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TO BE SOMETHING AND SOMETHING ELSE: DIALETHEIC TENSE LOGIC

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The paper is concerned with time and identity. In particular, it is concerned with the problem of fission and fusion. A number of people have considered the problem. One of them was Prior. Although he did not provide an alternative account to classical logic, Prior was sympathetic to the idea that fission (and fusion) is a counter-example to some of the classical principles of identity. In this paper, I will give a semantics that solves a problem that Prior described. It turns out that the semantics provides a simple and elegant solution to the problem of fission and fusion.

1. *Fission and Fusion*

Historically, fission and fusion have intrigued many philosophers, especially in the context of personal identity. They seem to appear from time to time as counter-examples to the theories of identity that have been proposed. Fission and fusion are troublesome cases of identity.

Among those who have considered fission and fusion was Prior.[2] He used fission (and fusion) to show that some of the classical principles of identity were inadequate. Before considering Prior's arguments and the problem that he faced, I will analyse the phenomena of fission and fusion so that the result of analysis will support Prior's intuition about fission and fusion.

[1]I would like to thank Graham Priest for his uncountably many comments on drafts of this paper. I would also like to thank Daniel Nolan for his thorough written comments that made me think about some issues that I was unaware of. I am also indebted to André Gallois for his verbal comments. The paper was read to the Philosophy Department of the University of Leeds and the University of Queensland, the 1997 meeting of the Society for Exact Philosophy, and the First World Congress on Paraconsistency. Many thanks go to the members of the audience for their comments.

[2]Prior (1968) ch. 8.

1.1. *Fission*

Fission is the case in which one thing becomes two things as a result of a split. For example, consider an amoeba, a, at a time t_0. At some time t_1, $t_0 < t_1$, a undergoes fission. So after t_1, e.g., at t_2, $t_1 < t_2$, there are two amoebas, b, c. Now where is a after t_1? There seems to be three possibilities:

(Γ) a is exactly one of b, c
(Δ) a is neither b nor c
(Θ) a is both b and c.

The problems associated with each option have been identified in many places.[3] So I will not present them here. Instead I will argue for Θ and show that Θ is realised at some time between t_0 and t_2 even under Γ and Δ. Incidentally, Prior took Θ seriously and put forward his argument against classical principles of identity.[4]

1.1.1. (Γ) a *is exactly one of* b, c

Let's start with option Γ. Γ has many problems. The main problem with Γ is that there is no plausible way to determine whether a is one or the other after t_1. For suppose that there are good reasons to support a's being b after fission. Then by symmetry, those reasons serve equally to support a's being c. The reasons to support a's being b are the exact reasons to support a's being c. For the only difference between b and c is a spatial one and so a bias cannot be given to one of them. Moreover, if b died after t_1, then it should be clear that a was c. But then extrinsic features have determined the identity. And this seems fallacious. For according to this, my identity is depending on my extrinsic features and so my smile (caused by someone in the department), for example, may change my identity. (If this were the case, my identity would change every day! Who would be writing the conclusion of this paper then!?) It might be thought that there are some examples other than my smile that justify that extrinsic features determine identity. However, any other examples are similar to the example of my smile. In particular, they do not offer anything more. For they lead to an equally dubious conclusion.

[3]For a brief summary of the standard problems, see, e.g., Patterson (1991).

[4]Note, however, that Prior did not provide any reasons why he subscribed to Θ.

