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POSSIBLE WORLDS WITH IMPOSSIBLE OBJECTS: THE IMAGINARY LOGIC OF N.A. VASIL'ÉV

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1. *Introduction*

The Russian logician Nikolaj Aleksandrovich Vasil'év (1880–1940) was one of the first logicians to introduce what he called an "*imaginary* (i.e. non-Aristotelian) *logic*" which constitutes an alternative to the classical one as it keeps the classical inferential structures but does not hold the principles of contradiction and of the excluded middle. He is able to do this on the basis of two conditions, one of which is ontological, the other logical: (*i*) the assumption of another world, *an imaginary world*, which is different from our real world as in it the negations, like the positive facts, are the objects of sensation, and in which other logical systems and logical operations are valid which are different from those of traditional logic; (*ii*) the interpretation of particular propositions in terms of modality, as well as the substantial independence of the proposition's and of the syllogism's structure from the principle of contradiction. The language used by Vasil'év is that of traditional formal logic. However, the contents and the results of his investigations —which are in a certain sense parallel to those conducted by Łukasiewicz and Peirce[1]— allow us to put him with full right among those authors who opened the way for the development of logical systems in which the principles of contradiction and/or of excluded middle do not hold, or have a limited application.

Though it is true that soon after their publication, Vasil'év's works aroused some interest in Russia[2], substantially his ideas remained largely

[1] Cf. RASPA (1989).

[2] Cf. SMIRNOV (1989:625).

unknown till the 50s in Russia and the 60s in the Western countries[3]. Since then, several readings of his work have been given. As this work does not constitute a complete system but a series of intuitions and proposals, which are sometimes only outlined and not further developed, several interpretations of it have been given, which often mutually disagree. Most of the work on Vasil'év has tried to show that he was a forerunner of modern non-classical logics. So after G.L. Kline and N. Rescher, followed by M. Jammer, who saw in Vasil'év one of the founders of many-valued logics[4], others interpreted him as a forerunner of paraconsistent logics[5], gave paraconsistent logical systems which tried to capture some important intuitions of Vasil'év's[6], or considered him as a founder of intensional logics[7]. In Russia, some gave a reconstruction of Vasil'év's system in terms of classical syllogistics, and interpreted his metalogic according to classical propositional calculus[8], while others enthusiastically considered him not only as a founder of many-valued logics and paraconsistent ones, but also as a forerunner of intuitionistic logic[9].

In our opinion, the controversy whether Vasil'év was a forerunner of many-valued logics or of the paraconsistent ones is not very important. The intuitions of an author may be developed in several directions, and the search for the forerunners began after the logics mentioned reached a certain level of development. Furthermore, there is in Vasil'év neither the idea of a third value besides true and false, nor is there the idea that an affirmation and its negation can both be true. Instead, what may be true —as we will see— is that for Vasil'év one and the same judgment states that a certain S possesses at the same time both the property P and the property non-P. Therefore, we will not deal with the place of Vasil'év in the field of the

[3]On the situation of Vasil'év studies in Russia as well as in the Western countries, cf. CAVALIERE (1991:66ff). We have extensively relied on the Russian edition of Vasil'év's work (Vasil'év 1989) since only small parts of it have been translated into other languages. However, many of Vasil'év's intuitions have influenced contemporary proponents of non-classical logic.

[4]Cf. KLINE (1965), and RESCHER (1969:4–7,14); cf. also JAMMER (1974:342–343).

[5]Cf. ARRUDA (1977:4,21; 1980:7; 1989:102).

[6]Cf. ARRUDA (1977:4,21; 1980:7; 1989:102); PUGA and da COSTA (1988).

[7]Cf. PRIEST and ROUTLEY (1989:29).

[8]Cf. SMIRNOV (1986:207; 1989); cf. also CAVALIERE (1991:68–69).

[9]Cf. BAZHANOV (1990:333; 1992:46), VERGAUWEN and SOLDATJENKOVA (forthcoming)

development of non-classical logics in the 20th century, but we will try instead to point out especially the relationship between Vasil'év's investigations and the theories of certain logicians and philosophers, who discussed already some years before him certain questions which were to be decisive in the birth of his own logic.

In Vasil'év's logical reflections different components come together which were already present in earlier logical and philosophical investigations, and which he arranges in a new and original manner, all of which lead to the postulation of an imaginary (non-Aristotelian) logic. First of all, (*a*) the discovery, in the first half of the 19th century, of non-Euclidean geometry has had a remarkable heuristic impulse on Vasil'év, as well as on many others contemporary authors. In this respect it should be noted that both Lobachevsky and Vasil'év were teaching at Kazan' University and Vasil'év explicitely admits having been deeply influenced by Lobachevsky's work on geometry. Secondly, (*b*) Vasil'év adopts, although he knew the recent developments of symbolic logic, the language of traditional formal logic, especially that of Ch. Sigwart's psychologistic logic, which he uses in an original manner.

2. *Sigwart's theory of judgments*

Vasil'év's analysis, intends to show the possibility and the rationality of a logic without the laws of contradiction and of excluded middle, starting from an analysis of the structure of the proposition —especially of particular propositions— and of the nature of its subject. Since Sigwart's logic, especially his theory of judgment and his criticism of particular judgments, plays a very important role in the rise of imaginary logic, it is worth analyzing what Sigwart says on both universal and particular judgments. According to Sigwart, the judgment is an active movement of thought that consists in stating something of something else. This presupposes two different *ideas [Vorstellunge]* which are present to consciousness, that of a subject and that of a predicate; the idea as the simple ideal form of the objects is the fundamental element of the judgment. Therefore, the real individual objects or their attributes, activities, and relations are, as they are ideas present in consciousness, the basis of the judgment, the fundamental form of which is that of the categorical judgment, or *simple judgment [einfaches Urteil]*, i.e., the judgment as «a synthesis of a subject and predicate accompanied by the consciousness of objective validity»[10], where the sub-

[10]SIGWART (1873–1878/1911[4]:§ 38, p. 316 [1895:233]); cf. also § 5, pp. 29ff [1895:25ff].

ject is intended as singular. Simple judgments are subdivided into two classes: the class of the *narrative judgments [erzählende Urteile]*, in which the subject is the idea of something which exists, and that of the *explicative judgments [erklärende Urteile] —analytical* in Kantian meaning—, in which the subject idea is the general meaning of a word and in which the judgment is not about any definite particular thing[11]. Since the function of a judgment is always the same, viz., that which is expressed by the categorical judgment, the differences among judgments depend only upon differences among their subjects and predicates, and not upon different functions of the judgment. Particularly, they depend upon whether the categorical synthesis is simple or complex, or upon whether the subject is a simple idea or a synthesis or combination of ideas[12]. Let us analyse this latter point, which concerns the distinction between singular and universal (or plural) judgment, which is that we are interested in here.

