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SURPRISING USER-FRIENDLINESS*

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Abstract

Some theorists are bewildered by the effectiveness of mathematical concepts. For example, Steiner attempts to show that there can be no rational explanation of mathematical applicability in physics. Others (notably Penrose) are concerned primarily with the unexpected effectiveness within mathematics. Both views consist of two parts: a puzzle and a positive solution. I defend their paradoxical parts against the sceptics who do not believe that the very problem of effectiveness is a genuine one. Utilising Horwich's theory of surprise, I argue that the central cases of effectiveness discussed by Steiner and Penrose are indeed surprising and call for an explanation.

1. *Steiner's puzzle*

There is a familiar problem of mathematical applicability which many theorists consider being *the* problem of mathematical applicability. Suppose I have two apples in one hand and two apples in another hand. I put all of the apples I am holding into the basket, and — lo and behold — there are four apples there. I repeat the procedure many times, and every time there are four apples in the basket, no less and no more. And suppose further I make a parody of Hempel, and think of my procedure as an experiment:

$$\frac{C_1 \qquad C_2 \qquad C_3}{E}$$

where C_1 is having two apples in one hand, C_2 is having two apples in another hand, and C_3 is putting them into the basket, whilst E is the quantity of apples in the basket. This would not perhaps appear an exceedingly exciting result, because it is so ordinary. But the exciting thing about it is the

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explanation we provide for it, when we try to establish the link between the antecedent conditions and the outcome. It is, we say, in virtue of the truth of the arithmetical statement

$$2+2=4 \tag{1}$$

that we get four apples in the basket. The antecedent conditions are empirical, the outcome is empirical, but the explanation is non-empirical, and herein lies the awkwardness of our explanation. For we believe that arithmetic is non-empirical, that arithmetical terms refer, that there are mathematical objects (numbers, in our case), and these objects are *remote*: they do not affect and they are not affected by the events of our physical world, they are not created and they are not destroyed.

Hence, the first exciting thing is the apparent gap between the physical events and the remote objects. The second exciting thing is of course the necessary link between $\{C_i\}$ and E. Once I know how many apples I had at the beginning and know how many there were in the basket, I do not seem to able to understand how it could be otherwise. This shows me that my apple experiment is not an ordinary one. If I jump from a sky-scraper and land on my two feet after a one-hour fall, then — assuming the weather conditions in the vicinity were usual — I can imagine a merciful god adjusting the gravitational constant. But I cannot imagine how

$$2+2=5$$

could be true. I also realize that its unimaginability is not a sign of my limited intellectual capacities, but rather a mark of the strong necessity between $\{C_i\}$ and E, deriving from the necessity of (1).

The third exciting thing about the experiment is that I cannot describe it — talk about it — without resorting to arithmetical concepts. These latter invariably appear in every version of the description I produce. Quite a different question is whether they commit me to the existence of numbers. Frege's view — endorsed by Steiner — identifies numerals with second-order predicates applying to ordinary concepts. The concepts themselves apply to objects. In this way, the objectual commitment is rules out. But even so the application of arithmetical laws to empirical laws occurs by necessity.

I make these familiar observations in order to have a better grip on the metaphysical applicability problem. It has, therefore, five theses:

(i) The truth of the physical laws demands the existence of remote objects;

(ii) There must be an explanatory link between the existence of remote objects and empirical events;

(iii) Remote objects have no explanatory power for empirical events (because they are remote);
(iv) The explanation they provide bears marks of a particularly strong kind of necessity;
(v) The application of mathematical objects is necessary.

Only the theses (i)–(iii) are at heart of the puzzle. For the physical laws, even the simplest ones, imply interaction between mathematical objects and empirical objects. In our primitive example, the law we are using is in effect this:

Law 1.1: *For every (stable, middle-sized) object, if there are m objects in one container and n objects in another container, bringing them together in the third container will result in $m + n$ objects being there.*

This law is unlikely to be found in any physical textbook, because so many details pertaining to the nature of objects have to be filled in it. It also seems to talk about specific circumstances — the presence of containers, for example, thus lacking the generality required. Despite being *ad hoc* and non-explanatory, it has predictive capacities and it suffices for our purposes: we are shown how mathematical objects feature in empirical predictions. Their existence is indispensable for the truth of these predictions, and therefore, we could expect some link to be exhibited between those objects and empirical events. But such a link is missing.

