Logique & Analyse **157** (1997), 45-66

SOME REMARKS ABOUT INDISTINGUISHABILITY AND ELEMENTARY PARTICLES

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Abstract

In the present survey we discuss some points about identity and indistinguishability in quantum physics. Our emphasis is on the possibility that quantum particles may have some kind of individuality, despite the theories which regard them as non-individuals in a sense. We also present some ideas which may conduct to a classical picture for physical phenomena that are usually described by means of quantum mechanics or quantum electrodynamics.

1. *Introduction*

The present survey mainly copes with foundational questions regarding identity and indistinguishability among elementary (quantum) particles. In this sense it is a philosophical work. We refer to the words of P. Suppes in [30]:

> We are no longer Sunday's preachers for Monday's scientific workers, but we can participate in the scientific enterprise in a variety of constructive ways. Certain foundational problems will be solved better by philosophers than by anyone else. Other problems of great conceptual interest will really depend for their solution upon scientists deeply immersed in the discipline itself, but illumination of the conceptual significance of the solutions can be a proper philosophical role.

Therefore, it is necessary to settle some philosophical terms. By *identity* we mean that if *a* and *b* are identicals, then they are the very *same* individual, that is, there are no 'two' individuals at all, but only one which can be named indiferently by either *a* or *b*. By *indistinguishability* we simply mean agreement with respect to attributes. We recognize that this is not a rigorous definition. Nevertheless such an intuition is better clarified in the next section.

Elementary particles that share the same set of state-independent (intrinsic) properties are usually said to be *indistinguishable*. Although classical particles can share all their intrinsic properties, we might say that they 'have' some kind of *quid* which makes them individuals. Hence, we are able to follow the trajectories of classical particles, at least in principle. That allows us to identify them. In quantum physics this is not possible, i.e., it is not possible, *a priori*, to keep track of individual particles in order to distinguish among them when they share the same intrinsic properties. In other words, it is not possible to label quantum particles.

The problems regarding individuality of quantum particles have been discussed in recent literature by several authors. Some few of them are [4] [6] [14] [16] [24] [26] [34]. On the possibility that the collections of such entities should not be considered as sets in the usual sense, Yu. Manin [21] proposed the search for a axioms which should allow to deal with collections of indistinguishable elementary particles. As he said,

> I would like to point out that it [standard set theory] is rather an extrapolation of common-place physics, where we can distinguish things, count them, put them in some order, etc.. New quantum physics has shown us models of entities with quite different behaviour. Even *sets* of photons in a looking-glass box, or of electrons in a nickel piece are much less Cantorian than the *sets* of grains of sand.

Other authors [6] [13] [14] have also considered that standard set theories are not adequate to cope with some questions regarding microphysical phenomena. These authors have emphasized that the ontology of microphysics apparently does not reduce to that one of usual sets, due to the fact that *sets* are collections of distinct objects.

Nevertheless, it has been recently proposed that standard set theory is strong enough to deal with collections of *physically* indistinguishable quantum particles [26], as we discuss with some details in the next Section. In Section 3 we make a very brief discussion on quasi-set theory in the context of quantum theory. In Section 4 we continue our dicussion about a classical picture for the quantum world by means of the particular description for photons proposed by Suppes et al. [31] [32] [33]. In Section 5 we present some final remarks.

2. *Indistinguishable particles and hidden variables*

We intend to show here that it is possible to distinguish, at least in principle, among particles that are '*physically* indistinguishable', where by 'physically indistinguishable' particles we mean, roughly speaking, those particles which have the same set of measurement values for their intrinsic properties. In a previous work [26] we assumed that '*physically* indistinguishable particles' are those which have the same set of measurement values for a correspondent complete set of observables. It seems clear that such a modification simplifies our conceptual framework and it is closer to the usual understanding about the meaning of (physical) indistinguishability. A kind of distinction is possible if we consider each particle as an ordered pair whose first element is the mentioned set of measurement values of the intrinsic properties and the second element may be viewed as a hidden property (a hidden variable) which corresponds to something which, intuitivelly speaking, was not yet measured in laboratory. The mentioned 'hidden property' may assume different values for each individual particle in a manner that it allows us to distinguish those particles which are 'physically' indistinguishable. Obviously such a hidden property appears to have a metaphysical nature, in a sense to be made precise. This is the kind of metaphysics that we advocate. The 'reasonable' metaphysics should be that one which could provide a hope for a future new physics. This future new physics may correspond to more extended physical systems that are not, untill now, measured in laboratories.

As remarked above, our concern here is only with the process of labeling physically indistinguishable particles. So, although we are not interested in describing here an axiomatic framework for quantum physics, quantum mechanics or even mechanics, we expect that our approach can be extended in order to encompass them. All that follows is performed in a standard set theory like Zermelo-Fraenkel with *Urelemente*[1] (ZFU).

Our picture for describing indistinguishability issues in quantum physics is a set-theoretical predicate, following P. Suppes' ideas about axiomatization for physical theories [29].

Hence, our system has five primitive notions: λ, X, P, m, and M. λ is a function $\lambda : N \to \Re$, where N is the set $\{1,2,3...,n\}$, n is a nonnegative integer, and $\Re$ is the set of real numbers; X and P are finite sets; m and M are unary predicates defined on elements of P. Intuitivelly, the images λ_i of the function λ, where $i \in N$, correspond to our hidden variables. We denote by λ_N the set of all λ_i, where $i \in N$. X is to be intuitivelly interpreted as a set such that its elements correspond to measurement values of the state-independent properties like rest mass, electric charge, etc.. The elements of X are denoted by x, y, etc. P is to be physically interpreted as a set of particles.

