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LINGUISTIC AND ONTOLOGICAL MEASURES FOR COMPARING THE INCONSISTENT PARTS OF MODELS*

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Abstract

Adaptive logics depend essentially on measures for the degree of abnormality of models. The linguistic approach to such measures compares the sets of abnormal, *e.g.*, inconsistent wffs verified by the models. The ontological approach compares models in terms of 'structural' properties that do not depend on the way in which the language is interpreted in the model.

While the linguistic approach has not been questioned up to now, present proposals for an ontological approach are affected by several weaknesses. The present paper argues for the attractiveness of an ontological approach and elaborates on the challenge to adequately define it. The final outcome is rather negative: the only sensible definition attained leads to a logic that does not seem to have any suitable application contexts.

1. *Aim of This Paper*

Several types of logics require that abnormalities of some sort are minimized. In inconsistency-adaptive logics, the feature appears in its purest form: the abnormalities to be minimized are inconsistencies.

From a *semantic* point of view, inconsistencies are minimized by selecting the models that interpret the premises as consistently as possible. There are several strategies to realize this effect —see Section 2. The most natural one (from a *semantic* point of view) is the minimal abnormality strategy.

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There are two kinds of approaches to the minimal abnormality strategy. The first, originated by me (in [3] and [2] for the propositional case, and in [5] for the full logics) will be called the *linguistic* approach. It selects models of the premises by comparing the abnormalities they verify. The second approach, originated by Graham Priest in [25], will be called the *ontological* approach. It compares models of the premises with respect to 'the inconsistencies of the model', say of its structure (see Section 4). Unlike what is the case for the linguistic definition, present ontological definitions are affected by several problems.

From the fact that I originated the linguistic approach, one should not conclude that I favour it. I consider both approaches as sensible in principle, and think it is essential to elaborate both and to delineate their respective domains of application. Apart from a brief comparison between both approaches, to make the underlying ideas and effects more perspicuous, the present paper mainly aims at defining a sensible ontological definition of minimal abnormality.

The (perhaps provisional) conclusion of the present paper will be that an ontological definition is either not sensible, or lacks appropriate application contexts. Nevertheless, it seems most instructive to clearly state the problem, to offer a scrutinized description of the reasons for the present failure, and to list the problems as well as attempted solutions on our way to the provisional conclusion. All this will certainly be useful to find an adequate ontological definition, if there is one, or to show that the ontological approach is not sensible, if it is not. The insights gained support the latter conclusion.

The interest of the problems discussed in this paper is not restricted to minimally abnormal models. Those problems are related to the nature and scope of semantic systems. The underlying issue is this: is a semantics merely a device to interpret a given language, or is it possible to distinguish between an ontological and a linguistic component of a semantics.

2. *Adaptive Logics*

Inconsistency-adaptive logics —see [9] for a survey— interpret a set of premises 'as consistently as possible', and hence define a consequence set that is (except for a border case) much richer than paraconsistent logics. If the premises are consistent, all now existing inconsistency-adaptive logics[1] deliver the same consequence set as Classical Logic —henceforth CL.

[1] I shall show in Section 6.1 that this fails for the original formulation of LP^m. However, Graham Priest changed the system in view of this.

Semantically, they do so by selecting a set of paraconsistent models of the premises (see [2] and [5]). Inconsistency-adaptive logics also have a dynamic proof theory —see [1], [3] (the oldest paper), and [5]— and this is their most striking and innovative feature. As the present paper deals with a problem that exclusively concerns the selection of the models, the proof theory will not be discussed.

The intended domains of application of inconsistency-adaptive logics are, first and foremost, scientific theories and scientific knowledge systems that were intended as consistent but turned out inconsistent. Other applications concern possibly inconsistent databases and possibly inconsistent everyday knowledge. The aim of the logics is *not* to resolve the inconsistencies. The replacement of inconsistent theories (etc.) by consistent alternatives is not a task for logic alone, but should depend on non-logical preferences (determined by the reliability of data, by methodological considerations, and so on) —see already [28, p. 37]. The adaptive logics considered in the present paper rely solely on logical considerations. They enable one to analyse the theory or database in its present state, to act on it (if we must), and to devise experiments and other ways to gain new relevant information.[2]

The strength of inconsistency-adaptive logics is also illustrated by the fact that all flat Rescher–Manor consequence relations (Free, Strong, Argued, C-Based, and Weak consequences —see [16]) were shown (in [11]) to be characterized by an inconsistency-adaptive logic. Similar results are forthcoming about prioritized such consequence relations (as described in [17]). It also was shown (in [4] and [20]) that several popular 'non-monotonic logics' are characterized by an adaptive logic combined with (non-logical) preferences.

Inconsistency-adaptive logics and other *corrective* adaptive logics[3] deliver consequence sets that are subsets of the CL-consequence set if the latter is trivial. *Ampliative* adaptive logics (compatibility, abduction, ...) deliver consequence sets that are supersets of the CL-consequence set, but still are not trivial (and are determined by some normality suppositions) —see [12] and forthcoming papers. In the present paper, I concentrate on inconsistency-adaptive logics. However, the problem discussed here arises just as well with respect to those other adaptive logics.

[2] So, the aim of adaptive logics is drastically different from the non-monotonic systems discussed, for example, in [21].

[3] Other corrective adaptive logics interpret premises as normally as possible with respect to logical abnormalities that are not inconsistencies —for example, gaps with respect to negation (negation-incompleteness), gluts with respect to conjunction ($A \wedge B$ true while some conjunct is false), etc., including combinations (see, e.g., [6]). Some corrective adaptive logics interpret premises as normally as possible with respect to ambiguities in the non-logical constants (see [29], [30] and [10]).

The phrase "as consistently as possible" is not unambiguous, and this leads to a multiplicity of inconsistency-adaptive logics. An inconsistency-adaptive logic is characterized by (i) a lower limit logic, (ii) an upper limit logic, and (iii) a strategy —see [9] for further possible variation. The upper limit logic is usually (and in this paper is always) CL. The lower limit logic is some paraconsistent logic. My preferred system for this purpose is $CLuN$ —see, *e.g.*, [5]— Graham Priest's preferred system is his LP —see, *e.g.*, [24]— others still prefer other systems (Joke Meheus prefers her AN from [22]). Let us use LLL as a variable name for the lower limit logic, and AL as a variable name for the adaptive logic obtained from LLL by the chosen strategy.

As I remarked before, the strategy determines which LLL-models of Γ are AL-models of Γ. A peculiarity has to be pointed out from the outset. "M is a CL-model" and "M is a LLL-model" are meaningful phrases —the CL-models of Γ being the CL-models that verify Γ, and similarly for the LLL-models. But "M is an AL-model" is not a meaningful phrase, whereas "M is an AL-model of Γ" is. Indeed, the AL-models of Γ are the LLL-models of Γ that are *selected* by the strategy in view of properties of all LLL-models of Γ.

For inconsistency-adaptive logics, the models are selected on the basis of their 'degree of inconsistency', or their 'inconsistent part'. Let me clarify this by briefly discussing some strategies. For any LLL-model M, we define the abnormal part of M, $Ab(M)$ —specific definitions are considered below. The minimal abnormality strategy refers, as expected, to minimally abnormal models of Γ, viz. LLL-models M of Γ for which it holds that

$$\text{no } LLL\text{-model } M' \text{ of } \Gamma \text{ is such that } Ab(M') \subset Ab(M)$$

Where AL is defined from LLL by the minimal abnormality strategy, $\Gamma \models_{AL} A$ iff A is verified by all minimally abnormal LLL-models of Γ.

In other strategies $Ab(M)$ plays an equally central role. The subsequent paragraphs of the present section illustrate this, and may be skipped (as the rest of the paper concentrates on the minimal abnormality strategy).

The other strategies require some preparatory steps. For example, for the *reliability* strategy we proceed as follows. Let $\exists A$ abbreviate A preceded by (in some preferred order) an existential quantifier over each variable free in A. Where $\Delta = \{A_1, \ldots, A_n\}$, let $Dab(\Delta)$ and $Dab\{A_1, \ldots, A_n\}$ abbreviate $\exists(A_1 \wedge \sim A_1) \vee \ldots \vee \exists(A_n \wedge \sim A_n)$. We call $Dab(\Delta)$ a minimal-Dab-consequence of Γ iff $\Gamma \models_{LLL} Dab(\Delta)$ and, for all $\Theta \subset \Delta$, $\Gamma \not\models_{LLL} Dab(\Theta)$. $U(\Gamma)$, the set of formulas that are unreliable with respect to Γ, is defined as the set of A such that $A \in \Delta$ for some minimal Dab-consequence $Dab(\Delta)$ of Γ. Finally, where AL is defined from LLL by the reliability strategy, $\Gamma \models_{AL} A$ iff A is verified by all LLL-models M of Γ for which $Ab(M) \subset U(\Gamma)$.

