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COMBINATION SEMANTICS FOR INTENSIONAL LOGICS PART I MAKINGS AND THEIR USE IN MAKING COMBINATION SEMANTICS

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Each proper semantics must be based on ontology

1. The above statement is a truism. But an important one. It is often forgotten, for in our time it is rather a common (and quite doubtful) conviction, that the only valuable ontology for contemporary semantics is the set-theoretical ontology.

In the past century set-theory indeed played the most important role in mathematics and logic and, in turn, in their philosophical applications. It is also true that the very paradigmatic case of a semantical analysis for formal languages, done by Alfred Tarski, is in fact a combination of set-theoretical and algebraic ideas.

Tarski-type semantics was extended in the sixties to the case of intensional languages providing us, as many believe, with a satisfactory method to deal with real philosophical problems.

2. In part, for sure, it is true. But only in part! If we distinguish, *inter alia*, between ontology of the being, including metaphysics (i.e., ontology of the world) on the one hand, and —on the other hand— the ontology of language and the ontology of mind (cf. [5]), then by their close connection with formal investigations of concepts, set-theoretical and algebraic ontologies are closely connected with two later types of ontology, but not with the first!

Real philosophy, however, is about the being. Therefore, we are still in need of a more suitable and subtle semantics for it.

3. In what follows I will try to outline such a semantics, based on *combination ontology*, which is a part of a deeply modal version of a general theory of analysis and synthesis.

To this end, I will start with rather general remarks concerning modalities, with particular emphasis put on ontological ones, passing next to a rather general description of a theory of analysis and synthesis.

Finally, the time will come for the proper topic of the paper.

Classification of modalities

4. Modalities are modifiers. For example, alethic modalities are modifiers of truth components, or —more generally— semantical, logical and ontological components of a judgement and objects involved in it.

5. Let us consider two conjugate classifications of modalities:

A. Based on a grammatical difference:
Noun-like (like *possibility*, etc.) vs. *Adjective-like* (possible, etc.)

B. Based on an ontological principle:
Logical modalities vs. *Superlogical* modalities.

6. LOGICAL modalities are used for collection and comparison: *possible, necessarily, contingently*, etc.

They are adjective-like and, in their depth, they are quantifiers (what is made clear in relational semantics).

7. SUPERLOGICAL modalities are used for expression and modification of very general conditions. They can be divided into several groups including:

A priori modalities, concerning what can be thought, used to delineate the realm of reason. Examples are *thinkable, understandable, reasonable, controvertible*.

Ontological modalities, kernel for our aim! They are useful for expression of the general and basic conditions for some families of objects or complexes. They are, *inter alia*, used for delineation of the *ontological space* of all possibilities, the most general field we can deal with.

Examples are: *possibility, necessity, contingency*, and *exclusion* taken in the sense of a condition; *compossibility, coexistence*, and *eminent existence* in the sense of Leibniz, *(formal) possibility* in the sense of Wittgenstein's *Tractatus*; *combinable, synthetizable* and *analyzable*; and several common philosophical modalities *de re: by necessity, essentially, by its very nature*, etc.

Makings are, in fact, the most fundamental between ontological modalities, and superlogical modalities in general. Ontological MAKINGS are as follows: *making possible, making impossible*, etc.

Finally let me mention also *metaphysical* modalities, concerning facts and existence, what is real or actual: *real, existing, actual, factual, true, false, to be a fact, to be true*.

Metaphysical MAKINGS are, among others, the following ones: *making true, making fact, making real, making actual*, etc.

Makings

8. They form a basic and very challenging family of modalities. We outline here some rules for makings in English, for the paper is written in English. We should, however, be careful in not too far and easy generalization of these rules. Makings, being a grammatical universal, are, for sure, realized in different ways in different languages (for example, in Polish).

A bit of English grammar

In English makings are of the form *Gerund* + *Noun*: *Making N*, for suitable N. The form is very general indeed.

9. Two basic cases are:

Making Possible	x *makes* y *possible*	$MP(x, y)$
Making Impossible	x *makes* y *impossible*	$MI(x, y)$.

They are basic for the following *purely apriori* reasons. The minimal combination can be obtained from two items only. If connection between them is either positive (which is denoted by putting arrow up) or negative (arrow down), then there are just four connections between these two items: if the connection is taken to be either positive, or negative, or both, or —finally— if both items under consideration are not connected at all, i.e., they are ontologically neutral, which is quite common an assumption in the case of *combinatorial ontology*.

x		y	
$\bullet$	$\uparrow$	$\bullet$	MP(x, y)
$\bullet$	$\downarrow$	$\bullet$	MI(x, y)
$\bullet$	$\uparrow\downarrow$	$\bullet$	MPI(x, y)
$\bullet$		$\bullet$	ON(x, y)

Observe that MPI(x, y) is apparently "incoherent" combination of the first two cases, whereas ON(,) means that both objects, x and y, are mutually neutral.