According to Sigwart *plural judgments [plurale Urteile]* are those in which a predicate is stated of a plurality of subjects such as e.g. copulative judgments as "*A* and *B* and *C* are *P*" and the plural judgments in the narrower sense, which are different from the former as in it *A*, *B*, and *C* fall under the same denomination *N*, so that we have the form "several *N*'s are *P*". The latter consist in «the intuition of a plurality of like or similar things which are named by the same word»[13], the quantity of which may be definite or indefinite. To this point, Sigwart distinguishes three different cases: (*a*) when the subject is taken in a distributive form, that is, when the predicate refers to each individual thing (e.g., "some stars are visible"); (*b*) when the subject is taken as a plurality (that is, collectively) and the predicate is stated of this plurality (e.g., "the trees grow thickly together", "numberless birds fill the wood with their song"), in which case the synthesis is of course simple; finally (*c*) when the real predicate is the numerical term (the quantifier), as in the judgment "many men are short-sighted", which does not refer to particular or definite individuals, but states only that the short-sighted are many[14].

The main thesis of Sigwart about plural judgments is that these are essentially simple judgments. Therefore the traditional distinction between universal and particular judgments has no citizenship right in logic, since it is possible to show that they both reduce in different way to simple judg-

[11]Cf. SIGWART (1873–1878/1911[4]:68ff [1895:53ff], 118ff. [1895:90ff].

[12]Cf. SIGWART (1873–1878/1911[4]:§ 38, pp. 315–316 [1895:232].

[13]SIGWART (1873–1878/1911[4]:§ 26, p. 217 [1895:158]).

[14]Cf. SIGWART (1873–1878/1911[4]:§ 26, pp. 215–219 [1895:157–160]).

ments: they are either a simple synthesis or a series of simple judgments about definite individual things. According to Sigwart, logic is a technical discipline of Thought whose epistemological purpose is to know that which is, and to formulate true and certain propositions, which are necessary and universally valid; it is from the Standpoint of the 'certainty of the judgment in the consciousness of the subject which judges' that Sigwart analyses and criticizes the traditional classification of judgments in universal and particular ones.

Let us consider the universal judgments, that is, those of the form "All *A*'s are *B*". According to the above-mentioned distinction between narrative and explicative judgments, Sigwart distinguishes between (*a*) *empirically universal judgments [empirisch allgemeine Urteile]* and (*b*) *unconditionally universal judgments [unbedingt allgemeine Urteile]*. The latter he further subdivides into (*b*.1) *explicative* (or *analytical*) ones and (*b*.2) *synthetical* ones. In (a) «the "all" by which the subject of the so-called universal judgments (...) is bound together, signifies, according to its original meaning, a definite number, and a judgment with "all" presupposes a limited number of particular objects which can be counted»[15]: in other words, it expresses the equality of two numbers, that is, of the number of the *A*'s and that of the *A*'s which are *B*; in this case, «"all" denies the exception».[16] The judgment "All *A*'s are *B*" refers to individual things and "all" occurs in it as a predicate: "those *A*'s which are *B* are all *A*'s", that is to say, there are many *A*'s, there are only *A*'s which are *B*, and there are no cases of *A*'s which are not *B*. Here it is clear —Sigwart argues— that the universal judgment originally refers to particular things taken together as an empirical generality which can be counted, and further that it presupposes the certainty of the subject about each judgment on each particular thing before one can state the same judgment of *all* subjects[17]. In (b), which concerns universal judgments which are unconditionally universal, we can have two distinct situations, according to the fact whether the judgment in question is analytical or synthetical. In (*b*.1), if we have to do with explicative (or analytical) judgments, then "all" has a secondary value, as it is simply the result of the analysis of the meaning of the subject idea: e.g., "all animals feel" arises from an analysis of the meaning of the word "animal", in which feeling is implicit. In fact, «because an *animal* feels, (*all*) *animals* feel».[18]

[15]SIGWART (1873–1878/1911[4]:§ 27, p. 220 [1895:160]).

[16]SIGWART (1873–1878/1911[4]:§ 27, p. 221 [1895:161]).

[17]Cf. SIGWART (1873–1878/1911[4]:§ 27, pp. 221–222 [1895:162]).

[18]SIGWART (1873–1878/1911[4]:§ 27, pp. 222 [1895:162]).

In ordinary language, the explicative judgment is translated into a narrative judgment concerning particular things; that is also why the inappropriate use of "all" has become common. As for (*b*.2), there are cases in which the judgment is not analytical but synthetical, as in: "all men are mortal". The synthetical judgment is the result of an inference; more precisely, it is the result of an inference either from all observed cases to all remaining others, or from the qualifications *[Bestimmunge]* included in the word, e.g. "man", to other qualifications which are necessarily connected to them. The unconditionally universal judgment does not concern directly the actual existence of the subjects: "all *A*'s are *B*" means only "what is *A* is *B*", or "if anything is *A* it is *B*". But then —Sigwart infers— the use of the plural is inadequate, since the adequate expression would then simply be "*A* is *B*", "man is mortal", "a square is equilateral", etc.[19] By falling back upon an infinite set of particular things, the quantifier "all" expresses a necessary connection *[notwendige Zusammengehörigkeit]* between the predicate *B* and the subject *A*, in the case that this does not refer to a countable set of particular empirical objects, but is the meaning of a general word.[20] In any case, we fall back upon the simple or singular judgment (*a*), if we intend as singular the subject denoted by a universal name (*b*.1); in the case of (*b*.2), this judgment can be reduced to either (*a*) or (*b*.1).

According to Sigwart, in traditional logic, if *B* is stated of the whole extension of the subject concept *A*, then the judgment is universal; while if it is stated only of a part of the extension of the subject concept, it is particular; if, finally, the subject-term is a proper name, or an equivalent expression, so that its extension is exhausted by a sole individual, then the judgment is singular. Sigwart thinks that this doctrine contains some obscurities, two of which are particularly prominent. First of all, in case of the distinction between universal and particular judgments, this doctrine does not distinguish between a judgment about a concept (that is, the meaning of the subject-word) and a judgment about empirical things. Furthermore, it neglects the differences between empirically universal judgments and unconditionally universal ones, as well as those between analytical and synthetical judgments. As far as the singular judgment is concerned, since its extension is exhausted by a sole individual, it is improperly assimilated to the universal judgment, since in the former the predicate belongs (or does not belong) to one subject, while in the latter it belongs to many sub-

[19]Cf. SIGWART (1873–1878/1911[4]:§ 27, pp. 222–224 [1895:162–163]).