[1] Although it does not matter if we have Urelemente in our formal framework, we consider that for future extensions of such a work we will need atoms or Urelemente. These atoms (in the set-theoretical sense) may be interpreted as elementary particles.

$m(p)$, where $p \in P$, means that p is a microscopic particle, or a micro-object. $M(p)$ means that $p \in P$ is a macroscopic particle, or a macro-object. Actually, the distinction between microscopic and macroscopic objects as mentioned here does not reflect, at least in principle, the great problem of explaining the distinguishability among macroscopic objects, since these are composed of physically indistinguishable things. As it is well known, Schrödinger explained that in terms of a *Gestalt* [27]. Nevertheless, this still remains as an open problem from the foundational (axiomatic) point of view.

Definition 2.1 $\mathfrak{D}_{\mathcal{O}} = \langle \lambda, X, P, m, M \rangle$ *is a system of* ontologically distinguishable particles, *abbreviated as* $\mathfrak{D}_{\mathcal{O}}$*-system, if and only if the following six axioms are satisfied:*

D1 $\lambda : N \to \Re$ *is an injective function, whose set of images is denoted by* Λ_N.
D2 $P \subset X \times \Lambda_N$.

We denote the elements of P by $p,q,r,\ldots$ when there is no risk of confusion.

Definition 2.2 $\langle x,\lambda_i \rangle \equiv \langle y,\lambda_j \rangle \leftrightarrow x = y \wedge i = j$.

Definition 2.3 $\langle x,\lambda_i \rangle \doteq \langle y,\lambda_j \rangle \leftrightarrow x = y$.

$\equiv$ is a binary relation which corresponds to our ontological indistinguishability between particles, while $\doteq$ is another binary relation which corresponds to the physical indistinguishability between particles.

Definition 2.2 says that two particles are ontologically indistinguishable if and only if they have the same set of measurement values for their intrinsic physical properties and the same value for their hidden variables. Definition 2.3 means that two particles are physically indistinguishable if and only if they have the same set of measurement values for their intrinsic (physical) properties.

D3 $(\forall x,y \in X)(\forall \lambda_i \in \Lambda_N)((\langle x,\lambda_i \rangle \in P \wedge \langle y,\lambda_i \rangle \in P) \to x = y)$.
D4 $(\forall p,q \in P)(M(p) \wedge M(q) \to (p \doteq q \to p \equiv q))$.
D5 $(\forall p,q \in P)(p \doteq q \wedge \neg(p \equiv q) \to m(p) \wedge m(q))$.
D6 $(\forall x \in X)(\forall \lambda_i \in \Lambda_N)((m(\langle x,\lambda_i \rangle) \vee M(\langle x,\lambda_i \rangle)) \wedge \neg(m(\langle x,\lambda_i \rangle) \wedge M(\langle x,\lambda_i \rangle)))$.

Axiom **D1** allows us to deduce that the cardinality of Λ_N coincides with the cardinality of N ($\#\Lambda_N = \#N$). Axiom **D2** just says that particles are represented by ordered pairs[2], where the first element intuitively corresponds to measurement values of all the intrinsic physical properties, while the second element corresponds to the hidden inner property that allows us to distinguish particles at an ontological level. Yet, axioms **D2** and **D3** guarantee that two particles that share the same values for their hidden variable are the very same particle, since our structure is set-theoretical and the equality = is the classical one. Axiom **D4** says that macroscopic objects that are physically indistinguishable, are necessarily identicals. Axiom **D5** says that two particles physically indistinguishable that are not ontologically indistinguishable (they are ontologically distinguishable) are both microscopic particles. Axiom **D6** means that a particle is either microscopic or macroscopic, but not both.

Axiom **D4** deserves further explanation. Let us observe that it was not postulated the existence of (in particular) micro-objects; but the axiomatic is compatible with such an hypothesis. Axiom **D4** entails that (ontologically) distinct macro-objects are always distinguished by a measurement value; if two particles are macro-objects, then there exists a value for a measurement which distinguish them. Then, macro-objects, in particular, obey Leibniz's Principle of the Identity of Indiscernibles and we may say that (according to our axiomatics) classical logic holds with respect to them while micro-objects may be physically indistinguishable without the necessity of being 'the same' object.

In [26] the axiomatic framework for a system of ontologically distinguishable particles is a little different from the present formulation. The main difference is on axiom **D5**, which does not exist in [26]. Such an axiom is necessary to prove the theorem that we present in the next subsection.

We discuss in [26] how our approach is out of the range of the well known proofs on the impossibility of hidden variables in the quantum theory, like von Neumann's theorem, Gleason's work, Kochen and Specker results, Bell's inequalities or other works where it is sustained that no distri-

[2]In [4] the authors discuss the possible representation of quantum particles by means of ordered pairs $\langle E,L \rangle$, where E corresponds to a predicate which in some way characterizes the particle in terms, e.g., of its rest mass, its charge, and so on, while L denotes an appropriate label, which could be, for example, the location of the particle in space-time. Then, even in the case that the particles (in a system) have the same E, they might be distinguished by their labels. But if the particles have the same label, the tools of classical mathematics cannot be applied, since the pairs should be identified. In order to provide a mathematical distinction between particles with the same E and L, these authors use quasi-set theory [13] [14]. In the present picture, according to axioms **D1–D3**, it is prohibited the case where two particles have the same (ontological) label.

bution of hidden variables can account for the statistical predictions of the quantum theory [3].