The *simple* strategy is only well-defined for some specific lower limit logics, for example AN from [22]. Where AL is defined from LLL by the simple strategy, $\Gamma \models_{AL} A$ iff A is verified by all LLL-models M of Γ for which $Ab(M) = \{B \mid \Gamma \models_{LLL} \exists(B \wedge {\sim}B)\}$.

From now on, I shall only deal with the minimal abnormality strategy.

3. *The Problem*

In [2], which only dealt with the propositional case, the abnormal part of a model was defined by

$$Ab(M) =_{df} \{A \mid v_M(A \wedge {\sim}A) = 1\}$$

In [5], this is generalized to the predicative case:

$$Ab(M) =_{df} \{A \mid v_M(\exists(A \wedge {\sim}A) = 1\}.$$

This definition is 'linguistic' in that it does not refer to properties of 'the model itself', but only to wffs verified by the model. Models that are elementary equivalent (verify the same wffs) have identical inconsistent parts.

Meanwhile, Graham Priest had generalized the first definition differently to the predicative case (in [25]). He defines the abnormal part of a model in terms of properties of 'the model itself', hence in an 'ontological' way. A simple way to explain the matter is as follows. Let the pseudo-language $\mathcal{L}^{\uparrow}$ be obtained by replacing the set of constants in $\mathcal{L}$ by a set of pseudo-constants, viz. one for each member of D —I shall use the members of D as names for themselves. The *inconsistent part* of M is then defined by:[4]

$$Ab^o(M) =_{df} \{A \mid v_M(A \wedge {\sim}A) = 1; A \text{ is a primitive wff of } \mathcal{L}^{\uparrow}\}$$

The important difference with the linguistic definition is that A is a wff of $\mathcal{L}^{\uparrow}$. Unlike $\mathcal{L}$, $\mathcal{L}^{\uparrow}$ enables one to fully describe the model. Whence I call the definition ontological.

In [25], Priest moreover introduces the *Domain Restriction* to the definition of minimal abnormal models:

> A model M of Γ is minimally abnormal iff no model M' of Γ *that has the same domain* as M is such that $Ab^o(M') \subset Ab^o(M)$.

The effect of the Domain Restriction may be illustrated by the following example.

[4] The superscript in $Ab^o(M)$ refers to "ontological". The reference to *primitive wffs* in the definition depends merely on properties of Priest's lower limit logic. The same restriction is required if the linguistic definition is applied to that or a similar logic —see below.

Example 3.1. Premise: $(\forall x)(Px \wedge {\sim}Px)$. For any model M of this formula, $\{P\mathsf{a} \mid \mathsf{a} \in D\} \subseteq Ab^o(M)$. So, where the domain of M is not a singleton, there is a model M' with a singleton domain $\{\mathsf{a}\}$ such that $Ab^o(M') \subset Ab^o(M)$. Hence, if the Domain Restriction were not imposed, all minimally abnormal models of the premise would have a singleton domain and hence

$$(\exists x)(\forall y)y = x$$

would be a consequence of the premise.

Both definitions lead to very different results, as I shall illustrate in Section 5. Moreover, the above ontological definition is inadequate, as we shall see in Section 6. But this does not entail that no sound ontological definition is possible. Moreover, there are at least two good reasons to look for one.

There clearly is a coherent notion behind the idea of an ontological measure. There is a clear sense in which a model *itself* (that is, its 'structure') may be inconsistent. Even if two models are elementary equivalent, they may in themselves have a different degree of inconsistency. So, a sound ontological definition seems at least possible.

The second reason is philosophically deeper. In view of present-day historical insights, we should be aware of the fact that our knowledge may be, and presumably forever will be, quite remote from the real structure of the world. This often forces one to reason about the world in ways that are partly independent of our present knowledge (and present languages). We not only do so, at least since Kant, in epistemological contexts. In a sense, this form of reasoning is present in many creative processes in the sciences. The idea of measuring the inconsistent part of a model in an ontological way —that is, from structural properties of the model rather than from properties of the set of wffs verified by the model— seems attractive from that point of view.

Let me put the matter in slightly different terms. Suppose that some of our theories is inconsistent, and that we are interested in interpreting it as consistently as possible. The linguistic approach will at best offer a minimally inconsistent interpretation of the theory *within* the bounds of our present conceptual system. If a sensible ontological definition may be found, it might transcend such bounds.

In the present paper, I discuss several proposals for an ontological definition. To avoid complications, I shall concentrate on the minimal abnormality strategy and moreover on lower limit logics in which all inconsistencies are reduced to the inconsistent behaviour of primitive formulas. The effect of the latter restriction is that the abnormal part of a model will be measured in terms of primitive formulas.[5] These logics will be *CLuNs* and (its fragment)

[5] See, however, Section 8.4 for open problems concerning the ontological definition for logics that isolate inconsistencies.

LP. In order to avoid confusion that might arise from reasoning in terms of the pseudo-language, I characterize (in Section 4) those logics in terms of a model-theoretic type of semantics: the inconsistencies in the models are measured without referring to the way in which the language $\mathcal{L}$ is connected to the 'structure' that underlies the model.

4. *A Semantics for CLuNs and LP*

A model[6] will consist of two clearly separated elements: a *structure* that does not contain any references to $\mathcal{L}$, and an *interpretation* that connects $\mathcal{L}$ to the structure.

A *structure* is a couple. Its first element, D, is a non-empty set. Its second element, $\{R_i,\ i \in I\}$, is a set of paraconsistent 'relations' on D, where I is a set of indices. For each $i \in I$, R_i has a certain adicity: $0, 1, 2, \ldots$. A relation of adicity 0 corresponds to a 'fact'; it may serve as the interpretation of a sentential letter. A 'relation' of adicity $n > 0$ may serve as the interpretation of a predicate of adicity n.

A paraconsistent 'relation' R_i on D is a couple $\langle E, A\rangle$, where (i) if the adicity of R_i is 0, $E, A \in \{0, 1\}$ with the restriction that $1 \in \{E, A\}$, (ii) if the adicity of R_i is 1, $E, A \in \wp(D)$ (where $\wp(D)$ is the power set of D), with the restriction that $E \cup A = D$, (iii) if the adicity of R_i is $n > 1$, $E, A \in \wp(D^n)$ (where D^n is the n-the cartesian product of D), with the restriction that $E \cup A = D^n$. Intuitively, E and A are the extension and the anti-extension of the sentential letter or predicate corresponding to R_i. These may overlap —thus causing inconsistency— and need to be exhaustive —thus avoiding negation-incompleteness.

We stipulate that $R_0 = \langle E, A\rangle$ is a relation of adicity 2 such that (i) $E = \{\langle \mathsf{a}, \mathsf{a}\rangle \mid \mathsf{a} \in D\}$, and (ii) $\langle \mathsf{a}, \mathsf{b}\rangle \in A$ whenever a and b are different elements of D.[7] R_0 will serve as the interpretation of identity. The requirement assures that $\alpha = \beta$ is true in a model whenever α and β have the same interpretation in it.[8]

A structure of this type is *appropriate* for a first order language, $\mathcal{L}$, with a set of sentential letters and predicates $\pi_j,\ j \in J$ iff, for any $j \in J$ there is an

[6] This part of the paper is inspired by a note by Graham Priest (electronic discussion).

[7] It is not excluded that $\langle \mathsf{a}, \mathsf{a}\rangle \in A$ for some $\mathsf{a} \in D$. Precisely this will cause inconsistent identities, for example $a = b \wedge {\sim} a = b$.

[8] The stipulation is asymmetric in that all inconsistencies with respect to identity are caused by the anti-extension of identity. This may be eliminated if one slightly changes the clause for $v^+_M(\pi_i\alpha_1 \ldots \alpha_n) = 1$ and $v^-_M(\pi_i\alpha_1 \ldots \alpha_n) = 1$ below.