Now against the assumption of symmetry, one may smuggle in asymmetry to the scene. Yet this implies, as Patterson (1991) points out, that there is no coherent solution to the question at issue. For symmetry is assumed in the argument. When an amoeba fisses, it is assumed that the resulting two amoebas are symmetrical in the sense that they share the same features. And this assumption makes the problem interesting in the first place. Otherwise, picking out one of them would provoke no disagreement, if it has some features that help identify it with the one before fission. Hence if there are good reasons to suppose that *a* is one of *b*, *c*, then those reasons serve equally to justify *a*'s being both *b* and *c*. And this is of course Θ.[5]

1.1.2. (Δ) a *is neither* b *nor* c

Now, option Δ. Suppose *a* ceases to exist at t_1 such that $t_0 < t_1 < t_2$. Then *a* no longer exists at t_2. But what about at t_1? At t_1, *a* is ceasing to exist and *b* and *c* are coming into existence. So t_1 is a state of change. The existence of states of change was advocated by Priest who consequently developed a logic that accommodates them.[6] His primary concern is with a state of change, called a *flux state*, in which there is a change between *p* being true and $\neg p$ being true. Although we are not concerned with a proposition *p* and its changes of truth value, this flux state gives us an insight into examining the state at t_1. So we start the discussion by explicating this state.

Priest analyses the flux state by appealing to a principle, which he calls *Leibniz' Continuity Principle (LCP)*. The most useful version of the principle to us here states that:[7]

[5]At this point, one may bite the bullet on my objection that Γ implies that there is no coherent solution to the question at issue. One may endorse Haecceitism and argue that there is a haecceity of *a* which, as a matter of fact, is preserved in one of and only one of *b* or *c*. Yet there are no empirical manifestations and so it is impossible to find out which of the two preserves the haecceity of *a*. Hence, one may argue, it is impossible to find any coherent solution to the question whether *a* is *b* and/or *c*.

Perhaps, there are some situations in which Haecceitism does have some force. However, I am, to make a confession, mystified by the notion of Haecceitism. Moreover, haecceitists seem to *assume* that haecceity of *a* is preserved only in one of *b* or *c* after fission. If one can do that, why can't we assume in a similar way that the haecceity of *a* is preserved in both? Of course, one may find that that is question begging in answering the question of identity of amoebas. But so is the argument of haecceitists. Haecceitism does not seem to have any force in the context at issue.

[6]Priest (1982) and Priest (1987) ch. 11.

[7]Priest (1982) p. 262.

> any physical state of affairs which holds arbitrarily close to a given time holds at that time.

In order to make this principle clear, consider the following application of the LCP. Suppose that prior to time t, a system S is in state S_0, and posterior to t, it is in state S_1. Since S is in S_0 anytime before and in particular arbitrarily close to t, S_0 holds at t. Similarly, S is in S_1 anytime after and in particular arbitrarily close to t. So S_1 holds at t. Hence the LCP entails that both S_0 and S_1 are realised at t.[8]

Now, the LCP can be applied to the case of fission in order to illuminate a problem with Δ. Prior to t_1, viz., t_0, a exists. Let S_0 be this state. Since the fission takes place at t_1, S_0 has to hold up to and in particular arbitrarily close to t_1. Hence S_0 holds at t_1, by the LCP. On the other hand, let S_2 be the state posterior to t_1, viz., t_2, that b and c exist. Then S_2 holds at any arbitrary time after t_1. Hence by the LCP, S_2 holds at t_1. Therefore, S_0 and S_2 are both realised at t_1. This means that a, b, and c all exist at t_1.

It is now easy to establish the argument that Θ is realised at t_1. Firstly, if a has to be exactly one of b or c, the problems that were encountered in considering Γ would arise. So there is a difficulty in deciding whether a is b or c. Secondly, it does not seem plausible to suppose that in this state of change a is neither b nor c. The LCP implies that a, b, and c all exist at t_1, the state of change. Thus, if a is neither b nor c, then there must be three distinct amoebas at t_1. However, this is absurd. For there are at most two amoebas in the process of fission. Hence if there is a state of change, a is both b and c in such a state.

In response to the above argument, one may argue in support of Δ that there is no instant of change, by rejecting the LCP. This means that the situation is said to change dramatically in no time at all. This line of argument has been criticised by Priest.[9] He argues that if changes occur dramatically, any event that takes some time is represented by a series of pictures patched together. So there are no changes in the world at all. For the things in a picture are at rest and so they do not change.