[20]Cf. SIGWART (1873–1878/1911[4]:§ 27, pp. 220 [1895:160]; cf. also § 33, p. 275 [1895:202–203]).

jects.[21] Therefore, the traditional classification of judgments into singular, universal and particular ones must be abandoned. This holds specifically for the particular judgment, which «is one of the most unfortunate and inconvenient creations of logic».[22]

We also find here the distinction between a judgment with an empirical subject and a judgment the subject of which is a general term. The particular judgment "some *A*'s are *B*", as translated from Aristotle's logic into the traditional one, has a meaning which is related to the empirically universal judgment since it denotes something which is particular, definite and countable; therefore, it presupposes a narrative judgment which treats of what is actually existent. Furthermore, it presupposes that each part of the extension of the subject contains a plurality of individuals. In this respect, it remains unclear why a single individual cannot constitute a part of the extension of the subject. Thus it seems that if particular judgments are narrative judgments, that is, judgments with empirical origin, then they can only mean that a definite predicate is stated of one or more subjects, which are not named individually, but are denoted in an indefinite way by a general word: «[plural judgments do] not seem to differ in their plurality from a number of judgments concerning single subjects, since the numerical determinateness is not emphasized».[23]

However, they differ from copulative judgments: in replacing the copulative judgment "John, and Peter, and Paul are colour-blind" with the particular one "some men are colour-blind", the individual definiteness of the statement is lost, while with the corresponding universal judgments the possibility of exceptions is stressed. As a plural judgment states an exception with respect to the corresponding universal judgment, it becomes particular. It is, then, clear that such a judgment can be expressed as well by a singular judgment, as soon as the subject is denoted by a general name instead of a proper name. Traditional logic teaches that particular judgments do not mean the exclusion of the universal ones: to say "some *A*'s are *B*" does not mean that "All *A*'s are *B*" is false; that is just another confirmation of the ambiguity of the particular formula. Actually, it should mean that *some A's are different from the other remaining A's*. In this sense, the particular judgment constitutes a preparation *[Vorbereitung]* for a universal judgment about the other remaining *A*'s. Let us consider the universal judgment "all fixed stars are immovable" and the copulative one "α Centauri, and 61 Cygni, and Sirius have a movement of their own", which can be

[21] Cf. SIGWART (1873–1878/1911[4]:§ 28, pp. 224–226 [1895:163–165]).

[22] SIGWART (1873–1878/1911[4]:§ 28, p. 227 [1895:166]).

[23] SIGWART (1873–1878/1911[4]:§ 28, p. 229 [1895:168]).

translated into the particular judgment "some fixed stars have a movement of their own". If we intend the particular judgment in the sense that α Centauri, and 61 Cygni, and Sirius are not fixed stars, the way is open for a universal judgment about the other remaining fixed stars; otherwise, the particular judgment in question establishes an exception to it.[24]

As for unconditionally universal explicative judgments, the adequate way of expressing them is a modal one. So, for particular judgments with an abstract or general subject the extension of which does not consist in a plurality of things, the adequate expression is not "some *A*'s are *B*", but "*A* can have *B*"; e.g., the judgment "some paralelograms have equal diagonals" is more adequately expressed as "a paralelogram can have equal diagonals". In fact, the knowledge that "some paralelograms have equal diagonals" does not come from an analysis of the concept "paralelogram"; further, if "some paralelograms" means to denote only a part of the extension of the paralelogram concept, that does not imply that the concept, just because it does not refer to definite countable individuals, would become more definite than before. "Some *A*'s" denotes, then, a part of the possible *A*'s; the traditional formulation can be kept only under the condition that the possible *A*'s, which are denoted, are not taken by mistake instead of the real *A*'s, which suggest the use of the plural[25]. Furthermore, later on Sigwart writes about particular judgments with a general subject: «A triangle may be acute, right-angled, or obtuse. No intuitable figure contains *only* the properties which I think under the word triangle. Before I can form the image of one, some definite relations of sides and angles are necessary; and when in constructing it I try different determinations or recall them from memory, the general idea presents me with a choice of various closer determinations. No definite colour is, e.g., necessarily connected with those attributes of an animal which constitute the content of the idea horse. The horse can be black, white, brown, etc. So far as concerns the content of my idea, these judgments are perfectly definite statements concerning the plurality of differences. So far as they are intended to apply to the nature of the existent, they express in the same way a material possibility which connects variation of colour with the organization of a certain animal. Only when applied to a definite particular thing does the judgment take upon itself the problematical significance of ignorance. When all I know of a thing is that it is a horse, I cannot state of it that it is black or white; when I know only that something is a triangle, I do not know whether it is right-angled or not. «When we are dealing with subjects thought of as general, then the judg-

[24]Cf. SIGWART (1873–1878/1911[4]:§ 28, p. 230 [1895:168]).

[25]Cf. SIGWART (1873–1878/1911[4]:§ 28, pp. 231 [1895:169]).

ment "*A* can be *B*", etc., is the adequate expression of the so-called particular judgment».[26]

Let us summarize. If judgments, either universal or particular, have an empirical subject, then they are about a countable series of simple singular judgments; on the other hand, if the subject is a general idea, then both universal judgments and particular ones mean that the conjunction of the subject with the predicate is, respectively, either necessary or possible: in the first case, the necessity is connected with the lack of exceptions, and it is given when the predicate comes from the analysis of the idea, that is, when the predication is essential; since the existence of exceptions is, in the second case, connected with a plurality of possibilities which results from the analysis of the possible predicates which are contained in an idea, and which are mutually compatible with each other, the predication is accidental[27]. The interpretation of particular propositions in terms of modality, is of fundamental importance for the birth and the development of imaginary logic, as we will soon see .

3. *Vasil'év on particular propositions and the law of the excluded fourth*

Vasil'év will explicitly take Sigwart's analysis of the particular judgment, and incorporates it into his new view of a non-Aristotelian logic. However his first article of logical character, "O častnych suždenijach, o treugol'nike protivopoložnostej, o zakone iskljucënnogo cetvërtogo [On Particular Propositions, the Triangle of Oppositions, and the Law of the Excluded

[26]SIGWART (1873–1878/1911[4]:§ 34, pp. 280–281 [1895:207]).

[27]Here it must be pointed out that Sigwart (1873–1878/1911[4]:239 [1895:176]) does not intend modality in the Kantian meaning: "The statement that a judgment is a possible one or a necessary one is not the same as the statement that it is possible or necessary for a predicate to belong to a subject. The former refers to the subjective possibility or necessity of judgment; the latter to the objective possibility or necessity of what is stated in the judgment. The Kantian distinction of the differing modality of judgments, according to which they are problematical, assertorial or apodectic, applies to the former; the Aristotelian proposition: Πᾶσα πρότασις ἐστιν η τοῦ ὑ πάρχειν ἢ του ἀνάγκησ ὑπάρχειν ἢ τοῦ ἐνδέχεσθαι ὑπάρχειν (*Anal. pr.*, I, 2, 25 a 1) to the latter". About the problematical judgment particularly, SIGWART (1873–1878/1911:242–243 [1895:178–179]) states that, since it expresses the uncertainty of what is asserted in the judgment, it is not properly a judgment, but merely an «unfinished attempt at a judgment».

