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MANY–VALUED DEONTIC PREDICATIONS

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Deontic predications require an indeterminately many-valued rather than classically bivalent prescription semantics for acts or practitions. Deontic logic must reflect a wide diversity of sources of moral obligations, which are sometimes in conflict. This is most clearly evident in what have become standard deontic logic modelings of consistency-preserving solutions to moral dilemmas. Many-valued interpretations are defined in a nonstandard propositional semantic foundation with truth value gaps to accommodate complex functions of mixed prescription values defined for many distinct deontic logics.

1. *Deontic Contrariety and Inconsistency*

Antigone in Sophocles' drama faces a moral dilemma. She is obligated by divine command of the gods to bury her slain brother Polyneices, and obligated not to bury him by the decree of King Creon. The contrariety poses a problem for rationalist axiology that threatens at least some deontic formalisms with logical inconsistency. The conflicting obligations imposed on Antigone in her predicament can be represented in the notation of naive deontic logic as:

$$O(A) \mathbin{\&} O({\sim}A)$$

This in itself is unproblematic, but it leads to logical inconsistency. Obligation standardly implies permissibility:

$$O({\sim}A) \supset P({\sim}A)$$

But by the duality between deontic obligation and permission operators:

$$O(A) \equiv {\sim}P({\sim}A)$$

By simplification and contraposition from this we further obtain:

$$P(\sim A) \supset \sim O(A)$$

From the above by hypothetical syllogism it follows that:

$$O(\sim A) \supset \sim O(A)$$

And from this and the original statement of Antigone's deontic contrariety in her moral dilemma, there occurs by conjunction rules and conditional detachment an outright logical inconsistency of the form:

$$O(A) \ \& \ \sim O(A)$$

2. *Relativizing Obligation Source and Circumstance*

The standard solution to the paradox corresponds with received wisdom about how the moral dilemma should be resolved as a matter of practical reasoning. It appears sensible to regard Antigone's obligation to perform act A (burying her brother) as holding under circumstances or deriving from an authority distinct from the circumstances or authority by which she is obligated not to perform or forbidden from performing A.[1]

The difference enters into the notation of a refined deontic logic as a relativization of obligation to distinct conditions or circumstances. Here we have, as a replacement for the original statement of deontic contrariety, the less naive formulation:

$$O(A/C_1) \ \& \ O(\sim A/C_2)$$

The advantage, where $C_1 \neq C_2$, is that there is no ostensive syntactical contrariety in the moral dilemma statement. Antigone is obligated to do A under circumstances C_1 where the gods generally demand burial, and obligated not to do A under distinct circumstances C_2 where the King forbids burial in this instance. From this dilemma restatement no logical inconsistency is forthcoming. By parity of construction, the modified inference sequence at most and at worst now yields:

$$O(A/C_1) \ \& \ \sim O(A/C_2)$$

Since there is no logical inconsistency in this expression, the consistency of deontic logic is preserved. The onus of uncovering an antinomy in applications of the formalism is shifted to the requirement of finding a

[1] Castañeda (1975), pp. 26-31.

univocal set of circumstances in which true deontic contrariety obtains.[2] Antigone's moral dilemma is avoided, as Sophocles describes the subsequent events of the play, when she ranks her obligations according to a hierarchy of values, in which obligations to the gods outweigh obligations to secular authorities. Where there is no plausible hierarchical ranking of conflicting obligation sources, moral dilemmas might be resolved, at least as obstacles to action, by random procedure, such as flipping a coin. Consequentialists may prefer this method in order to minimize the disutility that occurs when there is no basis for preferring any dilemma alternative to any other. Or, agents in some conceivable dilemma scenarios might break the moral indecision deadlock by walking away. Deontologists of a sufficiently radical streak, for whom the consequences be damned, may choose this course, if they believe that thereby they avoid dirtying their hands, or treating persons merely as means to an end, rather than as ends in themselves.

Although the slash 'A/C' convention, developed by G.H. von Wright and Bas C. van Fraassen for indicating the relativization of obligation sources or circumstances, is widely used in deontic logic, for convenience I shall make use of a variant proposed by Hector-Neri Castañeda. This is a device for regimenting W.D. Ross's informal distinction between actual and *prima facie* obligation.[3] Here obligations are indexed by a subscript as deriving from distinct sources. The translation of Antigone's dilemma into Castañeda's symbolism is thus:

$$O_g(A) \; \& \; O_k({\sim}A)$$

The gods g constitute an obligation source distinct from king k. The modified inference sequence at most and at worst now produces the noncontradictory proposition:

$$O_g(A) \; \& \; {\sim}O_k(A)$$

[2] See Marcus (1980); Jacquette (1991), pp. 44-45; (1994).