$i \in I$ such that the adicity of π_j is the same as that of R_i. An *interpretation* or assignment v of this language with respect to the structure, is such that $v(\pi_j) = R_i$, where R_i is a relation that has the same adicity as π_j, and $v(\alpha) \in D$ for all individual constants α.[9]

Where the interpretation of some schematic letter is a couple, it is handier to consider it as composed of two functions, v^+ and v^-, such that $v(\ldots) = \langle v^+(\ldots), v^-(\ldots)\rangle$. Similarly, the valuation v_M determined by the model M will be considered as composed of the functions, v^+_M and v^-_M, such that $v_M(\ldots) = \langle v^+_M(\ldots), v^-_M(\ldots)\rangle$. To extend the interpretation to a valuation determined by the model M, we first stipulate, as expected:

- if the adicity of π_j is 0, then $v_M(\pi_j) = v(\pi_j)$
- if the adicity of π_i is $n > 0$, then $v^+_M(\pi_i\alpha_1 \ldots \alpha_n) = 1$ iff $\langle v(\alpha_1) \ldots v(\alpha_n)\rangle \in v^+(\pi_i)$
- if the adicity of π_i is $n > 0$, then $v^-_M(\pi_i\alpha_1 \ldots \alpha_n) = 1$ iff $\langle v(\alpha_1) \ldots v(\alpha_n)\rangle \in v^-(\pi_i)$

As $v(=) = R_0$, the above handles identity in the suitable (paraconsistent) way. Extending the interpretation to the propositional connectives is straightforward:

- $v^+_M(\sim A) = v^-_M(A)$
- $v^-_M(\sim A) = v^+_M(A)$
- $v^+_M(A \wedge B) = min(v^+_M(A), v^+_M(B))$
- $v^-_M(A \wedge B) = max(v^-_M(A), v^-_M(B))$
- $v^+_M(A \vee B) = max(v^+_M(A), v^+_M(B))$
- $v^-_M(A \vee B) = min(v^-_M(A), v^-_M(B))$
- $v^+_M(A \supset B) = max(1 - v^+_M(A), v^+_M(B))$
- $v^-_M(A \supset B) = min(v^+_M(A), v^-_M(B))$

There seems no reason not to introduce the sentential constant $\bot$, defined syntactically by $\bot \supset A$. Its semantic definition is:

- $v^+_M(\bot) = 0$
- $v^-_M(\bot) = 1$

This allows us to explicitly define classical negation, henceforth written as $\neg$, as follows:

$$\neg A =_{df} A \supset \bot$$

[9] In other words, the structure is sufficiently rich to accommodate $\mathcal{L}$, but it may be much richer than $\mathcal{L}$ and several predicates from $\mathcal{L}$ may receive the same interpretation. Any other decision would undermine the ontological character of the ontological definition of a minimally abnormal model.

We leave it for the reader to check that, on this definition,[10]

- $v^+_M(\neg A) = 1 - v^+_M(A)$
- $v^-_M(\neg A) = v^+_M(A)$

Quantified formulas may be handled in terms of variant interpretations $v_{M^a_o}$ as defined in Chapter 9 of [18]:

- $v^+_M((\forall\alpha)A) = min(v_{M^\beta_o}{}^+(A_\alpha\beta))$
- $v^-_M((\forall\alpha)A) = max(v_{M^\beta_o}{}^-(A_\alpha\beta))$
- $v^+_M((\exists\alpha)A) = max(v^+_{M^\beta_o}(A_\alpha\beta))$
- $v^-_M((\exists\alpha)A) = min(v^-_{M^\beta_o}(A_\alpha\beta))$

This completes the semantics of $CLuNs$. The semantics of LP is obtained by removing the clauses for material implication and defining it by

$$A \supset B =_{df} {\sim}A \vee B$$

As a result, material implication is not detachable in LP, and $\neg$ cannot be defined in it.[11]

The linguistic definition for the abnormal part of a model reads:

$$Ab(M)$$
$$=_{df} \{A \mid v_M(\exists(A \wedge {\sim}A)) = 1; A \text{ is a primitive formula of } \mathcal{L}\}.$$

in which "formula" should be read as: open or closed formula.

The aim of the above construction obviously was to clearly spell out the ontological definition. For any relation $R_i = \langle E, A\rangle$, $i \in I$, the inconsistent part of R_i is $X_i = E \cap A$. As we have to compare models, I write X^M_i to refer to the sets X_i as determined by the model M. In agreement with Section 3,[12] the abnormal part of model M is defined in terms of properties of its structure as follows:

$$Ab^o(M) =_{df} \{\langle X^M_i, i\rangle \mid i \in I\}$$

Two models M and M' will be said to be *of the same type* iff they share D and, for all $i \in I$, the adicity of R_i is the same in both models. Let $Ab^o(M) < Ab^o(M')$ denote that, for some $i \in I$, $X^M_i \subset X^{M'}_i$, whereas, for all $i \in I$, $X^M_i \subseteq X^{M'}_i$. A model M of a set Γ will be said to be a *minimally abnormal* model of Γ iff no model M' of Γ is such that M' is of

[10] Remark that $v_M(\neg A) = \langle 0, 1\rangle$ if $v_M(A) = \langle 1, 0\rangle$ or $v_M(A) = \langle 1, 1\rangle$, whereas $v_M(\neg A) = \langle 1, 0\rangle$ if $v_M(A) = \langle 0, 1\rangle$.

[11] The above definition of $\neg$ makes the latter identical to $\sim$ in LP.

[12] In one respect, the following definition diverges from the one in Section 3 —see Subsection 6.5 for the justification of this.

the same type as M, and $Ab^o(M') < Ab^o(M)$. As desired, this definition of "minimally abnormal model of Γ" refers to the structure only, not to the interpretation.[13] Remark that the Domain Restriction was replaced by (what I shall call) the *Same Type Restriction*.

Where $v(P) = R_i$, I shall often write X_P^M instead of X_i^M.

5. *Comparing the two Definitions*

Let us consider some examples that illustrate the difference between both definitions.

Example 5.1. Premise: $Pa \wedge \sim Pa$. The ontological approach selects the models in which X_P^M is a singleton. The linguistic approach selects the models that verify only inconsistencies, $\exists(A \wedge \sim A)$, that are verified by *all* paraconsistent models of the premise —that is, $Pa \wedge \sim Pa$ and all formulas of the form $(\exists\alpha)(P\alpha \wedge \sim P\alpha)$.

So, according to the linguistic definition

$$(Pb \wedge \sim Pb) \supset \bot$$

and, equivalently,

$$\neg(Pb \wedge \sim Pb)$$

are consequences of the premise.[14]

The ontological definition does not lead to the above consequences. As it selects models in which X_P^M is a singleton, it results in the consequence

$$(\forall x)((Px \wedge \sim Px) \supset x = a)$$

The linguistic approach does not care whether X_P^M is a singleton, provided a is the only individual constant that is interpreted by a member of this set.

Example 5.2. Premises: $Pa \wedge \sim Pa,\ Qa$. On both approaches, these have the consequence

$$(Pb \wedge \sim Pb) \supset Qb$$

However, the reasons for this are rather different. On the ontological approach, X_P^M is a singleton. Hence, if $v(b) \in X_P^M$, then $v(a) = v(b)$, and

[13] However, the weight to be attached to this statement depends on a presupposition about the structure of the world. As is show in the Appendix, the logic LP is not only characterized by the semantics listed above in the text, but also by a semantics compounded from consistent structures and ambiguous interpretations.

[14] Given the way in which material implication is defined in LP, $(Pb \wedge \sim Pb) \supset \bot$ is an LP-theorem, viz. LP-equivalent to $\sim(Pb \wedge \sim Pb) \vee \bot$, to $\sim(Pb \wedge \sim Pb)$ and to $\sim Pb \vee Pb$.

hence Qb. On the linguistic approach, all minimally abnormal models of the premises verify $\neg(Pb \wedge \sim Pb)$, and hence $(Pb \wedge \sim Pb) \supset Qb$.[15]
Example 5.3. Premises: $Pa \wedge \sim Pa,\ Pb \wedge \sim Pb$. These have models in which a and b denote the same element of D, say a, and models in which they denote different elements, say a and b. On the ontological definition, all minimally abnormal models of the premises are amongst the former, and no model of the second sort is minimally abnormal. Hence, on the ontological definition,

$$a = b$$

is a consequence of the premises. On the linguistic definition, the minimally abnormal models have $\{Pa, Pb\}$ as their abnormal part, irrespective of the interpretation of a and b. So, $a = b$ is *not* a consequence of the premises (but, for example $\neg(Pc \wedge \sim Pc)$ is).
Example 5.4. Premises: $Pa \wedge \sim Pa,\ Pb \wedge \sim Pb,\ \sim a = b$. On the ontological definition, there are minimally inconsistent models of the premises in which $v(a)$ and $v(b)$ are different, and others in which they are identical, and hence $\langle v(a), v(b)\rangle \in X_=$. In the former models, X_P contains two elements, in the latter, both X_P and $X_=$ contain one element. On the linguistic definition, the models of the second kind are *not* minimally inconsistent. Indeed, they have at least $\{Pa, Pb, a = b\}$ as their abnormal part, whereas the minimal inconsistent models of the first kind have $\{Pa, Pb\}$ as their abnormal part.
Example 5.5. Premises: $Pa \wedge \sim Pa,\ Pb \wedge \sim Pb,\ \sim a = b,\ a = b \vee Qc$. The difference that surfaced in the previous example shows more clearly here. Only on the linguistic definition, Qc is a consequence of the premises.
Example 5.6. Premises: $Pa \wedge \sim Pa,\ Pb \wedge \sim Pb,\ Ra,\ \sim Rb$. From these premises,

$$a = b$$

is not derivable on either definition. Even on the ontological definition, there are minimally abnormal models that identify $v(a)$ and $v(b)$ —in these, X_P and X_R are singletons— and others that don't —in these, X_P contains two elements of the domain whereas X_R is empty.

