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# THE *ARISTOTELICITY* OF THOMASON'S SEMANTICS

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In recent articles, [4] and [5], S.K. Thomason develops a semantics for Storrs McCall's [2] representation of Aristotle's apodeictic syllogistic. [4] is largely an attempt to improve upon Johnson's [1] pioneering work. Thomason is inclined to think of Johnson's models as 'contrived' in that they are explicitly defined in terms of certain of McCall's axioms (Axioms 6-9 in Table 1). In [4] Thomason attempts to provide a semantics in which those axioms fall out, without having to stipulate explicitly that the interpretations satisfy them. In [5] Thomason extends those results, offering a more 'intuitive' explanation for them. In both [4] and [5] Thomason takes McCall's work as his base, and so the semantics he gives is specifically a semantics for McCall's L-X-M system. That this in turn makes it a good semantics for Aristotle's system depends on how well McCall represents Aristotle. In this paper I try to consider Thomason's semantics in terms of Aristotle's texts, showing how ways we might read them do make the semantics plausible.[1]

I set out the basics of Thomason's semantics as he gives them in [4], initially making only a few notational modifications. Ultimately, it will help to give modal LPC-translations of Thomason's formulas. The following is from pp. 113-114:

> McCall's system L-X-M (with inessential modifications...) is as follows. The language consists of *terms* (*x*,*y*,... will be used as meta-variables ranging over terms), *atoms*
>
> $Axy$ ("all $x$ are $y$"),
> $Ixy$ ("some $x$ are $y$"),
> $A^{\Box}xy$ ("necessarily all $x$ are $y$"),

[1] I have used the following editions and translations of Aristotle's works: W.D.Ross, *Aristotle's Prior and Posterior Analytics*: A Revised *Text with Introduction and Commentary*, Oxford: Clarendon, 1957. *Aristotle*, v.1, Loeb Classical Library, Cambridge, Mass.: Harvard UP, 1938. *Prior Analytics*, translated by Robin Smith, Indianapolis: Hackett, 1989. *Aristotle's Posterior Analytics*, translated by Jonathan Barnes, Oxford: Clarendon, 1975.

> $E^{\Box}xy$ ("necessarily all $x$ are non-$y$"),
> $I^{\Box}xy$ ("necessarily some $x$ are $y$"),
> $O^{\Box}xy$ ("necessarily some $x$ are non-$y$")
>
> and *formulas* $\bot$, $\alpha$ where $\alpha$ is an atom, and $\alpha \rightarrow \beta$ where $\alpha$ and $\beta$ are formulas...
> The axioms of L-X-M are the tautologies of classical propositional logic (with atoms, of course, in place of the propositional letters) together with all formulas of the forms A1-A14 of Table 1... and the only rule of inference is *modus ponens*.

The formulas A1-A14 of Table 1 are exactly the axioms of McCall's system L-X-M.

TABLE 1

| | | | |
|---|---|---|---|
| A1. | $Axx$ | | <1> |
| A2. | $I^{\Box}xx$ | | <1> |
| A3. | $Ayz \rightarrow Axy \rightarrow Axz$ | (Barbara XXX) | <1> |
| A4. | $Ayz \rightarrow Iyx \rightarrow Ixz$ | (Datisi XXX) | <1> |
| A5. | $A^{\Box}yz \rightarrow Axy \rightarrow A^{\Box}xz$ | (Barbara LXL) | <1> |
| A6. | $E^{\Box}zy \rightarrow Axy \rightarrow \mathrm{E}^{\Box}xz$ | (Cesare LXL) | <3> |
| A7. | $A^{\Box}yz \rightarrow Ixy \rightarrow I^{\Box}xz$ | (Darii LXL) | <2> |
| A8. | $\mathrm{E}^{\Box}yz \rightarrow Ixy \rightarrow O^{\Box}xz$ | (Ferio LXL) | <2> |
| A9. | $A^{\Box}zy \rightarrow O^{\Box}xy \rightarrow O^{\Box}xz$ | (Baroco LLL) | <3> |
| A10. | $O^{\Box}yz \rightarrow A^{\Box}yx \rightarrow O^{\Box}xz$ | (Bocardo LLL) | <1> |
| A11. | $I^{\Box}xy \rightarrow I^{\Box}yx$ | (LI-conversion) | <1> |
| A12. | $A^{\Box}xy \rightarrow Axy$ | (*A*-subordination) | <1> |
| A13. | $I^{\Box}xy \rightarrow Ixy$ | (I-subordination) | <1> |
| A14. | $O^{\Box}xy \rightarrow Oxy$ | (O-subordination) | <1> |

In [4] Thomason actually gives three semantics, increasing the strength from the first to the second to the third. The first semantics validates only a portion of McCall's system, the second validates more, and the third gets the whole of McCall's system. <1>, <2>, and <3> in the right-hand column of Table 1 correspond to the different semantic frameworks, indicating the stage at which each of the axioms above comes into play.