The first modality MP is crucial for ontologies based on the law of consistency (i.e., Aristotelian and Leibnizian), the second is its dualization, the third one —its quite Hegelian in spirit, whereas the last case, of ontological neutrality— is the most common assumption of so-popular combinatorial ontologies.

10. Notice that the first argument x is usually named *maker* (or *reason*), whereas the second argument y is a *result* of making.

11. Up to my knowledge a particular case of *making* for the first time (at least in analytical philosophy) was investigated by B. Russell and his followers. It is famous Russellian

Making True	x *makes* y *true*	$MT(x, y)$;

12. Let me consider also quite similar metaphysical makings:

Making Real	x *makes* y *real*	$MR(x, y)$
Making Actual	x *makes* y *actual*	$MAc(x, y)$
Making Fact	x *makes* y *to be a fact*	$MF(x, y)$

Also the following makings are quite useful:

Making Thought	x *makes* y *to be a thought*	*—thinking—*	$MTh(x, y)$
Making Act	x *makes* y *to be an act*	*—acting—*	$MA(x, y)$

13. To sum up, English form of makings is very general and formal. It can be done for any noun, without any clear limitation.

It is clear that the most investigated case of *making* is the case investigated in the theory of action, where *maker* is considered to be *an agent*, *making* itself is *doing an action*, whereas *what results from it* is sometimes named *patient*.

Finally let me emphasize, that in contemporary ontology positive makings are preferred over negative ones, whereas incoherent makings, like MPI(,), are usually knocked of into the logical hell.

A bit of onto\logic

14. Let me collect here the basic observations concerning *positive* makings. For more complex theory, including and comparing both positive and

negative modalities cf. [8].

(1) *Making possible and making impossible are two basic makings.*

It follows from purely apriori considerations sketched previously.

(2) *Making possible is, in a sense, ambiguous.*

Indeed, we should distinguish at least two extreme variants from a rather complex spectrum:

Strong variant: MP(x, y) means *x makes y and y is possible*, or *x makes y to be possible*: (MP\M) MP(x, y) $\leftrightarrow$ M(x, y) $\wedge$ M(y).

Weak variant: MP(x, y) means P(x, y): *x makes a necessary condition for y*, or *x excludes a barer for y*.

Hereafter MP is the common, general form of *making possible*, and similarly, *mutatis mutandis*, other makings as well. We can consider it as the combination of both variants mentioned above, as taken from the spectrum between these two extremities:

MP = M + P.

(3) *The strong variant $MP(\ ,\ )$ offers two natural ways to define other makings.*

For example:

(M\F) MF(x, y) $\leftrightarrow$ M(x, y) $\wedge$ F(y);
x makes y to be a fact, if it makes y and y is a fact; or

(MP\F) MF(x, y) $\leftrightarrow$ MP(x, y) $\wedge$ F(y)
x makes y to be a fact, if it makes y possible and y is a fact (or real).

(M\T) MT(x, y) $\leftrightarrow$ M(x, y) $\wedge$ T(y);
x makes y true, if it makes y and y is a proposition which is true; or

(MP\T) MT(x, y) $\leftrightarrow$ MP(x, y) $\wedge$ T(y), explained in a similar way:
x makes y true, if it makes y possible, where y is a true proposition, whereas x is its truth-maker (whatever it means)

(M\Th) MTh(x, y) $\leftrightarrow$ M(x, y) $\wedge$ Th(y)
x makes y to be a thought, if it makes y and y is a suitable mental representation (or picture)

(MP\Th) $\mathrm{MTh}(x, y) \leftrightarrow \mathrm{MP}(x, y) \wedge \mathrm{Th}(y)$
x makes y to be a thought, if it makes y possible and y is a suitable mental representation (or picture)

(M\A) $\mathrm{MA}(x, y) \leftrightarrow \mathrm{M}(x, y) \wedge \mathrm{A}(y)$
x makes y an act, if it makes it and y is an act

(MP\A) $\mathrm{MA}(x, y) \leftrightarrow \mathrm{MP}(x, y) \wedge \mathrm{A}(y)$
x makes y an act, if it makes it possible and y is an act; etc.

(4) *The weak version $P(,)$ is weaker indeed:*
(MP\P) $\mathrm{MP}(x, y) \rightarrow \mathrm{P}(x, y)$, or even $\mathrm{MP}(x, y) \rightarrow \mathrm{P}(x, \mathrm{M}(y))$
(MF\P) $\mathrm{MF}(x, y) \rightarrow \mathrm{P}(x, y)$, or even $\mathrm{MF}(x, y) \rightarrow \mathrm{P}(x, \mathrm{F}(y))$
(MT\P) $\mathrm{MT}(x, y) \rightarrow \mathrm{P}(x, y)$, or even $\mathrm{MT}(x, y) \rightarrow \mathrm{P}(x, \mathrm{T}(y))$
etc.