The differences remain if, in the above examples, the premises are added in conjunction and existential generalization is applied for all constants. Let us apply this to Example 5.5.

[15] The example is not very illustrative for LP because $(Pb \wedge \sim Pb) \supset Qb$ is an LP-theorem.

Example 5.7. Premise: $(\exists x)(\exists y)(\exists z)((Px \wedge {\sim}Px) \wedge (Py \wedge {\sim}Py) \wedge {\sim}x = y \wedge (x = y \vee Qz))$. By precisely the same reasoning as in the previous example, $(\exists x)Qx$ is derivable on the linguistic definition but not on the ontological one.

6. *Problems with the Original Ontological Definition*

Up to now, no objections were raised against the linguistic definition, whereas there are some serious problems with the above ontological definition. As there are nevertheless very good reasons to search for a sound ontological definition, I list the problems as systematically as possible. This will facilitate the discussion of attempted emendations of the definition.

6.1. *Failure of the Classical Recapture*

The least one might expect from a logic that pretends to interpret a set of premises as consistently as possible, is that it interprets consistent premises as consistent. For all logics considered in this paper, this is fulfilled iff the adaptive consequences of a consistent set of premises are its CL-consequences. In [3], it is proved that this obtains for (what is now called) the propositional fragment of $ACLuN1$. In [5], the proof is extended to the inconsistency-adaptive logics, $ACLuN1$ and $ACLuN2$, defined from the lower limit logic $CLuN$. In a forthcoming paper, the same is proved for two inconsistency-adaptive logics defined from the lower limit logic $CLuNs$, and for two inconsistency-adaptive logics defined from the lower limit logic LP. In [25], the same is stated (without proof) for LP^m, viz. as FACT 3. However, this statement is mistaken, as I now show.

Example 6.1. Premise: $(\exists x)(\exists y){\sim}x = y$. This consistent premise has models with a singleton domain, say $\{\mathsf{a}\}$. Where M is such a model, M is necessarily inconsistent in that $\langle \mathsf{a}, \mathsf{a}\rangle \in X_0^M$. Moreover, if $X_0^M = \{\langle \mathsf{a}, \mathsf{a}\rangle\}$ and $X_i^M = \emptyset$ whenever $i \neq 0$, then M is minimally inconsistent in view of the Same Type requirement (the premise has no consistent models of the same type). So, the definition fails to select the CL-models of the premise. Such examples are easily multiplied as appears from the next one.

Example 6.2. Let Γ contain all formulas (where $n \in \{2, 3, \ldots\}$)

$$(\forall x)(Px \wedge {\sim}Px) \vee (\exists x_1)(\exists x_2)\ldots(\exists x_n)({\sim}x_1 = x_2 \wedge \ldots \wedge {\sim}x_1 = x_n \\ \wedge {\sim}x_2 = x_3 \wedge \ldots \wedge {\sim}x_2 = x_n \wedge \ldots \wedge {\sim}x_{n-1} = x_n)$$

Γ is a consistent set of premises. Nevertheless, Γ has infinitely many finite models all of which are minimally abnormal and verify $(\forall x)(Px \wedge {\sim}Px)$.

Example 6.3. Premise: $(\exists x)(\forall y)x = y \vee (\exists x)(Px \wedge {\sim}Px)$. All its CL-models have a singleton domain. The premise has LP-models with larger domains, and *all* of these are inconsistent. According to the ontological definition, there are minimally abnormal models of the premise among the latter, viz. all those in which X_P is a singleton. So, in view of the Same Type Restriction, some LP^m-models of the wff are not CL-models. This example illustrates that Priest's definition sometimes fails to eliminate inconsistent models that have a larger domain than the consistent models of the premises.
Example 6.4. Premises: $(\exists x)Px,\ (\exists x){\sim}Px$.[16] All singleton models of the premises are inconsistent (in that the only object both has and has not property P) and some of them are minimally abnormal (viz. those in which this is the only inconsistency). Hence, even in the absence of identity, LP^m does not select the right models for consistent premises.

The failure of the Classical Recapture obviously results from the Same Type Restriction (the Domain Restriction), and there is no doubt that it has to go. Incidentally, the Same Type Restriction has other unwanted consequences as well. One of them is that the premises $(\forall x)(x = a \vee x = b \vee x = c),\ Pa \wedge {\sim}Pa,\ Pb \wedge {\sim}Pb,\ a = b \vee (\forall x)(Qx \wedge {\sim}Qx)$ fail to lead to the sound conclusion $a = b$. Indeed, some models of the premises have a three element domain, and the least inconsistent ones among these falsify $a = b$.

Although I have now shown that the original ontological definition is inadequate, it is worth pointing out some further independent difficulties. These will be helpful for finding a better ontological definition, if there is one.

6.2. *Failure of Strong Reassurance*

Strong Reassurance holds for an adaptive logic AL iff, for all sets of formulas Γ, if a LLL-model M of Γ is not an AL-model of Γ, then some AL-model M' of Γ is such that $Ab^o(M') < Ab^o(M)$ —$Ab(M') \subset Ab(M)$ for the linguistic definition.[17] Strong Reassurance is intuitive appealing: if a LLL-model M of Γ is 'defeated', then it should be defeated by a model of Γ that is itself non-defeated (that is minimally abnormal).[18]

[16] Graham Priest produced this nice example (personal correspondence) after I send him the previous ones.

[17] See the next to last paragraph of Section 4 for the definition of "$<$".

[18] Strong Reassurance is obviously closely related to the Smoothness Condition as defined, for example, in Definition 3.12 of [21].

If Strong Reassurance fails, some non-selected models are not selected because they belong to an infinite sequence(s) no member of which is minimally abnormal. In other words, every model in the sequence is defeated by a less abnormal one, but no model of the sequence is minimally abnormal.

It is shown in [7] that the linguistic measure warrants Strong Reassurance, but that Strong Reassurance fails for Priest's LP^m. The example in that paper is the following.

Example 6.5. Let Γ contain the formulas (where $n \in \{2, 3, \ldots\}$)

$$(\exists x_1)(\exists x_2)\ldots(\exists x_n)((Px_1 \wedge {\sim}Px_1) \wedge \ldots \wedge (Px_n \wedge {\sim}Px_n) \wedge {\sim}x_1 = x_2 \wedge \ldots \wedge {\sim}x_1 = x_n \wedge {\sim}x_2 = x_3 \wedge \ldots \wedge {\sim}x_2 = x_n \wedge \ldots \wedge {\sim}x_{n-1} = x_n)$$

Γ has very simple minimally abnormal models; for example the one in which $D = \{\mathsf{a}, \mathsf{b}\} = v^+(P)$, $v^-(P) = \mathsf{a}$, and $\langle \mathsf{a}, \mathsf{a} \rangle \in v^-(=)$. However, as was proved in [7], for some (actually, infinitely many) LP-models M of this Γ, no LP^m-model M' of Γ is such that $Ab^o(M') < Ab^o(M)$, and hence *Strong Reassurance fails for* LP^m.

This example illustrates the price to pay for the absence of Strong Reassurance. Γ has two kinds of LP-models: those in which $(\exists x){\sim}x = x$ is true and those in which it is false. All models of the *second* kind have an infinite domain, at least a denumerable subset of the domain consists of elements that both have and have not property P. *No* model of the second kind is minimally abnormal. Some models of the *first* kind are minimally abnormal, viz. those in which exactly one object is different from itself and both has and has not property P. So, the transition from LP to LP^m simply wipes out *all* models of the second kind.

This example moreover shows that the LP^m-consequence relation is counterintuitive: both

$$(\exists x)(x = x \wedge {\sim}x = x)$$

and

$$(\exists x)(\forall y)({\sim}(Py \wedge {\sim}Py) \vee y = x)$$

are LP^m-consequences of Γ. Γ states that infinitely many different objects both have and have not property P.[19] According to LP^m, it follows from this that some object is different from itself and that precisely one object both has and has not property P.[20]

[19] Γ states so on its 'normal' reading. It also states so on Priest's reading, even in view of the models.

[20] The conclusion should be qualified: the second formula is a theorem of LP.