2.1. *What about indistinguishability?*

It is natural to ask: what about the quantum phenomena of the microscopic world where indistinguishability plays an important role?

In order to get the quantum distribution functions, e.g., it is necessary to assume that quantum particles may be indistinguishable in some sense precisely established. In the case of fermions, it is also assumed *Pauli's exclusion principle*.

Loosely speaking, Pauli's principle states that two or more fermions cannot occupy the same state. This happens because a state like $| k' \rangle \, | k' \rangle$ is necessarily symmetrical, which is not possible for fermions. Different states also cannot be used to label fermions, since a fermion can change its state. In the case of bosons, the situation is more dramatic, since we may have several bosons occupying the same single state. Then, even if we have a collection of physically indistinguishable bosons or physically indistinguishable fermions, is it possible to find a reasonable way to express this into the framework of a standard set theory? In our picture, the answer is positive. But before we do that let us state the following theorem:

Theorem 2.1 If X is a unitary set and P has more than one element (the cardinality of P is greater than 1*) then* $(\forall p \in P)(m(p))$.

Proof: Consider $x \in X$ as the unique element of X. Since P has more than one element then there exist $p = \langle x,\lambda_i \rangle$ and $q = \langle x,\lambda_j \rangle$ such that $p \neq q$. Thus $\lambda_i \neq \lambda_j$, according to axiom **D3**. Therefore p and q are microscopic particles according to axioms **D5** and **D6**.□

In order to cope with, e.g., a collection of fermions we consider, as a first assumption, a $\mathfrak{D}_0$-system with X as a unit set and several particles, that is, all particles are microscopic. So, fermions are microscopic particles because they are physically indistinguishable objects. It should be emphasized that by considering fermions, we assume that the unique element $x \in X$ corresponds to the measurement values of all state-independent properties. Our second assumption is Pauli's exclusion principle. But before that, we need to establish the meaning of the symmetrical and the antisymmetrical states.

Without loss of generality, we consider a system of two physically indistinguishable particles, ontologically labeled as particles $\langle x,\lambda_1 \rangle$ and $\langle x,\lambda_2 \rangle$, or simply λ_1 and λ_2, if there is no risk of confusion. If we are concerned with a rigorous notation, we should denote the state of particle λ_1, in the Hilbert space formalism, as $| k', \langle x,\lambda_1\rangle\rangle$, where k' corresponds to the extra 'physical' information that is not available at x. But we prefer the abbreviate notation. The same state is denoted as $\left|k'_{\lambda_1}\right\rangle$. Likewise, we denote the ket of the remaining particle by $\left|k''_{\lambda_2}\right\rangle$. Hence, suppose that particle λ_1 is characterized by the state vector $\left|k'_{\lambda_1}\right\rangle$, where k' stands for a collective index for a complete set of observables (commuting or not). The state ket for the two particles system is

$$\left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle. \tag{1}$$

If a measurement is performed on this system, it may be obtained k' for one particle and k'' for the other one. But, actually, it is not possible to know if the state ket of the system is $\left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle$, $\left|k''_{\lambda_1}\right\rangle \left|k'_{\lambda_2}\right\rangle$ or any linear combination $c_1 \left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle + c_2 \left|k''_{\lambda_1}\right\rangle \left|k'_{\lambda_2}\right\rangle$. This is called the exchange degeneracy, which means that to determine the eigenvalue of a complete set of observables is not sufficient for uniquely specifying the state ket.

Using a notation similar to [25] we define the permutation operator P_{12} by

$$P_{12} \left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle = \left|k''_{\lambda_1}\right\rangle \left|k'_{\lambda_2}\right\rangle. \tag{2}$$

It is obvious that $P_{21} = P_{12}$ and that $P_{12}^2 = 1$. In the case of fermions:

$$P_{12} \left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle = - \left|k'_{\lambda_1}\right\rangle \left|k''_{\lambda_2}\right\rangle, \tag{3}$$

or, in the more general situations:

$$P_{ij} \mid n \text{ physically indistinguishable fermions}\rangle$$

$$= - \mid n \text{ physically indistinguishable fermions}\rangle, \tag{4}$$

where P_{ij} is the permutation operator that interchanges the particle ontologically labeled λ_i with the particle ontologically labeled λ_j, where i and j are arbitrary but distinct elements of N.

In our picture, it is possible to count fermions, since anyone of them is associated to a particular distinct label. So, we may deal with collections of fermions as sets. It is also clear what is meant by saying that a system of fermions is totally antisymmetrical under the interchange of any pair, since the meaning of the word 'interchange', according to equation (2), was made clear. Thus, we observe that by equation (2),

$$P_{12}\left|k'_{\lambda_1}\right\rangle\left|k'_{\lambda_2}\right\rangle=\left|k'_{\lambda_1}\right\rangle\left|k'_{\lambda_2}\right\rangle, \tag{5}$$

which contradicts equation (4). Hence, as expected, fermions cannot 'occupy' the same physical state, which is the exclusion principle in our language of hidden variables.