Picture

		MF		
		MT		
M	$\rightarrow$	MP	$\rightarrow$	P
		MTh		
		MA		

Making true

15. The only case of makings investigated up to now in a rather extensive way is the case of making true, MT. Its two basic clues are the following ones:

Russellian —Facts are left-side arguments of MT. They are *truth makers*. This implies

(BR) $\mathrm{MT}(x, y) \rightarrow \mathrm{F}(x);$

Let me add that too much attention put, mostly in vain, on truth makers is chiefly responsible for placing *makings in general* in the shadow of *makers*, covering thereby the modal character of makings.

Fregean and Tarskian —Making true means verification (satisfaction), i.e.,

(F\T) $\quad$ MT(x, y): $\leftrightarrow$ x $\models$ y,

fulfilling usual compossibility principles. As a matter of fact, Tarski's contribution to the theory of truth can be understood as a *partial* axiomatization of *making true* based on set-theory as its background ontology. Sound and complete axiomatization is still an open question.

Let me add, that it is a logical custom to differentiate between MT-arguments: MT(N, A), or MT(X, A), where the first is a *model* or *set of formulas*, whereas the second is a *formula*.

Few words of reflection

16. By the above analysis, in particular by (1), the general theory of makings must be based on (and, in fact, is a part of) combination ontology, or general ontology of analysis and synthesis.

Therefore time comes for ontological investigations.

Ontology

17. The most general theory of analysis and synthesis is one of two types: combination ontology or transformation ontology:

GAS = CO + TO.

In what follows I will, following Leibniz and Wittgenstein, concentrate my attention on combination ontology only, passing investigation of transformation ontology for another occasion.

Three approaches towards a General Theory of Analysis and Synthesis

Order Approach

It is natural, abstract and quite common. In the first steps let us define ontological spaces of three kinds and their simples (if any), in consequence also substance, i.e., the family of all simples.

18. *Ontological spaces.* Let OB be the class of all items (*objects*). Assume that the universe of a discourse U is included in OB.

We distinguish at least three natural types of an ontological space of analysis and synthesis:

The space of analysis: $<U, <>$ where $<$ is the relation *to be simpler than,*
The space of synthesis: $<U, \subseteq>$ where $\subseteq$ is the relation *to be a component of,*
The space of analysis and synthesis: $<U, <, \subseteq>$.

Notice the large number of questions concerning connections between the two basic ontological relations: *to be simpler than* and *to be a component of*. Are they coextensive? This option, however, seems to be a too far going oversimplification.

Obviously, an analysis passing from "bigger" to "smaller" is *down-oriented*, whereas a synthesis is *up-oriented*.

19. *Simples and substance.* The most important offsprings of the first, order approach are the notions of *simples* and *co-simples* (usually named *possible worlds*). Both are limit notions. Simples are limit-objects of the family of all proper ontological analyses; co-simples, on the other hand, are limits of suitable syntheses.

It is important to recognize that *at least six notions of simples can and should be distinguished* (cf. [9]). Let me recall here four of them, where $\prec$ is especially ambiguous, denoting hereafter either $<$ or $\subseteq$:

Superelements: SE(x) iff $\forall$y x$\prec$y
x is a superelement iff it is simpler than any object in the universe
Simples: S(x) iff $\neg\exists$y y$\prec$x
x is a simple iff there is no object in the universe which is simpler than it is
Atoms: A(x) iff $\forall$y (y$\prec$x $\rightarrow$ x=y)
x is an atom iff the only one element simpler than it (if any) is x itself
Elements: E(x) iff $\forall$y (y$\prec$x $\rightarrow$ x=y$\vee$SE(x))
Elements are weakenings of atoms, such that *the only objects simpler than them are they themselves or superelements*.

20. Now, following the long and very distinguished line of thinkers, including Anaximander, Anaxagoras, Democritus, Leibniz, Kant and Wittgenstein *substance* is defined as the family of all suitable simples. Observe that the substance must not be uniform, for it can be built up of different types of simples.

Notice also that in the case of so-called *unfounded* ontologies substance can be empty. In such a case it is, for sure, not so natural and easy to introduce a suitable inductive structure in the universe. Such alternative ontology must be treated quite seriously, however.

Finally let me stress that order approach offers an external description of synthesis. It does not, however, explain its mechanism!

