FIVE PHILOSOPHICAL PROBLEMS RELATED TO PARACONSISTENT LOGIC

Andrés BOBENRIETH M.[*]

Abstract

In this text I propose that, in order to study the relationship between philosophy and paraconsistent logic, rather than extracting philosophical extrapolations out of formal results, it is better to inquire what philosophical problems (with a well founded tradition) are affected by the emergence and development of this logical option.

Therefore, I will address important philosophical problems that are dramatically affected by the development of paraconsistent logic: (1) the effects of a contradiction in a deductive system; (2) what can be understood by negation and contradiction; (3) the formalisation of dialectics, (4) the notion of rationality, and (5) the referents of contradictions.

Then, I will go deeper into the last problem and as a conclusion, I will suggest that the main question within paraconsistency, rather than turning around the existence or non-existence of real and/or true contradictions (which has lead to conflicting positions among authors dealing with paraconsistent logic), is the possibility or impossibility of avoiding the repeated emergence of contradictions or inconsistencies within our cognitive systems. Rather than advocating a "paraconsistent world view", I will try to show that it is our world view which inevitably generates inconsistencies, and that paraconsistency is a way of accommodating this reality inherent to human knowledge.

1. Introduction

This paper will treat the relationship between philosophy and paraconsistent logic. First, three possible interpretations of this relationship will be

stated; and then, upon selecting the third, some philosophical problems that are deeply affected by the development of paraconsistent logic will be outlined. Finally, the problem of the existence of real and/or true contradictions, which, in a sense, has given rise to the different paraconsistent approaches will be considered, in order to show that this discussion has led to several misunderstandings that can be elucidated through a proper conceptual analysis[1].

2.   *Philosophy and paraconsistent logic*

In principle, to talk about the relationship between philosophy and paraconsistent logic can mean three different things: First, aspects of the philosophy of logic, such as the relationship between paraconsistent logic and classical logic, or the relationship among several non-classical logics. Second, it can refer to the philosophical views of the authors that created and/or developed paraconsistent logic systems, which, in some way, have affected their motivation, formulation and evolution. Finally, it can mean problems that have been discussed throughout philosophy and that are deeply affected by the emergence and evolution of paraconsistent logic. Although the importance of the first two can not be ignored, this paper will focus on the third.

Many reasons recommend this approach. The most important are that on the philosophy of logic much has been written already, so we will concentrate on the less studied aspects. Regarding the second topic, no major study has been undertaken, and explicit statements by paraconsistent logic authors about the philosophical foundations of the different paraconsistent logic systems are scarce. But paraconsistent logic is clearly a logical-formal development and, as such, it is not necessarily committed to a philosophical conception, beyond a certain agreement on the feasibility and usefulness of developing deductive systems that accept some types of inconsistencies without allowing their trivialization. In this sense, paraconsistent

---

[1]Many aspects will only be summarized in this paper. A much deeper development can be found in the book Bobenrieth, Andrés: *Inconsistencias ¿por qué no? Un estudio filosófico sobre la lógica paraconsistente* [Inconsistencies, Why Not? A Philosophical Study on Paraconsistent Logic] (Bogotá, Colcultura, 1996); especially in Chapter XII and in the Final Considerations.

logic is "philosophically neutral", as da Costa, Bueno and French, have said recently[2].

Then we will deal with some problems that in some way have been treated in the history of philosophy and that are directly affected by the development of paraconsistent logic. In this field, there is much to be examined thoroughly, provided that the implications of paraconsistent logic are acknowledged. Choosing this kind of question clearly agrees with what da Costa has stated in the sense that the relationship between philosophy and paraconsistent logic is very important, yet it is an indirect relationship[3].

Among the different problems that in this sense are relevant, there are five that can be considered as the most important: 1) the effects of including a contradiction in a deductive system; 2) the meaning of negation and contradictions and/or inconsistencies; 3) the formalisation of dialectic; 4) the notion of rationality; 5) the existence or non-existence of real and/or true contradictions. In what follows the main implications of the evolution of paraconsistent logic regarding these problems will be outlined.

3. *Contradictions in a deductive system*

By radically relativising the principle of Pseudo-Scotus, paraconsistent logic has weakened arguments that claimed, based on logical assumptions, that deductive systems could not embrace contradictions in any form. Presently, the question is no longer whether this can or cannot be done; paraconsistent logic has shown that it can. The question now is whether it is useful or adequate to accept some form of contradiction in a deductive system. So, reasons of another type must now be taken into consideration.

