Logique & Analyse **161-162-163** (1998), 135-143

C_1-COMPATIBLE TRANSITIVE EXTENSIONS OF SYSTEM *CT*

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Da Costa's paraconsistent systems of the series C_m (for finite m) (see [C1], [C2], and esp. [C3], pp. 237ff.) share important features with transitive logic, *TL* (which has been gone into in [P1] and [P2]), namely, they all coincide in that: (c1) they possess a strong negation, '$\neg$', a conditional, '$\supset$', a conjunction, '$\wedge$', and a disjunction, '$\vee$', with respect to which they are conservative extensions of *CL* or Classical Logic; (c2) they possess a non-strong negation, 'N' (notations are different for systems *C*) which does not possess all properties of classical negation, but for which the following schemata are theorematic (I use the letters 'p', 'q', etc as schematic letters; my notational conventions are basically Church's: associativity leftwards; a dot stands for a left parenthesis with its mate as far to the right as possible): $\ulcorner p\vee \mathrm{N}p\urcorner$, $\ulcorner \mathrm{NN}p\supset p\urcorner$, $\ulcorner p\supset \mathrm{N}p\supset \mathrm{N}p\urcorner$; (c3) they possess a monadic functor, '#', for which the following schemata are theorematic: $\ulcorner \mathrm{N}\#p\supset .p\wedge \mathrm{N}p\urcorner$, $\ulcorner \#p\wedge\#q\supset .\#(p\vee q)\wedge\#(p\wedge q)\wedge\#(p\supset q)\wedge\#\mathrm{N}p\urcorner$; (c4) they are almost unique anong paraconsistent logics in their having the three aforementioned features. (In general systems with features of that sort have been called '*extensional paraconsistent logics*' by Diderik Batens, who has also proposed systems bearing a kinship of sorts to those —even though they lack strong negation and a classicality operator, they can be easily extended in that way; see [B1].)

A difference between *TL* and the systems *C* is that in *C* functor '#' is defined through negation, 'N', and conjunction, '$\wedge$', and then strong negation, '$\neg$', is defined with those three functors, whereas in *TL* either '#' is taken as primitive, or else strong negation is taken as primitive (in which case $\ulcorner \#p\urcorner$ is defined as $\ulcorner \neg p\vee\neg \mathrm{N}p\urcorner$), or another primitive functor is introduced, one of strong afirmation, 'H', such that $\ulcorner \neg p\urcorner$ is then defined as $\ulcorner \mathrm{HN}p\urcorner$ and $\ulcorner \#p\urcorner$ as (e.g.) $\ulcorner \mathrm{H}(p\vee \mathrm{N}p)\urcorner$ (strong affirmation distributes over conjunction and also over disjunction).

There are other differences between *TL* and systems C_n. In *TL* some schemata hold which do not hold in *C*, such as $\ulcorner \mathrm{N}(p\wedge \mathrm{N}p)\urcorner$, $\ulcorner p\supset \mathrm{NN}p\urcorner$, $\ulcorner p\vee q\equiv \mathrm{N}(\mathrm{N}p\wedge \mathrm{N}q)\urcorner$, $\ulcorner p\wedge q\equiv \mathrm{N}(\mathrm{N}p\vee \mathrm{N}q)\urcorner$. Moreover, in *TL* there exist several primitive functors which do not exist in *C*, such as: (f1) an equivalential functor, '$\leftrightarrow$', for which the rule of inference $p\leftrightarrow q,\ r\vdash s$ holds, where

⌜r⌝ differs from ⌜s⌝ only by substituting to one or more occurrences of ⌜p⌝ as many occurrencess of ⌜q⌝; (f2) a functor of minimal affirmation, 'Y', such that this schema holds: ⌜$\text{Y}q\supset.q\wedge p\leftrightarrow q\equiv p$⌝ (or, abbreviating ⌜$q\wedge p\leftrightarrow q$⌝ as ⌜$q\rightarrow p$⌝: ⌜$\text{Y}p\supset.q\equiv.p\rightarrow q$⌝); a strong conjunction, '•', such that ⌜$p\bullet q\rightarrow.p\wedge q$⌝, but not conversely, '•' being endowed with the properties of classical conjunction save idempotence (the schema ⌜$p\bullet p\leftrightarrow p$⌝ does not hold).

