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INTERVAL SEMANTICS FOR DESCRIPTION OF CHANGE

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Abstract
The aim of this paper is to give a description of change in the framework of tense logic. Some different examples of using the intervals are considered. The main principles of a logic of inconsistent reasoning are presented in tense interval paraconsistent semantics.

Duration is the most important feature of time. There arise a lot of problems when we have to deal with objects, which do not possess a temporal extent. In his lecture "The Theory of Continuity" Bertrand Russell considered the philosophical aspect of the problem of continuity in the following words: "Space and time are treated by mathematicians as consisting of points and instants, but they also have a property, easier to feel than to define, which is called continuity, and thought by many philosophers to be destroyed when they are resolved into points and instants" (Russell: 135).

First of all it looks disputable to construct a period of time with instants which have not any duration. Aristotle used the intervals to analyse the problems of time and change. He insisted upon the continuity of movement, ascribed the attribute of continuity to time and assumed the infinite divisibility of the periods of time into parts (*Physics*: 219 a 10, 218 a 5). According to his interpretation of inconsistency one part of the changing thing is in one state, the other part is in another state (*Metaphysics*: 1005 b 20; *Physics*: 234 b 15). The inconsistent propositions correspond to different periods. However, Aristotle considered the change of quality to occur at once. He solved the problem of transition by assuming that the moment of transition was taken as the first moment of the next state (*Physics*: 253 b 25, 263 b 15–30).

Some recent investigations in different areas show that the moments are inapplicable to the study of the phenomenal continuums: for a number of quality changes there are no subdivision of object temporal existence which might separate clearly the state before change from the state after it. It is not possible to determine the last moment of a prior state and the first moment of a posterior one. Predicates in natural language are not valued relatively to moments of time (Hamblin: 414; Needham: 49). The beginnings and the ends of many states and processes are present somewhere at the intervals of time and have no presence at fixed points. In so far as there exists a vagueness of the boundaries between different states of being, propositions describing the states before and after change are either both true or their truth status is not determined.

J.N. Woodger in the thirties has built an original axiomatic theory with an elementary relation "to be a part" both in spatial and temporal senses (Woodger: 55). In terms of this relation and of the relation of time precedence the concepts of sum of parts, of moment, of organised units (such as a cell or an organism), of instant part of an organised unity were defined. These means allowed to formalise the biological relations, which arise from the model of cell division. The special kind of continuous change, a branching continuum, has been formally expressed.

J.-M. Laforge used as an initial concept "to be a fragment of a physical aggregate" (F) and the concept of the territory of a physical compound (T) for the description of physical space-time: "The territory is what completely overlaps a class of objects having a common mathematical or other well-defined property" (Laforge: 35). A propositional calculus extended by the constants F and T is considered as a logic of territories. A transformation of a physical compound, realised in the breaking up and a subsequent joining up of the parts, can break a topological structure of a physical compound, but doesn't break an invariance of F and T.

In terms of F and T the pseudo-topology of a physical compound is built. The concept of boundary, dividing set of points, is replaced by the concept of pseudo-boundary. The pseudo-boundary of a fragment A of a certain physical compound is a class of fragments that intersect A and its complement. An object belonging to the pseudo-boundary of the fragment A has properties of the objects from the fragment A and from its complement.

The temporal structures in two theories mentioned above contain the moments. C.L. Hamblin proposed an axiomatic theory of the intervals of time. An irreflexive, antisymmetric and transitive "later-then" relation was taken as a primitive constant (Hamblin: 415–419). The attempt to justify that Hamblin's "later-then" relation is something more than simply an order relation has led to another variant of the interval theories. Needham's aim was to build a theory of linear order in such a manner that the direction and the distinctions of future and past would be inexpressible: "Such a theory of linear order expresses no more than that certain times lie between others, the fundamental order concept being that of three-place betweenness relation. The procedure adopted in the theory presented here is to define a betweenness relation in term of which the linear ordering of time is expressed. A dyadic order relation can than be defined on the basis of the betweenness relation" (Needham: 51).

E. Lemmon introduced the notion of truthfulness relatively to the intervals as additional to the notion of truthfulness relatively to the moments. The basic relation of this logic of space-time zones is the four-dimensional relation "part-whole." The usage of a space-time structure allows to justify the distinctions between simple and continuous tenses of verbs, between perfective and imperfective aspects of verbs, and to construct a logic of certain tense adverbs (Lemmon: 101).