## 1. *The First Semantics*

At the heart of Thomason's semantics are various functions that assign extensions to the terms. Those functions are, in Thomason's notation, *Ext*, $Ext^{+}$, and $Ext^{-}$, which when applied to a term $x$ pick out the sets of things that are $x$, necessarily-$x$, and necessarily-not-$x$, respectively. For reasons that become clear later Thomason imposes certain conditions on the relations between these various extensions. The conditions on the first semantics are

(a) $\varnothing \neq Ext^{+}(x) \subseteq Ext(x)$, and
(b) $Ext^{-}(x) \cap Ext(x) = \varnothing$

A $model_1$ of the first semantics consists of a set of individuals, $W$, together with functions $Ext$, $Ext^{+}$, and $Ext^{-}$, subject to conditions (a) and (b). A *valuation* is a function $V$ from formulas to $\{T,F\}$ satisfying the following

(c) $V(\bot) = F$,
(d) $V(\alpha \rightarrow \beta) = T$ iff $V(\alpha) = F$ or $V(\beta) = T$.

If $M = (W, Ext, Ext^{+}, Ext^{-})$ is a $\text{model}_1$ then $V_M$ is the valuation satisfying:

$$\begin{array}{ll} V_M(Axy) = T & \text{iff } Ext(x) \subseteq Ext(y) \\ V_M(Ixy) = T & \text{iff } Ext(x) \cap Ext(y) \neq \varnothing \\ V_M(A^{\square}xy) = T & \text{iff } Ext(x) \subseteq Ext^{+}(y) \\ V_M(E^{\square}xy) = T & \text{iff } Ext(x) \subseteq Ext^{-}(y) \\ V_M(I^{\square}xy) = T & \text{iff } Ext^{+}(x) \cap Ext^{+}(y) \neq \varnothing \\ V_M(O^{\square}xy) = T & \text{iff } Ext^{+}(x) \cap Ext^{-}(y) \neq \varnothing. \end{array}$$

These semantic conditions can easily be expressed as rules for translation into modal-LPC as follows, using $A$, $B$, and $C$ instead of Thomason's $x$, $y$, $z$, and reserving $x$, $y$, $z$ for (bound) individual variables:

$$\begin{array}{lll} Axy & :: & \forall x(Bx \rightarrow Ax) \\ Ixy & :: & \exists x(Bx \& Ax) \\ A^{\square}xy & :: & \forall x(Bx \rightarrow LAx) \\ E^{\square}xy & :: & \forall x(Bx \rightarrow L{\sim}Ax) \\ I^{\square}xy & :: & \exists x(LBx \& LAx) \\ O^{\square}xy & :: & \exists x(LBx \& L{\sim}Ax) \end{array}$$ [2]

[2] In this translation method, $LA$ simply denotes the necessary $A$'s. Of course, in ordinary modal logic $L$ is a sentential operator whose semantics is typically given in terms of possible worlds: if $L\phi$ is true, then $\phi$ is true in all accessible worlds. This way of putting it sounds grossly unAristotelian, and for this reason many scholars eschew $L$'s and $M$'s. But we needn't do this. The possible worlds semantics certainly gives the right analysis and much more, but the 'much more' isn't relevant to Aristotle's modal logic. His logic concerns a language in which modal operators have scope over only simple terms, never over complexes, and so it can be given an analysis in terms of the extension of terms for necessary-$A$'s and necessary-not-$A$'s.

As my aim here is to see whether Thomason's semantics is a good interpretation of Aristotle, I want to pay close attention to the specific conditions Thomason imposes. Condition (b) is straightforward and uncontroversial. But, as it stands, condition (a) $\varnothing \neq Ext^{+}(x) \subseteq Ext(x)$, might occasion some controversy. Only *part* of (a) is suspect -- the part that says the set of things that are necessarily-*x* is not empty, $Ext^{+}(x) \neq \varnothing$. I will call this condition 'Strong Existential Import': if ordinary existential import tells us that whatever *A* may be $\exists xAx$ is true, then strong existential import tells us that given ordinary existential import $\exists xLAx$ is true. Thomason's semantics explicitly requires that strong existential import hold, and his work shows it is needed to get the logic to work. It is not clear, however, that it is a condition Aristotle uses. In fact, there is a very real question about whether the motivation for Strong EI is at all Aristotelian, or even *compatible* with Aristotle's discussions of modals. In the rest of this section I want to consider how we might try to justify Strong EI with Aristotle.