In (iv)-(v) we also notice two further aspects of the uneasy relationship between empirical events and remote objects. They do not constitute a puzzle, but will later be useful in assessing a different sense of applicability. You may also have qualms about the precise rôle mathematical entities play in the formulation of laws, or qualms about the intelligibility of (i). Let these qualms rest for a while until we introduce another applicability problem which is also Steiner's main concern.

Steiner claims that metaphysical applicability is unproblematic, or 'non-mystic' as he says (6, 22). His claim is correct, and one of my purposes will be to justify the absence of mystery there. Another sense of applicability — and a novel one, according to Steiner — is descriptive applicability. It is introduced thus:

> [D]escriptive applicability [is] the appropriateness of (specific) mathematical *concepts* in describing *and lawfully predicting* physical phænomena. …Applying [in this sense is] 'describing by means of'. ((6, 25) his italics)

The first thing distinguishing descriptive applicability from metaphysical applicability is its lack of generality. There is no single problem of descriptive applicability. Rather, there are many problems related to different concepts, each of which may require different solutions. Steiner gives the examples of some concepts whose applicability is unproblematic. One is the phænomenon of linearity. Many physical correlations have the form:

$$y = k_1 x_1 + k_2 x_2 + \ldots + k_i x_i + \ldots, \tag{2}$$

of which one instance would be the dependence of speed on time:

$$v = gt. \tag{3}$$

Linearity makes its other appearance in the principle of superposition which can be written e.g. for an electric field with several sources of electric charge:

$$\mathsf{E} = \mathsf{E}_1 + \mathsf{E}_2 + \mathsf{E}_3 + \ldots \tag{4}$$ [1]

The linearity of physical laws is explained once we notice that it works in the situations where physical processes are smooth. If they are not, linearity no longer holds.

Why does Steiner believe that there is no problem to tackle with linearity? On the face of it, he supplies a causal explanation of its applicability. The smoothness of natural processes was the *cause* for using the schema (2). The causal idiom can be understood only in the human context: because the nature is smooth and the scientists' experiences were such and such, they came to use (2). If we delete the reference to scientists, causation is lost. Surely, the formulation of the laws *in itself* could not be affected by nature. But in fact Steiner never mentions the human factor and the particular experiences responsible for the usefulness of (2). The question is rather that there must be some reason why (3) and (4) share the same mathematical form. The *prima facie* problem arises once you believe that this cannot be a mere co-incidence. The solution, then, is that both (3) and (4) describe smooth processes.

I suggest, however, that the problem cannot even be made intelligible without bringing in the human factor. Smooth processes cannot be identified unless with the aid of (2). The real issue seems to be in the necessity of mathematical applicability. Could there be laws not expressible in mathematical concepts? Or alternatively, could there be a world in which mathematics is not applicable — in which it is either true and not applicable, or just a

[1] Cf. (1, ch. XII-4) for details.

game? It is surely possible that not every mathematical concept applicable in our world should be applicable in every other possible world. A universe sufficiently poorer than our actual world will be described by significantly simpler laws. The actual example is difficult to devise: even the world consisting just of one molecule of hydrogen will have quantum phænomena, gravitational effects etc.; hence much of the actually applicable mathematics will be applicable in that world too. But we may think of a world consisting solely of a counterfactual element *hydrogen-minus* which does not generate, say, electromagnetic fields, and therefore, some actually applicable differential equations will not have physical meaning there. Similarly, a determinist world may render statistical concepts physically meaningless (at least for an objectivist about probability). But the world which is governed by laws employing no mathematical concept is simply indescribable.