6.3. *Not Constant with Respect to Isomorphic Models*

On the ontological definition, $Ab^o(M)$ is not constant with respect to isomorphic models, and hence some models are defeated by isomorphic models. In this sense, the original ontological definition of "inconsistent part of a model" is incoherent. The following is shown in [7]. Let M be a LP-model of the premise from Example 6.5 in which X_P^M as well as $D - X_P^M$ are infinite. By simply moving objects from one set to the other, one obtains isomorphic models that are more inconsistent than M, less inconsistent than M, or incommensurable with M.

6.4. *Failure of Reassurance*

Reassurance obtains for an adaptive logic AL iff any set Γ that has LLL-models also has AL-models —see [25]. The import of Reassurance is enormous. Where it is absent, the adaptive move involves the transition from a non-trivial LLL-consequence set to a trivial AL-consequence set, which would render the move inappropriate (if not nonsensical).

Reassurance is proven for LP^min [25]. However, it is not difficult to see that it fails for $CLuNs$, and would fail for LP if this system would contain a boolean negation. Here is an example:

Example 6.6. Let Γ contain the formulas (where $n \in \{2, 3, \ldots\}$),

$$(\exists x_1)(\exists x_2)\ldots(\exists x_n)((Px_1 \wedge {\sim}Px_1) \wedge \ldots \wedge (Px_n \wedge {\sim}Px_n) \wedge \neg x_1 = x_2 \\ \wedge \ldots \wedge \neg x_1 = x_n \wedge \neg x_2 = x_3 \wedge \ldots \wedge \neg x_2 = x_n \wedge \ldots \wedge \neg x_{n-1} = x_n)$$

This set has plenty of LP-models (supposing that LP is extended with "$\neg$"), but has no models in which X_P is finite, and hence has *no* minimally inconsistent models. The same result is obviously obtained by adding $\neg(\exists x){\sim}x = x$ to the premises of Example 6.5.

The conclusion is that LP^m is saved from drowning by lack of water. I do not mean the absence of classical negation (which Priest seems to consider as nonsensical). I mean that LP^m is only defined with respect to a fragment of Priest's 'true logic' —he believes that there is such an animal. Priest recognizes the need for a detachable (but not truth-functional) implication as well as for bottom —see for example [26]. So, Example 6 is all right for him if expressions of the form $\neg x_i = x_j$ are replaced by $x_i = x_j \rightarrow \bot$, in which the implication is detachable.[21] Even with this replacement, the premises have only models that *either* have an infinite domain *or* are trivial. As none

[21] Boolean negation, $\neg A$, cannot be defined by $A \rightarrow \bot$, for example because $A \vee (A \rightarrow \bot)$ is not a theorem if the implication is relevant or modal. However, $A, A \rightarrow \bot \models \bot$ obviously holds.

of the former is minimally abnormal, all minimally abnormal models of the premises are trivial.[22] So Reassurance fails if the ontological definition is extended to Priest's full logic: the above set (with the replacement) has non-trivial LP-models but no non-trivial LP^m-models.[23] Moreover, the example shows that Reassurance even fails if the Same Type Restriction is dropped.

6.5. *Failure to Identify Inconsistent Predicates*

In Section 4, I did *not* exclude that two different predicates of a first order language $\mathcal{L}$ have the same interpretation in a structure that is appropriate for $\mathcal{L}$ (and I justified the choice in a footnote). Priest's original ontological definition in [25] follows a different path. Phrased in terms of the semantics in Section 4, he requires that different predicates are mapped on different relations. So, each of the following sound inferences fails for the original definition.

Example 6.7. Premises: $Pa \wedge {\sim}Pa,\ Qa \wedge {\sim}Qa$. On the ontological definition, the only minimally abnormal models of these premises verify $v(P) = v(Q)$, whence

$$(\forall x)(Px \equiv Qx)$$

is a consequence of the premises. As this reduces to a rather weak claim in LP^m, due to the lack of a detachable implication, let me present a stronger example.

Example 6.8. Premises: $Pa \wedge {\sim}Pa,\ Qb \wedge {\sim}Qb,\ Ra$. On the ontological definition, the only minimally abnormal models of these premises still verify $v(P) = v(Q)$, whence

$$a = b$$

and hence

$$Rb$$

are consequences of the premises.

Such inferences may seem paradoxical. However, as soon as one grasps the idea behind the inconsistent models, their paradoxical flavour disappears. Just as the inconsistency of the structure is minimized by identifying $v(a)$ and $v(b)$ in Example 5.3, the inconsistency of the structure is minimized by identifying the interpretations of inconsistent predicates. It would be severely incoherent to allow for the one and not allow for the other. If the

[22] Priest stresses that his semantics comprises the trivial model.

[23] Incidentally, the Collapsing Lemma cannot be proved in the presence of a detachable implication and of bottom.

degree of inconsistency of a structure is independent of the names mapped to *objects* of the structure, then there can be no reason to make it dependent on the names mapped to the *relations* of the structure.

As this point is rather central in the sequel, let me briefly expand on it. Nothing prevents that the same element of the domain is named by different names. Similarly, nothing prevents that the same relation on the domain is named by different names. If two individual constants, a and b, name the same element of the domain, we have a standard logical operator to express this: identity. We have no standard logical operator to express that two predicates, P and Q, name the same relation. Of course, we can write (supposing that the adicity of the predicates is 1) $(\forall x)(Px \equiv Qx) \wedge (\forall x)(\sim Px \equiv \sim Qx)$. This warrants that $v(P) = v(Q)$, which means that P and Q are interpreted by the *same* relation.[24]

Let us return for a moment to Examples 5.3 and 5.6. In the latter example, some minimally abnormal models did not identify $v(a)$ and $v(b)$ because of the supplementary information provided about a and b, viz. Ra and $\sim Rb$. It is obviously be possible to obtain the same effect in Examples 6.7 and 6.8 by providing supplementary information on P and Q (or on a and b in Example 6.8). In both examples, there are minimally abnormal models (on the ontological definition) in which $v(P)$ is different from $v(Q)$ if one adds to the premises, for example, $Pc \wedge \sim Pc$ and $\neg(Qc \wedge \sim Qc)$. Alternatively, one may add Pc, Qd, and $\sim a = c$ in Example 6.7, and Sac and $\sim Sbc$ in Example 6.8.[25]

6.6. *Failure to Identify Inconsistent Sentential Letters*

The weakness mentioned in Section 6.5 obviously extends to sentential letters. Indeed, there are at most three relations of adicity 0 in the structure, viz. $\langle 1, 0\rangle$, $\langle 0, 1\rangle$ and $\langle 1, 1\rangle$. Hence, the inconsistent part of a model should

[24] The argument presupposes the standard extensional view. However, precisely this view underlies Priest's original semantics as well as the semantics from Section 4. Nearly everyone agrees today that P and Q may have a different meaning even if their extensions are identical. But a and b may just as well have a different meaning, even if they denote the same element of the domain. Typically, both sorts of non-extensional differences may be explicated by a worlds semantics, in which extensional identity is represented by identity at world w_0 (the real world), and intensional identity by identity at all worlds.

[25] The matter requires some attention. Adding Pc and $\sim Qc$ in Example 6.7 would simply result in $v(c) = v(a)$; adding Ra and $\sim Rb$ in Example 6.8, would simply result in $v(R) = v(P) = v(Q)$.

only depend on the question whether the relation $\langle 1, 1\rangle$ occurs in it, not on the *number* of sentential letters that are mapped on that relation.[26]

7. *Some New Proposals for an Ontological Definition*

We have seen that, on the ontological definitions discussed so far, two isomorphic models may have different inconsistent parts. A proposal that came up during the electronic discussion with Graham Priest was to stipulate that a model cannot be 'defeated' by an isomorphic model. The ontological definition of the abnormal part of a model remains as in Section 4, and so does $Ab^o(M) < Ab^o(M')$. The change comes here: a model M of a set Γ is a *minimally abnormal* model of Γ iff no model M' of Γ is such that (i) M' is of the same type as M, (ii) M' is *not isomorphic* to M, and (iii) $Ab^o(M') < Ab^o(M)$. Let us call the restriction in (ii) the *NI-Restriction* (non-isomorphic).