Here is one way we might take him. Among the things in Aristotle's metaphysics there are substances and their accidents. When we say 'Socrates is a man' we predicate essentially because being a man is part of Socrates' essence. When we say 'Socrates is white' we predicate accidentally. The predication in this case is accidental because being white isn't involved in what it is to be Socrates, it's accidental to him. In the *Categories*, Aristotle explains that for a subject to be white is for it to have a whiteness in it, (1a28-29). Matthews and Cohen [3] suggest a way of understanding this. They explain why when Aristotle talks about the whiteness in Socrates we should understand him to be talking about a particular whiteness that is in Socrates. This particular whiteness is not the same whiteness as, say, the particular whiteness that's in Callias. If we can call this whiteness white, and Aristotle seems to talk as though we can, then it's part of its essential nature to be white. Then Callias is only accidentally white because something in him (his whiteness) is essentially white. So if anything is white *at all* then something is essentially white, and this is just Strong Existential Import.

## 2. *The Second Semantics*

Even allowing the assumption of Strong EI, Thomason's first semantics still leaves the validity of Darii LXL and Ferio LXL, as well as Cesare LXL and Baroco LLL, unaccounted for. These are exactly the Axioms 6-9 (Table 1) that Johnson's interpretations are explicitly required to satisfy. In order to validate Darii LXL and Ferio LXL, Thomason introduces a new condition on his second semantics. (Cesare LXL and Baroco LLL fall under the third semantics which I look at in section 4 of this paper.)

Thomason defines a $\text{model}_2$ as a $\text{model}_1$ that satifies the additional condition

(1) $Ext(x) \cap Ext(y) \neq \varnothing \rightarrow Ext(x) \cap Ext^{+}(y) \neq \varnothing$.

Thomason takes this to mean:

(WP) If something satisfies both of two predicates then something both satisfies the first and necessarily satisfies the second, ([4], p.112).

(WP) is a weird principle to attribute to Aristotle. I will show why. This new condition, expressed in modal LPC, is

(1a) $\exists x(Bx \& Ax) \rightarrow \exists x(Bx \& LAx)$.

And this admits unwanted consequences. Given that for Aristotle all men are necessarily men, (1a) would allow the move, for example, from

(2) 'some man is white'

(which Aristotle says is true) to

(3) 'some man is necessarily white'

(which he repeatedly tells us is false).[3] If (WP) allows the move from (2) to (3) then we had better not attribute it to Aristotle since he would most certainly count that move as invalid. But (1) does gets the logic to work, and for this reason it is worth asking whether there is anything that *could* be

[3] (1a) takes us from $\exists x(Bx \& Ax)$ to $\exists x(Bx \& LAx)$. But since all men are necessarily men, all the *B*'s are *LB*'s, so we also have $\exists x(LBx \& LAx)$ which is Thomason's translation of 'some man is necessarily white.'

said for it as an interpretation of Aristotle. Any reasonable answer will need to block the move from (2) to (3).

Thomason, at any rate, is aware that his condition (1) is not an obvious Aristotelian doctrine. He offers a possible explanation:

> Someone who believes that predicates are, by their nature, necessarily non-vacuous, might well believe also that they interact in such a way that when they intersect they intersect necessarily (in the weak sense that each will be satisfied by something that satisfies the other necessarily), ([4], p. 120).

Thomason is careful not to actually attribute this notion to Aristotle; he just offers this as one way of accounting for the system McCall gives. We might try to justify Thomason's condition by appeal to Aristotle's talk in *Posterior Analytics* A22 about 'genuine predication.' Aristotle there explains that 'the white thing is a log' is not an example of genuine predication. The reason is that 'white' identifies a subject indirectly, or accidentally. Genuine predication doesn't allow picking out a subject in this way. If identifying the subject by a non-substantial term makes the predication not genuine, then only when we predicate something of a subject which is identified by a substance term do we predicate genuinely. For anything to be a substance is already for it to be essentially what it is. In this sense we might take 'man' as equivalent to 'necessary-man' since anything that is a man is so essentially. Horse would be equivalent to necessary-horse, and so on. This gives us a way around the invalid move above from (2) 'some man is white' to (3) 'some man is necessarily white'; instead, we would get validly from (2) 'some man is white' to (4) 'something that's a necessary-man is white.' Since that is *not* invalid, it seems a better way to go. But that would mean that Thomason's condition (1) cannot be quite so general a condition as he makes it out to be. Instead, we might try restricting it to the valid principle

(1b) $\exists x(Bx \& Ax) \rightarrow \exists x(LBx \& Ax)$, where $B$ identifies the subject term of the predication and the predication is *genuine*.[4]

It would be good to see what effect the restriction to (1b) might have on a particular syllogism. Darii LXL (A7, in Table 1) is part of McCall's axiom