One might sense a complication here. If mathematics is applicable necessarily, there might be nothing surprising in it. If ϕ is surprising, then at the very least $\neg\phi$ should be possibly true. Surprise occurs when our antecedent beliefs are revised in light of the new fact. But if we expected $\neg\phi$ all along instead of ϕ and the latter is necessarily true, we were a victim of a massive delusion. ϕ is true alright, we say, but think about it! — we erroneously add — it might have been that $\neg\phi$ is true (or: it might have turned out that $\neg\phi$). The illusion of surprise goes hand in hand with the illusion of ϕ's contingency, because surprise occurs when we locate ourselves in the world w (where ϕ) rather than u (where $\neg\phi$). However, when ϕ is necessary, there is no such u, and the surprise was illusory. These observations are crystallized into the following postulate:

Postulate 1.1: *If ϕ is surprising, then ϕ is contingent.*

We shall attempt to justify this claim to some extent later on. For the moment let us revert to Steiner's discussion. The two issues he does not always take care to keep separate are the usefulness of mathematical concepts in physics and the process of discovery of physical laws by a manipulation of mathematical concepts:

Fact 1.1: *Physical laws employ mathematical concepts.*

and:

Fact 1.2: *Mathematical concepts first invented for mathematical purposes later prove fundamental in physical laws.*

That this criticism is not entirely fair is evident from the following passage:

> My thesis concerns ... the successful deployment of a taxonomy, a scheme. Success is measured by whether the discoveries that scientists were looking for (the laws of atomic and subatomic particles; explanations for the various anomalies of the atomic and subatomic world) were in fact found in due time — using the scheme. (6, 73)
>
> [H]istorical Pythagoreanism was primarily metaphysics; I accent its epistemology. Thus, I shall not discuss whether the world 'is' numbers. (6, 60)
>
> Unlike Wigner, I shall explore the peculiar rôle of mathematics in scientific discovery. (6, 47)

Nevertheless, it is never clear why, by Steiner's lights, the Fact 1.1 does not deserve an explanation. Steiner claims that mathematics is anthropocentric, arising from 'the human aesthetic sense' (6, 64). If this is so, the Fact 1.1 should look surprising. Perhaps the application of *some* mathematical concepts is not surprising, but the application of *other* concepts is indeed.

2. *Penrose's puzzle*

We now turn to a different issue, that of the usefulness of mathematical concepts within mathematics. Like Steiner, Roger Penrose is fascinated with the remarkable utility of mathematical concepts in physical research. Of the utility of real numbers in measuring distance, time, energy he writes:

> The real number system is chosen in physics for is *mathematical* utility, simplicity, and elegance, together with the fact that it accords, over a very wide range, with the physical concepts of distance and time. It is *not* chosen because it is known to agree with these physical concepts over *all* ranges. ... As it turns out, *Nature is remarkably kind to us*, and it appears that the same real numbers that we have grown used to for the description of things at an everyday scale or large *retain their usefulness on scales much smaller than atoms* — certainly down to less than one-hundredth of the 'classical' diameter of a sub-atomic particle, say an electron or proton — and seemingly down to the 'quantum gravity scale', twenty orders of magnitude smaller than such a particle. ... Why is there so much confidence in these numbers for the accurate description of physics, when our initial experience of the relevance of such numbers lies in a comparatively limited range? This confidence — perhaps misplaced — must rest (although

> this fact is not often recognized) on the logical elegance, consistency, and mathematical power of the real number system, *together with a belief in the profound mathematical harmony of Nature*. ((4, 113–4), some italics added)

Nevertheless, Penrose does not dwell too much on the application of mathematics. His chief concern is to confront the idea that mathematical concepts are mental constructions and have no reality of their own. In order to defend realism about mathematical concepts he appeals to their remarkable usefulness within mathematics, and the most elaborate example he gives is the invention of complex numbers. They were introduced by Cardano for taking square roots of negative numbers and thus facilitating the solution of cubic equations. Later on, however, complex numbers were found to possess wonderful properties such as. In particular, the equivalences

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

are reinterpreted as:

$$e^{\imath A + \imath B} = e^{\imath A} e^{\imath B}, \tag{5}$$

since $e^{\imath z} = \cos z + \imath \sin z$. Similarly, Gauss proved that the algebraic equations with complex co-efficients:

$$a_0 + a_1 z^1 + \ldots + a_n z^n = 0$$

always have a solution for some complex number z.