A next step in the creation of the logic of space-time zones is a temporal interval logic. Usually the temporal logic calculi presuppose the concept of a time moment as a starting abstraction. An evaluation of the formulas is carried out relatively to the moments that are ordered by an earlier-later relation. A truth-value change of a formula testifies to a change in a state of affairs. The change of truth-value is considered here as instantaneous.

The first approach to the formal explication of change by that means is von Wright's formalism (Wright: 28–30). The values of the formulas are determined relatively to temporal structures with a discrete order of moments. An event is considered in this formalism as a pair of states of affairs, i.e., as a transition state. It is represented by the formula ATB. A and B designate propositions about states, T is the binary copula "and next." In von Wright's semantics the formula A is a semantic consequence of the formula ATB, while the formula B is not (Humberstone: 190). In virtue of the discreteness of a temporal structure there is no intermediary moment between the last moment of A's truth and the first moment of B's truth. So in this approach the moment of the transition coincides with the last moment of A's truth or with the first moment of B's truth. On the other hand, if the order of moments is dense, there is always a moment between A's and B's truth. Then the formula ATB is true at the moment, which is different from the truth of A and the truth of B (Humberstone: 192).

The introduction of two operators of negation was the first step in the construction of tense interval logic: "The strong negation of a formula would be true with respect to an interval if the formula itself was false through all subintervals of the interval, while the weak negation would be true merely if it was not the case that the formula negated was true with respect to the interval in question (even if it was true for some subintervals)" (Humberstone: 172). The interval model is a triple $\langle T, \subseteq, |\,| \rangle$, where T is a class of time intervals: t, u, v, ... t′, u′, v′ ... ; $\subseteq$ is a mereological subinterval relation; $|\,|$ is an interpretation function: $|\,|: F \times T \rightarrow \{0,1\}$ (F is a set of the formulas, 0 and 1 are the values "false" and "true" respectively). A propositional variable a is true relatively to an element of the model

$\langle T, \subseteq, |\ |\rangle$ if and only if $|a|t = 1$. A peculiarity of the interpretation of the propositional variables is the assumption of the steadiness of truth-value: $|a|t = 1$ entails $|a|t' = 1$ for any $t' \subseteq t$. This assumption is introduced to exclude the intervals at which a propositional variable is true and false simultaneously.

The second stage of this construction is an addition of tense operators into the formal language. The insertion of the earlier-later relation R into the above mentioned model turns it into a tense interval model: $\langle T, \subseteq, R, |\ |\rangle$. The minimal assumptions about the relations between $\subseteq$ and R are:

(a) for all t, u, u$'$ $\in T$, if u R t and u$'$ $\subseteq$ u, then u$'$ R t,

(b) for all t, t$'$, u $\in T$, if u R t and t$'$ $\subseteq$ t, then u R t$'$,

(c) for all w, t, u $\in T$, if u $\subseteq$ t and u R w, then either t R w, or for some v $\in T$, v $\subseteq$ w and v $\subseteq$ t. The assumptions (a) and (b) reveal a sense of u R t, (c) expresses the uninterruptedness of intervals (Humberstone: 184). An additional condition for an interpretation of the formulas is the infinite divisibility of intervals. This concept was introduced to express the continuity of time (*Physics*: 218 a, 263 a 10–15).

The infinite divisibility gives us intervals that combine the features of dense and discrete structures. In interval structures infinite divisibility is equivalent to the density of momentary structures. The discreteness of the intervals consists in the absence of an interval between two given intervals. This peculiarity of the interval structures may be used in the modelling of dynamic systems, if we consider the arising and disappearing of objects as momentary events, while their coexistence is a process which lasts in time. Such systems belong to a class of continuous-discrete systems. They are characterised by discrete and continuous types of changes. Namely, these systems behave as the continuous ones in the periods, which are between the discrete events.

An impossibility of point fixation of the arising or disappearing of an object demands an alternative approach to the analysis of changing systems: the arising and the disappearing are considered as uninterrupted processes. In this case, however, a problem of setting a boundary between the transitional states and a period of stable existence, remains unresolved. N.C.A. da Costa characterises the fundamental character of such a problem in the following words: "Here a countless set ... of continuums, temporal and nontemporal, would be mentioned, which arouse the same problem, such, for example, as a drawing of the hard boundary between children and adults, between the alive and the dead, and also between the other pairs of relevant changeable qualities in everyday language and even in empirical sciences" (da Costa N.C.A., 1982: 121).