Since we have characterized the permutation operator, the symmetrical and the antisymmetrical states, Pauli's exclusion principle and the labeling of quantum particles, now it is possible to deduce the quantum distribution functions.

Consider, for that, as exemplified in [10], a narrow energy range of about $10^{-33}J$, which is usually referred to as an *energy bin*. Such a bin contains about 10^{19} discrete energy levels. In order to get the quantum distribution function for bosons, we need to calculate the number of ways of distributing ν physically indistinguishable particles within K discrete energy levels, which is given by:

$$I=\frac{(K+\nu-1)!}{(K-1)!\,\nu!}. \tag{6}$$

We could also say that I corresponds to the number of ways of choosing K occupation numbers N_i, such that $\sum_i N_i=\nu$, where i denotes i-th discrete energy level.

In the case of fermions, N_i can be either 0 or 1 only. In this case, we have:

$$I=\frac{K!}{(K-\nu)!\,\nu!}. \tag{7}$$

The important point is that K and ν are interpreted as cardinalities of sets. K corresponds to the cardinality of the set of discrete energy levels within a certain bin and ν is the cardinality of the set of physically indistinguishable particles (either bosons or fermions) within a certain bin. These physically indistinguishable particles may be viewed as elements of a set, if we consider them as the ordered pairs, as presented in our axiomatic framework.

From now on, the calculation of quantum statistics is standard. For details see [10].

2.2. *The Helium Atom*

The helium atom is the simplest realistic situation where the problem of individuality plays an important role. In [26] we do not discuss this important point, but see [16]. With the identity question ignored, the wave function of the helium atom should be just the product of two hydrogen atom wave functions with Z = 1 changed to Z = 2. Nevertheless, the space part of the wave function for the case where one of the electrons is in the ground state (100) and the other one is in excited state (nlm) is:

$$\phi(\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{\sqrt{2}}[\psi_{100}(\mathbf{x}_1)\psi_{nlm}(\mathbf{x}_2) \pm \psi_{100}(\mathbf{x}_2)\psi_{nlm}(\mathbf{x}_1)], \tag{8}$$

where the + (–) sign is for the spin singlet (triplet)[3] and $\mathbf{x}_1$ and $\mathbf{x}_2$ are the vector positions of both electrons.

For the ground state, however, the space function must be necessarily symmetric. In this case, the problems regarding identity have no physical effect. The most interesting case is certainly the excited state. From a realistic point of view, equation (8) reflects our ignorance on which electron is in position $\mathbf{x}_1$ and which one is in position $\mathbf{x}_2$. Nevertheless, in the same equation there are terms like $\psi_{100}(\mathbf{x}_1)$, which correspond to a specific physical property of an individual electron, labeled by its position $\mathbf{x}_1$ in space.

We could interpret our hidden variables λ_i as the space-time positions of particles if we adopt Bohmiam mechanics to describe quantum phenomena. According to Bohm's causal interpretation for quantum mechanics, elementary particles do have well defined trajectories [3], despite the fact that the followers of Bohm's ideas do not consider space-time trajectories as legitime hidden variables [11].

2.3. *Hidden variables and the incompleteness of quantum physics*

Another natural question is the following: are we assuming that quantum physics is an incomplete theory, since we are using a sort of hidden variable formalism? The answer is: not necessarily.

There is a new technique in quantum optics called *interaction-free measurement*. This technique was predicted by Elitzur and Vaidman [8] and Kwiat et al. performed a preliminary demonstration of such an effect in laboratory [17]. It is basically a consequence of one of the most intriguing aspects of quantum mechanics, namely, nonlocality. Elitzur and Vaidman showed that it is possible to ascertain the existence of an object in a given region of space without interacting with it. Some people have questioned if such a technique violates Heisenberg's uncertainty principle [19]. We do not want to discuss the details about that. But certainly some foundational aspects of quantum theory should be revised.

[3] Spin singlet refers to total spin zero and spin triplet refers to total spin different of zero.

We question if it is possible to propose an experiment which allows us to keep track of individual electrons in order to distinguish among them by using the technique first proposed by Elitzur and Vaidman. If that is possible, then we could interpret our hidden variables as the space-time trajectories of the elementary particles. Following Schrödinger's terminology [28], that would be a manner to mark or to 'paint' electrons. Since our concept of physical indistinguishability refers just to the set of measurement values of intrinsic properties, it seems reasonable to consider that our hidden variables may correspond to the history of each particle, i.e., the space-time trajectory.

We recognise the problems about these ideas, if we restrict our discussions from the point of view of the *usual interpretation* for quantum physics. Nevertheless, we recall that there are other interpretations [2] [3] [31] [32] [33].

We return to this discussion in Section 4.

It seems clear that the way physicists usually do in labelling particles, despite the dificulties this procedure causes with respect to the foundations of physics [5] [14], it has a 'rationale' by itself.

Hence, by assuming some sort of distinguishability among elementary particles that share the same set of intrinsic properties, we do not contradict quantum distribution functions or other phenomena of the quantum world where (physical) indistinguishability plays a decisive role like the Helium atom.

The importance of indistinguishability among elementary particles in the case of interference is discussed at the end of the paper.