However, depending on how to conceptualise time, one may or may not find it plausible to represent time by a series of pictures. None the less, there is an independent counter-argument to Δ. Previously, I argued in the discussion of Γ, that extrinsic features do not determine identity. Now Δ

[8]The application of the LCP that Priest shows is this. A system S changes its state at t from S_0, at which p is true, to S_1, at which $\neg p$ is true. By the LCP, S_0 and S_1 are both realised at t. This means that $p \wedge \neg p$, i.e., a contradiction, is realised at t the state of change (the flux state).

[9]Priest (1982) and Priest (1987) ch. 11 & 12.

faces the same problem that was presented there. Suppose that *b* died after t_1. Then even those who endorse Δ would argue that *a* was *c* after fission. But if Δ is to be taken seriously when *b* does not die, then it must be that extrinsic features determine identity. Yet this has been rejected by even those who endorse Δ. Hence Δ is as dubious as Γ.

If it is argued that *a* is not *c* (nor *b*) after *b* dies, it must be that identity does not persist chronologically, unless one is prepared to accept the idea that identity is determined by extrinsic features. For there is nothing after fission that is identified with *a*. However, that is to undermine the philosophical perplexity of the problem of identity. The problem is to analyse the pre-theoretical intuition about how to be said to continue to be myself regardless of any changes, for example. Thus to argue that identity does not persist chronologically is to leave the entire question untouched.

However, there is another argument that supports Δ. That is the view that *a* is equivalent to *b* and *c* together. This view takes the *mereological* thesis that *b* and *c* together constitute an object, i.e., their mereological sum. Perhaps, there are some situations where the mereological thesis provides an appropriate solution. Yet in the context in question, it does not do it justice.

What the mereological thesis takes seriously is object-hood. Hence it is argued that the *object* exists after fission is the mereological sum of *b* and *c*. However, the problem in question is to identify an *amoeba a* with *amoebas b* and/or *c*. So our focus is on amoeba-hood. Now even mereologists agree that there are two *amoebas* after fission —never mind how many objects there are. Hence the mereologists have yet to give us their answer to the question at issue.[10]

1.2. *Fusion*

The situation is similar in fusion, which is symmetric to fission. Amoebas may not be a good example of fusion. For they do not fuse. None the less, it is easy to think of some simple creatures like amoebas. So consider two amoebas *a* and *b* at t_0. At t_1, they undergo fusion. So after t_1, e.g., at t_2, there is only one amoeba *c*. A similar question to that with fission arises, where are *a* and *b* after t_1? There seems to be three possibilities:

[10] Prior considered the mereological view as well. He as well, however, rejected the view by a similar argument to the one presented here. My argument was, in fact, inspired by Prior's. See Prior (1968), pp. 86–7.

(Γ') exactly one of a, b is c
(Δ') neither a nor b is c
(Θ') both a and b are c.

Since the situation is symmetric to fission, similar arguments apply to fusion. So if a is c after t_1, b must also be c after t_1, and *vice versa*. Hence there is no better reason for Γ' than for Θ'. Also even if neither a nor b is c at t_2, there must be a time at which both a and b are c between t_0 and t_2. Moreover, Δ' faces the problem that was put forward in the discussion of Δ above. Hence Θ' is realised under the option Δ', or there is a counter-argument to Δ'.

1.3. ... *and Formal System*

To sum up the preceding discussions, if there are plausible reasons to suppose that Γ and Γ', the same reasons serve equally to support that Θ and Θ' respectively. Also, although Δ and Δ' may be realised some time after fission and fusion, there is a time at which Θ and Θ' are realised. Hence if we develop a formal system that captures the phenomena of fission and fusion, the system has to be able to handle the cases of identity such as Θ and Θ'. And this was what Prior tried to do.

2. *Systems of Identity*

There are several ways to develop a formal system of identity. However, a formal system of identity that accommodates fission and fusion is hard to come by. This section concerns the difficulties associated with the standard systems of identity.