Many more applications of them are found in mathematics, and they clearly surpass any motives Cardano could have for introducing them originally. The observations over complex numbers are generalised and eventually the following fact emerges:

Fact 2.1: *The intra-mathematical use of mathematical concepts can be extended much wider than the original inventors could possibly contemplate.*

Parallel to Steiner's metaphysical fact, we could also formulate a fact relevant to Penrose's concerns:

Fact 2.2: *Many mathematical concepts are used in a wide range of fields within mathematics.*

The Fact 2.2 does not say anything specific about the way mathematicians discover the concepts. It only stresses the unity of mathematical discipline, or perhaps it serves a reductionist purpose by leading to a conclusion that every mathematical discipline is set theory in in disguise. Yet, as it happens, Penrose is not interested in the Fact 2.2 at all, occupying himself solely with the Fact 2.1. The latter grounds three claims:

Proposition 2.1: *Mathematical concepts are not inventions.*

Proposition 2.2: *Mathematical concepts exist in the world and are to be discovered.*

Proposition 2.3: *Mathematics is user-friendly.*

Consider first Proposition 2.1. It may appear to be a modus tollens and run as follows. Suppose complex numbers were Cardano's inventions (P); then they could not have had the properties beyond those known by Cardano (R); but they do ($\neg R$); therefore, they are not inventions ($\neg P$). There is, however, a justification missing for the transition from P to R. The missing step amounts to the claim that the properties of anything that is invented cannot be unknown by the inventor (Q). Then the argument is presented as a *reductio ad absurdum*:

$$\dfrac{\neg(P \supset R) \qquad \dfrac{P \supset Q \qquad Q \supset R}{P \supset R}}{\bot}$$

Or in the second-order language:

$$\dfrac{\exists x \exists F(Cn(x) \wedge Fx \supset \neg K(\mathsf{C}, Fx)) \qquad \dfrac{\forall x(Cn(x) \supset Inv(\mathsf{C}, x)) \qquad \forall x \forall F(Inv(\mathsf{C}, x) \wedge Fx \supset K(\mathsf{C}, Fx))}{\forall x \forall F(Cn(x) \wedge Fx \supset K(\mathsf{C}, Fx))}}{\bot}$$

We have arrived at a contradiction, but we have a choice: declare either $\forall x(Cn(x) \supset Inv(\mathsf{C}, x))$ or $\forall x \forall F(Inv(\mathsf{C}, x) \wedge Fx \supset K(\mathsf{C}, Fx))$ false. Penrose elects to reject the first premiss. Yet it remains unclear why the inventor (or a creator) has a special authority over his invention (creation). A typical instance of invention is some technological device. Suppose Hammurabi invented the wheel. Of course, he could not anticipate the use of the wheel in motor cars, windmills, or in the carousel. But we are not inclined to claim that *because of that* Hammurabi discovered the wheel, and not invented it. An even more convincing example is a game like soccer. We are certainly prepared to grant its invention. We do not assume, however, that the original inventor could claim superiority over Ronaldo thanks to the mere fact of his role in soccer's invention. Perhaps the sense of invention targeted by Penrose occurs in fictions. There we may reasonably insist that the author has a special, unchallenged authority over the properties of his characters.[2]

Further, Penrose apparently assumes that anything can be either invented (like a wheel or a novel) or discovered (like a planet). Since mathematical concepts are not invented, they are discovered[3]:

$$\frac{\forall x(Disc(x) \vee Inv(x)) \qquad \exists C \neg Inv(C)}{\exists C Disc(C)} \tag{6}$$

One might attempt to find a third alternative. Consider emotions. At the age of amoebas and bacteria there were no emotions. The first animal (or human) who experienced hatred, fear, or love was not discovering anything. But neither was he inventing them. So the case of emotions presents a useful challenge to (6). A parallel argument for the case of mathematical objects is made by Kant. We shall say a few words about it below.

