $\Box$ (for necessity), added to the language of M to yield a system N, satisfies replacement of provable equivalents, that $\vdash_N p \Leftrightarrow q$ entail $\vdash_N \Box p \Leftrightarrow \Box q$, where $p \Leftrightarrow q =_{df} (p \Rightarrow q)\&(q \Rightarrow p)$ and $p \Rightarrow q =_{df} \Box \phi(p, q)$, for some truth-functional combination ϕ of p and q; and
- if $\rightarrow$ is introduced into N by the definition $p \rightarrow q =_{df} p \Rightarrow q$;
- then if N extends S, it will be a non-conservative extension of S, that is, N will contain some theorem in the common vocabulary of N and S (truth-functions and $\rightarrow$) which is not a theorem of S.

The system defined by the 3-point Sugihara matrix is usually called $RM3$. It is the simplest Sugihara matrix designed to ensure that there is no strongest or weakest formula in a calculus, where p is the strongest formula in S if $\vdash_S p \rightarrow q$ for all formulae q, and p is the weakest if $\vdash_S q \rightarrow p$ for all formulae q. Takeo Sugihara [21] argued that no decent implication would allow the construction of such strongest and weakest formulae. The full Sugihara matrix, containing all the integers, is characteristic of the system RM, R-mingle. Thus Meyer's proof shows that in all those systems which respect the original Lewis intuition, that there be no strongest or weakest formula, implication cannot be captured by the MacColl-Lewis formula $\Box (\overline{p} \vee q)$ or

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[4] See [15]; cf. [2], §29.12, pp. 462-71. (On p. 462, §29.12 is attributed to Meyer.)

equivalently, $\sim(p\,\overline{q})$ (or any other modalization of a truth-function[5]) —now writing $\overline{p}$ for 'not-p' and $p \vee q$ for 'p or q'. The modality does not succeed in capturing relevance. Meyer wrote:

> Relevance is another ingredient in the analysis of logical consequence, unnoticed by Lewis and irreducible to those vital and fertile modal intuitions of Lewis for which we are all in his debt.[6]

We have seen that Meyer is historically inaccurate here: Lewis did notice, but later ignored, the demand for relevance. But there is a further problem with Meyer's formulation, as we will see, when he describes relevance as an *ingredient* in the analysis of consequence (or implication). In one sense, this is right: all correct implication is relevant; relevance is a necessary condition of valid consequence. But in another sense it is not: valid consequence cannot be truth-preservation plus relevance. Relevance cannot be a sieve on truth-preserving inference yielding relevant consequence.

2. *The Central Puzzle*

2.1. *Relevance and Truth-preservation*

Meyer is not the only writer to suggest that truth-preservation is inadequate to capture logical consequence and that consequence needs to satisfy a further condition of relevance. For example, Greg Restall writes in a recent article [19], p. 158: "it is not sufficient for B to be a consequence of A that, necessarily, given that A is true, so is B (or more crudely, that B is true in all worlds in which A is true)." Kielkopf, in his book [3], p. 12 claimed similarly that the Lewis paradoxes showed that formal validity (i.e., truth-preservation) was not sufficient for formal correctness (i.e., entailment or implication). Kielkopf traces the thought to G.E. Moore, in his introduction of the term 'entail' for the converse of 'follow from', as a synonym for 'implies' in the special sense of 'logically implies':

> A necessary condition for its being true that p implies q is that it shall be self-contradictory to assert that p is true but

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[5] In a recent article, Philip Kremer [4] shows that the $\rightarrow$ of R (see below) can be defined in an extension of *S4* with propositional quantifiers and definite propositional descriptions. This is not incompatible with Meyer's result, and in fact uses ideas of Meyer's and others' from the worlds semantics for R in the definition. In particular, Kremer's definition of $\rightarrow$ is not as the modalization of a truth-function.

[6] [2], p. 471.