Perhaps the simplest system of identity is the *Necessary Identity (NI)* system. Given the usual tense operators: P (it was the case that), F (it will be the case that), H (it has always been the case that), G (it will always going to be the case that), this system validates $a = b \models Ga = b$ and $a \neq b \models Ga \neq b$ (and also their mirror images: $a = b \models Ha = b$ and $a \neq b \models Ha \neq b$). In the face of fission and fusion, NI hits an iceberg. Consider, for example, a fission case. After fission, at t_2, $b \neq c$. So an NI system gives that $b \neq c$ at all times, in particular at t_0, i.e., before the fission. Yet at t_0, there is only one amoeba, a. So $b = a$ and $c = a$, and therefore $b = c$ at t_0. Hence the second principle: $a \neq b \models Ga \neq b$, is contradicted. A similar argument shows that $a = b \models Ga = b$ also fails. Thus fission is a counter-example to NI.

The system that rectifies this problem is *Contingent Identity (CI)*. In this system, as is expected, $a = b \models Ga = b$ and $a \neq b \models Ga \neq b$ fail. Then, as is clear, it is allowed that $b = c$ at one time, e.g., before fission, and $b \neq c$ at another time, e.g., after fission. Standard semantics for CI systems achieves this result by taking a member of the domain to have different parts (not in the sense of mereology) at different times. So a member of the domain is the sum of its parts. Formally, the standard semantics for CI systems can be given as follows.[11]

The language for CI systems is that of first order logic plus tense operators. Although it may be intuitively plausible to have a variable domain in considering fission and fusion, a constant domain suffices for our purpose. Hence we use a constant domain in this paper. An interpretation is a 5-tuple $\langle W, R, \Pi, D, I\rangle$, where W is a set of times, R is a binary relation on W, Π is a non-empty set (of parts), D is the domain containing functions from W to Π. For every constant, c, $I(c) \in D$; if necessary we augment the language so that every member of the domain has a name. For every $t \in W$, and n-place predicate, M, $I(t, M) \subseteq \Pi^n$. For the identity predicate $=$, $I(t, =) = \{\langle x, x\rangle : x \in \Pi\}$. The truth condition for atomic formulas is:

$$\nu_t(Mc_1 \dots c_n) = 1 \text{ iff } \langle (I(c_1))(t), \dots, (I(c_n))(t)\rangle \in I(t, M).$$

That is, $Mc_1 \dots c_n$ is true at t iff M is true of the parts of $c_1, \dots, c_n$ that exist at t.

The truth conditions for the connectives and quantifiers are as usual:

$\nu_t(\neg\alpha) = 1$ iff $\nu_t(\alpha) = 0$
$\nu_t(\alpha \wedge \beta) = 1$ iff $\nu_t(\alpha) = 1$ and $\nu_t(\beta) = 1$
$\nu_t(\alpha \vee \beta) = 1$ iff $\nu_t(\alpha) = 1$ or $\nu_t(\beta) = 1$
$\nu_t(\forall x\alpha) = 1$ iff for all constants, c, $\nu_t(\alpha(x/c)) = 1$
$\nu_t(\exists x\alpha) = 1$ iff for some constant, c, $\nu_t(\alpha(x/c)) = 1$.

The truth conditions for the tense operators are:

$\nu_t(P\alpha) = 1$ iff for some $t' \in W$ such that $t'Rt$, $\nu_{t'}(\alpha) = 1$
$\nu_t(F\alpha) = 1$ iff for some $t' \in W$ such that tRt', $\nu_{t'}(\alpha) = 1$
$\nu_t(H\alpha) = 1$ iff for all $t' \in W$ such that $t'Rt$, $\nu_{t'}(\alpha) = 1$
$\nu_t(G\alpha) = 1$ iff for all $t' \in W$ such that tRt', $\nu_{t'}(\alpha) = 1$.