[4] One might consider a further restriction which takes *all* terms in the apodeictic syllogistic to be substances. But then there will be no difference between Barbara XLL and Barbara LLL. And for Aristotle the first is invalid, the second valid.

system. Thomason interprets an LI-premise as $\exists x(LBx \& LAx)$. Darii LXL has an LI-conclusion, so Thomason's interpretation of Darii LXL is

$$\frac{\begin{array}{l}\forall x(Bx \rightarrow LAx)\\ \exists x(Cx \& Bx)\end{array}}{\exists x(LCx \& LAx).}$$

What's involved in getting the logic to work out right? First, take the minor premise in Darii above, this is $\exists x(Cx \& Bx)$. Assuming now that all premises are examples of genuine predication, then since $C$ is the subject term, we can legitimately apply (1b) to the minor premise to get $\exists x(LCx \& Bx)$. Now we have:

$$\frac{\begin{array}{l}\forall x(Bx \rightarrow LAx)\\ \exists x(LCx \& Bx)\end{array}}{\exists x(LCx \& LAx).}$$

and, so, simple transitivity gets the conclusion Thomason's interpretation requires. Thomason's version of Ferio LXL may also be derived from (1b).

The restrictions required in (1b) do not pose insurmountable problems for Thomason's second semantics. In fact, if Thomason simply restricts his (1) to (1b), he forfeits nothing other than a little extra generality. And doing so gets him a far more plausible interpretation. Thomason might not agree that he forfeits nothing here. He might think he must then forfeit Disamis XLL, because the restriction I suggest certainly doesn't allow Disamis XLL as McCall gives it.

## 3. *Disamis* XLL

McCall takes Disamis XLL to be

$$\frac{\begin{array}{l}Ica\\ LAcb\end{array}}{LIba,}$$

and this, plainly, is invalid. A simple falsifying model has terms brown, animal, and horse:

Some horses are brown
All horses are necessarily animals
———
Some animals are necessarily brown.

McCall gives no explanation for the apparent invalidity of the syllogism as he construes it. And this leaves McCall's interpreters in the rather awkward position of needing to explain the invalidity away.

Thomason, in the first place, takes an apodeictic I-premise, *LIba*, to be doubly modal: $\exists x(LBx \& LAx)$. (In a case where *B* is a substance term like 'animal,' *B* and *LB* are presumably equivalent.) So Thomason's interpretation of Disamis XLL will be

$$\frac{\begin{array}{l}\exists x(Cx \& Ax)\\ \forall x(Cx \rightarrow LBx)\end{array}}{\exists x(LBx \& LAx),}$$

where *A* is a predicate term. Here is where Thomason needs the Weird Principle (WP) that if something satisfies each of two predicates, then something satisfies the first predicate and of *necessity* satisfies the second. So if something is both *C* and *A*, then something is both *C* and *LA*. Consider how this affects the proof of Disamis XLL:

| | | |
|---|---|---|
| (1) | $\exists x(Cx \& Ax)$ | Given |
| (2) | $\exists x(Cx \& LAx)$ | (WP), 1 |
| (3) | $\exists x(LAx \& Cx)$ | Conversion, 2 |
| (4) | $\forall x(Cx \rightarrow LBx)$ | Given |
| (5) | $\exists x(LAx \& LBx)$ | Transitivity 3,4 |
| (6) | $\exists x(LBx \& LAx)$ | Conversion, 5 |

The Weird Principle is needed to get this to work because in premise (1) *A* is a predicate term, not a subject term. But there is good evidence that Aristotle himself would reject (WP). Nonetheless, the Weird Principle does seem necessary, since if we take Disamis XLL McCall's way, then Thomason's Weird Principle follows. On Thomason's interpretation the *B*-term in Disamis XLL is an essential *B*, so *B* and *LB* are equivalent in this case. Identifying the *C* and *B* terms gives

$$\frac{\begin{array}{l}\exists x(Bx \& Ax)\\ \forall x(Bx \rightarrow LBx)\end{array}}{\exists x(Bx \& LAx),}$$

So, $\exists x(Bx\&Ax) \rightarrow \exists x(Bx\&LAx)$.

But perhaps we don't want to take Disamis XLL McCall's way. When we look at the text of the *Prior Analytics*, we find that Aristotle's description of this syllogism is not completely clear:

> Thus, if it is necessary for *B* to belong to every *C* and *A* is below *C*, then it is necessary for *B* to belong to some *A*, (31b16-17).