> q is false. But I do not think that … the fact that it would
> be self-contradictory to assert that p is true but q is false is a
> *sufficient* condition for its being true that p implies q. ([16],
> p. 182)

Lastly, take Gerhard Schurz's programme of relevant deduction. In a recent
paper, he describes his project as belonging to the strategy of "*restricting the
given (classical) logic* by additional *relevance criteria*:"

> One should distinguish between *validity* in the sense of math-
> ematical logic and *appropriateness* with respect to *applied
> arguments* … So the paradoxes have to be avoided by defin-
> ing appropriate relevance criteria, singling out the relevant
> from the irrelevant deductions among all valid deductions.
> ([20], p. 399)

In all these writers, the claim is that mere truth-preservation is too coarse-
grained a notion to guarantee that the premises and conclusion are relevant
to one another. Truth-preservation (they say) is captured by the classical
condition that it be impossible for the premises to be true and conclusion
false. The paradoxes of strict implication show that that condition needs to
be supplemented by a further (relevance) condition. Of course, relevance
by itself is not sufficient, for it is symmetrical where consequence need not
be. Thus logical consequence is the intersection of truth-preservation and
relevance.

A contrary view was articulated by Arthur Prior ([18], p. 196). Recall
Lewis' comment that proof required "a connection of content or meaning
or logical connection." We are here analysing *logical* implication, or *logical*
consequence, Prior observed. Commenting on the claim by Nelson [17] "that
'entailment', i.e. the converse of 'following from', must consist in a definite
inner connexion between antecedent and consequent", he wrote:

> For some reason, [Nelson] does not consider the possibility
> that what Lewis's paradoxes show is precisely that necessary
> and impossible propositions as such *have* a definite inner con-
> nexion with all propositions whatever. (p. 196)

No better mark of a *logical* connection between one proposition and another
could be given than to be able to derive the one from the other. This position,
in contrast with the other, is at least coherent. Its fault is the same as that we
noted with Lewis' defence of the paradoxes of strict implication: it is empty
of content until we determine the mark of a correct derivation. That, we
will see, is why relevance is a necessary condition for valid inference. For
if we have shown that one proposition validly follows from another, then we
will *eo ipso* have shown they are logically connected and therefore mutually
relevant.

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2.2. *Truth-preservation*

Suppose, then, that we are presented with an argument for some conclusion
which is truth-preserving, that is, for which it is impossible that the premises
be true and the conclusion false. Moreover, suppose that we are persuaded
that the premises are true. It follows that it is impossible for the conclusion
to be false. Persuaded, for the present, by Restall, Moore, Schurz and others
that truth-preservation is not sufficient for valid inference, however, we jib
at conceding that the conclusion is true. For we have supposed only that
the argument is truth-preserving. Nothing has been said about its preserving
relevance. Accordingly, we refuse to infer the conclusion from the premises,
pending demonstration of relevance of premises to conclusion.

This looks to be a stony road to travel. The premises are true; and it is
impossible for the premises to be true and the conclusion false. So surely, we
exclaim, the conclusion must be true too. Yet is it coherent to concede that
the conclusion is true; to agree that the reason we concede it to be true is that
the argument is truth-preserving and that the premises are true; but refuse
to infer the conclusion from the premises on the ground that the premises
have not been shown to be relevant to the conclusion? Can we admit that the
argument shows that the conclusion is true and yet deny that the premises,
which we agree are true, imply or entail the conclusion? That seems absurd.
What more can we demand of a truth-preserving argument than that the truth
of its premises show that its conclusion is true?

The fact is that we are equivocating over the meaning of 'truth-preserva-
tion'. If an argument really is truth-preserving and has true premises then
there is no doubt that the conclusion is true. In other words, if the con-
clusion is true whenever the premises are, that is, the premises entail the
conclusion, and the premises are true, so too must be the conclusion. But
hitherto we have used 'truth-preservation' differently, to mean 'impossible
for the premises to be true and the conclusion false', and by that we mean,
in Lewis' symbols, $\sim(p\overline{q})$, where p represents the premises and q the con-
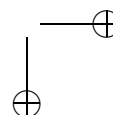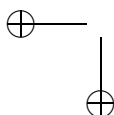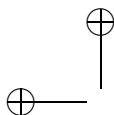clusion. So the form of argument is