Furthermore, paraconsistent logic has clarified that there are different types of trivialization, depending on the kind of expression that generates it, and especially on the operator that originates it. Also we must distinguish between the fact that most logical systems are finitely trivializable (such as the classic and most paraconsistent and relevant systems) and that

---

[2] da Costa, Newton C.A. / Bueno, Otávio A.S.: "Paraconsistency: towards a tentative interpretation", forthcoming; da Costa, Newton C.A. / Bueno, Otávio A.S. / French, S.: "The logic of pragmatic truth", *Journal of Philosophical Logic* 27 (1998) pp. 603–620.

[3] *Cf.* da Costa, Newton C.A.: "The Philosophical Import of Paraconsistent Logic", *The Journal of Non-Classical Logic* vol. 1, no. 1 (1982) p. 7.

some are only infinitely trivializable (such as da Costa's C_ω system). Major authors, like Popper, were very confused about this topic[4].

In general, the development of paraconsistent logic has made evident the limited value of arguing about such "logical impossibilities" in philosophical discussions.

4.   *The meaning of negations and contradictions*

Considering the second problem, paraconsistent logic has shown that it is possible to define several negations and that they would not be just formal devices, but that they affect what can be understood by a contradiction and/or an inconsistency. The classical conception of contradiction mixes two meanings that must be distinguished. One refers to the mainly syntactic situation in which a statement is the negation of another one. Another refers to the semantic situation in which two statements cannot be both true at the same time, and neither can they both be false. In fact, "contradictions" in most paraconsistent systems occur when asserting simultaneously a statement and its negation, yet assuming that both are true in the system (Lorenzo Peña's systems are a exception[5]). So when referring to contradictions (or inconsistencies) it is necessary to clarify their senses and which is the negation operator involved. Therefore, we have seen that when the classical perspective rejected "contradictions" many different things were rejected, and now it is clear that some can be accepted and others excluded.

---

[4] *Cf.* Bobenrieth, A. *Op. cit.* Cap. VI and Cap. XII (sec. 3).

[5] The exception lies in the fact that Lorenzo Peña defends a "gradualist" position about truth; thus, in his systems it is possible to assert some statements that are true in some degree and also false in some other degree. *Cf.* Peña, Lorenzo: *Rudimentos de Lógica Matemática* (Madrid: Consejo Superior de Investigaciones Científicas, 1991) p. 259, p. 279; *Introducción a las Lógicas No Clásicas* (México: Universidad Nacional Autónoma de México, 1993) p. 90. Also if a paraconsistent system (viz. Priest's systems) uses a semantics based on three values: $\{t\}$, $\{t,f\}$, $\{f\}$, then there will be sentences that are true, false, and both true and false (*cf.* Batens, D. "In defence of a programme for handling inconsistencies", forthcoming). Anyhow in these type of systems it can be said that their theorems "are in some degree true" (for Peña's systems) or "at least true" (for Priest's systems). This three valued semantics can easily be translated to the usual semantics for paraconsistent systems where both parts of the contradiction can be considered as true (and only true). (I owe these last clarifications to Prof. D. Batens).

5. *The formalisation of dialectic*

The formalisation of dialectic was attempted by several authors between 1975 and 1985[6], thinking that paraconsistent logic would bring together dialectic and formal logic. Several paraconsistent systems, known as "dialectic logic", were constructed[7]. Enthusiasm declined however when it was seen that such systems of logic failed to capture several fundamental elements of dialectic. Anyhow, the philosophical relevance of paraconsistent logic began to be clarified by these efforts, as the relation between logic and dialectic has been a widely discussed problem in the history of philosophy.

The main problem in this approach, I think, was that it did not endorse that dialectic is not, and cannot be, formal, at least in a Hegelian view. Asking a system of formal logic to capture something of the core of dialectic would be like asking it to stop being formal. This is so despite the misunderstanding created by Engels' "dialectic laws". Confrontations or oppositions in dialectic only exist if they are determined by the content of what is being confronted, in constant becoming, and not by an external form of such movement that could be isolated from it. Therefore, paraconsistent logic can provide some support for the dialectic proposal, insofar as it overcomes the classical rejection of all contradiction, but it should be a two-way relationship, as von Wright has suggested[8]. Thus, there should not only exist a logical space in which "dialectic regularities" could be included, but it would be necessary to establish some postulates that can capture certain characteristic aspects of the dialectic movement. However, it is not possible to include these postulates in a formal system while preserving its formal character. It would be necessary to build a deductive system

---

[6] A typical example of this enthusiasm was the publication of the book Marconi, Diego (ed.): *La Formalizzazione della Dialettica* (Torino: Rosenberg & Sellier, 1979).