[3] The 'slash' notation for circumstantial relativization of obligation appears in von Wright (1968), and van Fraassen (1972), pp. 417-438. See also Castañeda (1975), pp. 185-190; (1981), pp. 74-78; (1982), p. 37, n. 6. An alternative formalism involving conditionalization of obligation sources and circumstances is developed by Hintikka (1971). The adequacy of these solutions is criticized by al-Hibri (1978), pp. 94-97; and DeCew (1981), pp. 55-72. Ross distinguishes actual from *prima facie* obligation in (1930), pp. 19-47; (1939), pp. 84-86.

Moreover, at this level of analysis, there is as yet no need to postulate multiple prescription values in a nonstandard extrabivalent semantics. Indexed or circumstantially relativized obligation expressed as $O_i(A)$ ($O(A/C_i)$), for all that has been said so far, is either true or false, exclusively. The obligation to perform act A according to source or under circumstances i either holds or does not hold, *tertium non datur*.

3. *Obligation and Prescription Values*

Now, however, an interesting complication arises. The easiest way to explain the problem is by observing that intuitively there ought to be a precisely specifiable relation between the semantic truth value of $O_i(A)$ and what I shall henceforth call the *prescription value* of A. The singular term 'A' denotes a certain *act*, like *burying Polyneices*, or, in Castañeda's action theory and deontic logic, a *practition*, such as *to bury Polyneices*.[4] But if $O_i(A)$, then act or practition A, as obligatory according to source i, also has a special deontic value. It has whatever value the act or practition lacks, if, on the contrary, $\sim O_i(A)$, and the complementary value, if $O_i(\sim A)$.

An act or practition is *prescribed* or *not-prescribed* relative to an indexed obligation source. Castañeda refers to a prescription's *legitimacy* or *orthotic* values, which are not exactly, but more analogously, propositional, and which, as he maintains, are strictly isomorphic to truth values in propositional semantics.[5] The prescription value of A is *conferred* on it by or *supervenes* on its source-indexed obligation in $O_i(A)$. It is a fact about A that it is prescribed when $O_i(A)$. A is legitimized by the obligation to do it, and its prescription makes it obligatory to do it. Ideally, a complete exposition of deontic logic must make semantic provision, not only for truth values of obligation statements, and the legitimacy or orthotic values of prescriptions, but also for the prescription values of acts or practitions whose designators are embedded in deontic prescriptions as obligation statements, and for the logical connections they bear one to another.

The most obvious relation between the truth value of obligation statements and prescription values of act or practition terms is this:

(D) $O_i(A)$ is true iff A is prescribed$_i$

[4] Castañeda (1975), pp. 43-51.

[5] Ibid., pp. 119-123; 131; 146.

The biconditional combines the logical conditions for what might be called *conferral* of prescription or positive prescriptive value on acts or practitions by the obligation that they be performed, and the *supervenience* of obligation on prescription.

It might appear that prescription values are unnecessary. Instead of referring to the source-indexed prescription values of acts or practitions, perhaps we could substitute the truth of the proposition 'A is prescribed$_i$'. This is effective only if we know what it means for the proposition to be true without reintroducing prescription values. But there is only one imaginable way to do this, by interpreting 'A is prescribed$_i$' as implied by the truth of i. The proposal makes sense in some applications for some i, but not in others. Thus, if Kant's theory of the categorical imperative is true, then it might imply that an action is prescribed by that source. Yet, looking back at Antigone's dilemma, the solution seems implausible. The source of her obligation not to bury her brother is either the king or the king's decree. But both the king and his decree are nonpropositional, neither true nor false. As such, they are unable to support logical implication in the required sense. It appears that the best way to understand the truth of 'A is prescribed$_i$' is as related to A's having an i-indexed positive prescription value. These difficulties make it reasonable to conclude that source-indexed prescription values for acts or practitions are essential, ineliminable and irreducible, to a complete semantics of deontic logic.