L.A. Zadeh and his followers made a considerable contribution to the solution of this problem. A formal theory of fuzzy objects was developed for the study of the situations wich are vaguely defined. The fuzzy formal theories are based on the concept of a fuzzy set. A set *M* is fuzzy, if there is at least one object wich belongs to *M* in a degree wich isn't equal to 1. An apparatus of fuzzy programming was built on the basis of the theory of fuzzy sets. The possibility appears of task solutions related to decision making under parameter vagueness (e.g., of the duration of an operation fulfilment in some modelled control systems). In this case fuzzy programming methods are used for the solution of a task to determine the relevant controlling impact.

In the situations of transition from the state of affairs S_1 to the state of affairs S_2 there is a subinterval when both the description of S_1 and the description of S_2 are true and false at the same time. The resources of classical logic are insufficient for a formal expression of the inconsistent descriptions. Now a number of logical calculi with inconsistent propositions in their languages has been built. In these calculi the inconsistency does not spread out over the whole theory, and doesn't destroy it. So the claim of consistency is relative in some sense.

A theory is simply inconsistent for an operator of negation ~ if and only if for some expression C, C and ~C are theorems of that theory. A theory is inconsistent if and only if it is inconsistent for some of its operators of negation. A theory is contradictory if and only if it is an inconsistent theory and $A \wedge B$ is a theorem when A and B are theorems too. Every contradictory theory contains theorems of the form $A \wedge {\sim}A$. A theory is trivial or absolutely inconsistent if and only if each of its formulas is a theorem of that theory (Peña: 238–239).

In logical systems of inconsistent propositions non-derivability of some formula is not an earmark of consistency. The existence of such a formula testifies only the nontriviality of the system. Nontriviality is not related here to the absence of inconsistency as in classical logic. Classical two-valued Frege-Russell logic eliminates any paradoxical expression out of its formalisms. The logic of paradoxes has a different task: to infer all possible consequences from paradoxical expressions.

Two main approaches maintain this strategy: either a localization of the usage of the law of non-contradiction or its validity preservation. Paraconsistent logic realises the first approach. It is assumed that not all paradoxes are antinomies, and that not all paradoxes destroy a formal system. These are true paradoxes. The paradoxes, which are untrue on the basis of the paraconsistent logic principles, are selfinconsistent statements. Their attendance in a formal system testifies to its triviality. The second approach is represented by fuzzy contradictory logic. It treats some paradoxes as false, and others as true and false simultaneously: "... every contradiction or antinomy... *is false*, although *some* contradictions are true, since some sentences may be *both* true and false" (Peña: 240). An antinomy in such a case is not an earmark of the triviality of a formal system.

A formalisation of inconsistent reasoning needs a modification of an operator of negation. One of the ways to do this is through a weakening of classical negation. Such a modified operator does not submit to the law of contradiction. This method is justified by the fact that the classical operator of negation is not an adequate representation of the natural language negation. However, in order to keep the logical power of a formal system it is better not to weaken it by removing or by localising the law of non-contradiction and the law of excluded middle.

A principal advantage of the inconsistent fuzzy theories in comparison with ordinary fuzzy theories is the appreciation of the logical validity of the law of non-contradiction and the law of excluded middle. Contradictory infinitely-valued fuzzy logic gives a possibility to study the situation of fuzziness without a weakening of the logic by removing those laws. They are valid for each operator of negation, which functions in such a logic.

The construction of a logic that contains true contradictions has allowed to overcome some defects of the ordinary theory of fuzzy objects. One of the reasons for the development of fuzzy contradictory formal systems was the aspiration "To conceive every *body* as the set of its parts, that is to say: as an object which is individuated merely by the degrees of membership of all its parts to it, so that certain relations' holding between its members (its parts) is not a further requirement for the body's individuation, but a mere result arising from the existence of the body itself; when a body is broken up and then rearranged, the result may be *another* body, because (some of) the parts may belong to the new object to an extent different from the one to which they belonged to the former body" (Peña: 242–243).

To study the properties of time the crucial matter is the presence of a theory of change of objects, which may be represented as a change of a degree of membership of the parts of the changing objects. It is important namely in that case where an implementation of the relations between the parts of a body is not a presupposition of its individualization. That implementation itself is determined by the existence of the body.