Operator approach

21. It is based on investigation of two operators: *analyzer* α and *synthesizer* σ.

Analyzer α produces for a given x the family of all its pieces (parts or components):

$\alpha(x) := \{y : y \textit{ is obtained from } x \textit{ by } \alpha\}$;

whereas synthesizer σ collects for any x the family of all objects that *can be synthesized* from x (its substance):

$\sigma(x) := \{y : y$ *can be obtained by a combination involving* x*, or its substance* $S(x)\}$.

Operator approach gives also, as the order approach did, an external (or extensional) description of synthesis.

It opens, however, a way to its internal description.

Internal, or modal, approach

22. To describe (at least necessary) conditions of a successful synthesis it is convenient to use two basic ontological modalities introduced previously: *making possible* —MP(,), and *making impossible* —MI(.).

23. Recall § 9. For given two arbitrary objects x and y they can be considered as arguments for a *basic ontological connection* which, in turn, is either *positive* or *negative*. *A priori* there exist just four cases: positive connection —MP, negative connection —MI, both positive and negative, hence incoherent, connection —MPI, and the case of mutual ontological neutrality —ON.

The first case is taken here to be fundamental!

Explication for σ

24. Now we can offer the following, rather natural explication for synthesizer: *to be synthetizable from* x *is to be made possible from* x:

$$\sigma(\mathrm{x}) = \{\mathrm{y} : \mathrm{MP}(\mathrm{x}, \mathrm{y})\}$$

Notice that the above explication connects the second approach (operator one) with the third (internal) approach to the general theory of analysis and synthesis.

Wittgenstein’s insight

25. Let me quote one of the most mysterious theses of the *Tractatus*:

(2.033) Form is *the possibility* of structure.

Ask now what *the possibility* means? It has been pointed out by Frank Ramsey in his famous review of the *Tractatus* that it cannot be read as a logical modality (i.e., *form* cannot be treated as an alternative structure), for this reading would immediately make *Tractatus* inconsistent.

My own proposal (cf. [1], [2]) is the following one:

(5) Form of x is what *makes* the structure of y *possible*.

Formalization: MP(Form(x), Str(y)), hence —through suitable generalization— MP(x, y).

Further Wittgensteinian and Leibnizian clues make the nature of MP more clear: form of x is determined by its substance, whereas structure of y means a way in which a complex y is built up, the way, including order, in which its components are joined together into one object.

Using syntactical categorization of Leśniewski and Ajdukiewicz we obtain, on the other hand, that MP has the category of quantifier: s / n, s — which, as is easy to see, is of higher order and deeply modal.

Therefore MP is a modal quantifier, characterized after Wittgenstein’s clue by

(6) MP(x, y) $\leftrightarrow$ MP(S(x), y).

Conceptual framework of Combination Ontology

26. It is extremely rich, enough to define the basic notions of Leibnizian and Wittgensteinian ontologies. Hereafter I will cite only a few notions to illustrate the above claim as well as for further use.

Starting definitions and axioms

M(x): $\leftrightarrow$ MP(x, x)	ontological coherence (to be ontologically possible)
OF(x): $\leftrightarrow \exists$y MP(y, x)	ontological foundation
G(x): $\leftrightarrow \forall$y MP(x, y)	ontological generator (or God-like being of a theory)
FR(x): $\leftrightarrow \exists$y MP(x, y)	ontological fruitfulness
(SR) $\forall$x$\exists$y MP(y, x)	ontic *principle of sufficient reason*

Monotonicity principles with respect to $\prec$

MP(D,) $z \prec x \wedge MP(x, y) \rightarrow MP(z, y)$
Down oriented with respect to the first argument;
MP(U,) $x \prec z \wedge MP(x, y) \rightarrow MP(z, y)$
Up oriented with respect to the first argument. Similarly for the second argument
MP(,D) $MP(x, y) \wedge z \prec y \rightarrow MP(x, z)$
MP(,U) $MP(x, y) \wedge y \prec z \rightarrow MP(x, z)$
Also MP(D, D), MP(U, U), MP(D, U), MP(U, D) should be taken into account.

Consistency principles

Ontological ones

(OC) $\neg(MP(x, y) \wedge MI(x,y))$ consistency law
No item makes another one both possible and impossible!
(OEM) $MP(x, y) \vee MI(x, y)$ law of the excluded middle
Each item makes another one either possible or impossible!
(OBI) $\neg MP(x, y) \leftrightarrow MI(x, y)$ law of bivalence,
which is the conjunction of the two previous principles.

Notice that OBI simplifies the domain of investigation under consideration in a quite remarkable way: making possible and making impossible are interdefinable!

