SUPERTASKS AND MATERIAL OBJECTS

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Abstract

In this paper two arguments are presented to show that the infinite divisibility of time is incompatible with the existence of material objects as commonly conceived of. The first is based upon the Mark Supertask of van Bendegem, the second on a variant, the Urn Supertask.

Supertasks provide a way of exploring the consequences of the infinite divisibility of time. Thus in a series of articles Lauradogoita has shown that infinite divisibility leads to the possibility of Earman type anomalies not merely in Newtonian mechanics but even in Relativity.[1] In this paper I shall look again at the Mark Supertask of Van Bendegem, and then consider a variant, the Urn Supertask. In both cases we may show that infinite divisibility requires the abandonment of some common sense thesis concerning material objects.

It is not in fact infinite divisibility that is on trial here but something a little weaker, namely PIDS, the postulate of an infinite sequence of disjoint intervals of time I_n, where I_{n+1} is later than I_n, all contained in a single finite interval I. We could take I to be the interval represented by $(0, 1)$ and I_n to be represented by $(1 - 1/2^{2n}, 1 - 1/2^{2n-1})$.[2] Thus I_1 is the interval with end points $1/4$ and $1/2$, I_2 is the interval with end points $3/4$ and $7/8$, etc. These intervals have gaps between them, such as the gap between $1/2$ and $3/4$, so as to render irrelevant disputes over whether we should consider

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[1] By an Earman type anomaly I mean a physical process which occurs with pure spontaneity. This may be contrasted with ordinary randomness, which occurs when an earlier state necessitates that some event out of a range of events occurs without necessitating that any one of the events occurs. See John Earman *A Primer on Determinism.* Dordrecht: Reidel, 1986: 34. Lauradogoita provides an example of how something might spontaneously accelerate to the speed of light. See Jon Perez Lauradogoita, "Some Relativistic and Higher Order Supertasks", *Philosophy of Science* 65 (1998):502–517.

[2] (α, β) is the open set of reals $\{t : \alpha < t < \beta\}$. I talk of representation rather than identity because of scruples about identifying parts of time with sets of moments.

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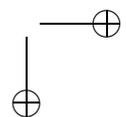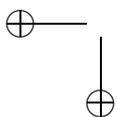
open or closed sets of moments. There are attractive accounts of time in
which infinite divisibility fails but PIDS holds. For instance we could deny
that there are point moments of time and insist that the intervals of time are
represented by all and the open intervals of real numbers (α, β) such that
$\alpha < \beta$. In that case no interval can be divided into two disjoint subintervals
but PIDS still holds.

The least radical way of denying PIDS is to assert that Time is discrete in
the sense that an interval of finite duration contains only finitely many mo-
ments of time. An alternative way in which PIDS could fail would be if there
is a lower bound Δ to the duration of intervals, which are then represented
by all the open intervals (α, β) such that $\beta - \alpha > \Delta$.

For a supertask we suppose that in each interval I_n a possible task is per-
formed, but the tasks are selected so that whole process has an astounding
product. Because we do not know the laws of nature when it comes to the
realm of the very small, the possibility in question is metaphysical not phys-
ical. We shall assume, however, that the very general features of Spacetime,
including PIDS are non-contingent, so that it is not just PIDS but the meta-
physical necessity of PIDS which is supposed for the reductio argument.

1. *The Mark Supertask*

The following example due to van Bendegem illustrates both the general
strategy and the difficulty in evaluating supertask arguments.[3]

> In I_n n marks are put down on a piece of paper and all the pre-
> vious marks erased. So the total number of marks increases
> throughout I. Yet at the end of I every mark has been erased
> so there are no marks left.

The supposition that someone making the marks is irrelevant. All that is
required is that the process described would be possible assuming the meta-
physical necessity of PIDS.

The Mark Supertask is possible only if the marks get smaller and smaller
with zero size as their limit. Otherwise the process involves something mov-
ing faster than the speed of light. And that nothing can do so is not some
law in addition to the structure of Spacetime, in which case we might say it
was metaphysically contingent in some sense, but is as much a consequence
of the supposed structure of Spacetime as PIDS itself. But if the marks get

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[3] See Van Bendegem, Jean Paul, "In Defence of Discrete Space and Time", *Logique et
Analyse* 38 (1995):127–150.

smaller and smaller then there is nothing peculiar about the end result in which there are no marks.