(P1) $\sim(p\overline{q}), p \vdash q.$

But this sequent cannot be conceded, on pain of re-introducing the paradoxes
of strict implication. For ' $\sim$ ' is a modal connective ('impossible'), and so
we can modally conditionalize, to obtain

$$\sim(p\overline{q}) \vdash p \dashv3 q$$

and since

$$\sim p \vdash \ \sim(p\overline{q}) \qquad\qquad (*)$$

follows immediately by Contraposition from Lewis' Consistency Postulate,[7] we have

(S1) $$\sim p \vdash p \prec q$$

by Transitivity.

(P1) is in fact a modal form of *Modus Ponendo Tollens*,

$$\overline{p\overline{q}}, p \vdash q,$$

in its turn trivially equivalent to *Modus Tollendo Ponens* (or Disjunctive Syllogism), notorious for its failure in relevance logics:

(P2) $$\overline{p} \vee q, p \vdash q.$$

What is needed to avoid this puzzle is a better account of truth-preservation. If we are determined to avoid the paradoxes of implication, we need to focus again on the major premise of the above inference, to discern its true logical form. The thought is this: we have a truth-preserving argument from true premises p to conclusion q. What is the least that the major premise (that the argument is truth-preserving) must say for this inference to be valid? It must say that p implies q. For if it is properly modal, that is, 'truth-preserving' means 'however things are, the conclusion is true whenever the premises are', it will support Conditionalization, to infer 'p implies q' from the major premise; and if p does imply q, then we can infer q from p. So the form of argument reads:

$$p \rightarrow q, p \vdash q,$$

where '$\rightarrow$' represents 'implies', and '$p \rightarrow q$' expresses the fact that the inference from p to q is truth-preserving.

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[7] The Consistency Postulate reads $\Diamond(pq) \prec \Diamond p$, where '$\Diamond$' means 'possible'. See [12], pp. 166-8. Alternatively, we can derive (*) from the correct half of postulate (1.8) in the *Survey*, *viz* $(p \prec q) \prec (\sim q \prec \sim p)$ (given the obviously valid $p\overline{q} \prec p$). On the incorrectness of (1.8) as given in [9], see [10].

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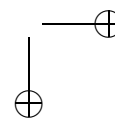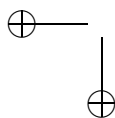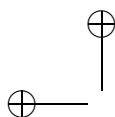
3. *Relevant Consequence*

3.1. *Fusion*

What, however, is the correct analysis of '$p \to q$'? We have seen that it is
not properly captured by Lewis' $\sim(p\bar{q})$; yet it is natural to read 'the truth
of p entails that of q' as 'it is impossible for p to be true and q false'. The
solution is found by reversing the order of explanation. Instead of trying to
explain '$p \to q$' by an analysis of Lewis' kind, we should instead go directly
for an account of '$p \to q$', and use that account of implication to explain the
formula concerning the impossibility of p and not-q.

We noted that Sugihara put forward a matrix as a way of screening out
irrelevant strongest and weakest formulas. However, the logic defined by the
(full) Sugihara matrix, *viz RM* (*R*-mingle),[8] still contains theses which are
not relevantly acceptable, for example,

$$(p \to q) \vee (q \to p).^9$$

If '$\to$' is to capture a notion of relevant implication, implication between
propositions between which there is a real "connection of content or mean-
ing or logical connection", in Lewis' phrase, then the suggestion that any
two propositions are so connected is clearly absurd. Although the Sugihara
matrix serves to exclude some unwanted implications, it is not sufficient to
exclude all.

Wilhelm Ackermann took up the challenge of Lewis' to provide the correct
analysis of the concept of implication, one, as Ackermann put it, "intended
to express the idea that a logical connection holds between [p] and [q], that
the content of [q] is part of the content of [p], or however else one wishes to

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[8] [2] I §29.3.