Semantic consequence is defined in terms of truth preservation at all times of all interpretations:

[11] See Hughes and Cresswell (1996).

$\Sigma \models \alpha$ iff for all interpretations $\langle W, R, \Pi, D, I\rangle$ and all $t \in W$, if $\nu_t(\beta) = 1$ for all $\beta \in \Sigma$ then $\nu_t(\alpha) = 1$.

Let's now consider a fission case. Let $I(a)$, $I(b)$, and $I(c)$ be functions associated with amoebas, x, y, z, in such a way that $I(a)(t_0) = x$, $I(b)(t_0) = x$, $I(b)(t_2) = y$, $I(c)(t_0) = x$, and $I(c)(t_2) = z$. Then the semantics for CI systems gives that $I(b)(t_0) = x = I(c)(t_0)$. Yet $I(b)(t_2) = y \neq z = I(c)(t_2)$. Hence CI systems allow identity to vary from time to time and therefore solve the problem for NI.

However, the standard CI systems do not fully respect fission and fusion. They face a problem that Prior (1968) described. Consider the following two logical principles:

$$a = a$$
$$a = b,\ \varphi(a) \models \varphi(b).$$

The first principle does not seem to excite any controversy. However, when it is put together with the second principle, called *substitutivity* or the *indiscernibility of identity*, it does not produce a desirable result. For from these two principles, immediately follows the law of transitivity:[12]

$$a = b,\ a = c \models b = c.$$

As Prior noticed, once Θ and Θ' are granted, this is not in accordance with fission and fusion. For at t_2, $b \neq c$ while $a = b$ and $a = c$. In a CI system, if $I(a)(t_2) = I(b)(t_2)$ and $I(a)(t_2) = I(c)(t_2)$ then $I(b)(t_2) = I(c)(t_2)$. Yet it must be that $I(b)(t_2) \neq I(c)(t_2)$. Thus standard CI systems hit another iceberg.

Consequently, Prior argued that fission (and fusion) was a counterexample to the principle of substitutivity. The problem Prior faced was that he was unable to provide a formal logical account in which the phenomenon of fission (and fusion) was accommodated and the principle of substitutivity did not hold. Prior was only able to give suggestions as to how to go about it:[13]

> ... it seems to me quite clear that the only way in which the ordinary logic of identity can be fully preserved is by maintaining that cases of this sort [fission] never occur or can occur, i.e., that it never is or can be the case that one individual thing becomes two individual things;

[12]For a proof of this, see Prior (1968) pp. 81–2.

[13]Prior (1968) pp. 84–5.

> and that whenever we are tempted to describe an empirical change in this way, what has really happened has been [(1)] the ceasing to be of one individual and the beginning to be of two others, or else [(2)] they were two all the time, only this was not apparent, or else [(3)] they are still, in spite of appearances, one.

The first option that he proposes, (1), does not solve the problem, for the reasons that were put forward in the consideration of Δ. (2) seems to be counter-intuitive and faces some problems.[14] (3) is a mereological view. As argued above, this too fails to have any force.

3. *Dialetheic Tense Logic*

It might be thought that it is impossible to build a formal logic that accommodates the phenomena of fission and fusion as is analysed earlier in the paper. This might be so if we work within the framework of classical or intuitionist logic. Yet the story is different if we use a paraconsistent logic. In this section, I will use a paraconsistent logic and give a semantics of tense logic that respects fission and fusion.

The paraconsistent logic that I will use has relational evaluations. Classically, semantic evaluations are taken to be functions that assign exactly one truth value to a formula. To have a paraconsistent logic, we may take the evaluations to be relations. A formula may then relate to no truth value, or it may relate to multiple truth values. This is essentially a semantics for First Degree Entailment (FDE).[15] By extending this idea, we develop a relational semantics with CI. The thought here is to modify the standard semantics for CI by defining the members of the domain as relations instead of functions as well.