This argument shows the usefulness of tense paraconsistent calculi, which have interval semantics. The sphere of application of existing tense interval logic is the propositions about states and processes only. The semantics of such a logic is standard in the sense that the propositions, which are interpreted relatively to the time intervals, submit to the law of non-contradiction. Contradiction is distributed over the distinct time periods. A proposition about a state before changing is true relatively to one period and a proposition about a state after changing is true relatively to the other period. A transition itself is excluded from logical description. The problem to explicate the origin and the end of a state remains unresolved. The attempts to demarcate clearly the states express an aspiration to eliminate inconsistency. Tense paraconsistent logic, on the contrary, allows to express not only a succession of states, but a tie between them as well.

There is a peculiarity in of paraconsistent logic concerning the analysis of propositions about transition states. The expression "changes value" is not interpreted as an indication of the absence of a value. It is assumed that change occupies some time period. The propositions are valued relatively to its subperiods. They are ascribed a degree of truthfulness in the interval [0,1]. It agrees with the fact that a changing object both retains, to a certain degree, the features of its previous state and acquires new ones, passing through a number of states or phases.

Later I will discuss a description of change in the framework of one branch of temporal logic, namely tense interval logic. The goal is to present an approach to the construction of tense paraconsistent logic in addition to the proposals of N.C.A. da Costa and S. French and G. Priest. In accordance with Hamblin's intention, I consider change as the idea of occupying an interval of time that has fuzzy boundaries. The description of change consists in the description of the states overlapping the interval of change. As a result such description contains inconsistent statements.

In order to give a formal representation of such description I offer to consider a formula to be true relatively to an interval, if and only if its negation is false over some subinterval. And if a negation of a formula is true over an interval, then the formula without negation must be false over some subinterval. The conjunction of two formulas is true over an interval if and only if there is some subinterval and these formulas are true over all subintervals of that subinterval. The reference to all subintervals is stipulated by the above intuition about truth of a formula over an interval. Without this restriction the formulas considered as connected, may be true over different subintervals of the evaluation interval.

These intuitions are embodied in the semantics of interval paraconsistent logic. It is defined relative to a model I of the form $\langle T, \subseteq, |\,| \rangle$, where T is a class of time intervals; $\subseteq$ is mereological subinterval relation; $|\,|: P \times T \rightarrow \{0,1\}$ is a value function for propositional variables. The function $|\,|$ is extended inductively to supply values for the full set of the formulas of the language of interval paraconsistent logic.

Definition 1.
The truth conditions for the formulas of the language of interval paraconsistent logic are:

T1. $|{\sim}A|t = 1$ iff $\exists t'(t' \subseteq t$ and $|A|t' = 0)$;
T2. $|A \wedge B|t = 1$ iff $\exists t'(t' \subseteq t$ and $\forall t''($if $t'' \subseteq t'$, then $|A|t'' = 1$ and $|B|t'' = 1))$;
T3. $|A \vee B|t = 1$ iff $\exists t'(t' \subseteq t$ and $\forall t''($if $t'' \subseteq t'$, then $|A|t'' = 1$ or $|B|t'' = 1))$;
T4. $|A \supset B|t = 1$ iff $\exists t'(t' \subseteq t$ and $\forall t''($if $t'' \subseteq t'$, then $|A|t'' = 0$ or $|B|t'' = 1))$;
F1. $|{\sim}A|t = 0$ iff $\forall t'($if $t' \subseteq t$, then $|A|t' = 1)$;
F2. $|A \wedge B|t = 0$ iff $\forall t'($if $t' \subseteq t$, then $\exists t''(t'' \subseteq t'$ and $|A|t'' = 0$ or $|B|t'' = 0))$;
F3. $|A \vee B|t = 0$ iff $\forall t'($if $t' \subseteq t$, then $\exists t''(t'' \subseteq t'$ and $|A|t'' = 0$ and $|B|t'' = 0))$;
F4. $|A \supset B|t = 0$ iff $\forall t'($if $t' \subseteq t$, then $\exists t''(t'' \subseteq t'$ and $|A|t'' = 1$ and $|B|t'' = 0))$.

Validity is defined in the following manner:

Definition 2.
A formula A of the language of interval paraconsistent logic is valid iff for any model I $|A|t = 1$ for every $t \in T$.