Proposition 2.2 obviously follows from Proposition 2.1 and the argument (6). Proposition 2.3 is never defended by Penrose explicitly. Realism announced in Proposition 2.2 is deemed to be a fully satisfactory explanation of the Fact 2.1. One might, however, take a further step and question the causes of, e.g., Cardano's enormously successful discovery. There must be a user-friendly link between the discoverer and the discovered. Secondly, there is no warrant why mathematical objects discovered with one purpose by mind should later prove useful in the 'wide extensions' mentioned in the Fact 2.1. Realism *by itself* does not help in explaining this sort of success. Such an argument is appealing in so far as we believe in the surprisingness of the Fact 2.1.

3. *An account of surprisingness*

If the surprisingness of the usefulness of mathematical concepts is at stake, we need a more reliable guide in evaluating surprisingness[4]. The first thing

[2] Cf. Cervantes in *Don Quixote de la Mancha*: 'For me alone was Don Quixote born, and I for him: he knew how to act, and I how to write: we were destined for each other.'

[3] We face a potentially dangerous retort that our quantifiers in the following proof are all objectual, whereas we need second-order quantifiers to quantify over concepts. Let us ignore this difficulty.

[4] I borrow heavily here from the published and unpublished work of Roger White, as well as from (2).

to do is to distinguish between surprising and improbable facts. One might believe that the probability of a monkey typing 'I want a banana' (the outcome E) is extremely low, whereas the probability of it typing 'tyue ojn48 iwl;' (the outcome $E*$) is high. In reality, meanwhile, these probabilities are very much the same. Both of them are random sequences of fifteen characters. But whereas the typing of the second sequence appears ordinary and dull, the typing of the first would be surprising. Hence surprisingness cannot be identified merely with low probabilities. Looking at the case more carefully, we notice one particular phænomenon: the typing of a meaningful phrase undermines our initial view of the situation. Initially we thought we were dealing with an animal unable to understand human language, and a typing machine, whose performance does not affect the probability of the outcome E as against the outcome $E*$.

With the monkey typing a meaningful phrase, this assumption was put under threat. Let us designate by H our initial hypothesis and by $H*$ the rival hypothesis, which could be either that the monkey understands human language, or the typing device is rigged, or both. We assume that H and $H*$ are exhaustive. Then on the one hand, from Bayes's theorem we have:

$$P(H|E) = \frac{P(H)P(E|H)}{P(E)}$$

but on the other:

$$P(E) = P(H)P(E|H) + P(H*)P(E|H*).$$

Putting that together, we get:

$$P(H|E) = \frac{P(H)P(E|H)}{P(H)P(E|H) + P(H*)P(E|H*)}. \tag{7}$$

The surprisingness effect occurs if there is a significant decrease of the probability of H in light of the new evidence supplied by E: $P(H|E) \ll P(H)$. The fraction above shows that this is achieved when $P(E|H)P(H) \ll P(E|H*)P(H*)$ — when, in other words, there is an alternative hypothesis, itself not highly implausible, which makes E highly probable.

This condition clarifies the difference between the two equally improbable events E and $E*$. The difficulty, however, arises when we need to assess the cases in which the hypothesis $H*$ is itself incredible.[5] For instance, we might rule out from the outset the possibility of the rigged keyboard, itself a

[5] This drawback in Horwich's account was noticed by White.

not too implausible assumption. The remaining hypothesis — that monkeys understand human language — is still wildly implausible. Yet the surprisingness of the phænomenon does not disappear, whereas the probability of E on the assumption of $H*$ increases:

$$P(E|H) < P(E|H*)$$

where $H*$ is consistent with $\neg H$. The new surprisingness condition has, therefore, little to do with the implausibility of the rival hypothesis. Surprising phænomena induce a shift in our antecedent beliefs, however improbable the new beliefs might be.

4. *No surprising necessary statements*

We are now going to pay our debt and defend the Postulate 1.1. Wittgenstein's *Tractatus* contains a couple of gnomic remarks denying the surprisingness of logical propositions:

> 6.125 It is possible—indeed possible even according to the old conception of logic—to give in advance a description of all 'true' logical propositions.
> 6.1251 Hence there can *never* be surprises in logic.