[7] *Cf.* Routley, Richard / Meyer, Robert: "Dialectical Logic, Classical Logic and the Consistence of the World", *Studies in Soviet Thought* 16 (1976) pp. 1–25; Routley, Richard: "Dialectical Logic, Semantics and Metamathematics", *Erkenntnis* 14 (1979) pp. 301–331; da Costa, Newton C.A. / Wolf, Robert G.: "Studies in Paraconsistent Logic I: The Dialectical Principle of the Unity of Opposites", *Philosophia (Philosophical Quarterly of Israel)* vol. 9 no. 2 (1980) pp. 189–217; da Costa, Newton C.A. / Wolf, Robert G.: "Studies in Paraconsistent Logic II: Quantifiers and The Unity of Opposites", *Revista Colombiana de Matemáticas* vol. 19 (1985) pp. 56–67; Alves, Elias H.: "On the decidability of a System of Dialectical Propositional Logic", *Bulletin of the Section of Logic, Polish Academy of Sciences* vol. 7 (1978) pp. 17–32.

[8] *Cf.* von Wright, G.H.: "Truth, Negation and Contradiction", *Synthèse* 66 (1986) p. 5.

using as underlying logic some system of formal logic, which will be para-consistent without any doubt, but it would also require some extralogical postulates that could capture in some way the dialectic movement. This situation is analogous to that in which one intends to built an axiomatic structure for a scientific theory: one thing is the underlying logic, and an-other one is the set of extralogical postulates that belongs to the theory itself, which must have a specified content. The deductive system is a prod-uct of the joint articulation of both. Summing up, the development of para-consistent logic has shown that those assertions of logical impossibilities against dialectic have no basis; but this is not enough to "formalise the dialectic", because it would be necessary to develop, not a system of "dia-lectic logic", but a deductive theory for dialectic that will have as under-lying logic an adequate paraconsistent system. But this task, as far as I know, is still to be accomplished.

6. *The notion of rationality*

The fourth problem concerns how the development of paraconsistent logic affects the notion of rationality. Clearly the assumption that total consis-tency must be always required in any rational structure, becomes question-able as long as sound logical systems can be developed so that they can appropriately handle certain inconsistencies. This has been stated by sever-al authors, especially da Costa[9], Miró Quesada[10], Priest / Routley[11] and Rescher[12]. This reintroduces the topic of what would be the minimal re-quirements that allow us to consider a cognitive system rational. In this sense, the principles of reason, stated by da Costa[13], as well as Rescher's

---

[9] *Cf.* da Costa, Newton C.A.: *Ensaio sobre os Fundamentos da Lógica* (São Paulo: HUCITEC y Editora da Universidade de São Paulo, 1980) pp. 42ff.

[10]*Cf.* Miró Quesada, Francisco: "Paraconsistent Logic: Some Philosophical Issues", in Priest / Routley / Norman (eds.): *Paraconsistent Logic, Essays on the Inconsistent* (München, Hamden, Wien: Philosophia Verlag, 1989) pp. 627–652.

[11] Cf. Priest, Graham / Routley, Richard: "Applications of Paraconsistent Logic", in Priest / Routley / Norman (eds.): *Paraconsistent Logic, Essays on the Inconsistent* (München, Hamden, Wien: Philosophia Verlag, 1989) pp. 377ff.

[12]*Cf.* Rescher, Nicholas: *Rationality. A Philosophical Inquiry into the Nature and the Rationale of Reason* (Oxford: Clarendon Press, 1988) chap. 5.

[13] *Cf.* da Costa, Newton C.A.: *Ensaio sobre os Fundamentos da Lógica* (São Paulo: HUCITEC / Editora da Universidade de São Paulo, 1980) pp. 42ff.

reflection on rationality[14], are specially interesting. We are, then, faced with a profound field for philosophical reflection, as long as we can search for what Miró Quesada has described as "to recover the vision of the whole towards which all authentic philosophy aims"[15].