Onto\logical ones

They hold in the proper domain of logic, where at least the second argument is a proposition (or formula), whereas the negation is classical.
(OLC) $\neg(MP(x, \neg A) \wedge MP(x, A))$ meta-consistency law
No proposition is made possible, together with its negation, by the same item

(OLEM) $\mathrm{MP}(x, \neg A) \vee \mathrm{MP}(x, A)$ meta-excluded middle law
Each item makes possible either A or $\neg A$
(OLB) $\neg\mathrm{MP}(x, A) \leftrightarrow \mathrm{MP}(x, \neg A)$ meta-bivalence law

Notice that the above laws govern usual semantics. They are, in a sense, metalogical!

Compossibility and Compatibility

27. Let us finally consider *compossibility* —the most eminent ontological modality, used by Leibniz as the main notion of his great metaphysics. First I will recall Leibniz's original construction, which is metalogical in its depth, passing next to its onto\logical counterpart in combination ontology.

Leibniz's metalogical construction

28. Recall first that Leibniz believed that a suitable logical calculus of concepts enabling its user to solve any rational question can be and will be discovered. Assuming that it is done he was in power to sketch the full ontological system —from monads and qualities to the real world.

Thus let some logical calculus of concepts (names?, predicates?) be given. Cn is the connected consequence operator, whereas —for any x— Th(x) is its Cn-theory.

Leibniz defined modal concepts by the following metalogical conditions:

$\mathrm{M}(x) :\leftrightarrow \bot \notin \mathrm{Th}(x)$ — x is possible *(its theory is consistent)*
$\mathrm{L}(x) :\leftrightarrow \bot \in \mathrm{Th}(\neg x)$ — x is necessary *(its negation is impossible)*
$\mathrm{C}(x, y) :\leftrightarrow \bot \notin \mathrm{Cn}(\mathrm{Th}(x) \cup \mathrm{Th}(y))$ — x and y are compossible *(their common theory is consistent).*

Immediately we obtain Leibnizian "soundness" conditions:

(7) $\mathrm{C}(x, y) \leftrightarrow \mathrm{C}(y, x)$ *Compossibility relation is symmetric.*
(8) $\mathrm{M}(x) \leftrightarrow \mathrm{C}(x, x)$ *Possibility means self-compossibility.*
(9) $\mathrm{C}(x, y) \rightarrow \mathrm{M}(x) \wedge \mathrm{M}(y)$ *Compossibility implies possibility.*

When can the above implication be reversed?

Onto\logical construction

29. Observe that in the framework of combination ontology we have already, in § 26, defined M(x) in a way respecting (8).

On the other hand, the previous question suggests that between MP(,) and C(,) there is another relation, more fundamental than compossibility one. It is so-called *compatibility* relation. Indeed, putting

CP(x, y):$\leftrightarrow$ MP(x, y)$\wedge$MP(y, x) —for compatibility, and
C(x, y):$\leftrightarrow$ M(x)$\wedge$M(y)$\wedge$CP(x, y) —for compossibility

we obtain a manageable compossibility relation obeying the above Leibniz's "soundness" conditions.

30. Clearly, *wholes* are combinations of compossible collections, whereas *possible worlds* are obtained by maximalization of wholes.

Finally, observe that in our approach we start with basic ontological makings —modalities more fundamental than Leibnizian compossibility, for they are definable in two steps from our two basic makings: *making possible* and *making impossible*.

Now time comes for executing these ideas in the framework of logic.

Logic

Combination semantics of logical modalities obtained by means of makings combination

Definitions

31. An ontological frame U is the triple <U, MP, MT>; where U is any collection, whereas MP and MT are appropriate binary relations on U, respectively *making possible* and *making true*.

Without any loss of generality we can split U into the extralanguage domain U′ and the language domain FOR: U = U′$\cup$FOR, with suitable restrictions of MP and MT to these subdomains: MP, MT $\subseteq$ U′$\times$FOR.

32. For classical connectives we assume the well-known Fregean Tarski's conditions characterizing MT:

($\neg$)	MT(x, $\neg$A)	iff	$\neg$MT(x, A)
($\wedge$)	MT(x, A$\wedge$B)	iff	MT(x, A)$\wedge$MT(x, B)
($\vee$)	MT(x, A$\vee$B)	iff	MT(x, A)$\vee$MT(x, B)
($\rightarrow$)	MT(x, A$\rightarrow$B)	iff	MT(x, A) $\rightarrow$ MT(x, B)

32.1 Logical modalities are interpreted, however, by means of their conjugate onto\logical modalities. In particular

($\Diamond$) MT(x, $\Diamond$A) iff MP(x, A) !

It is possible that A holds at x (or x is making that A is possible true) iff x makes A possible.

Observe, that by dualization

($\Box$) MT(x, $\Box$A) iff $\neg$MP(x, $\neg$A).

32.2 For any ontological frame we put as usual:

$U \models$ A iff $\forall$x $\in$ U$'$ MT(x, A).