Fortunately the Mark Supertask can be modified to avoid this difficulty. Think of the "sheet of paper" as a region of space, and think of a "mark" as a certain kind of particle which is "on the sheet" just in case the particle is zero distance from the region. We may also stipulate that no particle moves in (or between) any of the I_n except as a result of the operations described. Then although the particles might move by less and less as n increases we still have the peculiar result that throughout the process the particles that are zero distance from the region increase in number, but at the end there is no particle zero distance from the region. Hence either particles move discontinuously or cease to exist.

To obtain a reductio ad absurdum of PIDS we require not merely that this result is peculiar but that it is metaphysically impossible. Now it is reasonable to assert the metaphysical impossibility of an uncaused annihilation or discontinuous motion of material objects. So *provided* we are prepared to deny that the supertask is the right sort of process to count as the cause of annihilation or discontinuity, we may assert treat the Mark supertask as a reductio ad absurdum of the joint assumption that PIDS holds and there are material objects. The latter is required because if, contrary to common sense, there are no continuing material objects but only a sequence of momentary events, then whether or not there is some further event in a sequence would be a matter of physical rather than metaphysical necessity.

2. *Supertasks and Zeno's Principle*

There is considerable intuitive support for Zeno's Principle that nothing can have completed an infinite sequence of consecutive processes. So why should we not use Zeno's Principle to argue that supertasks are impossible? My answer is in two parts. First, unless it is qualified, Zeno's Principle already provides an argument against PIDS, namely that when combined with that postulate it leads to a version of Zeno's Paradox, in which, whenever Achilles gets to where the tortoise was the last time he looked, he pauses to look (but for no longer than he has just run) and then runs on (not looking until he gets to where the tortoise had been). Provided Achilles runs more than twice as fast as the tortoise walks he should still catch up. Second, even if in some way that argument is resisted, the unqualified version of Zeno's principle in no way prevents the supertask occurring. For if the whole process is impossible there must be some integer n for which the sub-process occurring in I_n is impossible, but that is not the case. What the unqualified version of Zeno's principle would show is the set up no longer exists for any $t \geq 1$. For otherwise it would have completed the infinity of processes. It

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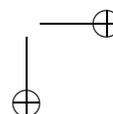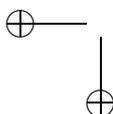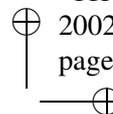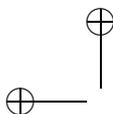
may not be clear just what the set up is. In fact it might be the whole Universe, but in any case that the occurrence of a supertask brings the whole set up to an end would by itself be a reductio ad absurdum of PIDS. So either way PIDS cannot be defended by appeal to Zeno's Principle.

3. *The Urn Supertask*

The chief purpose of this paper is to introduce a variant on the Mark Supertask, which I call the Urn Supertask, which, like the Mark Supertask is a reductio of the joint suppositions that PIDS holds (non-contingently) and that there are material objects as we commonly conceive of them. Recall that the Mark Supertask was based upon the proviso that a supertask is not the sort of process which could cause annihilation or discontinuity of the marks. The Urn Supertask has the advantage of not being subject to that proviso. We shall, however, be ignoring Zeno's Principle which would make supertask arguments against PIDS quite redundant.

Initially there is precisely one ball in the urn. In each interval I_n first a new ball is put in, then the urn is shaken and one of the two balls is taken out. The balls, we are to suppose, are qualitatively identical. Hence at each stage there are two possible qualitatively identical outcomes: either the ball which has just been put in is taken out, or the ball which was already there is taken out. Hence, it is asserted, there are infinitely many qualitatively identical histories of the whole process. Of these just two are: (1) the initial ball remaining there the whole time and each new ball being taken out straightaway and (2) the initial ball being taken out in the first interval, and thereafter each ball being taken out in the interval after it was put in. In case (1) there is one ball left at the end. In case (2) there is no ball left at the end. Yet the two processes were, right up to the last moment, qualitatively identical.

Here we have supposed the Persistence Thesis, namely that:

(1) All material objects persist for an interval of time of positive duration;

(2) If A is an object existing throughout interval J_1 and B an object existing throughout J_2 then there is a fact of the matter as to whether A and B are the same;

(3) Although objects can cease to exist they do not do so merely as a result of ordinary interaction with other objects.