3. *Quasi-set theory and quantum theory*

Quasi-set theory is based on Zermelo-Fraenkel axioms and permits to cope with collections of indistinguishable objects by allowing the presence of two sorts of atoms (*Urelemente*), termed m-atoms and M-atoms [13] [15]. A binary relation of indistinguishability between m-atoms (denoted by the symbol $\equiv$), is used instead of identity, and it is postulated that $\equiv$ has the properties of an equivalence relation. The predicate of equality cannot be applied to the m-atoms, since no expression of the form $x = y$ is a formula if x or y denote m-atoms. Hence, there is a precise sense in saying that m-atoms can be indistinguishable without being identical.

The universe of quasi-sets is composed by m-atoms, M-atoms and *quasi-sets*. The axiomatics is adapted from that of ZFU (Zermelo-Fraenkel with *Urelemente*), and when we restrict the theory to the case which does not consider m-atoms, quasi-set theory is essentially equivalent to ZFU, and the corresponding quasi-sets can then be termed 'ZFU-sets'. The M-atoms play the role of the *Urelemente* in the sense of ZFU.

In order to preserve the concept of identity for the 'well-behaved' objects, an *Extensional Equality* ($=_E$) is introduced for those entities which are not m-atoms on the following grounds: for all x and y, if they are not m-atoms, then $x =_E y$ corresponds to say that $\forall z(z \in x \leftrightarrow z \in y) \vee (M(x) \wedge M(y) \wedge x \equiv y)$. It is possible to prove that $=_E$ has all the properties of classical identity.

It seems reasonable to assume that the hidden variables formalism presented above may be an interpretation for quasi-set theory. The M-atoms and m-atoms of quasi-set theory should be interpreted, respectively, as the 'macroscopic particles' and 'microscopic particles' of the hidden variables formalism. The binary relation $\equiv$ of quasi-set theory should be interpreted as the 'physical indistinguishability' relation of the hidden variables formalism, which has a second binary relation called 'ontological indistinguishability' with no correspondence in quasi-set theory.

In [16] a quasi-set theory for bosons and fermions is presented. The authors obtain the quantum distribution functions (Fermi-Dirac and Bose-Einstein) and discuss the Helium atom. Unfortunatelly, we cannot give all the details about [16] since that paper is still being prepared. Besides, to use quasi-set theory for dealing with collections of quantum particles means that we consider non-individuality right at the start and this is an assumption that we intend to avoid in this paper. As we remarked in the beginning of our text, our intention is to work on the possibility that elementary particles may be considered as individuals of some sort.

Nevertheless, even in quasi-set theory non-individuality may be naturally interpreted as an individuality which is somehow 'veiled'.

4. *Virtual photons*

Consider two electrons at points A and B in space-time, ending up at points C and D. Such an event can happen in several different ways. There can be, for example, a photon exchange between the two electrons, which is not detected in the initial or final conditions of the experiment. Such an exchanged photon is usually called a 'virtual photon', as remarked by R.P. Feynman [9]. Due to the fact that we cannot label electrons, there is no way to know if the electron at point C is the same one which was at point A. So, there is no way to know if actually a virtual photon was exchanged or not.

Returning to our previous discussion about interaction-free measurements, we wonder if it is possible to keep track of both electrons in order to distinguish between them by using some variation of the experiment proposed by Elitzur-Vaidman. That is just a conjecture, but motivated by experimentation and by the formalism of quantum mechanics itself.

There are other kinds of 'virtual photons' in the literature. We call the virtual photon refered to above as f-virtual-photon. The letter f accounts for *Feynman*.

Suppes and de Barros [31] began a foundational analysis on diffraction of light. It was formulated a probabilistic theory of photons with well-defined trajectories. The wave properties come from the expectation density of the photons. The photons are also regarded as *virtual*, because they are not directly observable, including their anihilation of each other (according to the assumptions given below). What can be detected is the photon-matter interaction. The meaning of *virtual* used here is not the same as in the case of f-virtual-photons. The assumptions are:

- Photons are emitted by harmonically oscillating sources;
- They have definite trajectories;
- They have a probability of being scattered at a slit;
- Absorbers, like sources, are periodic;
- Photons have positive and negative states (+-photons and −-photons) which locally interfere, when being absorbed;
- Photons change their states when reflected by a perfect conductor.

In this particular description for photons, we call virtual photons as s-virtual-photons.

The assumptions given above are supposed to be somehow natural for particles. From these assumptions we can define fields as we do below, as a purely probabilistic concept. Consequently, the probabilistic properties of the defined field are derived in a manner familiar from stochastic processes from the properties of collections of the sample paths, i.e., trajectories of photons. It remains to be seen whether this reduction of fields to distributions of particles with linear trajectories can be carried through for all phenomena usually described by QED, like Aharanov-Bohm effect, anomalous magnetic moment of the electron, Lamb shift, Casimir effect, etc.. For now, such a conjecture seems unlikely, since there is no process of quantization to justify our assumption that the vacuum state has energy $\frac{1}{2}\hbar\omega$ and momentum $\frac{1}{2}\hbar k$, where $k = \omega/c$, and ω is the frequency of the source which 'generates' the s-virtual-photons.

In the case of the Casimir effect, the particle model presented is used to calculate pressure exerted on certain plates in the spirit of the particle theory of classical statistical mechanics. Nevertheless, we do not use (in principle) any quantization process to get the reduced Planck's constant $\hbar$. It is one of our proposed open problems to define a transformation group from which Planck's constant may naturally arise. It could be, e.g., the conformal group, since from the Poinçaré group we get only the speed of light c.