No unique vision on rationality exists, or has to exist, supporting or related to paraconsistent logic. However, reflection on rationality has to be affected in some way by what has been achieved in paraconsistency. Yet, consistency can no longer be considered a necessary requirement of rationality. We have given it too much importance. Nevertheless, paraconsistent logic does not eliminate the requirement of consistency, but locates it and qualifies it, making it necessary to weigh consistency together with other requirements of rationality.

7. *The substratum of contradictions*

This paper concludes by pointing out a further problem but closely related with the others, especially with the second and the fourth. Since the mid-eighties, it has become increasingly evident that the different researchers in paraconsistency have very diverse approaches towards the existence of real and/or true contradictions. This has led to different paraconsistent positions ranging from what could be called a "weak paraconsistent" position, such as Batens'[16], to a much more radical position, held by Priest and Routley, which they have called a "dialethic" position[17], with an intermediate position sustained by most authors such as Newton da Costa in several writings[18]. In fact the latter, together with Bueno and Béziau, has recently restated this position postulating what is called an "agnosticism" concern-

[14]*Cf.* Rescher, Nicholas: *Rationality. A Philosophical Inquiry into the Nature and the Rationale of Reason* (Oxford: Clarendon Press, 1988) chap. 5.

[15]*Cf.* Miró Quesada, Francisco: "Paraconsistent Logic: Some Philosophical Issues", in Priest / Routley / Norman (eds.): *Paraconsistent Logic, Essays on the Inconsistent* (München, Hamden, Wien: Philosophia Verlag, 1989) p. 646.

[16]*Cf.* Batens, Diderik: "Paraconsistent Extensional Propositional Logics", *Logique et Analyse* vol. 90–91 (1980) pp. 195–234 (pp. 196ff.); "Against Global Paraconsistency", *Studies in Soviet Thought* vol. 39 (1990) pp. 209–229.

[17]Cf. Priest / Routley / Norman (eds.): *Paraconsistent Logic, Essays on the Inconsistent* (München, Hamden, Wien: Philosophia Verlag, 1989) p. xx.

[18]*Cf.* da Costa, Newton C.A.: *Ensaio sobre os Fundamentos da Lógica* (São Paulo: HUCITEC y Editora da Universidade de São Paulo, 1980) pp. 205ff.

ing the existence of real contradictions, while not preventing the "existence" of "true" contradictions within formal systems[19].

This controversy is, in my opinion, misleading in some senses. Two assertions made by authors related to paraconsistent logic help to clear the whole problem. The first, made, separately, by Lukasiewicz[20] and Vasiliev[21], states that there is no such thing as a "negative perception"; a position also held, from different perspectives, by authors such as Piaget[22]. There seems to be nothing in reality such as the referent of the cognitive operation of negating; what we have is a series of perceptions, that could be call "positive" ones, and within the development of our cognitive structures we state that some are incompatible with some other, and thus negation emerges as the most direct form of expressing this proposed incompatibility. Therefore, it does not seem appropriate to speak of "perceiving" a contradiction, since in the mainly syntactic sense of contradiction (i.e. an assertion and its negation) it would imply "perceiving" a negation. Moreover, in the semantic sense (i.e. two sentences for which it is taken for granted that they can not be both true and neither both be false), it would imply simultaneously perceiving, about the same thing, two determinations which were previously assumed as incompatible, and in this case we would not be "perceiving" a contradiction, but we would only be realising that the alleged incompatibility was wrong. Wondering whether there are "real contradictions", or not, would be pointless unless first it is clarified what is meant by this term.

It is important to recall another assertion here. Carnielli and Lima Marques start their presentation of some paraconsistent systems based upon the notion of truth by default, stating that, for their purposes, it will be assumed that "contradictions are linguistic facts"[23]. Although this aspect is not further developed, it points to a necessary clarification. What we under-

---

[19] Cf. da Costa, Newton C.A. / Bueno, Otávio A.S.: "Consistency, Paraconsistency and Truth (Logic, the Whole Logic and Nothing but the Logic)", Ideas y Valores 100 (1996) pp. 48–60.

[20] Cf. Lukasiewicz, Jan: "On the principle of contradiction in Aristotle", Review of Metaphysics XXIV (1971) pp. 507f.