Let's call the logic that captures the above insight *Dialetheic Tense Logic (DTL)*. The language of DTL is that of first order logic plus tense operators. An interpretation for the language is a 5-tuple $\langle W, R, \Pi, D, I\rangle$, where W is a set of times, R is a binary relation on W, Π is a non-empty set (of

[14] One of the problems is the one that is closely related to the problem of "overpopulation". If it is said that there were in fact two things all the time, it would also be the case that there were in fact an infinite number of things. For if three things come into existence as a result of fission, then it must be that there were in fact three things all the time. And if four things come out, then ..., and so on. Hence there is no coherent method of determining the number of things at any time.

[15] For a semantics for FDE, see Dunn (1976).

parts), D is the domain containing relations between W and Π, I is a function such that for every constant c, $I(c) \in D$; if necessary we augment the language so that every member of the domain has a name. For every n-place predicate M and $t \in W$, $I(t, M) = \langle E_t, A_t \rangle$ (extension and anti-extension), where $E_t \subseteq \Pi^n$ and $A_t \subseteq \Pi^n$. For the identity predicate $=$, $I(t, =) = \langle E_t, A_t \rangle$ such that $E_t = \{\langle x, x \rangle : x \in D\}$.

Let ρ be a semantic relation between a formula and a truth value. Then the truth/falsity conditions for atomic formulas are:

> $Mc_1 \ldots c_n \rho_t 1$ iff $I(t, M) = \langle E_t, A_t \rangle$, and for $1 \leq i \leq n$, there are $d_i \in \Pi$ such that $\langle t, d_i \rangle \in I(c_i)$ and $\langle d_1, \ldots, d_n \rangle \in E_t$
> $Mc_1 \ldots c_n \rho_t 0$ iff $I(t, M) = \langle E_t, A_t \rangle$, and for $1 \leq i \leq n$, there are $d_i \in \Pi$ such that $\langle t, d_i \rangle \in I(c_i)$ and $\langle d_1, \ldots, d_n \rangle \in A_t$.[16]

These conditions allow a member of the domain to have multiple parts at a time. For there may be more than two $d \in \Pi$ such that $\langle t, d \rangle \in I(c)$. These may also give rise to inconsistency. For if a constant c relates to multiple parts at a time, an atomic sentence Mc may have multiple truth values. For example, if one part of c has the property of being M and another has the property of being $\neg M$, Mc is both true and false. Hence relationalisation of the members of the domain requires a logic to have multiple truth values and therefore the logic has to be paraconsistent.

On the other hand, a part of c may lack both the properties M and $\neg M$. Further suppose that every part of c lacks those properties at a time. In this case, Mc has no truth value at the time. Also if c has no parts at a time, then Mc has no truth value at that time. Hence DTL has a FDE semantics.[17]

We now extend the truth/falsity conditions to formulas by the following conditions:

- $\neg \alpha \rho_t 1$ iff $\alpha \rho_t 0$
  $\neg \alpha \rho_t 0$ iff $\alpha \rho_t 1$

[16] As is often argued in the context of personal identity, whether these conditions give rise to three-dimensional or four-dimensional ontology is an interesting question. However, the main objective of this paper is to show a formal semantics that solves a problem that Prior described. Since the problem arises in order to answer the question whether an amoeba a at t_0 is identical with amoebas b and c at t_2, solving the problem may require three-dimensional ontology. However, I leave the entire question for another occasion. For this is, though important, another issue to consider.

[17] In order to obtain an LP semantics, we place the condition that $E_t \cup A_t = D^n$ where $I(t, M) = \langle E_t, A_t \rangle$ for each n-place predicate M.

- $\alpha \wedge \beta\rho_t 1$ iff $\alpha\rho_t 1$ and $\beta\rho_t 1$
  $\alpha \wedge \beta\rho_t 0$ iff $\alpha\rho_t 0$ or $\beta\rho_t 0$

- $\alpha \vee \beta\rho_t 1$ iff $\alpha\rho_t 1$ or $\beta\rho_t 1$
  $\alpha \vee \beta\rho_t 0$ iff $\alpha\rho_t 0$ and $\beta\rho_t 0$.