Fourth]"[28], is devoted just to an analysis of the nature of the particular judgment. From here he comes to deny the law of excluded middle, to replace it with that of excluded fourth, and to reconsider the opposition relations among propositions in the traditional syllogism.

First of all, Vasil'év divides, following Sigwart, all judgments in *judgments about facts* (Sigwart's narrative judgments), which presuppose a spatial-temporal moment, and *judgments about concepts* (Sigwart's explicative judgments), which do not presuppose such a moment. The former have as a subject something which is actual, i.e. sensations and ideas, the latter have classes or concepts as their subject. The law of excluded middle can be applied only to judgments about facts, but not to judgments about concepts. This refers us back to Sigwart's just quoted passage: as long as I speak about the concept of the triangle, I cannot say if it is right-angled or not. If, however, I form an image of one triangle, then I am compelled to make a choice. Of *a definite triangle* I can say that it *is equilateral* (or not), while of *the triangle* I can say only that it *can be equilateral.* The latter kind of judgment —we have seen— corresponds to the particular judgment with a general subject, that is, not an empirical one, as in "some triangles are equilateral", and it is part of those judgments which Vasil'év calls judgments about concepts. There is no doubt, that Vasil'év took the distinction between judgments about facts and judgments about concepts from Sigwart. That this distinction is very important for him, can be seen not only in the course of his argumentation, but also in the opinion which he expresses on the German logician concerning his meaning for the contemporary development of the logic; according to Vasil'év, «Sigwart (...) was a revolutionary in logic»[29], especially as he criticizes and reformulates some traditional doctrines such as: the modality of judgments, the theory of negative judgments (cf. *infra*), and the one on the affirmative categorical judgment as the basic form of judgment.

Still in accordance with Sigwart, who criticized the traditional meaning of the particular judgment and interpreted "some, and maybe all, *S*'s are *P*" as *some S's are different from the other remaining S's*, Vasil'év interprets "Some *S*'s are *P*" as "only some *S*'s are *P*", that is, "some *S*'s are *P* and

[28]Originally, this article was published in Russian in *Učënie zapiski Kazan'skogo Universiteta* [*Scientific Papers of Kazan' University*] 77 (1910), 1–47. Together with the other two logical articles of 1912 and 1912–1913, and some minor papers, it was reprinted in N.A. VASIL'ÉV (1989); on this, cf. the Review-essay of CAVALIERE (1991). A brief summary of this article is given in a long note of VASIL'ÉV (1912–1913/1989:100–101, n. 6 [1993:333–334, n. 7]); but cf. also CAVALIERE (1991:55–57), ARRUDA (1984:473–476), and SMIRNOV (1986:206–208; 1989:629–631).

[29]VASIL'ÉV (1912–1913/1989:122 [1993:350]).

some *S*'s are not *P*". According to Vasil'év, the traditional interpretation of the particular judgment "some, and maybe all, *S*'s are *P*" shows an oscillation between two alternative hpotheses: "either all *S*'s are *P*, or just some *S*'s are *P*". Such a disjunction may constitute the starting point for the cognitive process, at the end of which we have only one of the two judgments in question. These are both intended as universal. Indeed, also the judgment "*Only some S's are P*" (which is called *M* by Vasil'év) extends over the whole of the *S*-class, as it means that a part of *S* possesses the predicate *P* and the remaining part does not possess *P*. This means that a particular judgment presupposes a particular negation —"some *S*'s are *P*" presupposes that "some (the remaining) *S*'s are not *P*"— and vice versa. We will then write:

1. *M* (or *SuP*) for: $SiP \wedge SoP$.

Vasil'év gives two interpretations of *M*, corresponding to the distinction between judgments about facts and judgments about concepts. The first is in terms of a disjunctive judgment: for any *s* which belongs to the class *S*, "only some *S*'s are *P*" means that each *s* is *P* or *Q* or *R*, etc.; in other words: "each *s* is *P* or is not *P*"[30], and *s* is a definite empirical individual. The other interpretation considers *s* to be a concept and *P* as an accidental predicate of *s*; accordingly, "only some *S*'s are *P*" means, "*S* can be *P*" —where the analogy with Sigwart is obvious. This latter type of judgment —which, according to the symbolism usually used for Aristotelian logic, we can write as *SuP*— is called *accidental* by Vasil'év, because it means that *P* can belong to *S*, but does not belong necessarily to it, as it does not belong to all *S*'s. The accidental judgment is also a synthesis of two judgments, that is, "some *S*'s are *P* and some *S*'s are not *P*": "In an accidental proposition the particular affirmative proposition presupposes a particular negation and *vice versa*"[31]. Now, it is clear that Vasil'év —as well as Sigwart— refuses particular judgments and accepts only universal judgments, which correspond to the three forms *A*, *E* and *M*, that is, *SaP*, *SeP* and *SuP*.

Let us consider —in agreement with Vasil'év's purpose to construct a non-Aristotelian logic— only such judgments which have a concept as subject. We have the three kinds of judgments above mentioned, which correspond to Sigwart's plural judgments with abstract subject: (*i*). Affirmative judgments about concepts, or affirmative universal judgment, which express the necessity that a given predicate belongs to a given concept, as in

[30]Cf. SMIRNOV (1989:629); CAVALIERE (1991:56).

[31]VASIL'ÉV (1912-1913/1989:100, n. 6 [1993:333, n. 7]).

"a triangle must be closed" or "all triangles are closed" (*SaP*); (*ii*) Negative judgments about concepts, or negative universal judgments, which express the impossibility that a given predicate belongs to a given concept, as in "a triangle cannot be square" or "no triangles are square" (*SeP*)[32]; (*iii*) accidental judgments about concepts *M*, which express the possibility that a given predicate belongs to a given concept, as in "a triangle can be equilateral" or "some triangles are equilateral" (*SuP*). Only one of these three forms is true in every given case; this is the *law of the excluded fourth*: "A predicate relates to every concept in such a way that it is either characteristic of it as its *proprium* (necessary property) or as its *accidens* (accidental property), or it is not characteristic at all. But there is no fourth possibility"[33].

According to the Aristotelian logical square, if two universal propositions *A* and *E* are both false, then the particular propositions *I* and *O* are true, but since Vasil'év unifies *I* and *O* in the sole accidental proposition, the square of opposites does not hold in the imaginary logic, but the *triangle of opposites* holds: each proposition is false, if one of the remaining two propositions, but not both, is true[34]. Consequently, the following propositions all hold:

2. a. $\neg(SaP \wedge SeP)$
   b. $\neg(SaP \wedge SuP)$
   c. $\neg(SeP \wedge SuP)$
   d. $SaP \vee SeP \vee SuP$
   e. $\neg SuP \rightarrow (SaP \vee SeP)$
   f. $\neg SeP \rightarrow (SaP \vee SuP)$
   g. $\neg SaP \rightarrow (SeP \vee SuP)$.