[21] Cf. Arruda, Aida I.: N.A. Vasiliev e a Lógica Paraconsistente (Campinas: Centro de Lógica, Epistemologia e História da Ciencia-UNICAMP, 1990) p. 44.

[22] Cf. Piaget, Jean: Investigaciones sobre la contradicción (Madrid: Siglo Veintiuno, 1978) pp. 1f.

[23] Carnielli, Walter / Lima Marques, Mamede: "Razão e irracionalidade na representação do conhecimiento", Trans/Form/Ação vol. 14 (1991) p. 169.

stand by contradictions, or by inconsistencies, emerges only as a result of applying our conceptual structures to reality. Getting back to the Latin etymology[24], there is not much sense in saying that facts "contradict" each other (in Latin *contradicere*, where *dicere* means to tell, to speak, to say), since facts do not "say" anything, they do not "speak" of each other or of themselves. Incompatibilities emerge within our discourse about reality. A contradiction emerges whenever two assertions considered incompatible are simultaneously stated about a unique entity and in the same sense. Our knowledge is categorical and it is by virtue of our categories that contradictions occur. Therefore, the problem of contradictions does not lie in reality. It arises as a result of our use of categorical schemes, and it is precisely there where its huge significance lies.

On the other hand, to talk of "true contradictions" would require a clarification of which sense we are using the adjective *true*, since there does not seem to be such a thing as a "real referent" for those contradictions. Alternatively, it could be asserted that our categorical schemes are, in some way, unequivocally determined; a thesis that would be assertable both from a "radical realism" and, paradoxically, from a "radical idealism". However, both positions would have to explain why we, human beings, have been able to use, and in fact do use, very different categorical schemes to explain the same phenomena. And why, in so many cases, they do turn out to have equivalent explanatory power.

Another sense in which talk of "true contradictions" could be defended, would be to assert that there are certain contradictions that have no solution. Regarding this, it would be necessary to point out that the history of Western knowledge contains many examples of contradictions that seemed unsolvable, but later turned out to be "solved" under a different explanatory scheme. In this sense, it is not enough to argue that a given contradiction has not been solved yet, since there is nothing preventing it being "solved" in the future, within some other categorical scheme. From this perspective, it seems clear that no categorical scheme can, in any way, exhaust reality, since it is always possible to build a finer scheme able to make distinctions not made by the previous one. As if we were throwing different nets over reality, there is no warrant that any particular net could appropriately capture and classify all the phenomena to be explained.

However, to underestimate the significance of the problem of contradictions would be a serious mistake. If we assume that contradictions emerge because our knowledge inevitably uses categorical schemes that establish distinctions among phenomena, and that these distinctions are a necessary requirement for the emergence of contradictory predicates, then it becomes

---

[24] This is also clear in the German terms "Widerspruch" and "sprechen".

evident that the problem of contradictions is fundamentally linked with the comprehension of human knowledge.

Contradictions seem to emerge mainly in the so called "limit cases", and when such a situation occurs, that does not mean that this limit case is *inherently* contradictory, but it turns out to be so due to the categorical scheme that has been applied. It is always possible, in principle, to build a new categorical scheme in which some "limit cases" cease to have that characteristic, although this does not prevent other "limit cases" from emerging in the new scheme. One of the greatest contributions of paraconsistent logic is to make it possible to handle, in an appropriate way, the categorical schemes in which some "limit cases" have emerged, without being forced in any way to try to substitute them. Therefore, there is no longer such a thing as a "logical imperative" leading us to keep searching for a scheme able to "solve" in all cases the contradictions that have emerged. So doing will now depend on several considerations of different kinds, mainly pragmatic.

In short, in principle, *any* contradiction is solvable, but this in no way solves the problem of contradictions, since it seems impossible to solve *all* contradictions. Indeed, every scheme that solves some contradictions brings in new ones. This has been so throughout the history of Western knowledge, but we can now realise that, although no particular contradiction has to be always with us, diverse and changing contradictions will indeed be always with us. Our notion of rationality has attempted to reject contradictions, but, if we accept that they emerge precisely because we use rational schemes, then we will be able to acknowledge that contradictions come along with rationality. Paraconsistent logic has made it possible to take, or even retake, this path.

Instituto de Estudios Humanísticos
Universidad de Valparaíso (Chile)
abobenri@uv.cl

Department of Philosophy
University of Leeds (UK)
phlab@leeds.ac.uk