[9] The proof is straightforward. The Sugihara matrix consists of two copies of the integers,
J_1 and J_2 with (everything in) $J_1 \leq$ (everything in) J_2 and where J_2 are the designated
elements. '$\vee$' is interpreted as $\max$ and

$$v(p \to q) = \begin{cases} \max(v(p)^*, v(q)) & \text{if } v(p) \leq v(q) \\ \min(v(p)^*, v(q)) & \text{otherwise} \end{cases},$$

where if $v(p) = j \in J_1$ (resp. J_2) then $v(p)^* = -j \in J_2$ (resp. J_1). Then if $v(p) \leq v(q)$,
$v(p \to q) \in J_2$. So if $v(p \to q)$ is undesignated, then $v(p) > v(q)$, so $v(q \to p)$ is
designated.

put it."[10] His calculus Π' became the calculus E of entailment of Anderson and Belnap on dropping the admissible rule (γ),[11] E in turn gave rise to the calculus R of relevant implication on dropping the modal aspect of $\rightarrow$ in E, and E itself finally gave way to $R^\square$, the result of adding modality explicitly to R, once E and $R^\square$ were shown distinct.[12] The semantics for E and R then showed that interest should focus on the connective/operation of fusion, $\circ$, connected with $\rightarrow$ by residuation:[13]

$$p \circ q \rightarrow r \dashv\vdash p \rightarrow (q \rightarrow r).$$

One can then show in R that

$$p \circ q \dashv\vdash \overline{p \rightarrow \overline{q}}$$

and conversely,

$$p \rightarrow q \dashv\vdash \overline{p \circ \overline{q}}.$$

In $R^\square$, let $p \Rightarrow q =_{df} \square(p \rightarrow q)$, and let $\sim p =_{df} \square\overline{p}$. It follows that

$$p \Rightarrow q \dashv\vdash \ \sim(p \circ \overline{q}),$$

that is, p entails q just when '$p \circ \overline{q}$' is impossible. Is it plausible to read this as 'it is impossible that p and not-q'?

3.2. *Residuation*

The natural reading of 'and' is as extensional conjunction, such that 'p and q' is true iff p is true and q is true. That this homophonic truth-condition captures the truth-function of conjunction depends, clearly, on the reading of the occurrence of 'and' on the right-hand side. (If it meant 'or', for example, then it would be false unless the occurrence of 'and' on the left also meant 'or'.) Continuing to represent this sense of 'and' by concatenation, it is clear

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[10] [1], p. 113: "Die strenge Implikation, die wir durch $A \rightarrow B$ wiedergeben, soll ausdrücken, daß zwischen A und B ein logischer Zusammenhang besteht, daß der Inhalt von B ein Teil des Inhaltes von A ist, oder wie wir es sonst ausdrücken wollen."

[11] [2] II §41; that (γ) is admissible for E, see [2] I §25.

[12] [2] I §28.1.

[13] The connective $\circ$ matches an operation $\circ$ in the algebraic semantics and an operation $\circ$ in the worlds-semantics.

that its residual is material implication:

$$pq \supset r \dashv\vdash p \supset (q \supset r)$$

that is,

$$\overline{pq} \vee r \dashv\vdash \overline{p} \vee \overline{q} \vee r.^{14}$$

Consider *Modus Ponens*, in the sense of detachment for $\rightarrow$. From

$$p \rightarrow q \vdash p \rightarrow q$$

we infer by Residuation, since $\rightarrow$ is (by definition) the residual of $\circ$ (fusion):

$$(p \rightarrow q) \circ p \vdash q.$$

Assuming that $\rightarrow$ is a restriction of $\supset$, we have

$$p \rightarrow q \vdash p \supset q$$

whence

$$(p \rightarrow q)p \vdash q.$$

Indeed, in general, if $p \circ q \rightarrow r$ then $pq \rightarrow r$, i.e., $pq \rightarrow p \circ q$. But they are not equivalent, since, as we saw in §1.3 above, residuation for $\supset$ (i.e., Exportation) leads straight to (M2). In particular, Simplification must fail for $\circ$: $p \circ q \not\vdash p$ and $p \circ q \not\vdash q$.