Since according to Vasil'év the law of contradiction is a consequence of the law of the excluded middle[35], the elimination of the latter drives to the elimination of the former; so the way is opened to the formulation or proposal of an imaginary (non-Aristotelian) logic, that is, *a logic without the law of contradiction.*

[32]We agree with KLINE (1965:324), who replaces in Vasil'év's example 'virtuous' with 'square', because the latter predicate expresses better than the former the logical impossibility.

[33]VASIL'ÉV (1912-1913/1989:100, n. 6 [1993:333, n. 7]).

[34]Cf. SMIRNOV (1986:207; 1989:627).

[35]Cf. VASIL'ÉV (1912/1989:77–78). Cf. also KLINE (1965:318).

Later on Vasil'év's discourse will concern only the judgments of concept because for the judgments of fact the usual logical laws hold. The imaginary logic —we will see— is a logic of the concepts, insofar as only on this level, and not in reality, there are contradictory concepts. Vasil'év assumes, then, an imaginary world, by means of which (and in which) he gives a reference to contradictory concepts and negations.

4. *Negation and the principle of contradiction in imaginary logic*

According to Vasil'év, that the law of contradiction may be eliminated results very clearly from the analysis of its meaning: «*The law of contradiction expresses the incompatibility between an affirmation and a negation*»[36]. By an affirmation Vasil'év understands a judgment about objects or facts, which is based on perceptions, sensations or ideas of these objects or facts. Such perceptions and sensations can only be positive, because there are only positive objects and facts. Thus the negation may be defined only as «what is incompatible with the affirmation»[37]. In such a definition of negation the law of contradiction is already included. In order to explicate this central point of Vasil'év's reflection, let us consider more in detail his concept of negation, which again is developed with regard to Sigwart's view.

According to Sigwart, negation is an act of the subject which is directed against an affirmative judgment of which it states the falsity; the negation is, then, incompatible with the affirmation but it is also in connection with and subordinate to the latter, without which it could not even subsist. This has an ontological foundation, viz. the fact that there are no sensations or ideas of negative facts; that is why negation, to which nothing real corresponds, can be only a "refusal" by the subject of something positive. The law of contradiction, according to which "*A* is *B*" and "*A* is not *B*" cannot

[36]VASIL'ÉV (1912/1989:59). KLINE (1965:318, n. 7) points out that it is possible to express formally this incompatibility relation by means of the Peirce-Sheffer stroke '|' ('alternative denial'), which may be interpreted as meaning 'not both' (cf. PEIRCE 1880: *CP* 4. 12–20; SHEFFER 1913; WHITEHEAD and RUSSELL 1910–13[1]/1925–27[2]: I, xvi ff.). Using this function we have that:

$$A \mid A \equiv \neg A$$

and

$$A \mid B \equiv \neg(A \wedge B) \equiv \neg A \vee \neg B;$$

therefore

$$A \mid (A \mid A) \equiv \neg(A \wedge \neg A) \equiv \neg A \vee A.$$

[37]VASIL'ÉV (1912/1989:59).

be true at the same time, expresses then the essence and the meaning of the negation[38].

According to Vasil'év, if there are only positive facts to which sensations, perceptions, and ideas which are positive as well, correspond, then there is a negation only where there is an incompatibility and not just a simple difference or a lack of predication. Moreover, if only an affirmative judgment is immediate, insofar as it is based directly on sensations and perception of facts and objects, then negation is always *deduced.* In the first case, Vasil'év explicates that the negation of e.g. *blue* cannot be something as *sweet* or *ugly* but only something which falls under *non-blue*, that is, red, white, orange, and such. Here it seems that Vasil'év intends the relation of incompatibility also to include Aristotelian contrariety as well. This agrees also with the triangle of opposites, in which there are only universal propositions under which the relation of opposition is just that of contrariety: the opposite propositions may be both false, so that, insofar as the law of excluded middle is not valid, a third proposition is true. Concerning the second case, Vasil'év states at first that it is not possible to ascertain immediately that *A* lacks the predicate *B*, because it is not possible to have the sensation of an absence; it can be perceived only in a mediate manner, that is, through the comparison of the object *A* with the predicate *B*. However, «*the simple lack of the predicate B in my perception or idea of the object A cannot serve as a logical foundation for a negative judgment*»[39], because the absence of a thing —intended in the sense that the subject does not find it— does not imply the non-existence of the same thing: this can be though it is possible that it is not perceived. Furthermore —and that is fundamental for imaginary logic— Vasil'év denies that there are 'negative capacities': to not see something means to see some other thing, and to make a comparison. It is to note a difference between what I see and the predicate I thought: «But the simple difference between the real image of the object and the supposed one, as well as each simple difference, cannot be a foundation for negation. Only if there are characteristics in the real image of the object which conflict with the supposed image, I can say that the supposed image does not exist[40]. Then, «a lack can serve as foundation for a negative judgment only when it is linked with an incompatibility. *In general*

[38]Cf. SIGWART (1873–1878/1911[4]:§ 20, pp. 158ff. [1895:119ff.]; § 23, p. 191 [1895:139]).

[39]VASIL'ÉV (1912/1989:59).

[40]VASIL'ÉV (1912/1989:60).

—Vasil'év concludes— *we can state that the sole logical foundation for negation is incompatibility*»[41].

In confirmation of what has been just said there is also the fact that negative judgments are not original but they are conclusions of *inferences*. Since there are no negative perceptions, the negative judgment —Vasil'év argues— can only be a *deduction* from positive perceptions: I do not see *non-white*, but I see *red* and *blue*, and I know that red and blue are not compatible with white. I can deny that a predicate *P* belongs to an object *S* only if I know that a predicate *N*, which is incompatible with *P*, belongs to *S*. The negative judgment "*S* is not *P*" is then the result of a syllogism of the first figure:

3. *N* is incompatible with *P*

   S is *N*

   S is not *P*.

Negative judgments in traditional logic are of two kinds: the one is the judgment of the major premiss ("*N* is incompatible with *P*"), the other is the conclusion ("*S* is not *P*") of a syllogism[42].

Now, if we look at the definitions of the law of contradiction and of negation given above, it results that the latter implies the former, that is, that the law of contradiction is already contained in the negation: insofar as the negation is incompatible with the affirmation, the law of contradiction can state the incompatibility of both. But then —Vasil'év argues— «to construct a logic which is free from the law of contradiction means to construct a logic without the negation, that is, without incompatibility»[43]. Therefore, the imaginary logic needs a different concept of negation.