Alas, this does not work. Suppose that M and M' are models of the same set of premises, and that, for both M and M', $D = \{\mathsf{a}, \mathsf{b}_1, \mathsf{c}_1, \mathsf{b}_2, \mathsf{c}_2, \ldots\}$, $v^+(P) = \{\mathsf{a}, \mathsf{b}_1, \mathsf{b}_2, \ldots\}$, and $v^-(P) = \{\mathsf{a}, \mathsf{b}_1, \mathsf{c}_1, \mathsf{b}_2, \mathsf{c}_2, \ldots\}$, and that $X_i = \emptyset$ whenever $R_i \neq v(P)$. Let M differ from M' in that $v^+(Q) = \{a\}$ and $v^-(Q) = \emptyset$ in M, whereas $v^+(Q) = \emptyset$ and $v^-(Q) = \{a\}$ in M'. Let Σ be the set of models that are obtained from M by moving finitely many c_i from $v^-(P)$ to $v^+(P)$. Similarly, let Σ' be the set of models that are obtained from M' by moving finitely many c_i from $v^-(P)$ to $v^+(P)$. All members of Σ are isomorphic, and so are all members of Σ'. It is easily seen that (i) for any $M'' \in \Sigma$, some $M''' \in \Sigma'$ is such that $Ab^o(M''') < Ab^o(M'')$, and (ii) for any $M''' \in \Sigma'$, some $M'' \in \Sigma$ is such that $Ab^o(M'') < Ab^o(M''')$. It follows that the NI-Restriction fails to warrant Strong Reassurance.

In [7], I presented a crude but effective means to warrant Strong Reassurance. Instead of identifying the adaptive models of a set of premises (in the case of the minimal abnormality strategy) with the minimally abnormal models of the premises, one defines them as follows:

AD A LLL-model M of Γ is an AL-model of Γ iff there is no *minimally abnormal* LLL-model M' of Γ such that $Ab^o(M') < Ab^o(M)$.

This definition warrants Strong Reassurance by brute force: a model is selected unless some minimally inconsistent model 'defeats' it. Let us call the italicized restriction the *AD-Restriction*. It warrants that all models of an infinitely descending sequence are adaptive models.

[26] When writing [25], Graham Priest apparently did not recognize the linguistic character of my solution for the propositional case —he refers to [2]— and generalized it to a halfway house between the linguistic and the ontological approach.

The AD-Restriction solves the problem from Subsections 6.3 and 6.2, and hence also the one from Subsection 6.4. It does not solve the other problems, the most central of which concerns the Classical Recapture (Subsection 6.1).

The former version of this paper contained a definition I had cooked up in an attempt to remove the Domain Restriction (in terms of Section 4: the Same Type Restriction) while still avoiding the effects that the Domain Restriction was intended to avoid. I shall not present the definition here because I now think there are some problems with it. However, the underlying idea might be useful for other people attempting to find an adequate ontological definition. This idea was not only to compare the inconsistent part of models, but also their *consistent* part. This move seems a natural one: the loss in inconsistency should be matched by a gain in consistency (rather than by a decrease in the domain). Where X_i^M denotes the inconsistent part of R_i in M, C_i^M will denote the consistent part of R_i in M and will be defined as follows: where the adicity of R_i is n, $C_i^M = D^n - X_i^M$.

Recently, Graham Priest told me that, in the galley proofs of [27], he modified his original definition by simply *dropping* the Domain Restriction. I shall call this the Handbook Definition. This definition warrants the Classical Recapture. Indeed, by dropping the Domain Restriction, only consistent models are minimally abnormal models of a consistent set of premises. At this moment, Graham is convinced that the argument for the Domain Restriction, illustrated in Example 3.1, does not hold water. Writes he: "But if $[(\forall x)(Px \wedge {\sim}Px)]$ is *all* the information we have, and inconsistencies are to be minimized, perhaps it *is* correct to infer that there is just one thing." (last footnote to Section 7 of [27]).

And perhaps he is right. There is a clear idea behind selecting the minimally abnormal models in this sense. The results may sound counterintuitive at first sight, but intuitions in such matters are not very reliable.

Of course, four other problems remain. Those from Subsections 6.2, 6.3 and 6.4 are solved by the AD-Restriction. The problem from Subsection 6.5 may be solved as in Section 4, viz. by not requiring that different predicates are interpreted by different relations.

Apparently, then, we have found an unobjectionable ontological definition: Priest's Handbook Definition with two emendations. Unfortunately, we are not home yet. The definition seems technically unobjectionable but, as is shown in the next section, it has very weird properties and seems to lack any sensible application contexts.

8. *Problems with the Emended Handbook Definition*

8.1. *Collapsing Predicates*

I already quoted from a footnote of [27]: "But if $[(\forall x)(Px \wedge {\sim}Px)]$ is *all* the information we have, and inconsistencies are to be minimized, perhaps it *is* correct to infer that there is just one thing." Priest continues, "Note that $[\{(\forall x)Px \wedge {\sim}Px, (\exists x)Qx \wedge (\exists x){\sim}Qx\} \not\models (\exists x)(\forall y)x = y]$." and next gives an example of a minimally abnormal model M of the premises that falsifies $(\exists x)(\forall y)x = y$. However, M is only a minimally abnormal model of the premises because Priest *presupposes* that P and Q are bound to name different relations. As I showed in Subsection 6.5, this presupposition cannot be justified. So let us see what happens if the presupposition is dropped.
Example 8.1. Premises: $(\forall x)Px \wedge {\sim}Px,\ (\exists x)Qx,\ (\exists x){\sim}Qx$. Let M be a model with domain $\{\mathsf{a}, \mathsf{b}\}$, and let $v(P) = R_1 = \langle\{\mathsf{a}, \mathsf{b}\}, \{\mathsf{a}, \mathsf{b}\}\rangle$ and $v(Q) = R_2 = \langle\{\mathsf{a}\}, \{\mathsf{b}\}\rangle$. Let all other relations R_i be such that $X_i = \emptyset$. Hence:

$$Ab^o(M) = \{\langle\emptyset, 0\rangle, \langle\{\mathsf{a}, \mathsf{b}\}, 1\rangle, \langle\emptyset, 2\rangle, \langle\emptyset, 3\rangle, \ldots\}$$

This is a least inconsistent model with a two-element domain, and actually the model described in the footnote in [27].

Let M' be a model with domain $\{\mathsf{a}\}$, and let $v(P) = v(Q) = R_1 = \langle\{\mathsf{a}\}, \{\mathsf{a}\}\rangle$. Let all other relations R_i be such that $X_i = \emptyset$. Hence:

$$Ab^o(M') = \{\langle\emptyset, 0\rangle, \langle\{\mathsf{a}\}, 1\rangle, \langle\emptyset, 2\rangle, \langle\emptyset, 3\rangle, \ldots\}$$

As $Ab^o(M') < Ab^o(M)$, M is not minimally abnormal. Any model with a non-singleton domain is similarly 'defeated' by a model with a singleton domain. Hence,

$$(\exists x)(\forall y)x = y \tag{8.1.1}$$

is still a consequence of the premises. In other words, the 'supplementary information', $(\exists x)Qx,\ (\exists x){\sim}Qx$, fails to prevent that only singleton models are minimally abnormal.

Of course, Priest might have presented an example in which the supplementary information concerns a predicate of an adicity larger than 1, in which case (8.1.1) would not have been a consequence of the premises. This is not much consolation, as I shall show after presenting two further instructive examples.
Example 8.2. Premises: $Pa \wedge {\sim}Pa,\ Qb \wedge {\sim}Qb,\ Sa,\ {\sim}Sb$. The minimally abnormal structures are those in which $v(P) = v(Q) = v(S)$ and X_P is a

singleton (viz. $v(a) = v(b)$). So, each of the following are consequences: $a = b$, $(\forall x)(Px \equiv Qx)$, $(\forall x)(Px \equiv Sx)$, $a = b$, $\sim Sa$, Sb.[27]
Example 8.3. Premises: $Pa \wedge \sim Pa,\ Qb \wedge \sim Qb,\ Pc \wedge \sim Pc,\ \sim(Qc \wedge \sim Qc)$. As $\sim(Qc \wedge \sim Qc)$ is an LP^m-theorem, its presence has no effect. The minimally abnormal structures are those in which $v(P) = v(Q)$ and X_P is a singleton (viz. $v(a) = v(b) = v(c)$). Each of the following are consequences: $a = b$, $a = c$, $b = c$, $(\forall x)(Px \equiv Qx)$.

Suppose that you hold the philosophical conviction that all relational properties are parasitic on individual properties. This means that you will populate your structures with relations of adicity 1 only. Let us restrict the domain of discourse to humans to simplify the example. Suppose that you find out that all humans have some inconsistent property ($(\forall x)(Px \wedge \sim Px)$). Of course, humans have many other properties, and these vary widely to make them all distinct: some are women while others are not, some are blue-eyed while others are not, some are friendly while others are not, etc. Still, the Emended Handbook Definition results in the consequence: all humans are identical (there is only one human being) and "to be a women," "to be blue-eyed," "to be friendly," etc. all name the same inconsistent relation (as does P). If you cannot imagine worse, go on reading.
Example 8.4. Premises: $Pa \wedge \sim Pa,\ Qb,\ \sim Qc$. As there need not be any relation between P and Q, nor between a, b and c, the sensible conclusion seems to be that $\sim b = c$. Alas, this does not follow on the Emended Handbook Definition. Indeed, the premises have a minimally abnormal model M in which $v(a) = v(b) = v(c)$, and $v(P) = v(Q) = R_1 = \langle \{v(a)\}, \{v(a)\} \rangle$. Of course, they also have minimally abnormal models in which $v(b) \neq v(c)$. Still, $\sim b = c$ is *not* a consequence of the premises as M falsifies it.

