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REMARKS ON INDIVIDUATION, QUANTUM OBJECTS AND LOGIC

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Abstract

Some general points regarding *individuation* are considered in conformity with the so called 'negative' theories concerning the concept of substance. We discuss in what sense logic may play an important role in describing what might be considered to be an 'individual'. In the particular case of individuation and bundle theories, the necessity of considering the validity of Leibniz Law is emphasized. Then, taking into account a plausible view regarding elementary particles according to which they are characterized by no more than a certain collection of properties (intrinsic properties), we conclude that if we agree in considering both the particles as entities devoid of individuation and quantum mechanics as a theory which is not 'incomplete' with regard to individuation, then in its underlying logic Leibniz Law cannot be valid.

1. *Basic Assumptions*

Let us assume, *a la* Russell, that an individual is a bundle of qualities.[1] So, there is no any kind of *substratum, haeceitty, primitive thisness* [26], *quid* of some kind of 'transcendental individuality' [22], [11] being supposed. Furthermore, by Leibniz Law we understand the principle which informally says that 'if individuals a and b have all the same qualities, then $a = b$ and conversely', where $a = b$ means that there are not two distinct items, but only one which can be referred to indifferently as either a or b. In other words, 'two' individuals which have all the same qualities are the very

[1] See [20, p. 9]. We are avoiding the discussion if relational properties must or not be considered among the qualities of a thing since our arguments do not depend on this point. We still use the terms 'property' and 'attribute' as synonymous for 'quality'. All the discussion below can be done in a higher-order denumerable language [15], and we still suppose that all the attributes (formulas of that language with just one free varible) are universal in the sense that for a certain object there is not a peculiar property which individualizes it (see [13]).

same individual, and vice-versa. In a second-order language with identity, this principle may be written:[2] for all a and b,

$$a = b \leftrightarrow \forall P\,(P(a) \leftrightarrow P(b)) \quad (1)$$

Suppose now that we have a non-empty set Δ and let $\mathcal{P}$ be the class of the attributes of the elements of Δ, that is, for every $x \in \Delta$, if there exists P such that $P(x)$ holds, then $P \in \mathcal{P}$. Taking into account what was said in the footnote above about the language, without loss of generality we may think of $\mathcal{P}$ as a denumerable set, so that we may list its elements: $P_1, P_2, \ldots$. Then we state the following definition:

Definition 1.1

1. *For every $x \in \Delta$, the set of the attributes of x is denoted $\mathcal{P}_x$. In other words, $\mathcal{P}_x \subset \mathcal{P}$ and $P_j(x)$ for every $P_j \in \mathcal{P}_x$, while $\neg P_k(x)$ when $P_k \notin \mathcal{P}_x$.*[3]
2. *For every $x \in \Delta$, we call* rank(x) *the least integer λ (if it exists) such that there exist λ predicates $P_{x_1}, P_{x_2}, \ldots, P_{x_\lambda}$ in $\mathcal{P}$ whose collection* individualizes *x. So, rank(x) might be viewed as the cardinal of the collection of the 'essential qualities' of x.*
3. *If x and y share the same set C of predicates, we say that x and y are* indistinguishable with respects to *that set of attributes, and write $x \equiv_C y$.*

The sense of the words 'individualizes x' at item 2 should be understood by observing the concept of 'individual' being considered; for every x and y, if they have the same 'essential' attributes (which entails that $rank(x) = rank(y)$), then they are the same individual; alternatively, if they disagree in what respects to at least one of their essential attributes or have distinct rank, then they are distinct individuals. In symbols, if $x \neq y$, then $rank(x) \neq rank(y) \vee \exists P\,(P(x) \wedge \neg P(y))$, where P range over the collections of attributes of both x and y. This result is a version of Leibniz' Principle of the Identity of Indiscernibles.

[2] Perhaps the most famous use of this expression as defining the concept of identity is Whitehead and Russell's *Principia Mathematica* (Section B, *13). It should be recalled that Ramsey considered such a definition as one of the 'serious defects' in Whitehead and Russell's work [21, pp. 180ff].

[3] Note that if we admit that for a certain x, $\mathcal{P}_x \subseteq \mathcal{P}$ then, if the equality holds, x would be identical to every element of Δ, hence, by Leibniz Law (1), Δ would have just one element.

Let us remark that the philosophical literature sometimes makes restriction on predicates like $I_a \in \mathcal{P}$ defined in such a way so that for a certain $a \in \Delta$, $I_a(x) \leftrightarrow x = a$. Then, for every b in Δ, if b shares with a the predicate I_a, it results that $b = a$. In our case, we assume that that if $b \neq a$, then among the attributes of b there are no predicates like I_a.