Notice that the above receipt works, *mutatis mutandis*, for any intensional logic!!

Correspondence

33. It is easy to check the following correspondence list for modal and onto\logical formulas:

(T) $A \rightarrow \Diamond A$ — *Axiom of Gödel-Feys-von Wright*

It is characterized by the implication MT(x, A) $\rightarrow$ MP(x, A), i.e., by the inequality MT$\leq$MP: *Making truth implies (or is included in) making possible.* In short: *Truth implies possibility.*

It is, in fact, the original Aristotelian explication of the axiom, obviously more transparent and clear than the alternative explication offered by relational semantics: xRx, i.e., the alternativity relation between possible worlds is reflexive.

Axiom of noncontingency (for its nonsymmetric case)

(TV) $A \rightarrow \Box A$

is characterized by the reverse inclusion: MP$\leq$MT. Its ontological characterization is thus given by the implication: MP(x, A) $\rightarrow$ MT(x, A), reverse to the previous one. Thus noncontingency means that *making possible implies making truth*, in short: *possibility implies truth*.

Finally, the conjunction of both axioms, i.e., *Triviality axiom*

(TR) $\Diamond A \leftrightarrow A$

corresponds to the equality: MP = MT, saying that *making possible* is simply *making truth*. In short: *possibility* means *truth*.

To conclude, the above three closely connected axioms are explained in our semantics in a clear, natural and intuitive way.

33.1 Let us pass now to three well-known axioms connected with *ontological rationalism*.

Consider first the *axiom of ontological rationalism of Leibnizian type*, saying that *nothing is contingent (in a symmetric version of contingency)*

(R) $\Diamond A \rightarrow \Box A$

It is characterized by the implication MP(x, ¬A) → ¬MP(x, A), which is equivalent to the *ontological consistency law* (in its metalogical version, cf. § 26):

$$\neg(\mathrm{MP}(\mathrm{x}, \neg\mathrm{A}) \wedge \mathrm{MP}(\mathrm{x}, \mathrm{A})).$$

It is indeed the soundest explication of Leibnizian axiom, for it it well-known that ontological rationalism is based on the law of noncontradiction!

Recall that in the case of relational semantics the axiom is connected with the condition of functionality for alternativity relation, which is also very rationalistic in spirit.

Its dual version is the famous axiom of *standard deontic logic* of (Aristotle)-von Wright-(Makinson):

$$(D) \quad \Box A \rightarrow \Diamond A$$

It corresponds with the implication ¬MP(x, A) → MP(x, ¬A), which is equivalent to the metalogical version of the ontological excluded middle law:

$\mathrm{MP}(\mathrm{x}, \mathrm{A}) \vee \mathrm{MP}(\mathrm{x}, \neg\mathrm{A})$: *x makes possible A or its negation.*

Joining both axioms together we obtain the *axiom of strong rationalism*

$$(DR) \quad \Box A \leftrightarrow \Diamond A$$

which corresponds to the principle of metalogical bivalence:

$\neg\mathrm{MP}(\mathrm{x}, \mathrm{A}) \leftrightarrow \mathrm{MP}(\mathrm{x}, \neg\mathrm{A})$: *x makes possible either A or $\neg A$.*

Recall that the relational semantics shows another side of strong rationality: (DR) corresponds to the restriction of the alternative relations to functions. Recall that, in fact, rationalistic description of the universe is often purely mathematical, hence it is indeed done in terms of functions.

Anyway, at least in the above cases (but not only in them) combination semantics demonstrated in a fairly clear way its power of intuitive philosophical characterization of an important family of modal axioms!

33.2 Let us now pass to several well-known modal axioms. Consider first the *Brouwersche axiom of symmetry* introduced by Becker and Kripke:

$$(B) \quad \Diamond\Box A \rightarrow A$$

It is characterized by the implication MP(x, □A) → MT(x, A), saying that *to make necessity of A possible means to make A true*. Indeed, to make necessity of A possible is to guarantee verification of the formula in (at least some family of) possible worlds, hence guarantee that it is true as well.

Consider now the *transitivity axiom of Lewis*

$(4) \quad \Diamond\Diamond A \rightarrow \Diamond A$

It is characterized by the ontological implication: MP(x, $\Diamond$A) $\rightarrow$ MP(x, A), which says that *to make possibility of A possible means to make A possible*.