In fact, from the text it appears there are two possible responses to McCall. In the lines leading up to this, at 31b12-16, Aristotle pretty clearly means to describe a syllogism with one universal and one particular premise. But from the text it would appear that '*A* is below *C*' at 31b16-17 must mean 'some *A* is *C*,' with *A* as the subject term. Following McCall, Thomason gives the *AC* premise as 'some *C* is *A*,' with *C* as the subject term. But, as Aristotle sets out the *AC* premise at 31b17-20, the subject term of the premise is *A* and *C* is really the predicate. That suggests the syllogism at 31b16-17 might really be

$$\begin{array}{l} \exists x(Ax\&Cx) \\ \forall x(Cx \rightarrow LBx) \\ \hline \exists x(LBx\&LAx), \end{array}$$

where *A* is the subject term of the first premise. If we take it that way, then Thomason doesn't need the Weird Principle at all. He only needs the restricted version of (1) -- i.e., (1b) -- that says we can put an *L* on a term in subject position.

Perhaps a still better move would be to say that '*A* is below *C*' at 31b16-17 really is a universal. The syllogism in question would turn out to be

$$\begin{array}{l} \forall x(Ax \rightarrow Cx) \\ \forall x(Cx \rightarrow LBx) \\ \hline \exists x(LBx\&LAx). \end{array}$$

All Thomason needs to get that to come out right is Strong Existential Import together with the assumption that no terms are empty:

| | | |
|---|---|---|
| (1) | $\forall x(Ax \rightarrow Cx)$ | Given |
| (2) | $\forall x(LAx \rightarrow Ax)$ | Given |
| (3) | $\forall x(Cx \rightarrow LBx)$ | Given |
| (4) | $\forall x(LAx \rightarrow LBx)$ | Transitivity 2,1,3 |
| (5) | $\exists xLAx$ | Strong EI |
| (6) | $\exists x(LAx\&LBx)$ | 4,5 |

(7) $\exists x(LBx \& LAx)$ Conversion, 6

Either of these two readings would, I think, help Thomason. But because his interest is to give a good semantics for McCall, Thomason simply admits Disamis XLL as McCall gives it, and so is left needing to validate reasoning that Aristotle himself might have rejected.

The situation is complicated by the fact that Disamis XLL seems to be derivable by conversion of the conclusion from Datisi LXL. In Thomason's account Datisi LXL is

$$\begin{array}{l} \forall x(Cx \rightarrow LAx) \\ \exists x(Cx \& Bx) \\ \hline \exists x(LBx \& LAx). \end{array}$$

According to the genuineness requirement, the *B* term in Datisi LXL must be a substance term for it to be the subject of the LI-conclusion. So if we apply the genuineness requirement in this place, Thomason's Datisi LXL comes out as valid. Also, the LI-conclusion validly converts. But we must be careful here, because when Disamis XLL is obtained in this way from Datisi LXL both the *A* and the *B* terms are substance terms.The conversion doesn't guarantee the *unrestricted* validity of Disamis XLL because Disamis allows an accidental term in the conclusion. So, unlike Datisi, genuineness allows us instances of Disamis with true premises and false conclusion, which shows that care must be taken even using the valid conversion principles.

As it happens the Datisi LXL just given may not in fact be the syllogism discussed in Aristotle's text at *An.Pr.*, 31b19-20. Aristotle gives the minor premise as '*B* is below *C*' (31b20) which if it's a particular would seem to be $\exists x(Bx \& Cx)$, not $\exists x(Cx \& Bx)$. If '*B* is below *C*' is *a genuine* premise, then *B* must be a substance term. So we can represent the syllogism as

$$\begin{array}{l} \forall x(Cx \rightarrow LAx) \\ \exists x(LBx \& Cx) \\ \hline \exists x(LBx \& LAx). \end{array}$$

This, of course, is valid. Disamis XLL as Thomason gives it cannot be derived from this, but the valid (particular) version of Disamis XLL that I suggest can be.

So while at first it might appear that Thomason has to give up a valid syllogism, a close examination of Aristotle's text shows this isn't the case. Part of the apparent difficulty might come from the fact that there has been too great a reliance on a traditional listing of the valid modal syllogisms, resulting in attempts to tackle problems that are not Aristotle's. Disamis XLL and Datisi LXL as they are represented in Thomason's system are not clearly Aristotle's; certainly, they are not the syllogisms with which he is concerned at 31b16-17 and 31b19-20. On my reading of Aristotle's text, there is nothing that would seem to require principles like Thomason's (WP). And since (WP) itself seems to admit unAristotelian results, any attempt to capture Aristotle's meaning ought not require such a principle.

## 4. *The Third Semantics*

Cesare LXL and Baroco LLL remain to be validated, and these Thomason treats in his Third Semantics. He defines a $model_3$ as a $model_2$ that satisfies the conditions

(e) $Ext(b) \subseteq Ext^{-}(a) \rightarrow Ext(a) \subseteq Ext^{-}(b)$
(f) $Ext(b) \subseteq Ext^{+}(a) \rightarrow Ext^{-}(a) \subseteq Ext^{-}(b)$.