Such complicated pattern of relations has led us to find out a system with all the properties shared by *TL* and systems C_m (for m finite) and to see how it can be strengthened with properties of *TL* while remaining *C*-compatible.

Definition: a system *S* is C_m-compatible iff C_m is an extension of a system of which *S* is also an extension and, if the schemata which are theorematic in C_m but not in *S* are added to *S*, the result is a paraconsistent system (for negation 'N'), i.e. it does not have the inference rule: $p, \text{N}p \vdash q$.

I shall show below that system *CT* —to be sketched out in a moment— is such that both *TL* and C_m (for $m<\infty$) are extensions thereof. (I shall also show that, if we reinforce *CT* by adding certain functors and theorematic schemata of *TL*, we get systems some of which are C_1-compatible.) My hypothesis (which will not be proved here) is that system *CT* is the strongest system with such a feature.

System *CT*
Primitive symbols: $\vee$, $\wedge$, N, #.
Definitions: ⌜$\text{S}p$⌝ abbreviates ⌜$p\wedge\text{N}p$⌝; ⌜$\neg p$⌝ abbr. ⌜$\#p\wedge\text{N}p$⌝; ⌜$p\supset q$⌝ abbr. ⌜$\neg p\vee q$⌝; ⌜$p\equiv q$⌝ ⌜$p\supset q\wedge.q\supset p$⌝.
Inference rule: *modus ponens* ($p, p\supset q \vdash q$)

Axiomatic schemata:

(*Export*)	$p\wedge q\supset r\supset.p\supset.q\supset r$
(*Transit*)	$p\supset q\wedge(q\supset r)\supset.p\supset r$
(*L-simpl.*)	$p\wedge q\supset p$
(*R-simpl.*)	$p\wedge q\supset q$
(*L-addition*)	$p\supset.p\vee q$
(*R-addition*)	$q\supset.p\vee q$
(*Conv2Neg*)	$\text{NN}p\supset p$
(*Chrisippus*)	$\#p\vee\text{S}p$
(*hered*)	$\#p\wedge\#q\supset.\#(p\vee q)\wedge\#(p\wedge q)\wedge\#\text{N}p$
(*Clas-Clas*)	$\#\#p$
(*Conj2Disj*)	$p\supset r\wedge(q\supset r)\supset.p\vee q\supset r$

(*Conj2Conj*) $p\supset q\wedge(p\supset r)\supset.p\supset.q\wedge r$

(Notice that this axiomatization is heavily indebted to the one proposed for *CL* by Prof. Hubert Hubien in his paper [H1].)

This system is stronger than C_ω but less strong than C_m (for m finite).

Now I am going into some crucial points concerning system *CT*.

It seems to be in order to propose readings of our symbols. Not that da Costa has always bothered to provide us with such readings; more often than not he hasn't, except in so far as 'N', '$\vee$', '$\wedge$' and '$\supset$' are concerned. Oddly enough he has failed to offer any natural-language reading for either '$^{(m)}$' (our '#') or '$\neg$', or strong negation. "#p" can be read as "It is a classical matter whether or not p", where a classical matter is a disjunction between two entirely opposite situations each of which either completely holds or else does not hold at all. Likewise "$\neg p$" can be read as "It is not the case that p at all"; "Hp" as "It is fully the case that". '$\equiv$' is read 'if, and only if'.