The expected density of ±-photons emitted at instant t in the time interval dt is given by

$$s_{\pm}(t) = \frac{A_s}{2}(1 \pm \cos \omega t), \tag{9}$$

where ω is the frequency of a harmonically oscillating source, A_s is a constant determined by the source, and t is time. If a photon is emitted at t', $0 \leq t' \leq t$, then at time t the photon has traveled (with speed c) a distance r, where

$$t - t' = \frac{r}{c}. \tag{10}$$

The conditional space-time expectation density of ±-photons for a spherically symmetric source with given periodicity ω is:

$$h_{\pm}(t,r|\omega) = \frac{A}{8\pi r^2}\left(1 \pm \cos \omega\left(t - \frac{r}{c}\right)\right), \tag{11}$$

where A is a real constant.

The scalar field defined in terms of the expectation density $h_{\pm}(t,r|\omega)$ is

$$\varepsilon = \varepsilon_0 \frac{h_+ - h_-}{\sqrt{h_+ + h_-}}, \tag{12}$$

where ε_0 is a scalar physical constant. Using (11), (12) may be rewritten for a spherically symmetric source as:

$$\varepsilon = \varepsilon_0 \sqrt{\frac{A}{4\pi r^2}} \cos \omega\left(t - \frac{r}{c}\right). \tag{13}$$

Suppes et al. [32] applied the particular description for photons in order to explain the Casimir effect [23]. We do not discuss here the details about the particular description for diffraction of light because it does not provide any insight about the very nature of photons with respect to individuality. Since the explanation for the Casimir effect is presented in [32] by using some usual techniques derived from statistical mechanics, we concentrate our discussions on this topic.

In the cited paper it was first considered the simplest case, where two perfectly conducting parallel plates, stand face to face in vacuum at a distance d much smaller than their lateral extensions. It is well known that such plates attract each other with a pressure due to the vacuum energy, as predicted by Casimir, given by

$$P = -\frac{\pi^2 \hbar c}{240 d^4}. \tag{14}$$

Usually, such an attraction is explained in terms of the vacuum field. We consider that in the vacuum there is a non-vanishing energy which could be associated to virtual photons in a sense to be made precise. A random distribution of oscillating sources of photons, in the vacuum, which do not interfere with each other, is used to derive (14), as it follows.

The photons outside the plates that strike such surfaces act to push the plates together, while reflections of the photons confined between the plates push them apart. This idea was proposed by Milonni, Cook, and Goggin [22] and also presented in [23], but not actually developed from a pure particle viewpoint. Milonni presented virtual photons as particles associated to the quantum vacuum state. Such particles are called, in this paper, as v-virtual-photons, where the letter v accounts for *vacuum*.

The particles that we are considering should satisfy a probability density $f(k) \geq 0$. Rather than assume an explicit expression for $f(k)$ (which requires some assumptions about the virtual photons), it is preferred to state some properties that $f(k)$ must satisfy:

(i) $\int_0^\infty \int_0^\infty \int_0^\infty f(k) dk_x dk_y dk_z = 1$, and the mean and the variance of $f(k)$ are finite;

(ii) There exists a constant H such that $h(t, r_S | k) < H$;

(iii) $h(t, r_S | k) f(k)|_{k=0} = 1$, and all derivatives of this expression vanish at $k_z = 0$.

From (i)~(iii) and assumptions made earlier, it may be inferred that the number of v-virtual photons is finite for any bounded region of space-time. It may be also inferred that $h(t, r_S | k) f(k)|_{k=\infty} = 0$, which is intuitively an expected property of a cutoff function.

The *xyz* space is divided into parallelepipeds of sides L_x, L_y, and L_z, as in the usual description of QED. So, all k_x, k_y, and k_z must assume discrete values, as it is explained in the next paragraphs.

A photon changes its state from positive to negative and vice versa when it is reflected. This implies that the defined scalar field, given by (12), vanishes at the reflecting surface. So, according to (13) and recalling that $k = \omega/c$, we have at the wall:

$$\cos\left(\omega t - \omega \frac{r}{c}\right) = \cos(\omega t - kr) = 0. \tag{15}$$

If we set $\omega t = \pi/2$, which corresponds to a convenient choice for the origin of time, it is easy to see that the values of k_x, k_y, and k_z that satisfy the boundary condition in $x = L_x$, $y = L_y$, and $z = L_z$ are:

$$\frac{k_{x,y,z}}{\pi} = \frac{n}{L_{x,y,z}}. \tag{16}$$

Notwithstanding, one natural question arises: what about the photons with linear momenta that do not satisfy (16)? We recall that reflectors, like absorbers [31], behave periodically, since the photons are continuously hitting the plates. Thus, the probability of reflecting a photon is given by:

$$p = C(1 + \cos(\omega t + \psi)), \tag{17}$$

where ψ is a certain phase. If $p = 0$, then there is no interaction with the plates, which means that no momentum is delivered to it.

As an example, consider the first strike of a photon on a plate perpendicular to the z axis. Such a surface is not oscillating before the strike. But after reflection, the wall oscillates with the same frequency ω associated to the linear momentum $k = \omega/c$. The particle reflects on the other wall and returns to the first wall with a phase $2L_z k_z$. But we must have $2L_z k_z = 2n\pi$, from (17), if the particle is to be reflected again on its return to the first wall. Obviously, $\cos(\omega t - 2L_z k_z) = \cos(\omega t)$ if and only if $2L_z k_z = 2n\pi$.