2. *Individuals*

Let $\mathcal{A}$ be a structure

$$\mathcal{A} = \langle \Delta', P_k \rangle k \in K$$

where $\Delta' \subseteq \Delta$ and K encompasses only indices $x_1, ..., x_k$ for elements $x \in \Delta'$ with $x_k < rank(x)$. In other words, the considered qualities are chosen so that they do not individualize the elements of Δ'.

If we have a language L which has names $a, b, ...$ for the elements of Δ, then if we interpret L in $\mathcal{A}$, we may say that $\mathcal{A}$ acts as a 'partial structure' for the elements of Δ with respect to individuation.

Let us suppose in addition that $\mathcal{A}$ can be 'extended' to a *total* structure

$$\mathcal{B} = \langle \Delta, \mathcal{P} \rangle$$

where $\mathcal{P}$ is, as above, the set of all qualities of the elements of Δ.[4] From the above suppositions, in such a structure every element of Δ can be individualized;[5] hence it makes sense to state the following definition:

Definition 2.1 The elements $x \in \Delta'$ are individuals *if the partial structure $\mathcal{A}$ can be extended to a total structure $\mathcal{B}$ as above.*

This raises the following question. Why should such an $\mathcal{A}$ not be extended to $\mathcal{B}$?[6] In order to understand this question, let us analyse the relationship between the nature of the elements of Δ', the possibility of extending $\mathcal{A}$ to $\mathcal{B}$ and logic.

First of all, let us make the following 'Assumption':

[4] The idea of 'partial' and 'total' structures are taken in the sense of [18]; see also [3], [4].

[5] But see the restrictions below.

[6] Let us remark that when we say that $\mathcal{B}$ *extends* $\mathcal{A}$, we are not supposing necessarily that the collection of attributes of the objects of Δ can be effectively described, but that they exist *in principio*.

Assumption: The objects of Δ are something like the elements of a *set* of usual set theories or physical objects such as those described by classical mechanics.[7] In other words, our Assumption admits that the elements of Δ can be considered as *individuatable* entities,[8] that is, as objects which can be identified in time (in the second case), have *genidentity* or whatever you like that confers them an *identity*. Then we may consider the following cases:

1. $\mathcal{A}$ can be extended to $\mathcal{B}$. Two subcases may occur:

   (a) If Leibniz Law holds, then the elements of Δ' are individualized by their attributes (contained in $\mathcal{P}$) and in this case we may say that they are individuals in the sense of Def. 2.1.

   (b) If Leibniz Law does not hold,[9] then even in the case where a and b have all the same attributes, we cannot conclude that they are the same entity. In other words, the attributes are not sufficient to individualize the objects: something more is needed, namely, Leibniz Law. In this case, if a and b are distinct objects, some kind of 'transcendental individuality' might be considered as providing the distinction. This case could also be analysed in the following way: the negation of Leibniz Law says that it may be the case that $a \neq b$ but there is no $P \in \mathcal{P}$ such that $P(x)$ but $\neg P(y)$. In this case we may say that a and b differ *solo numero*. Leibniz Law can be violated also by assuming that there exists P as above but $a = b$, but this does not occur in the bundle theories, since such a P would be a quality of a but not of b, and this contradicts the fact that a and b are the very same object.

2. $\mathcal{A}$ cannot be extended to $\mathcal{B}$. Then:

   (a) If Leibniz Law is valid, then, although we cannot individualize the elements of Δ', there is a sense in saying that they can be thought of as if they are individuals, since *if* a total structure could be achieved, *then* they would be individualized. We

[7] As remarked by Redhead and Teller, 'classical' physical objects have a well-defined identity [22].

[8] The expression *individuatable* was coined by M. Redhead and P. Teller to mean entities which have well defined identity [22].

[9] We have presented some logical systems of this kind; see [16], [5], [17].

might say that the elements of Δ' are individualized only 'conceptually' (see [22]). That is, we could reason in the following way: if for a certain $\mathcal{A}$ the total structure $\mathcal{B}$ cannot be reached, that is, it is not possible to know all the properties of all the objects of Δ',[10] then we may say that all we have is 'partial' information on the objects of Δ', since they cannot be distinguished from one another even if Leibniz Law holds in the underlying logic. In this case, the $\mathcal{A}$ is an 'incomplete' structure; then a kind of a transcendental individuality might be considered, since the individuation of a thing would depend on some 'hidden' attribute.

(b) If Leibniz Law does not hold, then the elements of Δ' cannot be supposed as individualized even conceptually. In this case, the above Assumption is to be called into question, since the intuitive idea of an 'individual' apparently makes no sense. In other words, we cannot assure that the individuality of the elements of Δ' can be achieved and we have here an example of 'genuine' indistinguishable entities.