According to Vasil'év, in traditional logic, the negative judgment "*S* is not *P*" includes two moments: a formal one, which «asserts that the truth of a negative judgment implies the recognition of the falsity of the affirmation but leaves open the question about the foundation according to which it is possible to state the truth of negative judgments»[44]; and a material one,

[41]VASIL'ÉV (1912/1989:60).

[42]Cf. VASIL'ÉV (1912/1989:61). Cf. also KLINE (1965:319–320).

[43]VASIL'ÉV (1912/1989:62).

[44]VASIL'ÉV (1912/1989:62).

which explicates that «the negative judgment is based on the incompatibility of the predicates and is either the assumption of such an incompatibility or a deduction from such an assumption»[45]; in other words, the negation is either the major premiss or the conclusion of a syllogism; «Therefore, the formal moment speaks about the properties of the negation, the material one about the foundation of the negation. We can keep the formal moment and change the material one: so we obtain a different negation»[46]. Since in traditional logic, while the affirmative judgment is immediate the negative one is always deduced, in order to have a different negation we have to suppose that all negative judgments are as immediate as the affirmative ones; this is possible only by postulating a world in which such judgments as "*S* is not *P*" can be derived from experience. In this case, such judgments would continue to be negative, insofar as they state the falsity of affirmative judgments but they would be different from the negative judgments of traditional logic, because they are based on a different material moment, that is, on the immediate perception and not on assumptions of incompatibility or deductions from such assumptions. This "different negation" is «the judgment which asserts the falsity of the affirmation but is not based on the incompatibility»[47]. A logic in which the negative judgment is immediate and not deduced is just the imaginary logic. The separation of the negation from the property of incompatibility implies the abandonment of the law of contradiction. Imaginary logic is, then, a logic without such a law.

At this point a clarification and a distinction are necessary. The law of contradiction which is here denied is the law which affirms the impossibility that two incompatible predicates can belong to the same object, that is, the impossibility that there are contradictory objects which can serve as a foundation for the coexistence of an affirmative judgment and its negation. Such a law corresponds to the Kantian formulation «No thing has a predicate which contradicts it»[48]. From this law we must distinguish another law which is, the *law of the absolute difference between truth and falsehood*, according to which «one and the same judgment cannot be true and false at the same time»[49]. Sigwart expresses this as «both judgments "*A* is *B*" and

[45]VASIL'ÉV (1912/1989:62).

[46]VASIL'ÉV (1912/1989:62–63).

[47]VASIL'ÉV (1912/1989:63).

[48]KANT (1781[1]–1787[2]:A 151 = B 190).

[49]VASIL'ÉV (1912/1989:64).

"*A* is not *B*" cannot be true at the same time»[50]. Such a formulation, «forbids to accept at the same time the judgment "*A* is *B*" by means of the affirmative statement and to deny the judgment "*A* is *B*" by means of the negative statement "*A* is not *B*"»[51]. In relation to the fundamental distinction between these two laws, Vasil'év expresses himself, following Sigwart, still in psychologistic terms and asserts that the law of the absolute difference between truth and falsehood concerns the knowing subject and forbids him to contradict himself, while the law of contradiction concerns the objects and forbids that they can contain contradictions: "This latter law banishes the contradictions from the world as well as the former banishes them from the subject. The law of contradiction has an objective meaning and the law of the absolute difference between truth and falsehood a subjective one. Therefore, it is true that it is possible to violate or to eliminate the law of contradiction without infringing on the law of the absolute difference between truth and falsehood. If I affirm that a certain *N* is and is not a man at the same time, of course I violate the law of contradiction, but, if I affirm this always and remain convinced of my opinion without contradicting myself, I do not violate the law of the absolute difference between truth and falsehood»[52].

In other words, we can say that the logical principle of contradiction is still valid, but not the ontological one. In imaginary logic each judgment has only one truth value, but in the imaginary world there are also negative facts and there are objects which possess contradictory properties. Therefore, «it is possible that in the same object coexist the presuppositions for both an affirmative judgment and a negative one»[53]. In the imaginary logic a judgment can never be true and false at the same time, but the same object can have opposite properties. Consequently, a third kind of judgment is required besides affirmation and negation. «Let us suppose that the fact *a* is the foundation of the affirmative judgment "*S* is *A*" and the fact *b* the foundation of the negative judgment "*S* is not *A*". The relation between the facts *a* and *b* is not an incompatibility relation as in our logic. That is why it is perfectly possible that *a* and *b* coexist. What is happening in such a case? On the grounds of the fact *a*, it is possible to affirm that the affirmative judgment "*S* is *A*" is true, while on the grounds of the fact *b*, it is false. On

[50]VASIL'ÉV (1912/1989:64).

[51]VASIL'ÉV (1912/1989:64; cf. also 1912–1913/1989:106–107 [1993:338]).

[52]VASIL'ÉV (1912/1989:65–66).

[53]VASIL'ÉV (1912/1989:66).

the other hand, on the grounds of the fact *a*, the judgment "*S* is not *A*" is false, while on the grounds of the fact *b* it is true. So in this case the affirmative judgment and the negative one result to be both true and false. But this is forbidden by the law of the absolute difference between truth and falsehood. That is why in the case that both the facts *a* and *b* exist at the same time a third judgment must exist which is true in such a case. We will call this third kind of judgment, which expresses the presence of the contradiction in the object *S*, that is, the coexistence in it of the foundations of both an affirmative judgment and a negative one, a judgment of contradiction, or better an indifferent judgment, and we will indicate it in this way: *S is and is not A at the same time*»[54].

In the imaginary logic there are then three kinds of qualitatively different judgments: affirmative ones (*S is A*), negative ones (*S is not A*) and indifferent ones (*S is A and is not A at the same time*). The latter two kinds of judgments are negative in the sense that they assert the falsity of the affirmation. Each of them is false if one of the other two is true; this is the *law of the excluded fourth*. Therefore, it is possible to say that "*S* is and is not *A* at the same time". Vasil'év himself underlines the relation between the accidental judgment and the indifferent one: «The indifferent judgment of the imaginary logic finds its counterpart in the accidental judgment about concepts: "*S* can be *P*". Such a judgment can be seen as an original synthesis of affirmation and negation. In fact, the accidental judgment "*S* can be *P*", "the triangle can be equilateral" is equivalent to the form: "some *S* are *P*, some *S* are not *P*", "some triangles are equilateral, some triangles are not". That is why it is possible to say that *the logic of concepts is analogous to the imaginary logic*»[55].

While for objects and facts either the affirmation or the negation is valid, for concepts one of the three above-mentioned judgments is valid. The law of the excluded fourth, which is a law of the imaginary logic, holds in our 'terrestrial logic' of concepts as well. From the discussion on the distinction between the law of contradiction and the law of the absolute difference between truth and falsehood it emerges that «some logical truths are absolute, others are not»[56]. Such an affirmation has a match in the distinction between on the one hand empirical and real laws, which are valid for our world, and purely formal laws on the other hand: among the former there is the law of contradiction, among the latter the law of the absolute difference between truth and falsehood. The former is an empirical law, since it is

[54]VASIL'ÉV (1912/1989:66).