If we consider $L_{x,y,z}$ very large compared with any physical dimensions of interest, we can assume that the $k_{x,y,z}$ approach a continuum. This is what holds for photons outside the plates.

Now, the inward pressure: the expected number of photons that strike the area dS of one of the plates, i.e., have trajectories in the direction of the plates, within the time interval dt is

$$h(t, r_S|k) f(k) \frac{1}{\pi^3} dk_x dk_y dk_z \cos\gamma \, c \, dt \, dS, \tag{18}$$

where γ is the angle of incidence of the photons on the plate with respect to the normal of the surface, i.e., $\cos\gamma = k_z/k$, where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. Thus, the element of volume that we are taking into account is $\cos\gamma \, c \, dt \, dS$. The factor $\frac{1}{\pi^3}$ is justified by (16), since outside the plates we approach the continuum as an idealization or a limit.

The momentum delivered to the plate by a single reflected photon is equal to the negative of the change in the momentum of the photon. Hence, the momentum is equal to $2\frac{1}{2}\hbar k_z$, if we consider the plate perpendicular to the z component of the xyz system of coordinates. Therefore, the expected linear momentum transfered to an area dS on the plate during the time interval dt is

$$\frac{\hbar}{\pi^3}\frac{k_z^2}{k}h(t,r_S|k)f(k)dk_x dk_y dk_z c\, dt\, dS. \tag{19}$$

The force on the plate is obtained by dividing (19) by dt. The pressure is obtained by dividing the force by dS. We denote the inward pressure as P_{in} and the outward pressure as P_{out}. Hence:

$$dP_{in} = \frac{\hbar c}{\pi^3}\frac{k_z^2 h(t,r_S|k)f(k)}{\sqrt{k_x^2+k_y^2+k_z^2}}dk_x dk_y dk_z. \tag{20}$$

Integrating over momentum:

$$P_{in} = \frac{\hbar c}{\pi^3}\int_0^\infty dk_x \int_0^\infty dk_y \int_0^\infty dk_z \frac{h(t,r_S|k)f(k)k_z^2}{\sqrt{k_x^2+k_y^2+k_z^2}}. \tag{21}$$

The equation given above is identical to a result due to Milonni, Cook and Goggin [22], if we consider that $h(t,r_S|k)\,f(k)$ has the role of the usual cut-off function.

To get the expression for the outward pressure we use similar arguments. Nevertheless, because of the small distance d between the plates, we must take into account the periodicity given in (16), at least for the z component. For this small distance d, the continuum approach does not hold. Thus:

$$P_{out} = \frac{\hbar c}{\pi^2 d}\sum_{n=1}^{\infty}\int_0^\infty dk_x \int_0^\infty dk_y \frac{h(t,r_S|k)f(k)\left(\frac{n\pi}{d}\right)^2}{\sqrt{k_x^2+k_y^2+\left(\frac{n\pi}{d}\right)^2}}. \tag{22}$$

We note that it follows from (i)~(iii) that P_{in} and P_{out} are both finite.

The resultant pressure is given by:

$$P_{out} - P_{in} = \frac{\pi^2\hbar c}{4d^4}\sum_{n=1}^{\infty} n^2 \int_0^\infty dx \frac{h(t,r_S|x,u)f(\sqrt{x+n^2})}{\sqrt{x+n^2}} - \frac{\pi^2\hbar c}{4d^4}\int_0^\infty du\, u^2 \int_0^\infty dx \frac{h(t,r_S|x,u)f(\sqrt{x+u^2})}{\sqrt{x+u^2}}, \tag{23}$$

where, by change of variables, $f(k) = f(\sqrt{x+u^2})$, $x = x'^2 = \frac{k_x^2 d^2}{\pi^2} + \frac{k_y^2 d^2}{\pi^2}$, $u = k_z \frac{d}{\pi}$, $\theta = \tan\left(\frac{k_y}{k_z}\right)$, and $dk_x dk_y = x'dx'd\theta \frac{\pi^2}{d^2}$. The expression $h(t,r_S|x,u) f(\sqrt{x+u^2})$ corresponds to a cutoff function. In this conceptual framework $h(t,r_S|k)$ is bounded and $f(\sqrt{x+u^2})$ has the physical interpretation of a probability density of the frequencies of the photons.

Frequently it is assumed that the cutoff function has the property of going to zero as k approaches infinity and going to one when k approaches zero. This is justified physically with the hypothesis that the conductivity of the reflecting conductors decreases to zero as the frequency gets high. Since $h(t,r_S|k)$ is bounded, it is easy to see that the product $h(t,r_S|k)\, f(k)$ must assume a similar role with respect to the cutoff, from a mathematical standpoint. According to the mathematical assumptions that were made about $h(t,r_S|k)$ and $f(k)$, all the properties of a cutoff function are satisfied for $h(t,r_S|k)\, f(k)$.