A formula is $valid_3$ if it is true in all $models_3$. Since a $model_3$ does satisfy conditions (e) and (f), Thomason gets Axioms A6 (Cesare LXL) and A9 (Baroco LLL) to come out valid because all $models_3$ are carefully restricted in a way that guarantees their validity. Consider the justification for Cesare LXL as Aristotle gives it at 30b9-13:

| | | |
|---|---|---|
| $\forall x(Bx \rightarrow L{\sim}Ax)$ | let it not be possible for *A* to belong to any *B*[5] | (i) |
| $\forall x(Cx \rightarrow Ax)$ | let *A* merely belong to *C* | (ii) |
| $\forall x(Ax \rightarrow L{\sim}Bx)$ | neither is it possible for *B* to belong to any *A* | (iii) |
| $\forall x(Cx \rightarrow Ax)$ | but *A* belongs to every *C*, consequently | (iv) |
| $\forall x(Cx \rightarrow L{\sim}Bx)$ | it is not possible for *B* to belong to any *C* | (v) |

Cesare LXL does not seem valid if accidental terms are allowed in subject position. For take *A* to be animal, *B* white, and C man, and suppose that the

[5] A word about the text and my LPC-translations here: Aristotle often expresses necessary universal privative (LE) premises as denials of possibility in the *Prior Analytics*. He very clearly considers his expressions 'it is not possible for *A* to belong to any *B*' and '*A* does not belong to every *B* of necessity' to be equivalent -- he repeatedly offers each as examples of his necessary universal privatives. For '*A* does not belong to every *B* of necessity' and similar constructions, see 25a30-31; 30a17-23; 30b26-27; 31a37. For 'it is not possible for *A* to belong to any *B*,' see 30b10-12; 30b14-15; 31a5-10; 31b33-36. To reflect the fact that these are apodeictic premises (not problematic premises), I use $\forall x(Bx \rightarrow L{\sim}Ax)$ to represent a necessary universal privative.

only white things are plants, which by necessity are not animals. The illegitimate move of course is the principle that gets from (i) to (iii), a principle, perhaps, most commonly called 'LE-conversion.' In modal LPC this is just $\forall x(Bx \rightarrow L{\sim}Ax) \rightarrow \forall x(Ax \rightarrow L{\sim}Bx)$. And this is equivalent to condition (e) above. As Aristotle first states LE-conversion, at 25a29-30, and when he uses it in Cesare LXL above, the conversion seems to be fully general -- that is, it appears that (e) captures just what Aristotle means without the need for any additional restrictions on terms. But that is only part of the story that emerges from the text. As Aristotle goes on to elaborate, it becomes clear that (e) is really too general for his purposes. Here is why.

I have already referred to *Posterior Analytics* A22, in which Aristotle explains that accidents can never take the subject position, only the predicate position. What's more, *were* an accident to take subject position, then the result for Aristotle either is not predication at all, or is predication only by courtesy, *An. Post.*, 83a15-17. If this holds true in the *Prior Analytics* as well, then any instance of (e) involving an accidental term presents a problem since (e) puts both the *A* term and the *B* term in subject position. There is some evidence that Aristotle is aware in the *Prior Analytics* of the difficulties this raises: In 43b1-6, he sets out a method for selecting syllogistic premises that would appear to rule out necessary privative premises in general:

> So one must select the premises about each subject in this way, assuming first the subject itself, and both definitions and whatever is peculiar to the subject; next after this, whatever follows the subject; next, whatever the subject follows; and then, whatever cannot belong (*me endechetai...huparchein*) to it. (Those to which it is not possible (*me endechetai*) for the subject to belong need not [or, perhaps, must not[6]] be selected, because the privative converts), (43b1-6).

If this method applies to to universal premises[7], then, taken strongly, it would appear to mean that LE-conversion is altogether irrelevant to the syllogistic. Even in its weakest sense, it shows that in the syllogistic Aristotle doesn't regard every instance of LE-conversion as relevant.

6 Robin Smith (p. 151) notes the Greek is open to either reading.

7 One might suppose that 43b1-6 is only about particular premises. But it seems that Aristotle means to set out a general method for selecting syllogistic premises. In fact, it would appear that 43b6 must be about universal premises. This is because Aristotle has earlier explained (25a35-36) that necessary particular privatives (LO-premises) do *not* convert, since even non-modal O-premises do not convert in the syllogistic. The point of 43b6 is that premises about what is not possible *do* convert, so unless Aristotle is contradicting himself, the line must refer to universals.

Following the argument in *Posterior Analytics* A22, one would suppose the irrelevant LE-conversions are those that involve some accidental term. LE-conversion, and hence, condition (e), does work when both terms are required to be substance predicates, as for instance when *A* is 'horse' and *B* is 'man': 'all men are necessarily-non-horses' goes to 'all horses are necessarily-non-men.' That seems unproblematic, especially if we think that all men are necessarily men and all horses are necessarily horses, and that what it is to be a horse, i.e., the nature of horse, excludes being a man, and vice versa.