And again:

> 6.126 The proof of the logical propositions consists in the following process: we produce them out of other logical propositions by successively applying certain operations that always generate further tautologies out of the initial ones.
> 6.1261 In logic process and result are equivalent. (Hence the absence of surprise.)

Wittgenstein can be understood in two mutually related ways. The simplest one is to focus on the fact that logical propositions (i.e. theorems of propositional logic) are all tautologies. However complicated, they are equivalent to the simplest tautologies, such as $p \vee \neg p$ or $p \supset p$. So the surprisingness of their truth should be assigned the same degree as the surprisingness of the simple tautologies.

A more ambitious view which can be ascribed to Wittgenstein concerns the role of proof. According to it, one can never express surprise over the truth of S unless one understands what S says. Such a condition is perfectly trivial. But in the logical/mathematical case the meaning of S is recognised by the speaker only on the condition that the speaker knows the proof of S (this is the impact of the claim that in logic the process and result are identical). But the proof of S eliminates its surprisingness, since it should allegedly consist in a series of tautological equivalences. A fairly complicated tautology is reduced by a series of equivalences to a less complicated tautology, and ultimately, to some simplest tautology R. The fact that $S \equiv R$ may appear surprising only to someone who is ignorant of this series of equivalences in the proof: every step in the series is by itself not surprising.

The gaps existing in the Tractarian solution are obvious. There is, first, no attempt to clarify the conditions of genuine surprise. And secondly, the solution might seem plausible for the case of propositional tautologies. Its significance for a wider range of necessary statements (in predicate logic, traditional mathematics, or metamathematics) rests on the controversial assumption that all these areas are reducible to propositional logic.

Nevertheless there is an appealing element in the Tractarian view. S is surprising for X only if X had an antecedent ground to believe that $\neg S$. But when S is necessarily true, it is unclear on what basis S could believe that. To take again Penrose's example (5), it is surprising by virtue of its beauty and simplicity. Then it should be surprising no more than the necessarily false proposition $e^{\imath A + \imath B} = e^{\imath A} + e^{\imath B}$.

5. *The truly surprising user-friendliness...*

5.1. *... in mathematical applications in physics*

Let us see how this account fares with the Facts 1.1, 1.2, 2.2, and 2.1 when they play the rôle of the outcome E. Steiner operates with two rival hypotheses:

Hypothesis 5.1: *Manipulations with physically meaningless mathematical concepts (or: with mathematical notation) should not yield physically meaningful statements.*

and:

Hypothesis 5.2: *Nature conforms to mathematics, which is itself a human creation.*

The Hypotheses 5.1 and 5.2 express naturalism and anthropocentrism respectively. Naturalism is the initial hypothesis H, whereas anthropocentrism is the rival hypothesis $H*$. An instant objection would be that these are metaphysical doctrines which hold or do not hold necessarily. If naturalism is true of the actual world, it is true of other worlds too, and similarly for anthropocentrism.

But this argument is unconvincing. We can imagine a world which has particularly intractable laws. Suppose further that mathematics developed by counterfactual humans there is limited to arithmetic and Euclidean geometry. There will be then no evidence in favour of anthropocentrism in that world. Equally we can imagine a world with particularly simple laws, nevertheless inhabited by an immensely talented population. In such a world, mathematics developed may far surpass the needs of an adequate physical theory.

(It is again crucial to recognise that naturalism and anthropocentrism as they appear in the Hypotheses 5.1 and 5.2 are *epistemic* claims. We might not be able to conceive a world with the laws indescribable by mathematics, but this impossibility does not preclude the limited strength of particular mathematical theories developed by the populations in different worlds.)

Therefore, the probabilities assigned to naturalism and anthropocentrism will be between 0 or 1. As we have seen, the new evidence (expressed here by Fact 1.2) can lend support to the rival hypothesis on the assumption that that hypothesis is not itself improbable. But how are we to assign specific values to the probabilities naturalism and anthropocentrism? The sceptical reaction to Steiner's arguments stems, it seems, from the extremely low probability of anthropocentrism. And according to (7) above, Steiner's Fact 1.2 will not be seen as surprising in case the hypothesis $H*$(=Hypothesis 5.2) is highly improbable. However, the surprisingness of the Fact 1.2 will be sustained if we limit ourselves to the denial of H(=Hypothesis 5.1) without committing ourselves to any of its alternatives.