In general, if, whenever a predicate of adicity n occurs in Γ, some predicate of the same adicity behaves inconsistently on Γ, then one obtains a minimally inconsistent model of Γ by choosing a structure with a one element domain D, by mapping any predicate of adicity n to the relation $R_i = \langle D^n, D^n \rangle$, and by making all other relations consistent. The ensuing trouble is either that all minimally abnormal models have a singleton domain, as in Examples 8.1–3, or that some minimally abnormal models have a singleton domain, as in Example 8.4.

[27] The divergence between the ontological and the linguistic definitions is striking. The classical negation of the last three formulas are consequences of the premises according to the linguistic definition: $\neg a = b$, $\neg\sim Sa$, $\neg Sb$.

8.2. *Collapsing Sentential Letters*

The trouble from Section 8.1 affects also sentential letters. Here, the dramatic and unpalatable effects are even more clear, as appears from the following standard example.
Example 8.5. Premises: $p, {\sim}p, q, {\sim}q \vee r$. From these r is not derivable because the model in which $v(p) = v(q) = \langle 1, 1\rangle$ and $v(r) = \langle 0, 1\rangle$ is not more inconsistent than the one in which $v(p) = \langle 1, 1\rangle$, and $v(q) = v(r) = \langle 1, 0\rangle$.

Let me generalize this. If Γ is an inconsistent set of propositional formulas, the relation (of adicity 0) $\langle 1, 1\rangle$ occurs in any model of Γ. It follows that, on the Emended Handbook Definition, *all* paraconsistent models of Γ are minimally abnormal at the propositional level. Hence, on that definition, $Cn_{AL}(\Gamma) = Cn_{LLL}(\Gamma)$ whenever Γ is inconsistent.

8.3. *Isomorphic Models and Strong Reassurance*

Introducing the AD-Restriction warrants Strong Reassurance (and hence Reassurance). Alas, it also leads to problematic results. Reconsider for a moment the premises of Example 6.5: the formulas (where $n \in \{2, 3, \ldots\}$)

$$(\exists x_1)(\exists x_2)\ldots(\exists x_n)((Px_1 \wedge {\sim}Px_1) \wedge \ldots \wedge (Px_n \wedge {\sim}Px_n) \wedge {\sim}x_1 = x_2 \\ \wedge \ldots \wedge {\sim}x_1 = x_n \wedge {\sim}x_2 = x_3 \wedge \ldots \wedge {\sim}x_2 = x_n \wedge \ldots \wedge {\sim}x_{n-1} = x_n)$$

On the original ontological definition, Γ had only minimally abnormal models in which X_P is a singleton. On the Emended Handbook Definition, there also are minimally abnormal models in which X_P is an infinite set. These models are easily identified: the models in which both X_P and $D - X_P$ are infinite.

Γ states that infinitely many different objects both have and have not property P. According to the original definition, it follows that some object is different from itself and that precisely one object both has and has not property P. According to the Emended Handbook Definition, it follows that *either* some object is different from itself and that precisely one object both has and has not property P, *or* there are infinitely many objects that behave *consistently* with respect to P. That doesn't look good either.

The AD-Restriction prevents absurd consequences by not ruling out any member of infinitely descending sequences. Thus, part of the adaptive character of the logic is traded in. This is the price of the way out. In the presence of adaptive logics based on the linguistic definition, the price seems too high.

8.4. *Problems For Generalizing The Proposal*

$CLuNs$ and its fragment LP reduce *all* inconsistencies to the level of primitive wffs; many other logics do not. Examples are all of Newton da Costa's C_i-logics —see, e.g., [19]— and my preferred lower limit logic $CLuN$ —see, e.g., [5]— which is defined syntactically by full positive CL, including $\perp$, together with $A \vee {\sim}A$ (equivalently, $(A \supset {\sim}A) \supset {\sim}A$).

In such logics, some inconsistencies are isolated in that they do not derive from primitive inconsistencies —in $CLuN$ all inconsistencies are isolated: no inconsistency entails an inconsistency in terms of superformulas, and an inconsistency entails only an inconsistency in terms of subformulas iff the latter are contained in its 'positive part'.[28] The advantage of isolating all inconsistencies is that, as inconsistencies are not spread, more classical consequences are delivered by the inconsistency-adaptive logic.

It is typical for logics that isolate inconsistencies (fully or in part) that negations of (closed and open) *formulas* are to be introduced as verified directly by the interpretation or assignment. How an ontological approach might be generalized to such logics is unknown today. This is especially serious as there are arguments (see [3], [5], and [8]) for preferring poorer paraconsistent logics as the lower limit logic of inconsistency-adaptive logics.

A different generalization problem concerns other strategies, some of which were mentioned at the end of Section 2. Some of these have serious advantages (over the minimal abnormality strategy) from a proof theoretic point of view (see [5, § 7]), or are suitable in specific contexts —see, for example, [11]. What becomes of such strategies on the ontological approach is fully unknown. It is even unclear whether the problem is sensible.[29]

9. *A Provisional Conclusion*

A fair rendering of the situation seems the following. Whatever one's attitude with respect to dialetheism —the claim that there are *true* inconsistencies— one will try to interpret an inconsistent theory as consistently as possible —see [9]. I compared the linguistic and the ontological approach from a semantic point of view, concentrating on the minimal abnormality strategy.

[28] Thus $((p \wedge {\sim}p) \wedge q) \wedge {\sim}((p \wedge {\sim}p) \wedge q) \vdash_{CLuN} (p \wedge {\sim}p)$. However, one may prove that, if $(A \wedge {\sim}A) \vdash_{CLuN} (B \wedge {\sim}B)$, then $A \vdash_{CLuN} (B \wedge {\sim}B)$.

[29] The main reason for this is that the ontological definition has never been connected to a proof theory or tableau method.

The linguistic definition has not been shown to have problematic aspects or lead to problematic results. This definition also has a number of strong points. First and foremost, it leads to a (dynamic) proof theory (nearly any of the quoted papers deals with it) and to a tableau method (see [13] and [14]). Neither of these is present for logics that rely on the ontological approach. It is even doubtful whether a proof theory or tableau method for the ontological approach are possible. The linguistic definition was shown to be attractive for several adaptive logics that do not reduce all inconsistencies to the level of primitive formulas. It was proved attractive for several adaptive logics that allow for other logical abnormalities than inconsistencies, and for logics that allow for ambiguities in the non-logical constants. It moreover has variants (that rely on the same definition of the abnormal part of a model) that are adequate for strategies different from minimal abnormality. All Rescher–Manor consequence relations (see Section 2) were characterized by an inconsistency-adaptive logic relying on the linguistic definition. The incorporation of 'non-monotonic logics' (see again Section 2) occurred fully in terms of logics relying on the linguistic definition. None of these results is paralleled by the ontological approach.

Especially in view of the problems spelled out in Sections 8.1 and 8.2, one can only conclude that a sound ontological definition is not available. The Emended Handbook Definition is both clear and coherent. However, it is hard to see any application contexts in which the features described in Sections 8.1 and 8.2 would be suitable. Whether an adequate ontological definition is possible is an open question, but the available arguments support a negative answer. Apparently, any sensible approach to minimal abnormality should treat inconsistencies in different propositional letters and in different predicates as different. This seems to support Priest's halfway house, the Handbook Definition, provided it is corrected by imposing the AD-Restriction. If we go that linguistic, however, in counting inconsistent sentences and predicates rather than their extensions, then there seems no possible justification for not counting inconsistent constants rather than their extensions. And if we do so, we obtain the linguistic definition or a variant of it, not a definition that can sensibly be called ontological.

The above questions lead to a deeper one. It is obviously sensible to distinguish between the structure of a model and its interpretation of the language $\mathcal{L}$. The central question, however, considers the status of the distinction. One view conceives the distinction as merely technical; models are merely devices to interpret a language 'from within'. This suits the Vienna Circle (primarily Rudolf Carnap, who did all the hard work): possibilities are merely combinations of linguistic entities (state descriptions). Another view connects the distinction to that between the structure of the world and the realm of language. On this view, we are able to talk about ontology in a way that is not determined by our present best scientific insights.