In *CT* we easily prove the four following results:

(R1) Chrisippus's principle —namely $\ulcorner\#p\vee Sp\urcorner$ — is, in the presence of the other axiomatic schemata and inference rules, equivalent to $\ulcorner N\#p\supset Sp\urcorner$, i.e. the assertion that whatever fails to stand by classical strictures is contradictory. Proof: first we prove $\ulcorner p\supset p\urcorner$, hence (by definition) $\ulcorner p\vee\neg p\urcorner$ and $\ulcorner p\vee q\supset.q\vee p\urcorner$. By instantiation we have $\ulcorner\neg p\vee\neg\neg p\urcorner$, hence $\ulcorner p\supset\neg\neg p\urcorner$. We thus prove $\ulcorner p\supset q\supset.\neg q\supset\neg p\urcorner$. We also prove de Morgan: $\ulcorner\neg(p\vee q)\supset.\neg p\wedge\neg q\urcorner$ and associativity: $\ulcorner p\vee q\vee r\supset.p\vee.q\vee r\urcorner$. Then we prove distributivity: $\ulcorner p\vee q\wedge r\supset.p\wedge r\vee.q\wedge r\urcorner$ and $\ulcorner p\supset.q\supset.p\wedge q\urcorner$. Hence $\ulcorner p\vee q\supset.\neg p\supset q\urcorner$. Then from $\ulcorner\#p\vee Sp\urcorner$ we prove $\ulcorner\neg\#p\supset Sp\urcorner$. But since $\ulcorner\#\#p\urcorner$ is theorematic, we have $\ulcorner N\#p\supset\neg\#p\urcorner$; hence $\ulcorner N\#p\supset Sp\urcorner$. Q.e.d. The converse proof is also straightforward: from $\ulcorner N\#p\supset Sp\urcorner$ we get $\ulcorner\neg N\#p\vee Sp\urcorner$, hence $\ulcorner\#N\#p\wedge NN\#p\vee Sp\urcorner$, hence $\ulcorner NN\#p\vee Sp\urcorner$, hence $\ulcorner\#p\vee Sp\urcorner$. Q.e.d.

(R2) *CT* contains, among others, the following theorem-schemata: $\ulcorner p\supset q\vee p\urcorner$ (*Funnel*), $\ulcorner p\supset q\supset p\supset p\urcorner$ (Peirce), $\ulcorner p\wedge\neg p\supset q\urcorner$ (Cornubia for strong negation). The proof is trivial: with $\ulcorner p\wedge q\supset r\supset.p\supset.q\supset r\urcorner$ plus $\ulcorner p\wedge q\supset p\urcorner$ and $\ulcorner q\wedge p\supset p\urcorner$ prove, first, $\ulcorner p\supset.q\supset q\urcorner$ and hence $\ulcorner p\supset p\urcorner$; then prove (by definition) $\ulcorner p\vee\neg p\urcorner$, hence (thanks to $\ulcorner p\supset.p\vee q\urcorner$ and $\ulcorner q\supset.p\vee q\urcorner$) $\ulcorner\neg\neg p\vee\neg p\vee q\urcorner$, i.e. *e [prorsus] falso quodlibet*, namely: $\ulcorner\neg p\supset.p\supset q\urcorner$; whence Funnel follows thanks to $\ulcorner p\supset r\wedge(q\supset r)\supset.p\vee q\supset r\urcorner$ and exportation; now take a particular case of Funnel, namely $\ulcorner p\supset q\supset q\vee.p\supset q\urcorner$. Whence conjunctive assertion (namely $\ulcorner p\supset q\wedge p\supset q\urcorner$) follows (again thanks to $\ulcorner p\supset r\wedge(q\supset r)\supset.p\vee q\supset r\urcorner$). Peirce is proved as follows: by exportation we get (once we have proved $\ulcorner p\supset p\urcorner$) $\ulcorner p\supset q\supset p\supset.p\supset q\vee p\supset p\urcorner$ (again thanks to $\ulcorner p\supset r\wedge(q\supset r)\supset.p\vee q\supset r\urcorner$) and by transitivity (and Funnel)

⌜$p\supset q\supset p\supset p$⌝. Cornubia follows from *e prorsus falso quodlibet* plus the lemma ⌜$p\supset(p\supset q)\supset.p\supset q$⌝, i.e. absorption, which can be easily proved from conjunctive assertion.