[55]VASIL'ÉV (1912/1989:82).

[56]VASIL'ÉV (1912–1913/1989:98 [1993:332]).

derived from the existence in our world of incompatible predicates, and it is real, «because it does not concern thought but reality, not the judgments but the objects»[57]. It affirms that there are no contradictions or contradictory objects (as, e.g., round squares) in reality. The latter is a formal law of thought, insofar as it concerns «*only thought and not reality, only judgments and not objects*»[58]. Formal laws (such as the law of the absolute difference between truth and falsehood, the laws of identity and of sufficient reason) are presupposed by any logic, while empirical laws (such as the laws of contradiction and of excluded middle) may change according to the logic and the possible world they are connected with. Imaginary logic is a logic which keeps all formal laws but abandons the empirical ones; it is a logic which denies the law of contradiction (and the law of excluded middle) without denying the law of the absolute difference between truth and falsehood (and that of excluded fourth).

That the law of contradiction is empirical in character is also proved by the fact —and here Vasil'év mentions the Russian neo-Kantian philosopher and psychologist Aleksander I. Vvedenskij (1856–1925)— that we are able to think but not to represent contradictions as, for e.g., the *round square*. An imaginary logic becomes possible only when we do not any longer try to accord thought and representation. Here Vasil'év goes against Sigwart, who identifies both. Vasil'év clearly separates thought and representation thus indicating that, though his own theory is expressed in psychologistic terms, it nevertheless contains the potential to become free of the very same psychologism. After having shown the conceivability of a logic without the law of contradiction Vasil'év goes on to show in which way it is possible to operate with such a logic.

5. *Applying imaginary logic: the reconstruction of the syllogism*

Vasil'év does not want to reinvent all of logic but only to show that the introduction of certain changes in our logic which were believed to be self-destructive for logic itself, nevertheless still allow one to reason and to infer. An essential condition for reasoning and inferences to continue to be valid is the following: even though the law of contradiction (and the law of excluded middle) is eliminated, the other laws must continue to hold; therefore, the independence of the inferential processes from the law of contradiction must be shown. In order for this to be the case the following condi-

[57]VASIL'ÉV (1912/1989:67).

[58]VASIL'ÉV (1912/1989:68).

tions must be fulfilled: first, the knowing subject with its mental organisation, must continue to be the same as in our world. Here Vasil'év pays off his debt to Sigwart, since he uses the dependence of logic on the knowing subject to justify a nodal point of his argumentation: however, the connection between the invariability of the knowing subject and the fact that in other worlds other laws hold can only mean that the changed (or eliminated) logical laws do not depend on the knowing subject but depend on the objects, on reality; therefore, they are empirical and not merely logical laws. Second, the imaginary world(s) too must be the same as our world, with the sole exception that in it there are negative facts, so that to the same *S* can belong *P* and non-*P* at the same time. Finally, the (other) logical laws must be the same as in our logic, while the empirical laws are refuted. In short, all that is purely logical is unchangeable, while all that is empirical is changeable or eliminable. In imagining other worlds we do not break the logical laws but only the laws of reality[59].

In imaginary logic judgments may be classified according to their quality as affirmative, negative and indifferent and according to their quantity as singular judgments or judgments about a class or a concept, which may be either singular or accidental. Firstly, Vasil'év analyses those judgments which have their origin in sensation, i.e. judgments about a single *S*, which may be in accordance with one of the three forms mentioned above: "*S is A*", "*S is not A*" and "*S is and is not A at the same time*". Secondly, Vasil'év examines universal judgments about a concept or a class *S*; here he resumes of course the distinction between judgments about facts and judgments about concepts (see above). So in this case a judgment may be either universally affirmative ("*every single S is P*"), universally negative ("*no single S is P*") or universally indifferent ("*every single S is and is not P at the same time*"). Among the universal judgments there are exclusion forms to the extent that if one of them is false, then one of the other two is true. In case that not every *S* of a given class belongs to the predicate *P* we have accidental judgments, which may be of four kinds: (*i*) "*some S are P and all the others are not P*", (*ii*) "*some S are P and all the others are and are not P at the same time*", (*iii*) "*some S are not P and all the others are and are not P at the same time*" and, finally, (*iv*) "*some S are P, some are not P, and the remaining S are and are not P at the same time*". These seven kinds of judgments, three universal and four accidental ones, exhaust all possible cases concerning the relation between a class *S* and a predicate *P*[60].

[59]Cf. VASIL'ÉV (1912/1989:69–70; 1912–1913/1989:101–103 [1993:334–335]).

[60]Cf. VASIL'ÉV (1912/1989:70–71).

Now Vasil'év has to show that using such kinds of judgments syllogisms can be constructed. He first concentrates on the first figure of the syllogism. We mentioned earlier that in the imaginary logic the syllogism's structure is independent from the principle of contradiction; in fact, since the law of contradiction implies negation, it is sufficient to build syllogisms in which negative judgments do not appear, in order to prove that it is possible to reason and infer without the law of contradiction. Since in *Barbara* there are no negative judgments, the law of contradiction has no influence there. We have also said that in the imaginary world there are objects which possess both *P* and non-*P*. Let us consider a class of objects such that "every *M* is and is not *P* at the same time". If we construct a syllogism in *Barbara*, the major premiss of which is such a universal indifferent judgment and the minor premiss is an affirmative judgment, then we obtain as a conclusion an indifferent judgment:

4. Every *M* is and is not *P* at the same time

   S is *M*

   S is and is not *P* at the same time.

Vasil'év points out that this inference is valid: on the basis of "*S* is *M*", what is true of *M* is true of *S* as well; therefore, if it is true to say that *M* is and is not *P*, the same must be true of *S*. Otherwise, if we do not accept the conclusion "*S* is and is not *P* at the same time", we would have two possibilities: either "some, but not all, S are P" or "some, but not all, S are not P". Clearly, these judgments contradict the major premiss, so that in both cases we would violate the law of the absolute difference between truth and falsehood; therefore, we are compelled to accept the universally indifferent judgment as a conclusion[61]. Both rules of the first figure are kept in the imaginary logic: the major premiss must be universal and the minor premiss affirmative; otherwise, we would not be able to infer at all. Consequently, Vasil'év adds two new forms to the traditional four of the first figure, viz. (*a*) *Mindalin* and (*b*) *Kindirinp*:

[61]Cf. VASIL'ÉV (1912/1989:72ff.; 1912–1913/1989:107–108 [1993:339]).

5.a. Every *M* is and is not *P* at the same time

Every *S* is *M*

Every *S* is and is not *P* at the same time.

5.b. Every *M* is and is not *P* at the same time

Some *S* are *M*

Some *S* are and are not *P* at the same time.