The truth/falsity conditions for the quantifiers are:

- $\forall x\alpha\rho_t 1$ iff for all c, $\alpha(x/c)\rho_t 1$
  $\forall x\alpha\rho_t 0$ iff for some c, $\alpha(x/c)\rho_t 0$

- $\exists x\alpha\rho_t 1$ iff for some c, $\alpha(x/c)\rho_t 1$
  $\exists x\alpha\rho_t 0$ iff for all c, $\alpha(x/c)\rho_t 0$.

For the tense operators:

- $P\alpha\rho_t 1$ iff for some $t' \in W$ such that $t'Rt$, $\alpha\rho_{t'} 1$
  $P\alpha\rho_t 0$ iff for all $t' \in W$ such that $t'Rt$, $\alpha\rho_{t'} 0$

- $F\alpha\rho_t 1$ iff for some $t' \in W$ such that tRt', $\alpha\rho_{t'} 1$
  $F\alpha\rho_t 0$ iff for all $t' \in W$ such that tRt', $\alpha\rho_{t'} 0$

- $H\alpha\rho_t 1$ iff for all $t' \in W$ such that $t'Rt$, $\alpha\rho_{t'} 1$
  $H\alpha\rho_t 0$ iff for some $t' \in W$ such that $t'Rt$, $\alpha\rho_{t'} 0$

- $G\alpha\rho_t 1$ iff for all $t' \in W$ such that tRt', $\alpha\rho_{t'} 1$
  $G\alpha\rho_t 0$ iff for some $t' \in W$ such that tRt', $\alpha\rho_{t'} 0$.

Semantic consequence is then defined in terms of truth preservation at all times of all interpretations:

> $\Sigma \models \alpha$ iff for all interpretations $\langle W, R, \Pi, D, I\rangle$ and all $t \in W$, if $\beta\rho_t 1$ for all $\beta \in \Sigma$ then $\alpha\rho_t 1$.

DTL given above has some novelties. The most notable novel effect is on the Prior's problem, i.e., substitutivity and identity. In particular, DTL does not validate the law of transitivity.

Suppose that $\langle t, x\rangle \in I(a)$, $\langle t, y\rangle \in I(a)$, $\langle t, x\rangle \in I(b)$, $\langle t, y\rangle \in I(c)$, where $x, y \in \Pi$, and $I(t, =) = \langle E_t, A_t\rangle$ where x and y are distinct. Then there is a $v \in \Pi$, viz., x, such that $\langle t, v\rangle \in I(a)$ and $\langle t, v\rangle \in I(b)$, and $\langle v, v\rangle \in E_t$. So $a = b\rho_t 1$. Similarly, $a = c\rho_t 1$. However, there is no $v \in \Pi$ such that $\langle t, v\rangle \in$

$I(b)$ and $\langle t, v\rangle \in I(c)$. Hence $b = c\rho_t 1$ does not hold. Therefore, the law of transitivity fails.

In the above semantics, the account of identity is different from that of standard CI systems. The relational semantics of DTL defines identity in such a way that a and b are identical at a time if a and b relate to at least one common part at the time in question; whether or not a and b relate to something else, is irrelevant. And a and b fail to be identical at a time if a and b do not relate to anything in common at that time. Let us call the identity defined in this way *relational identity*. This relational identity captures the intuitions about identity in fission and fusion, in the forms of Θ and Θ', as we will now see.