Similarly, the *Euclidean axiom of Lewis*

$(5) \quad \Diamond\Box A \rightarrow \Box A$

It corresponds to the implication MP(x, $\neg$A) $\rightarrow$ $\neg$MP(x, $\Box$A): *Making possible not-A excludes making possible that A is necessary, or it is excluded that x makes both A necessary and negate it.*

Analogously, the *axiom of distribution*

$(Dis) \quad \Diamond(A \vee B) \leftrightarrow \Diamond A \vee \Diamond B$

which is characterized by MP(x, A$\vee$B) $\leftrightarrow$ MP(x, A)$\vee$MP(x, B)

33.3 Pass now to the famous axiom of Kripke, which limits normal modal logics from the down

$(K) \quad \Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

It is characterized by the following formula:

$\neg\mathrm{MP}(\mathrm{x}, \neg\mathrm{A}) \wedge \mathrm{MP}(\mathrm{x}, \neg\mathrm{B}) \rightarrow \mathrm{MP}(\mathrm{x}, \neg(\mathrm{A} \rightarrow \mathrm{B}))$,

giving, in a sense, a condition for falsification of an implication.

33.4 Consider also

Negative counterpart logic — $(nT) \;\; \neg A \rightarrow \Diamond A$

It is characterized by a rather nice condition:

MT(x, A)$\vee$MP(x, A)

—any x either makes A true or makes it possible.

33.5 Finally, let me characterize the remaining three "crucial" logics:

Verum logic — $(VER) \;\; \neg\Diamond A$

It means that MP is empty, i.e., that nothing is possible for nothing is made possible.

Falsum logic — $(FALS) \;\; \Diamond A$

It means that MP is full, saying that everything is possible for everything can be made possible (for example, during *Meditationes de prima philosophia*, by the Cartesian Demon).

Negation logic — $(NEG)\ \Diamond A \leftrightarrow \neg A$

It means that MP = $-$MT, i.e., making possible is the complement of making true.

Observe that the above list two kinds of axioms were discussed: purely formal —like 4, 5, Dis, K; and ontological —like T, D, R, nT, B, TR and NEG, VER as well as FALS.

Completeness

34. Now, it would be fine to prove that the above conditions fully characterize logics in question.
The above semantics is indeed quite general and broad.

Let P be a modal logic, C —the classical consequence operator based on detachment and the classical logic CL. L(P) denotes the family of all Lindenbaum oversystems of P.

The *canonical frame* <L(P)∪FOR, MT, MP> is defined now by putting the following definitions of makings:

$MT(X, A)$ *iff* $A \in X$
$MP(X, A)$ *iff* $C(X, \Diamond A)$ *is consistent*

Immediately from the definition we see that the canonical valuation, i.e., the *making true relation* MT is simply the characteristic function of the Lindenbaum system X.

By quite standard argument we have

(10) THE COMPLETENESS THEOREM (Cf. Perzanowski [2], theorem (38)).
For any modal logic P, P is characterized by the class of all ontological P-frames.

Proof. We work with the classical logic expressed in the modal language. Therefore all Boolean conditions put on MT are clearly satisfied. P being logic is also closed on substitution.
Therefore we must check the only remaining case —for possibility:

($\Diamond$) MT(X, $\Diamond$A) iff MP(X, A).

But MP(X, A) means that C(X, $\Diamond$A) is consistent. X, being Lindenbaum set, is maximally consistent. Hence $\Diamond$A must belong to X. This means, however that MT(X, $\Diamond$A), what should be checked.

Rest of the proof is standard. Q.E.D.

Many corollaries follow immediately, including

(11) *CL is complete with respect to all ontological frames.*

(12) *KT is complete with respect to all KT-frames.*

(13) *DR is complete with respect to all DR-frames.*
ETC.

Comments

35. Clearly, the crucial relation MP of canonical frames can be defined also in a more familiar, Tarskian, way:

MP(X, A) iff X $\vdash$ $\Diamond$A.

Both definitions provide us with metalogical explications of *making possible* in the case of canonical models: X *makes* A *possible* means either

- in the spirit of Leibniz —that X *is consistent with the claim that* A *is possible*, or
- in the spirit of Tarski —that X *infers that* A *is possible*.

36. Notice, however, that quite a lot of reasonable modifications of MP-definition limit seriously the application of canonical frames.

In particular, following Leibniz's clue of § 28 in the literal way:

($\Diamond$) MP(X, A) iff C(X, A) is consistent

we obtain a semantical characterization of the Post-complete logic TR, for MT(X, $\Diamond$ A) iff $\Diamond$ A $\in$ X, whereas C(X, A) is consistent iff A $\in$ X; hence ($\Diamond$) means that $\Diamond$ A $\in$ X iff A $\in$ X, i.e., X $\in L(TR)$.

37. Similarly, putting MP(X, A) iff C(X, $\neg$A) we obtain adequate semantics for the negative logic NEG.

Generalizations

38. The combination ontological semantics works for all intensional logics!

38.1 Let us outline first its generalization to the case of an arbitrary n-ary functor F(A_1,...,A_n). As a matter of fact there are at least three ways of generalization:

I Frames: <U∪ FOR, MT, MF>, where MF⊆U×FOR^n.
(F) MT(x, F(A_1,...,A_n)) iff MF(x, <A_1,...,A_n>);
whereas in the case of canonical frames we use Tarskian trick
MF(X, <A_1,...A_n>) iff X $\vdash$ F(A_1,...,A_n).