It follows that fusion represents a manner of composing the premises of an argument weaker than extensional conjunction, yet sufficient to allow the premises to act together to entail the conclusion. What is distinctive about this manner of composition is that a fusion can contain no unnecessary part. Note that, although

$$p \rightarrow p \circ p,$$

$\circ$ is not idempotent, since $p \circ p \rightarrow p \dashv\vdash p \rightarrow (p \rightarrow p)$, and the right-hand side of this is the Mingle axiom, from which follows that unwanted $(p \rightarrow q) \vee (q \rightarrow p)$ (see §3.1 above).

The identification of fusion in the 1960s was clearly a theoretical advance of the first order, marking a distinction from extensional conjunction which deepened our understanding of the implicational paradoxes and of implication itself. The question is: how should '$p \circ q$' be read? For two reasons, the reading 'p and q' is most appropriate. First, if $\circ$ is genuinely novel, we can at best press into service whatever word is closest among those we have. Since $\circ$ expresses the composition of premises in entailing their joint conclusion,

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[14] The commutativity of pq ensures that $\supset$ is both right and left residual.
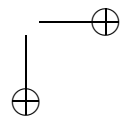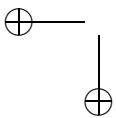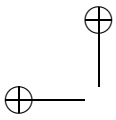
nothing comes closer than 'and'. To be sure, we now need to exercise care to avoid ambiguity and misunderstanding, so that we know when 'p and q' expresses 'pq' and when '$p \circ q$'. Were the dangers of such confusion too bad—and where they are—we might consider a neologism, 'p fuse q'.

But secondly, is '$\circ$' so very novel? Consider the equivalence between $p \Rightarrow q$ and $\sim(p \circ \overline{q})$. We already use 'and' to express truth-preservation: that it be impossible for p to be true and q false. Taking 'and' to mean extensional conjunction led early workers on the theory of implication to the mistaken theories of material and strict implication—mistaken, that is, as theories of implication. If we use 'and' to express $\sim(p \circ \overline{q})$, we find that the existing formula exactly captures the new theory of implication. In other words, the novel theoretical identification of '$\circ$' reveals an existing ambiguity in the word 'and': when we say that p and q entail r, for example, do we mean that pq entail r, or that $p \circ q$ entail r? The difference is not always significant. But when it is (when, for example, pq entail r but $p \circ q$ don't—when r is p, say), then we can now mark that difference and understand its significance.

4. *Conclusion*

Relevance is a necessary condition of implication. Nothing in this paper should suggest otherwise, and indeed, the whole paper is premised on that belief. However, the concept of relevance is not an ingredient in the concept of implication. I think Meyer did not mean to suggest it was (see footnote 6 above). For he wrote:

> It would be a mistake to think that because certain logics are called 'relevant' they give an all-around account of relevance ... The persistent error of previous attempts to find a relevant implication was in the thought that we already understood implication, and only needed to understand relevance in addition. ([15], pp. 228-9)

So the correct inference from his demonstration that implication (that is, entailment) is not the modalization of a truth-function is not that we need to analyse the concept of relevance to obtain a sieve by which to separate out the valid inferences from those that are merely truth-preserving. Rather, it shows that strict implication and its like do not properly capture the concept of truth-preservation at all. If an advocate of relevance as a necessary condition of entailment gives his opponents the concept of truth-preservation, he will be faced with the unattractive prospect of refusing to infer a conclusion from certain premises which at the same time, given the truth of

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those premises, he has to admit is true. Meyer's proof shows that truth-preservation (that it be impossible for the premises to be true and the conclusion false) is not properly analysed as the modalization of a truth-function. Truth-preserving arguments are those whose premises really, relevantly, entail their conclusions, that is, the truth of whose premises logically implies the truth of their conclusions.

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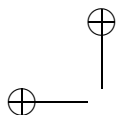
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