As far as the other syllogistic figures are concerned, the second poses some problems, while the third is valid and has three new indifferent forms[62].

So, now Vasil'év has reached the aim which he set himself: «not to give a system of imaginary logic —that is a task for an absolutely different work— but to show the principle on which it is constructed»[63] and, furthermore, to demonstrate that «the imaginary logic keeps the coerciveness of the reasonings and the rigor of the logical rules»[64]. However, he will never construct a complete system of imaginary logic. The reasons for this are to be searched for in his biography, but that was not the object of the present paper.

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REFERENCES

ARRUDA, A.I. (1977): "On the Imaginary Logic of N.A. Vasil'év", in Arruda, da Costa and Chuaqui (1977, eds.: 3–24).

ARRUDA, A.I. (1980): "A Survey of Paraconsistent Logic", in Arruda, Chuaqui and da Costa (1980, eds.: 1–40).

ARRUDA, A.I. (1984): "N.A. Vasil'év: A Forerunner of Paraconsistent Logic", *Philosophia Naturalis* 21: 472–491.

[62]Cf. VASIL'ÉV (1912/1989:75–76). In his (1912-1913/1989:109) he even claims that the second figure is impossible.

[63]VASIL'ÉV (1912/1989:78).

[64]VASIL'ÉV (1912/1989:78).

ARRUDA, A.I. (1989): "Aspects of the Historical Development of Paraconsistent Logic", in Priest, Routley and Norman (1989, eds.: 99–130).

da COSTA, N.C.A. and R. CHUAQUI (1977, eds.): *Non-Classical Logics, Model Theory and Computability*, Amsterdam/New York/Oxford: North-Holland.

CHUAQUI, R. and N.C.A. da COSTA (1980, eds.): *Mathematical Logic in Latin America*, Amsterdam/New York/Oxford: North-Holland.

BAZHANOV, V.A. (1990): "The Fate of one Forgotten Idea: N.A. Vasil'év and his Imaginary Logic", *Studies in Soviet Thought* 37: 143–151.

BAZHANOV, V.A. (1992): "C.S. Peirce's Influence on the Logical Work of N.A. Vasiliev", *Modern Logic* 3: 45–51.

CAVALIERE, F. (1991): Review-essay of N.A. VASIL'ÉV, *Voobražaemaja logika*, ed. by V.A. Smirnov, Moscow: Nauka, 1989, *Modern Logic* 2: 52–75.

FENSTAD, J.E., I.T. FROLOV and R. HILPINEN (1989, eds.): *Logic, Methodology and Philosophy of Science*, Amsterdam/New York/Oxford/Tokyo: North-Holland.

JAMMER, M. (1974): *The Philosophy of Quantum Mechanics*, New York: John Wiley and Sons, 1974.

KANT, I. (1781^1–1787^2): *Kritik der reinen Vernunft*, Riga: Hartknoch.

KLINE, G.L. (1965): "N.A. Vasil'év and the Development of Many-valued Logic", in Tymieniecka (1965, ed.: 315–326).

PEIRCE, Ch.S. (1880/1931): "A Boolean Algebra with One Constant", in *Collected Papers of Charles Sanders Peirce*, vol. IV ed. by C. Hartshorne and P. Weiss, Cambridge: Harvard University Press, 12–20.

PRIEST, G. and R. ROUTLEY (1989): "First Historical Introduction. A Preliminary History of Paraconsistent and Dialectic Approaches", in Priest G., R. Routley and J. Norman (1989, eds.: 3–75).

PRIEST, G., R. ROUTLEY and J. NORMAN (1989, eds.): *Paraconsistent Logic. Essays on the Inconsistent*, ed. by G. Priest, R. Routley and J. Norman, München/Wien: Philosophia.

PUGA, L.Z. and N.C.A. da COSTA (1988): "On the Imaginary Logic of N.A. Vasil'év", *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 34: 205–211.

RASPA, V. (1999): "Łukasiewicz on the Principle of Contradiction", *Journal of Philosophical Research* 24: (to appear).

RESCHER, N. (1969): *Many-valued Logic*, New York: McGraw-Hill.

SHEFFER, H.M. (1913): "A Set of five Independent Postulates for Boolean Algebras, with Application to Logical Constants", *Transactions of the American Mathematical Society* 14: 481–488.

SIGWART, Ch. (1873–$1878/1911^4$): *Logik*, ed. by H. Maier, 2 vols., Tübingen: Mohr [Engl. trans.: SIGWART (1895)].

SIGWART, Ch. (1895): *Logic*, trans. by H. Dendy, London/New York: Swan Sonnenschein & Co./MacMillan & Co.

SMIRNOV, V.A. (1986): "Modality de re and Vasiliev's Imaginary Logics", *Logique et Analyse* 29: 205–211.

SMIRNOV, V.A. (1989): "The Logical Ideas of N.A. Vasiliev and Modern Logic", in Fenstad, Frolov and Hilpinen (1989, eds.: 625–640).

TYMIENIECKA, A.T. (1965, ed.): *Contributions to Logic and Methodology; in Honor of I.M. Bocheński*, Amsterdam, North-Holland.

VASIL'ÉV, N.A. (1910): "O castnych suždenijach, o treugol'nike protivopoložnostej, o zakone iskljucënnogo cetvërtogo [On Particular Propositions, the Triangle of Oppositions, and the Law of the Excluded Fourth]", *Ucënie zapiski Kazan'skogo Universiteta* [*Scientific Papers of Kazan' University*] 77: 1–47; repr. in VASIL'ÉV (1989: 12–53).

VASIL'ÉV, N.A. (1912): "Voobražaemaja (nearistoteleva) logika [Imaginary (Non-Aristotelian) Logic]", *Zurnal Ministerstva Narodnogo Prosvešcenija* 40: 207–246; repr. in VASIL'ÉV (1989: 53–94).

VASIL'ÉV, N.A. (1912–1913): "Logika i metalogika [Logic and Metalogic]", *Logos* 1–2: 53–81; repr. in VASIL'ÉV (1989: 94–123) [Engl. trans.: VASIL'ÉV (1993)].

VASIL'ÉV, N.A. (1989): *Voobražaemaja logika*, ed. by V.A. Smirnov, Moscow: Nauka.

VASIL'ÉV, N.A. (1993): "Logic and Metalogic", trans. by L. Vasyukov, *Axiomathes* 4: 329–251.

VERGAUWEN, R. and SOLDATJENKOVA, T. (forthcoming): "The Worlds of Logic and the Logic of Worlds: N.A. Vasil'év's *Voobražaemaja Logika*. Introduction, Translation, Discussion". *Logique et Analyse.*

WHITEHEAD, A.N. and B. RUSSELL (1910–13[1]/1925–27[2]), *Principia Mathematica*, Cambridge: Cambridge University Press.