(R3) The fragment of *CT* expresible only with '$\supset$', '$\wedge$', '$\vee$', '$\neg$' is exactly *CL*. Proof: take any standard presentation of *CL* and show the equivalence between its set of axioms and that of *CT* when symbol 'N' is omitted. In fact Hubien's axiomatization is but a variant of the well-known axiomatization of Hilbert & Ackermann, which is clearly equivalentent to our positive system of axioms plus classical negation endowed with ⌜$p\supset q\supset.\neg q\supset\neg p$⌝, ⌜$p\supset\neg\neg p$⌝ and ⌜$\neg\neg p\supset p$⌝. The three are provable in *CT* (since ⌜$p\supset q$⌝ abbreviates ⌜$\neg p\vee q$⌝). Therefore, *CT* contains *CL*. The converse can also be proved quite easily, since in *CL* ⌜$\neg p\vee q\equiv.p\supset q$⌝. Thus replace the set pf Hilbert & Ackermann's primitive symbols {$\neg$, $\supset$, $\vee$, $\wedge$, $\equiv$} with {$\neg$, $\vee$} and define '$\supset$' and '$\equiv$'. The three nonpositive axioms then become redundant or idle.

(R4) *CT* is stronger than C_ω (since C_ω lacks Peirce and Funnel). Proof: C_ω is positive (intuitionistic) logic enlarged with a very weak negation satisfying just ⌜$\mathrm{NN}p\supset p$⌝ and ⌜$p\vee\mathrm{N}p$⌝. *CT* is of course stronger, since it includes the whole classical positive calculus. (See (R2) above.)

We can strengthen *CT* by adding one or several among the following principles. (That by so doing we obtain proper strengthenings can be shown through a da Costa's valuation semantics, which is two-valued but not truth-functional: we can easily devise such a semantic for *CT* failing to satisfy any one of the following schemata; devising it is left as an exercise to the reader):

(*2negation*)	$p\supset\mathrm{NN}p$
(*DeMorgan-1*)	$p\vee q\supset\mathrm{N}(\mathrm{N}p\wedge\mathrm{N}q)$
(*DeMorgan-2*)	$p\wedge q\supset\mathrm{N}(\mathrm{N}p\vee\mathrm{N}q)$
(*DeMorgan-3*)	$\mathrm{N}(p\wedge q)\supset.\mathrm{N}p\vee\mathrm{N}q$
(*DeMorgan-4*)	$\mathrm{N}(p\vee q)\supset.\mathrm{N}p\wedge\mathrm{N}q$

Let *LTL*, or *lean transitive logic*, be the fragment of *TL* expressible with symbols occurring in the *CT* language. *LTL* is the result of adding to *CT* all those five axioms plus the principle of contradiction or *Ænesidemus*, namely: ⌜$\frac{1}{2}\equiv\mathrm{N}\frac{1}{2}$⌝, where '$\frac{1}{2}$' is a sentential constant with whatever meaning. We can call *JTL* (*jejune transitive logic*) the result of adding to *CT* the just mentioned principles except Ænesidemus. *CL* is *JTL* plus the axiomatic schema: ⌜$\#p$⌝. *TL* is of course a conservative extension of *CL*, but it cannot be classically "strengthened" (once Ænesidemus has been added, no classical meaning can be given to 'N').