If we consider:

$$F(u) = u^2 \int_0^\infty dx \, \frac{h(t,r_S|x,u) f(\sqrt{x+u^2})}{\sqrt{x+u^2}}, \tag{24}$$

it is clear that the Euler-MacLaurin sum formula [1] may be applied to (23). The factor that is multiplying $\frac{\pi^2 \hbar c}{4d^4}$ in (23) may be written as:

$$\sum_{n=1}^{\infty} F(n) - \int_0^\infty du F(u) = -\frac{1}{2} F(0) - \frac{1}{12} F'(0) + \frac{1}{720} F'''(0) \ldots \tag{25}$$

for $\lim_{u\to\infty} F(u) = 0$, since $\sum_{n=1}^{\infty} F(n)$ is finite and so $\lim_{n\to\infty} F(n) = 0$. Note that $F(0) = 0$, $F'(0) = 0$, $F'''(0) = -12h(t,r_S|0)$, and all higher derivatives $F^{(n)}(0)$, where n is odd, vanish in accordance with assumption (iii). Since by (iii) $h(t,r_S|0)f(0) = 1$:

$$P_{out} - P_{in} = -\frac{\pi^2 \hbar c}{240 d^4}. \tag{26}$$

Equation (26) is identical to (14), which completes our derivation of the Casimir effect for parallel plates.

In [32] the case for the solid ball is also discussed. Actually, the literature about the Casimir effect is huge. We do not intend to discuss about this in the present paper.

4.1. *Remarks*

One of our points in this Section is about the identity issues of 'virtual photons'. If we refer to f-virtual-photons, it is obvious that we are talking about a possible exchanging of photons (in the usual sense). Nevertheless, if we are talking about virtual photons in the sense of particles associated to the quantum vacuum state, as in [22], the question is: are these virtual photons indistinguishable? Milonni et al. [22] refer to those virtual photons as particles. Nevertheless, their calculations for the Casimir effect are performed in terms of the vacuum field and not in terms of particles. In [32] Suppes et al. make all the calculations for the Casimir effect by assuming a particular description. Their method is very similar to the usual treatment given for statistical dynamics. So, those photons should satisfy a probability density $f(k) \geq 0$. Rather than assume an explicit expression for $f(k)$ (which requires some assumptions about the virtual photons), it was preferred by the authors to state some few general properties that $f(k)$ should satisfy.

Let us analyze some possibilities:

1. The particular description by Suppes et al. is classical in the sense that there is no process of first or second quantization. So, one reasonable assumption is to consider that s-virtual-photons and, in particular, v-virtual-photons, are classical particles, i.e., distinguishable particles. Thus, they should satisfy Maxwell-Boltzmann statistics. Nevertheless, there is no choice of parameters in Maxwell-Boltzmann statistics which allows an agreement between equation (23) and the usual descriptions for the Casimir effect. Besides, there are experimental facts which cannot be ignored. Lamoreaux [18], e.g., measured the attraction between a gold-plated sphere and a gold plate, agreeing with the usual theory to within 5%. So, it seems that the usual descriptions for the Casimir effect are very reasonable.

2. If s-virtual-photons do not satisfy Maxwell-Boltzmann statistics then we could admit the possibility that they are indistinguishable, despite the classical picture. There are two usual statistics for indistinguishable particles, namely, Bose-Einstein and Fermi-Dirac. Bose-Einstein applies for photons, for example. Nevertheless, there is again no choice of parameters in Bose-Einstein statistics which allows an agreement between equation (23) and the usual description. In certain cases Fermi-Dirac may be used. We mean by that that Fermi-Dirac distribution function may be used in equation (23) in order to get equation (26). Nevertheless, it seems risky to assume

that s-virtual-photons do satisfy Fermi-Dirac statistics. Usually, such an statistic is associated to particles with half integer spin.

3. One could investigate the possibility that s-virtual-photons do satisfy other statistics than Maxwell-Boltzmann, Fermi-Dirac or Bose-Einstein.

We should recall that this particle description is a local theory of photons which violates Bell's inequalities [33]. Our intention is not to advocate the particular description of photons as a better alternative than QED. Our intention is to compare alternative ideas and to try to understand possible consequences of such ideas. These ideas may be seen as tools for a better understanding of the microscopic world and even of QED. The possibility of a classical explanation for elementary particles seems at least interesting, mainly for those people who are concerned with questions like non-locality, wave-particle duality and indistinguishability.

5. *Final remarks*

1. It is usually considered that the interference produced by two light beams is determined by both their mutual coherence and to the indistinguishability of the quantum particle paths. Our present discussion in this paper is focused only on the 'corpuscular' features of quantum particles, in the sense that we are not dealing with coherence. Mandel has proposed a quantitative link between the wave and the particle descriptions by using an adequate decomposition of the density operator [20]. He considers, in his calculations, a density operator corresponding to an incoherent mixture of states, which would be associated, in principle, to some experimental setup that allows to identify the source of the detected photon. Mandel does not present such an experimental setup. In the particular description of photons by Suppes et al. indistinguishability is not assumed and all wave properties come from the expectation density of the photons. So, in principle, there is no link between indistinguishability and coherence.

2. The unique difference between the electron and the negative muon is their rest mass [12]. This intriguing physical phenomenon has motivated Dirac [7] to propose a model for the electron in terms of a membrane which should allow to assume that the muon is an electron in an excited 'state'. There are two important points about this.

First, if an electron is a membrane, then it may be individualized in some sense. The second point is a bit more critical. If the muon is an excited electron then it is possible, using Schrödinger's terminology [28], to paint electrons, at least in principle. In other words, it is possible, in principle, to mark or individualize electrons by changing their intrinsic properties, mainly for those cases where there are just two electrons.

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ACKNOWLEDGMENT

I would like to thank D. Krause for extensive comments on the present work.

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