This last remark perhaps causes some perplexity. In fact, it seems that the very nature of the entities might depend on logic. In fact, if we are apt to accept only 'numerically distinct' objects, we may reject Leibniz Law and postulate that $\mathcal{A}$ can be extended to $\mathcal{B}$. But, if Leibniz Law does not hold, then the impossibility of extending $\mathcal{A}$ (into the scope of the negative theories concerning substance) is incompatible with any (intuitive) idea of an 'individual'. In the next section we will see that apparently this is the case with quantum particles.

3. *Quantum Objects*

The concept of physical object, that is, those entities which are treated by physical theories, has been recognized not only as of the fundamental importance for philosophical discussions on the foundations of physics but also as a very problematic one [27], [29], [7], [2], [22], [23], [26]. Of course there are particular instances of a more general concept of 'object' which have been largely discussed in literature by several authors (see the above references).

[10] By simplicity, let us suppose that this occurs for all the elements of Δ'.

Leaving aside physical objects in general, let us pay attention exclusively to the so called elementary particles, which constitute a very particular and intriguing kind of physical objects. The discussion could begin by questioning their existence, from those which sustain some kind of realist position to those which think of them as mere 'fictions', as a *façon de parler* we use in our theories only. Our hypothetical arguments of course do not depend on these topics. We take for granted, as present physics teaches us, that the concept of 'elementary particle' is defined by prescribing to these entities some characteristic features (such as rest-mass, spin, electric charge, angular momentum, ...) which characterize them in groups: all electrons have the same set of features which characterize the entity 'electron', and the same occurs with all protons, all neutrinos, and so on with their corresponding properties. This is the motive for considering bundle theories.

Ever since the first developments of quantum theory it has been recognized that this theory leads us far from the old (say, 'materialistic') concepts of ancient physics and intuition. Schrödinger [24], [25] and Heisenberg [14, p. 13], among other important authors, have mentioned this point. In short, elementary particles are entities to which even the term 'particle' seens to be dubious; contrarily to the usual macroscopic objects of our environment, which can be 'analysed' in order to describe their properties, elementary particles cannot be 'taken' in this way. Furthermore, there are 'particles' that cannot be observed, such as quarks and virtual particles. Even the 'properties' ascribed to these entities differ from those of the ordinary objects, since it does not make sense to say of an electron, for instance, that it is blue; in particular, it has been recognized that elementary particles have no *genidentity*, so as that in general they cannot be distinguished one from the others in certain situations and so on. A fundamental point is that their 'properties' are prescribed by physical laws: quantum objects are *nomological* [27], [28, p. 222]. Even those attempts to characterize quantum entities by other devices, such as by using transformation groups (as proposed by Wigner and Weyl), which is based on 'grupal' properties of symmetry transformations, permit us of distinguishing among classes of the objects of the theory, but not among objects themselves (see [2]).

Perhaps we may agree with Schrödinger in saying that it is impossible to describe reality as it 'objectively' is (cf. [1]) and then all we have are the concepts we describe in our theories, which in certain sense 'picture' the reality. In the particular case of quantum objects, all we have are the *ab initio* prescribed properties. In this sense, quantum mechanics is not an 'incomplete theory' (regarding individuation). In this sense, the impossibility to distinguish between photons is not due to our incapacity or due to the limitations of our measurement apparatus. Perhaps entities themselves are such that they *in fact* cannot be distinguished from one another in all the situations.

The discussion on this topic is of course very subtle, and we will not pursue it further here. Let us only mention that regarding quantum objects, it is well known that it has been much discussed if there is a concept of primitive thisness involved, and that there is a view according to which quantum objects (in the general case) cannot be distinguished even conceptually [26], [22], [23]. Then, if we agree that the intrinsic properties are all we have and that there is no 'incompleteness' in the theory, we are faced here with a situation similar to that one described in the preceding paragraph: the structure $\mathcal{A}$ (where the properties are the intrinsic properties and in the domain we have the elementary particles) cannot be extended to $\mathcal{B}$ and Leibniz Law is not valid (otherwise, as we remarked above, we would be committed to conceptually individualizable entities). In other words, if we are apt to accept firstly that elementary particles are entities devoid of individuality, having no *genidentity*, as entities which cannot be individualized even conceptually and, secondly, that quantum mechanics is not an incomplete theory (concerning individuation), then it seems to us that the underlying logic of such a theory cannot postulate the validity of Leibniz Law. In essence, perhaps also with regard to quantum ontology we might agree with Einstein in saying that "Erst die theorie entscheidet darüber, was man beobachten kann".

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