II Now, we can also work with the family of n-ary relations $MF_x \subseteq FOR^n$.
Frames: <U∪FOR, MT, (MF_x: x ∈ U)>,
(F) MT(x, F(A_1,...,A_n)) iff MF_x(A_1,...A_n).

III We can also try to work with F-fusion Π_F(x, A_1,...A_{n-1}), returning again, but in a different way to the case I, to the case of binary relation MF which characterize the functor F:
(F) MT(x, F(A_1,...,A_n)) iff MF(Π_F(x, A_1,...A_{n-1}), A_n).

Observe that in each case, under appropriate proviso, we can obtain the paradigmatic case MP := M$\Diamond$.

38.2 The semantics also covers the case of given finite families of intensional functors (F_i: i ∈ I).
Frames: <U∪FOR, MT, ($MF_{<i,x>}$: i ∈ I, x ∈ U>;

(F_i) MT(x, F_i(A_1,...,A_{ni})) iff $MF_{<i,x>}$(A_1,...,A_{ni})

To define $MF_{<i,x>}$ in canonical models we use again Tarskian trick.

The example

39. An important and interesting example of use, in fact, of combination semantics in the realm of deontic logic was independently developed by Kazimierz Świrydowicz in his analysis of norms and conditional duties, cf. [10], [11].

Consider the logic of norms N of K. Świrydowicz (connected with the logic of conditional obligation of G. H. von Wright).

Its language is based on ¬, ∧, !; where the last functor is used to express A norm: A!B read: *if* A *do* B.

Rules of the logic N:
(R→) A!B, B→C ⊢ A!C
(DK) A!B, A!C ⊢ A!(B∧C)
(→R) A!B, C→A ⊢ C!B
(DA) A!B, C!B ⊢ (A∨C)!B

The logic is characterized by *making duty* model <U∪FOR, MT, (MD$_x$: x∈U)>, where MD$_x$ ⊆ FOR^2. We put

(!) MT(x, A!B) iff MD$_x$(A, B)

Abbreviations: B≤C iff ∀x MT((x, B→C) (i.e. ⊨ B→C); and
MD(A, B) iff ∀x MD$_x$(A, B).

ŚW-frames are making duty frames fulfilling the following conditions:
MD(,up) B≤C and MD(A, B), then MD(A, C)
MD(dn,) C≤A and MD(A, B), then MD(C, B)
MD(,∧) MD(A, B) and MD(A, C), then MD(A, B∧C)
MD(∨,) MD(B, A) and MD(C, A), then MD(B∨C, A).

(15) K. ŚWIRYDOWICZ'S COMPLETENESS THEOREM.
ŚW-frames are adequate for N.

Conclusion

40. First, makings form an uniform family of meta-modalities, with the classical making MT as the paradigmatic case.

It is, however, not the basic one. The basic making in the present positive approach is the onto\logical making MP.

41. Combination of makings produces adequate semantics for intensional logics. The semantics is, in a sense, a union of matrix semantics —because of MT, and relational semantics— by MP (and other makings, if necessary); what explains its power.

In result, our semantics has the generality of matrix semantics, and the power of explication characteristic for relational semantics!

42. Similar investigation can be done in the case of two further basic makings: MI and MPI, leaving us into an unorthodox parainconsistent realm.

43. To be not purely formal and artificial the ontological combination semantics has to be based on real ontology. Its explanatory power depends on the previous onto\logical theory of suitable superlogical modalities, like the theory of MP offers an explanation for logical modalities $\Diamond$ and $\Box$; or suitable Świrydowicz's conditions on making duty offer a sensible semantics for von Wright's logic of norms.

Combination semantics for a given family of modalities put difficulty with the place of semantical analysis where it really is —not on a rather artificial problems connected with proving suitable completeness theorem, but *in looking for an appropriate background ontological analysis of modalities under consideration, for discovering their fundamental metatheory*.

44. The receipt therefore for finding a sound and essential semantics for a given logical modality (both positive and negative) is as follows:

Make first so adequate as possible analysis of ontological preassumptions concerning the subject under investigation. Using results of this analysis try next to outline a suitable ontological frame, in particular to find makings appropriate to describe on the metalevel the investigated realm.

Use in turn the semantical apparatus described in §§ 31–37. Finally, compare results of the formal analysis with our preformal starting intuitions.

In short, following the old and good advice, evaluate the semantics and its methods by their fruits.

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