LTL and even *JTL* are not C_m-compatible (for *m* finite). Proof: with *DeMorgan-1* or $\ulcorner p\vee q\supset N(Np\wedge Nq)\urcorner$ we prove a variant of noncontradiction ($\ulcorner N(Np\wedge NNp)\urcorner$) from the principle of excluded middle, which is theorematic in *CT*. Thus with the help of *2negation* or $\ulcorner p\supset NNp\urcorner$ (the converse of which is theorematic in *CT*) we prove the general principle of noncontradiction, which of course is incompatible with da Costa's systems (it collapses them into *CL*).

In fact starting with *CT* we obtain C_1 by adding *BF* —or the *Back-to-the-Fold* principle—, which is the converse-Chrisippus principle, namely: $\ulcorner NSp\supset\#p\urcorner$ —what is not contradictory is classical. (Again the proof is trivial but tedious, and is thus left to the reader. Hint: in C_1 prove all axioms of *CT* (defining $\ulcorner\#p\urcorner$ as $\ulcorner NSp\urcorner$ and thus getting *BF* quite cheap); then in *CT* plus *BF* prove any set of axioms of C_1.) In order to obtain C_2 instead, we add $\ulcorner NSp\wedge NSSp\supset\#p\urcorner$. In general C_m is *CT* plus $\ulcorner NSp\wedge NSSp\wedge...\wedge NS_mp\supset\#p\urcorner$, where '$S_m$' stands for a string of m occurrences of 'S'.

(Da Costa's original axiomatization was of course different: with $\wedge$, $\supset$, $\vee$, N as primitive, if we define 'S' in such a way that $\ulcorner Sp\urcorner$ abbr $\ulcorner p\wedge Np\urcorner$ and we have *Modus Ponens* as the only inference rule, the axioms are: $\ulcorner p\supset .q\supset p\urcorner$; $\ulcorner p\supset q\supset .p\supset(q\supset r)\supset .p\supset r\urcorner$; $\ulcorner p\supset .q\supset .p\wedge q\urcorner$; $\ulcorner p\wedge q\supset p\urcorner$; $\ulcorner p\wedge q\supset q\urcorner$; $\ulcorner p\supset r\supset .q\supset r\supset .p\vee q\supset r\urcorner$; $\ulcorner p\supset .p\vee q\urcorner$; $\ulcorner q\supset .p\vee q\urcorner$; $\ulcorner NNp\supset p\urcorner$; $\ulcorner p\vee Np\urcorner$; $\ulcorner NSp\supset .q\supset p\supset .q\supset Np\supset Nq\urcorner$; $\ulcorner NSp\wedge NSq\supset .NS(p\supset q)\wedge NS(p\wedge q)\wedge NS(p\vee q)\wedge NSNp\urcorner$.)

The main idea behind adding *BF* —and of course, philosophically, da Costa's chief motivation— is the *noninconsistency assumption*, namely that denying a contradiction entails accepting that the situation therein involved is a classical one. In other words, if and when it is not the case that both *p* and not-*p*, then *p* is a classical situation. Whatever is noncontradictory is classical.

The noninconsistency assumption has of course been questioned by many other paraconsistent logicians —including the present author—, who have argued that, if contradictions can be true, one of those true contradictions may well be that *p*-and-not-*p* both obtains and does not obtain.

Yet da Costa's approach enjoys two significant characteristics, or perhaps advantages. The first one is that, when somebody claims, for a certain particular situation, *p*, to accept both *p* and not-*p*, his interlocutors are likely to rejoin: 'Then you do not accept the principle of non-contradiction!'. Needless to say, other paraconsistent schools regard such a rejoinder as stemming from a classicist confusion —mistaking 'not to accept *s*' for 'to accept not-*s*', or 'to accept *r*', if *s* = N*r*. Even so, da Costa's point is not entirely devoid of *prima facie* plausibility. That constitutes the first advantage of the approach implemented in the C systems.