Let's turn now to Thomason's condition (f). In modal LPC (f) becomes $\forall x(Bx \rightarrow LAx) \rightarrow \forall x(L{\sim}Ax \rightarrow L{\sim}Bx)$. Consider an instance of (f) in which the *B* term is an accident. Let *A* be man and *B* be white: if every white is a necessary man, then every necessary-non man is necessarily-not white, which seems not to follow. But if the antecedant in this case is a premise at all, it is only a premise by courtesy, so it isn't clear whether in this case we can legitimately or meaningfully convert according to (f). Thomason introduces (f) in order to validate Baroco LLL. In modal LPC, his translation of Baroco LLL will be:

$$\begin{array}{l} \forall x(Bx \rightarrow LAx) \\ \exists x(LCx \& L{\sim}Ax) \\ \hline \exists x(LCx \& L{\sim}Bx). \end{array}$$

This would seem to fail when *A* is animal, *B* is white, and *C* is plant. One reply would be to point out that the only term here that is not modally qualified is the *B*-term, and that is a subject term. If Aristotle in the modal syllogistic really is concerned only with predication that is genuine, then *B* is already implicitly modal since it must be a substance term. This would mean that the terms set out here -- animal, white, and plant -- cannot be used to provide a counter-example to Baroco LLL, and so, presumably, it is valid.

Maybe this cannot be the whole story. In establishing the invalidity of Camestres LXL (30b20-40) and Baroco LXL (31a10-15), Aristotle chooses terms which obviously allow for premises which are not genuine. His terms in those cases are animal, man, and white. The question for principle (f) is why these terms should not be chosen there. I have some ideas about why, but my purpose in the present paper is to see what can be said in favor of Thomason's semantics on the basis of Aristotle's text. In discussing these issues I have accepted Thomason's translations and considered the extent to which the assumptions he makes can be textually justified.

## 5. *Thomason's Relational Semantics*

In [5] Thomason aims to make his semantics more intuitive. What he does is neat, and it raises a new question: does what makes it more intuitive also make it any more Aristotelian in flavor. So far, in the earlier sections of this paper I have tried to find ways of restricting Thomason's semantic conditions in order to make them better fit Aristotle. The relational models are based on Thomason's unrestricted semantics, and so the relational account will be likely to admit some unwanted consequences, such as the Weird Principle.

In his relational models, in place of $Ext(x)$ and $Ext^{+}(x)$ Thomason uses $|x|$ and $|x^{\Box}|$. He also introduces $|x^{\Diamond}|$ to denote the possible $x$'s. $|x^{\Diamond}|$ is the complement of $Ext^{-}(x)$. If $Ext^{-}(x)$ denotes the things that are necessarily not $x$'s -- the things that couldn't be $x$'s -- then $|x^{\Diamond}|$ denotes the things that could be $x$'s -- things whose nature doesn't rule out their being $x$'s. Thomason takes $|x^{\Box}|$ to be basic and non-empty. He then defines $|x|$ and $|x^{\Diamond}|$ in terms of $|x^{\Box}|$ using two relations $R$ and $S$. $|x|$ is defined in such as way that for any individual $a \in |x|$ iff $\exists b(b \in |x^{\Box}|$ and $bRa)$. The possible extension is defined similarly: for any individual $a \in |x^{\Diamond}|$ iff $\exists b(b \in |x^{\Box}|$ and $bSa)$. These can be understood as saying, less formally, that for any individual $a$ to be $x$ is for $a$ to be $R$-related to an individual $b$ that is necessarily-$x$. For $a$ to be possibly-$x$ is for $a$ to be $S$-related to an individual $b$ that is necessarily-$x$.

Expressing this in LPC with $A$, $B$, etc., for predicates representing terms, and $x$, $y$, etc., for individual variables (not terms as in Thomason), then the idea is that we take as basic for any term $A$ the set of things which are necessarily $A$. We might justify this by thinking of the necessary $A$'s as the things whose nature it is to be $A$, and all the other $A$'s as things which are $A$ by some kind of association with a necessary $A$. So we might say, then, that Socrates is white in virtue of his association with a whiteness, where this whiteness is white by nature.