Clause (3) is based upon an intuitive idea of an ordinary interaction, but we are to suppose that the sort of process by which one ball is picked out of the urn is quite compatible with the balls surviving.

Persistence is not metaphysically necessary for even if there are material objects there might have been nothing physical or there might have been

a universe with only physical but non-material fields in it. But what we may assert is that of metaphysical necessity if there are material objects (as commonly conceived of) then they satisfy Persistence. Indeed this might well be an analytic truth. Hence under the supposition that PIDS holds non-contingently and that material objects are possible then, intuitively, the Urn Supertask should occur in some possible world. But the Urn Supertask leads to a violation of Impotence of Individuality, namely the principle that if qualitatively identical processes result in products which are not qualitatively identical then the results are not determined by which process is which. That we may claim to be a metaphysically necessary principle. Likewise we may claim that it is metaphysically necessary that the mere occurrence of a possible supertask does not cause the destruction of the objects involved, in this case the balls. This results in a reductio ad absurdum of the combination of PIDS and the existence of material objects.

4. *Versions of the Urn Supertask*

If we interpret the Urn Supertask literally then it fails because the histories are not strictly qualitatively identical. For as the urn is shaken each ball follows a distinct trajectory and this influences which one is taken out. To ensure that the two possible histories being considered are indeed qualitatively identical at each stage we need some minor variation. in which the way the "urn" is "shaken up" results in two qualitatively identical situations.

Suppose, first, the "balls" are point particles and that they move towards each other horizontally —one coming from the right and one the left— they collide without fusing into a single particle and move apart along the vertical axis —one going up and one down. Then there is no qualitative difference between the situation in which the particle from the right goes downwards and the one in which the particle from the right goes upwards. Call this the point particle interpretation of the Urn Supertask. We may note that there is also no qualitative difference between either of the above and the situation in which collision causes annihilation followed by creation of two new particles, or the situation in which collision results in momentary fusion followed by fission. But by the Persistence Thesis we are entitled to interpret the situation as persistent particles bouncing off each other without being destroyed.

Here is another version, the homogeneous matter interpretation. The objects being considered are parcels of homogeneous matter. They come together form a bigger object for a while and separate. While together although there is fusion into a single material object there are still two distinct parcels of matter with a geometric boundary between them which changes in various random ways so that when the parcels of matter separate again. Which
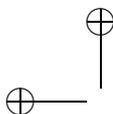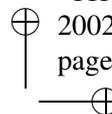
is which is determined entirely by the histories of the individual parcels of matter and not by any qualitative feature. Hence by the Impotence of Individuality it should make no difference to the outcome which is which.

Yet again there is the composite object interpretation, in which two, possibly macroscopic, objects of the same size and shape, made of, say, gold, come together much like the parcels of homogeneous matter considered above with a very complicated and changing boundary. At some time during their temporary union every constituent particle lies near enough to the boundary for it to interact with a constituent particle from the other piece of gold as point particles bouncing off each other or as small parcels of homogeneous matter interacting as in the homogeneous matter interpretation. Of the infinitely many possible outcomes most are ones in which the two pieces of gold get too mixed up to survive as distinct objects. But there are two, qualitatively identical, special cases. One is that in which the particle interactions at the boundary result in no overall swapping and the complex changes in the boundary eventually result in the same macroscopic objects as before. In the other case all the particles are swapped over so once again we have the same objects as before, but swapped over. Here we are assuming that for two lumps of gold, unlike living organisms, the identity over time is determined by their constituents.

If material objects such as lumps of gold are not made up of either of point particles or of small parcels of homogeneous matter, what are they made up of? One answer might be that they are made of something physical but non-material like a field. In that case either: (1) the so-called material objects are more like waves and there are no material objects as commonly conceived of, or (2) there is identity over time for field-strengths at points, perhaps thought of as tropes, in which case we can still set up the Urn Supertask, treating the persistent point field strengths as like point particles.

Another answer to the question of what lumps of gold might be made of is to suppose that there are no fundamental particles because every object is composed of smaller ones —with structure all the way down. In that case we may rely on the composite object interpretation all the way down.

Conclusion

If Zeno's Principle holds then we have an independent case against PIDS and the supertask case is redundant. But if Zeno's Principle does not hold then the Mark Supertask and the Urn Supertask provide two different arguments to show the incompatibility of PIDS with the possibility of material objects as ordinarily conceived of.