Moreover —and this constitutes the second advantage of da Costa's preferred approach—, the noninconsistency assumption succeeds —in the absence of *2negation*, *DeMorgan-2*, *DeMorgan-3* and *DeMorgan-4*— in enforcing an important constraint, viz. the *confinement of contradictions* (not to multiply contradictions beyond absolute necessity): thanks to the noninconsistency assumption —or equivalently to the *BF* principle— (plus the non-endorsement of involutivity and De Morgan), a given contradiction not only fails to render the theory deliquescent (trivial) but also fails to trigger an infinite chain of further contradictions, whereas, upon other paraconsistent underlying logics (such as a relevant or a transitive logic), once, for a certain constant 'a', a given theory contains both 'a' and 'Na', it is bound to also contain infinitely many different contradictions (⌜$a \wedge \mathrm{N}a \wedge \mathrm{N}(a \wedge \mathrm{N}a)$⌝, ⌜$\mathrm{N}(a \vee \mathrm{N}a)$⌝, etc).

Now, da Costa's is not the only paraconsistent approach to have implemented the confinement constraint. In fact the late Richard Sylvan's approach often (although perhaps not always) leaned towards some sort of containment policy; but especially Graham Priest's approach is arguably a containment view (see the present writer's Critical Notice: 'Graham Priest's "Dialectheism" — Is It Altogether True?', *SORITES* # 7 (November 1996) (ISSN 1135-1349), pp. 28–56). Admittedly, those other approaches put the containment constraint to serve different purposes.

Both advantages (if they are such) are of course closely related. Probably what is implicitly assumed by those interlocutors who equate asserting a contradiction with denying (and in fact rejecting) the principle of noncontradiction is that nobody is so unreasonable as to both swallow a contradiction and yet also espouse the very same denial of that contradiction. Contradictions are assumed to be bad and even irrational. First-level contradictions are bad enough as they are, but adding second-level contradictions and so on is still more irrational. Now, all approaches implementing the confinement constraint somehow or other assume as much —namely that contradictions are bad and thus not to be endorsed except as an extreme measure, when nothing else works to solve a difficulty, and even so perhaps only temporarily.

Whatever our final views on such a debate (and my own opinion is that, infinite chains being harmless, no serious mishap ensues from advocating both noncontradiction and also certain contradictions), the present discussion (or digression) makes out a case for the claim that climbing up to the *C* systems is not a whimsical choice.

Let me explain. System *CT* is classical logic plus a very weak negation endowed with only two principles: converse double negation and excluded middle. *CT* does not prejudge any additional principles as regards negation. In fact *CT* can be strengthened into classical logic (thus collapsing 'N' into a notational variation of '$\neg$') by adding the schema ⌜$\mathrm{S}p\supset q$⌝ (or ⌜$\#p$⌝). C_1 is instead obtained by adding ⌜$\mathrm{SS}p\supset q$⌝ (or ⌜$\#\mathrm{S}p$⌝); C_2 is obtained by adding ⌜$\mathrm{SSS}p\supset q$⌝ (or ⌜$\#\mathrm{SS}p$⌝); and so on. The idea behind the strengthening into classical logic is that all contradictions —even first degree contradictions— are bad and unacceptable; the one behind C_n is that contradictions of $(n+1)$th degree are bad; and one (perhaps the) reason that can be adduced for that is that an infinite chain of deeper-level contradictions seems baffling: you can admit "*p* and not *p*" but not "*p* and not-*p* and not (*p* and not *p*) and ...". At some level or other you are bound to stop, or else nobody will really understand what your point amounts to. Perhaps that level is not the first level, but it must be some finite level or other.

On the other hand, instead of climbing to any of those systems, you can choose to accept contradictions of any level of complexity. Then, for some particular "*p*", you will espouse ⌜$\mathrm{S}...\mathrm{S}p$⌝ for any finite sequence of 'S' and reject any of ⌜$\#p$⌝, ⌜$\#\mathrm{S}p$⌝, etc. But then by the same token you will also accept ⌜$\mathrm{NS}p$⌝, ⌜$\mathrm{NSS}p$⌝, etc, that is to say all corresponding instances of the principle of noncontradiction. Now, for all other formulae "*p*" such that you do not accept ⌜$\mathrm{S}p$⌝, there is no valid ground on which you will base a rejection of ⌜$\mathrm{NS}p$⌝. Thus, for every "*p*" you will then accept ⌜$\mathrm{NS}p$⌝. Which means that then you accept the principle of noncontradiction.