If the necessary $A$'s determine the (actual) $A$'s , then prefixing an $L$ to $A$ is at least intuitively backwards. So instead of $LA$, let $A^*$ denote those things that are $A$ by nature. So $A^*$ corresponds to $LA$ except that it is not defined by prefixing a modal operator to $A$. Then Thomason's rule would be that instead of writing $LAx$, one would write $A^*x$; and instead of writing $Ax$ one would write $\exists y(A^*y \,\&\, yRx)$; and instead of writing $MAx$ one would write $\exists y(A^*y \,\&\, ySx)$. Consider $\exists y(A^*y \,\&\, yRx)$. Let $yRx$ be true iff $y=x$ or $x$ is 'in' $y$ -- 'in' in the sense of the *Categories.* Then '$x$ is $A$' would be true iff either $x$ is $A$ by nature or there is in $x$ a $y$ that is $A$ by nature.

In order to get all of McCall's syllogisms, Thomason imposes certain restrictions on the relational models:

(g) $|x^{\Box}| \neq \varnothing$

guarantees that there are some necessary $x$'s. That is, Strong Existential Import still holds. With $A^*$ for $LA$, it becomes $\exists x A^* x$.

(h) $R$ is an equivalence relation.

This validates the 'dubious' inference from $\exists x(Bx\&Ax)$ to $\exists x(Bx\&LAx)$ -- that is, it validates (WP) -- as follows:

Suppose $\exists x(Bx\&Ax)$. Then, in terms of Thomason's translation, this means that $\exists x(\exists y(B^*y\&yRx) \& \exists z(A^*z\&zRx))$. That is, for some $x$, $y$, $z$, we have $(B^*y \& yRx \& A^*z \& zRx)$. But $R$ is an equivalence relation, so since $(yRx \& zRx)$, then $yRz$, and so $B^*y\&yRz \& A^*z$ for that same $y$ and $z$; that is, $\exists z(\exists y(B^*y\&yRz) \& A^*z)$. So, with $x$ for $z$, we have $\exists x\exists y(B^*y\&yRx) \& A^*x)$, and this is just Thomason's relational interpretation of $\exists x(Bx\&LAx)$.

If $R$ is an equivalence relation then it's difficult to give an Aristotelian justification -- in fact, the symmetry requirement on $R$ introduces problems here. Suppose that Socrates is white by virtue of the fact that he stands in relation $R$ to something (his whiteness) that's essentially white. Then by symmetry his whiteness is $R$-associated with Socrates and so his whiteness is human -- since it is $R$-associated with something essentially human, namely Socrates. So there is something (Socrates' whiteness) which is accidentally human and is essentially white. This is an odd justification for the principle. But again, the principle at issue here is the Weird Principle, $\exists x(Bx\&Ax) \rightarrow \exists x(Bx\&LAx)$, and that, as we have seen, is something Thomason may not need.

If $R$ is the relation between a substance and an accident that holds when the substance has the accident in it, then $S$ is the relation between a substance and an accident if the accident is the kind of thing the substance could have in it. According to the relational semantics, it will be true that 'some man is possibly white' if some man is associated by $S$ with an essential white (a whiteness). But Thomason wants to make the $S$-relation symmetrical. This would have the whiteness $S$-associated with the man. Following Thomason's translations, then, some whiteness would be a possible man. The plausibility of this depends upon what, if anything, Aristotle would count as a possible man; and it is not at all obvious that a whiteness would count. Again, the symmetry requirement gets unintuitive results.

One of the neat things about Thomason's $R$ and $S$ relations is the way they link with Aristotle's account of predication in the *Categories*. But Thomason wants both $R$ and $S$ to be symmetrical relations, and in each case that gets him into trouble. There's no reason to think that these relations should be symmetrical. In the case of $R$, this is easy to see if we take $R$ to be the converse of the 'in'-relation in the *Categories*. In the *Categories* the truth of 'Socrates is white' is explained by the fact that he has 'in' him something that is essentially white. That is to say, something that is essentially white stands in relation $R$ to him. But the 'in'-relation for Aristotle is a relation of ontological dependence, and if that were symmetrical then Socrates (a primary substance) would have to be in a subject. Aristotle is very explicit that no substance will ever be that: 'That it never is present in a subject holds good of all substance whatever,' *Categories*, 3a7.

In developing a semantics for McCall, Thomason answers many of the important interpretive questions McCall leaves open. For the few respects in which Thomason's results sound unAristotelian, I suggest ways we might try to make his semantics better fit Aristotle. The *Categories* suggests a plausible justification for the Strong Existential Import that Thomason's semantics requires. Several others of Thomason's conditions are so general that they admit invalid readings. For all but one of these conditions, restricting premises to what Aristotle calls genuine predication gives a way of blocking the unintended readings and validating Thomason's conditions. In order to validate Disamis XLL as McCall represents it, Thomason introduces a Weird Principle. But McCall's account of Disamis XLL is, I think, not obviously right. Finally, in his Relational Semantics, Thomason gives a clear, intuitively plausible account of Aristotelian statements of necessity and possibility. But the symmetry of the relations he uses is too strong.

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