By using the formal language of the DTL, the state at t_2 in fission can be represented as: $\langle t_2, x\rangle \in I(a)$, $\langle t_2, y\rangle \in I(a)$, $\langle t_2, x\rangle \in I(b)$, and $\langle t_2, y\rangle \in I(c)$. Then it can be checked that $a = b\rho_{t_2}1$ and $a = c\rho_{t_2}1$. So at t_2, i.e., after fission, a is identical with b and c. Yet b is not identical with c. How to achieve this conclusion puzzled Prior who failed to show how it would be possible. However, thanks to the relational identity, DTL achieves this conclusion and solves the puzzle. For the semantics shows that $b = c\rho_{t_2}1$ does not hold, although $a = b\rho_{t_2}1$ and $a = c\rho_{t_2}1$. Hence the relational semantics of DTL, that gives rise to relational identity, is just what we need in solving the problem of fission and fusion.

4. ... *and Multiple Denotation*

Priest (1995) has also shown a semantics that *prima facie* solves the Prior's problem. He extends the idea of relationalisation of truth value assignments (to formulas) to denotation assignments. Then the logic of this semantics allows not only multiple truth values but also multiple denotations. So in a case of fission, 'a' denotes b and c after t_1. Hence $a = b$ and $a = c$ after t_1. Yet, as in the case of DTL, the law of transitivity breaks down in the case of multiple denotations. So we cannot infer that $b = c$ after t_1, as is required to solve Prior's problem.

However, the logic of multiple denotations that Priest has shown is not equipped with tense operators. Since there is overwhelming agreement that tense operators are necessary in analysing fission and fusion, the adequacy of an application of multiple denotation by itself to fission and fusion is dubious.[18]

Moreover, the logic of multiple denotation does not verify the rule of Existential Generalisation (EG). In this, it can be shown that $a \neq a$ may be

[18] For some problems of the logic of multiple denotation, see Tanaka (1997).

true (and false) but $\exists xx \neq x$ is not true.[19] The problem with the logic of multiple denotation is that it allows multiple denotations for constants but not for variables. While having the same effects as multiple denotations on Prior's problem, denotations of DTL are, in some sense, classical. As a result, the rule of EG holds in DTL.

5. *... and Leibniz' Law*

As a reaction to the semantics presented above, one may question the plausibility of relational identity. One may ask what relational identity really is. In particular, one may ask whether relational identity is a correct account of identity.

Classically, identity is defined to be reflexive, symmetric and transitive. Moreover, and more importantly, it satisfies *Leibniz' Law*: *a* and *b* are identical iff they both have and lack the same properties. Now it is often assumed that a correct account of identity has to confirm to Leibniz' law. Relational identity, however, does not satisfy Leibniz' law. Hence relational identity, it may be argued, cannot be a correct account of identity.

This Leibniz' law based argument appeals to the assumption that Leibniz' law is a norm for identity. However, this assumption is a strong one. It seems that Leibniz' law is an overwhelmingly accepted law of identity. Yet when it comes to justifying Leibniz' law, people often beg the question. They argue that two things being identical means that they have and lack the same properties. Thus, they conclude, Leibniz' law has to be satisfied. Clearly, this is question-begging. Hence the Leibniz' law based argument does not have much effect on relational identity. For there is no legitimate reason to suppose that a correct account of identity need confirm to Leibniz' law.

Perhaps there are some arguments that justify Leibniz' law and are not question begging. However, if one enforces Leibniz' law and so classical identity, then it does not seem possible that the problem of Prior be solved, as Prior himself observed. Note that the principle of substitutivity is a part of Leibniz' law. Hence, in resolving the problem put forward by Prior, there is a good reason to give up Leibniz' law and classical identity.

[19] For the details of this, see Priest (1995) p. 365.

6. *Conclusion*

A formal system that accommodates the phenomena of fission and fusion is hard to come by. Fission and fusion provide problems for the standard systems. These problems can be rectified by modifying the standard semantics of CI systems. The technique is to take a member of the domain to be a relation instead of a function. The semantics of CI systems developed in this way solves the problems for standard systems. Since the semantics is paraconsistent, paraconsistent logics provide a technique to capture the phenomena of fission and fusion.

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