By addition of the earlier-later relation R the model I is turned into a tense interval paraconsistent model TI: $\langle T, \subseteq, R, |\ |\rangle$. The truth conditions for the formulas of tense interval paraconsistent logic are T1-F4 supplemented by the ordinary conditions for the formulas with tense operators "it will always be the case, that ..." (G), "it has always been the case, that ..." (H), "it will be the case, that ..." (F) and "it has been the case, that ..." (P).

T5. $|GA|t = 1$ iff $\forall t' \in T$ (if $t\,R\,t'$, then $|A|t' = 1$);
T6. $|HA|t = 1$ iff $\forall t' \in T$ (if $t'\,R\,t$, then $|A|t' = 1$);
T7. $|FA|t = 1$ iff $\exists t' \in T$ ($t\,R\,t'$ and $|A|t' = 1$);
T8. $|PA|t = 1$ iff $\exists t' \in T$ ($t'\,R\,t$ and $|A|t' = 1$);
F5. $|GA|t = 0$ iff $\exists t' \in T$ ($t\,R\,t'$ and $|A|t' = 0$);
F6. $|HA|t = 0$ iff $\exists t' \in T$ ($t'\,R\,t$ and $|A|t' = 0$);
F7. $|FA|t = 0$ iff $\forall t' \in T$ (if $t\,R\,t'$, then $|A|t' = 0$);
F8. $|PA|t = 0$ iff $\forall t' \in T$ (if $t'\,R\,t$, then $|A|t' = 0$);

and by the special conditions for the operators "it always be the case, that ..." (L), "it sometimes be the case, that ..." (M).

T9. $|LA|t = 1$ iff $\forall t'$(if $t' \subseteq t$, then $|A|t' = 1$);
T10. $|MA|t = 1$ iff $\exists t'(t' \subseteq t$ and $|A|t' = 1$);
F9. $|LA|t = 0$ iff $\exists t'(t' \subseteq t$ and $|A|t' = 0$);
F10. $|MA|t = 0$ iff $\forall t'$(if $t' \subseteq t$, then $|{\sim}A|t' = 1$).

Then $\neg A = L \sim A$, where "$\neg$" is Humberstone's strict interval negation.

We can use this semantics for the study of many-valued logic. D. Bochvar has offered a calculus with internal and external connectives. The truth matrices for some of them are:

I

A	$\neg$A	+A
0	1	0
1	0	1
*	*	0

II

A $\cap$ B:

A\B	0	1	*
0	0	0	*
1	0	1	*
*	*	*	*

where $\neg$A is intrinsic negation of A, +A is extrinsic assertion of A ("A is true"), $\cap$ is an intrinsic conjunction, * is the value "meaningless". The connectives $\neg$, + and $\cap$ make a functionally complete system of connectives (Finn).

Theorem.
Bochvar's matrices are recoverable by the truth conditions of interval paraconsistent semantics.

Proof. The matrix I is represented by conditions:

1. $|A|t = 1$ iff $\forall t'$(if $t' \subseteq t$, then $|\neg A|t' = 0$);
2. $|\neg A|t = 1$ iff $\forall t'$(if $t' \subseteq t$, then $|A|t' = 0$);
3. $|A|t = 0$ iff $\forall t'$(if $t' \subseteq t$, then $|\neg A|t' = 1$);
4. $|\neg A|t = 0$ iff $\forall t'$(if $t' \subseteq t$, then $|A|t' = 1$);

5. $|A|t = *$ iff $\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg A|t' = 1 \text{ and } |A|t' = 1)$;
6. $|\neg A|t = *$ iff $\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg A|t' = 1 \text{ and } |A|t' = 1)$;
7. $|+A|t = 1$ iff $\forall t'(\text{if } t' \subseteq t, \text{ then } |A|t' = 1)$;
8. $|+A|t = 0$ iff $\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg A|t' = 1)$ or
$\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg A|t' = 1 \text{ and } |A|t' = 1)$.

The matrix II is represented by conditions:

9. $|A \cap B|t = 1$ iff $|A|t = 1$ and $|B|t = 1$;
10. $|A \cap B|t = 0$ iff $|A|t = 0$ or $|A|t = 0$;
11. $|A \cap B|t = *$ iff $\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg A|t' = 1 \text{ and } |A|t' = 1)$ or
$\forall t'(\text{if } t' \subseteq t, \text{ then } |\neg B|t' = 1 \text{ and } |B|t' = 1)$.

Thus the interval interpretation of Bochvar's calculus is derived.

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