Those are two legitimate, plausible options: either (1) only some low-level contradictions and no higher-level contradiction, and no general principle of noncontradiction; or else (2) contradictions of every level plus the principle of noncontradiction. (The classicist's choice is the former, with admission of contradictions of 0 level only, i.e. no contradictions at all.)

Even though I personally happen to think that the latter choice is better, more elegant, I nevertheless acknowledge the rationality and the motivation of da Costa's own choice.

Anyway, are there extensions of *CT* which are C_m-compatible (in the technical sense of the term we are using)? There probably are. If to *CT* we add one among *2negation*, *DeMorgan-2*, *DeMorgan-3*, *DeMorgan-4*, the result can probably be shown to remain C_m-compatible. But then why has da Costa kept clear of them all, thus impoverishing his weak negation beyond necessity? The probable reason is that, if you add e.g. *DeMorgan-2*, then in a contradictorial theory wherein, for some particular constant $\odot$, we have '$\odot\wedge\mathrm{N}\odot$', *DeMorgan-2* will yield '$\mathrm{N}(\mathrm{N}\odot\vee\mathrm{NN}\odot)$', i.e. a negation of an instance of excluded middle. And da Costa tries to confine (unavoidable) contradictions to atomic sentences, as far as possible. In some of his systems every nonatomic formula must be classical; that's not always the case as regards the modelizations of his main systems of the *C* series, though; but even so, he clearly leans towards taking (most) nonatomic formulae to stand by classical standards —at least once disjunction has been entered— and thus to thinking that, whatever the behaviour of p —classical or not—, 'p or not-p' ought to behave classically: if admitted as true, its negation must be thoroughly rejected as purely and unmixedly false.

Not only is it possible to obtain C_m-compatible systems by strengthening *CT* with at least one among double negation and the DeMorgan principles (except *DeMorgan-1*), but, what is more, apparently all those principles can be added together, at the same time, —again with the exception of *DeMorgan-1*— without the resulting system losing its C_m-compatibility.

While a study of da Costa's preferred semantical account of his systems through the method of two-valueed non-truthfunctional valuations invented by da Costa (and developed by I. Arruda, E. Alves and others) needn't concern us here, adapting the technique to the envisaged enrichments is rather straightforward. Thus double negation (⌜$p \supset \mathrm{NN}p$⌝) requires that, for every valuation v, $v(p)=v(\mathrm{NN}p)$.

Our examination of a different way of setting up systems C_m shows that da Costa's whole logical enterprise —as carried out in the construction of the C systems— must not be reduced to espousing *BF*; that you can perfectly well reject *BF* while keeping many of the programmatic points implemented in the C systems; that your path and da Costa's can bifurcate without your being bound to part company with his orientation right from the start. Even without *BF* a lot of the significance and usefulness of [something close to] the C systems remains.

Thus, our main result has been to clarify the true relations between Transitive Logic and da Costa's C systems, a clarification which was hard to attain within the framework of da Costa's original presentation of his systems. We now see that *TL* and the C systems are built up on an underlying common ground, system *CT*, i.e. classical logic plus: (a1) a weak nonclassical negation enjoying at least converse double negation and excluded middle; and (a2) a symbol for classicality (or classical well-behavedness), which we have written as '#', enjoying the expected properties (hereditariness, Chrisippus, and the classicality of classicality-judgments —or what, from a gradualistic viewpoint [not da Costa's] can be termed *the two-valuedness of two-valuedness-attributions*, i.e. *whether a situation is classical or not is a classical matter*).

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