There is a third possibility. Even the distinction is connected to that between the structure of the world and the realm of language, the former may still be determined by the conceptual schemas that underly our scientific language. Whenever one adapts the conceptual structure in terms of which the world is approached, the structures of the models have to be adapted as well. In other words, those structures (and the results of minimizing abnormality) will not survive a scientific revolution.

It seems to me that this third possibility is the only sensible one. We know that our conceptual schemas do (presumably) not coincide with the world itself, but we have no possibility to conceive the world independently of our present insights. The history of philosophy shows convincingly that an a-historic ontology is beyond human reach. In view of this, presumably any ontological definition of minimal abnormality is ill-directed.

If the inconsistency of theories derives from the fact that linguistic entities refer ambiguously, then only a linguistic definition is sensible. What one wants to minimize are the ambiguous interpretations, not any inconsistencies in the world out there. Interestingly, it is impossible in principle to decide whether an inconsistency derives from the structure of the world or from the ambiguous interpretation (that is, from the fact that our language refers ambiguously). Even the logic we use cannot discriminate between the two possibilities. This is easily seen by comparing the semantics from Section 4 with the one from the Appendix. Both semantic systems characterize the same logic, viz. $CLuNs$ as well as LP.[30]

Appendix: Inconsistent Interpretations

The semantics from Section 4 presupposes that the non-logical constants of the language unambiguously refer to objects and 'relations' in the world, and hence that all inconsistencies derive from the world itself. This presupposition is obviously wrong for inconsistencies that can be removed. In this appendix, I devise a semantics for $CLuNs$, and hence for LP, in which inconsistencies are caused by the *interpretation* of linguistic entities, leaving it to the reader to spell out the mixed case.

Let a structure be defined as in Section 4, except that all relations are classical: $X_i = \emptyset$ for all $i \in I$. As their anti-extension is a function of their extension, it need not be mentioned separately. Also, there is no need to reserve a relation for the interpretation of identity.

[30] I spell the matter out in the Appendix, which also contains a natural variant for the linguistic definition.

Where $\alpha \in \mathcal{C}$, $v(\alpha) \subseteq D$. Where $\pi \in \mathcal{P}^n$ (a predicate of adicity n), $v(\pi) \subseteq \{R_i \mid i \in I; R_i$ has the same adicity as $\pi\}$.[31] To accommodate the quantifiers, we introduce a set of pseudo-constants $\mathcal{O}$ (of at least the same cardinality as the largest domain we want to consider), and stipulate that $v(\alpha) \subseteq D$ for all $\alpha \in \mathcal{O}$.[32] Here is how the valuation function handles primitive formulas:

- if the adicity of π is 0, then
  - $v^+_M(\pi) = 1$ iff $R_i = 1$ for some $R_i \in v(\pi)$
  - $v^-_M(\pi) = 1$ iff $R_i = 0$ for some $R_i \in v(\pi)$
- if the adicity of π is $n > 0$, then
  - $v^+_M(\pi\, \alpha^1 \ldots \alpha^n) = 1$ iff $\langle \mathsf{a}_1, \ldots, \mathsf{a}_n \rangle \in R_i$ for some $R_i \in v(\pi)$, $\mathsf{a}_1 \in v(\alpha_1)$, …, and $\mathsf{a}_n \in v(\alpha_n)$
  - $v^-_M(\pi\, \alpha^1 \ldots \alpha^n) = 1$ iff $\langle \mathsf{a}_1, \ldots, \mathsf{a}_n \rangle \notin R_i$ for some $R_i \in v(\pi)$, $\mathsf{a}_1 \in v(\alpha_1)$, …, and $\mathsf{a}_n \in v(\alpha_n)$
- for identity:
  - $v^+_M(\alpha = \beta) = 1$ iff $v(\alpha) = v(\beta)$
  - $v^-_M(\alpha = \beta) = 1$ iff $v(\alpha) \neq v(\beta)$, or $\mathsf{a} \neq \mathsf{b}$ for some $\mathsf{a}, \mathsf{b} \in v(\alpha)$.

The logical constants are handled as in Section 4, except for the quantifiers. Let A^β_α be the result of replacing any free occurrence of the individual variable α by the individual constant or pseudo-constant β.

- $v^+_M((\forall\alpha)A) = 1$ iff $v^+_M(A^\beta_\alpha) = 1$ for all $\beta \in \mathcal{C} \cup \mathcal{O}$
- $v^-_M((\forall\alpha)A) = 1$ iff $v^-_M(A^\beta_\alpha) = 1$ for some $\beta \in \mathcal{C} \cup \mathcal{O}$
- $v^+_M((\exists\alpha)A) = 1$ iff $v^+_M(A^\beta_\alpha) = 1$ for some $\beta \in \mathcal{C} \cup \mathcal{O}$
- $v^-_M((\exists\alpha)A) = 1$ iff $v^-_M(A^\beta_\alpha) = 1$ for all $\beta \in \mathcal{C} \cup \mathcal{O}$

Let π be a predicate with adicity n. The consistent part of π consists of the n-tuples that are π in all respects, $\bigcap v(\pi)$, and of the n-tuples that are π in no respect, $D^n - \bigcup v(\pi)$.[33] The rest of the domain, $\bigcup v(\pi) - \bigcap v(\pi)$, contains the inconsistent extension of π.[34]

[31] Whenever $v(\alpha)$, respectively $v(\pi)$ is not a singleton, α, respectively π, is interpreted ambiguously.

[32] The pseudo-constants name the values of the individual variables.

[33] Remember that, where π is a predicate with adicity n, $v(\pi)$ is a set of sets of n-tuples.

[34] There are two relations with adicity 0, viz. 0 and 1. A set of such relations is consistent if $\bigcup v(\pi) - \bigcap v(\pi)$ is $\emptyset$ and inconsistent if it is $\{0, 1\}$.

The linguistic definition distinguishes three sources of inconsistencies. (i) A linguistic entity may be interpreted abnormally.[35] (ii) The (normal or abnormal) interpretation of a constant may overlap with an n-tuple that belongs to the abnormal interpretation of a predicate with adicity $n > 0$.[36] (iii) The abnormal interpretation of a constant or pseudo-constant may cause it to relate inconsistently to the normal interpretation of a predicate.[37]

The present models suggest an alternative linguistic definition: the abnormal part of a model is the set of linguistic entities that have an abnormal interpretation. This is clear and sensible, appears directly applicable,[38] and does not seem to lead to any computational problems.[39]

The closest approximation to an 'ontological' definition defines the abnormal part of a model as the set of abnormal interpretations (the non-singleton sets in the domain of the interpretation function). This definition clearly does not coincide with any of the ontological definitions from previous sections. Although the definition is clear in itself, it is affected by the problems from Sections 8.1 and 8.2, and moreover requires that one combines it, in an ad hoc manner, with the AD-Restriction.[40]

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[35] If $A \in \mathcal{S}$ and $v(A)$ is not a singleton, then $v_M(A \wedge \sim A) = 1$. If $v(\alpha)$ is not a singleton then $v_M(\alpha = \alpha \wedge \sim\alpha = \alpha) = 1$ if $\alpha \in \mathcal{C}$, and $v_M((\exists x)(x = x \wedge \sim x = x)) = 1$ if $\alpha \in \mathcal{O}$. Where the adicity of π is $n > 0$, $v_M((\exists\alpha_1)\dots(\exists\alpha_n)(\pi\alpha_1\dots\alpha_n \wedge \sim\pi\alpha_1\dots\alpha_n)) = 1$ whenever $v(\pi)$ is not a singleton.

[36] Example: if P has adicity 2, $v(a) = \mathsf{a}$, $v(P) = \{R_1, R_2\}$, and $\langle \mathsf{a}, \mathsf{b}\rangle \in R_1 - R_2$, then $v_M(Pa \wedge \sim Pa) = 1$.

[37] Let $v(P) = R_1$ have adicity 1, $v(\alpha) = \{\mathsf{a}, \mathsf{b}\}$, $\mathsf{a} \in R_1$ and $\mathsf{b} \notin R_1$. If $\alpha \in \mathcal{C}$, then $v_M(P\alpha \wedge \sim P\alpha) = 1$; if $\alpha \in \mathcal{O}$, then $v_M((\exists x)Px \wedge \sim Px) = 1$.

[38] At the propositional level, it coincides with the aforementioned linguistic definition.

[39] In *CLuNs*, for example, it is easily expressed that $Pa \wedge \sim Pa$ is equivalent to: either a behaves abnormally and P normally, or a behaves normally and P abnormally, or both a and P behave abnormally.

[40] Most unpublished papers in the reference section (and many others) are available from the internet address http://logica.rug.ac.be/centrum/writings/index.html.

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