(Indeed, the alternatives to naturalism are not limited to the version of anthropocentrism espoused by Steiner. One other alternative, incidentally mentioned by Steiner, is the Kantian approach[6]. According to Kant, nature as the subject of physical research consists of phenomena. These latter are describable in spatio-temporal notions. On the other hand, arithmetic and geometry conceptualize our intuitions of space and time. Therefore, mathematics is (necessarily) applicable, simply because its link with experience is never broken. Yet another alternative is offered by Spinoza[7]. He purported to establish that 'the order and connection of ideas is the same as the order and connection of things'. This follows not from any anthropocentric argument, but rather from the view that mind and the universe are not two distinct things. Of course, both of these alternatives have to be explained further in order to make them workable; yet we can see that the denial of naturalism does not lead us necessarily to the 'user-friendly' anthropocentrism.)

[6] Cf. (3, §38).

[7] Cf. (5, II:7, Scholium to II:8).

5.2. ... *within mathematics*

Penrose has his own duo of hypotheses:

Hypothesis 5.3: *Mathematical concepts are created by the humans.*

and:

Hypothesis 5.4: *Mathematical concepts are discovered by the humans.*

They express idealism and platonism respectively. One difficulty, already mentioned before, is the unclarity in the link between the evidence and the hypotheses. There is simply no obvious sense in which the application of a mathematical concept in a variety of fields can lend support to either its invention by a particular human, or its discovery by the same human. The second difficulty is that unlike the Hypotheses 5.1 and 5.2, idealism and platonism are metaphysical doctrines, being true or false by necessity. Hence, the analysis of surprise given above is not applicable in this case.

However, the second objection requires some defence. The truth of idealism (i.e. of the hypothesis 5.3), in particular, may seem to depend on the existence of humans. Given the necessity of origin, in a possible world w where no life has evolved on Earth we may claim that no humans exist. Yet some other creatures in w, as a matter of fact, can use the arithmetic of natural numbers. There is nothing implausible in such a scenario. Then the concept of natural number, whatever its origin in w is, was not invented in w by the humans. On the other hand, according to idealism, at least in our actual world @ idealism is true: mathematical concepts have verily been invented by the humans. Therefore, idealism is contingent.

Our version of idealism is clearly defective, but not unexpectedly so. For the claim Penrose is attempting to rebut is not about the unique capacities of the humans. The idealist commitment is conditional: *if* there were humans in w, *then* they would have created mathematical concepts. But if w contains no humans, then no doubt some *other* creatures might have invented mathematics. And surely the humans and the extra-terrestrials might have invented mathematics independently of one another. Lastly, in the world u, where there is no life at all (but only helium and hydrogen, say), there are no mathematical concepts either, simply because there is no one to invent them. Spelled out in full, the whole idea of idealism may seem queerly irrelevant, partly because it is obscure whether there actually are theorists willing to subscribe to it.

Be that as it may, we get a more cogent formulation of idealism:

Hypothesis 5.5: *For any mathematical concept C, necessarily: there is an agent who created C.*

This is rendered, perhaps less misleadingly, in the language of possible worlds: For any mathematical concept C and for any world w, necessarily: if C exists at w, then there is an agent who created C in w.

It is also obvious that platonism has nothing to do with the fortunes of the humans. In fact, in order to claim that X was discovered at t, rather than invented at t, we ought to have some evidence of X's existence prior to t. Penrose presents no such evidence, and it is difficult to see what such evidence might consist in. The true contention of platonism is rather that a mathematical concept C is not part of the world and its existence is not dependent on the events in the particular world, including the agents who may, or may not, come to possess it. We get:

Hypothesis 5.6: *For any mathematical concept C and for any world w, C exists at w.*

Hence, the necessity of platonism and idealism is re-affirmed.

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