4. *Many-Valued Deontic Semantics*

If prescription values are adopted, then there must be an indeterminate number of distinct sources of obligation and prescription. Antigone is obligated not only by the gods and King Creon, but also by her parents, teachers, peers, local governmental authorities, and, arguably, the principle of utility or categorical imperative, or both. There is an open-ended list of real and potential sources of qualified *prima facie* obligation, as many as the institutions and principles that prescriptively impose obligations on responsible agents.[6]

While the resolution of Antigone's dilemma seems to require only two prescription values for the two conflicting obligations that impinge on her, other possible n-ary obligation conflicts of increasing order evidently require a many-valued prescription semantics. Unlike the classically

[6] The open-endedness of potential *prima facie* or qualified sources of obligation indicate the need for multiple source-indexed prescription values under principle (D).

bivalent *T*/*F* truth value semantics to which it is related by principle (D), the prescription values of acts or practitions cannot simply be determined as corresponding to *P*/*N* values (where '*P*' stands for positive, and '*N*' for negative prescription values, reading '*N*' as the value 'not-prescribed', intuitively less strong than being 'proscribed' or forbidden). If more complex moral dilemma paradoxes are to be forestalled by relativization to distinct obligation sources, then there must be an unlimited plurality of prescription values and their complements, $P_1, \ldots, P_n / N_1, \ldots, N_n$.

To provide an exact formal semantics for a family of many-valued deontic logics exemplifying this conception, we define a model $\mathbb{M} = \langle \mathbb{D}, \mathbb{V} \rangle$, where L is the set of all wffs in a propositional logic to which is added infinitely many deontic obligation operators $O_1, \ldots, O_n, \ldots$, and $\mathbb{D}$ and $\mathbb{V}$ are functions such that, for all i:

$$\mathbb{D}: L \rightarrow \{N_i, P_i\}$$
$$\mathbb{V}: L \rightarrow \{\mathrm{T}, \mathrm{F}, \_\_\_\}$$

such that:

$$\mathbb{V}(O_i(A)) = \begin{cases} T \ \ \textit{if} \ \ \mathbb{D}(A) = P_i \\ F \ \ \textit{if} \ \ \mathbb{D}(A) = N_i \\ \_ \ \ \text{otherwise} \end{cases}$$

Then we require that, for all i,j:

I. $\mathbb{D}(A) = P_i \equiv \mathbb{D}(\sim A) = N_i$
II. $\mathbb{D}(A_1 \ \& \ A_2) = P_i \equiv (\mathbb{D}(A_1) = P_i \ \& \ \mathbb{D}(A_2) = P_i)$
III. $(\mathbb{D}(A_1) = P_i \ \& \ \mathbb{D}(A_2) = P_j) \supset \mathbb{D}(A_1 \ \& \ A_2) = P_{f(i,j)}$

The last condition interprets the prescription value of a conjunction of acts or practitions with mixed or compound prescription sources. The generality of the condition permits application of many different functions *f* by which a determination of prescription source is established. This in turn makes it possible to define many different deontic logics by which mixed or compound prescription sources are calculated. The effect of the semantics can be seen in the following matrix of prescription values and truth values with truth value gaps.

We consider a simple function for a plausible definition of conjunction for acts or practitions A_1 and A_2, needed to determine the prescription value of $A_1 \ \& \ A_2$, and the truth or legitimacy values of $O_i(A_1 \ \& \ A_2)$, $O_j(A_1 \ \& \ A_2)$, and $O_k \ (A_1 \ \& \ A_2)$. The function assigns the next largest prescription source to the obligation to do both conjoined acts or prescriptions, according to the following rule:

(C) For any conjunction A_1 & ... & A_n of n act or practition conjuncts, with potentially distinct prescription values, if any conjunct has value N_i ($1 \leq i \leq n$), then (i) if the obligation sources of the prescription values of any act or practition conjuncts in A_1 & ... & A_n are distinct, then the prescription value of A_1 & ... & A_n is N_{n+1}; (ii) if the obligation sources of the prescription values of all act or practition conjuncts in A_1 & ... & A_n are identical, then the prescription value of A_1 & ... & A_n is N_i; (iii) if no conjunct in A_1 & ... & A_n has value N_i, then, if the obligation sources of the prescription values of any act or practition in A_1 & ... & A_n are distinct, then the prescription value of A_1 & ... & A_n is P_{n+1}; otherwise, (iv) the prescription value of A_1 & ... & A_n is P_i.

The combinatoric for these many-valued evaluations is $4m^2$, where m is the number of original prescription values. Where $m = 2$, $4(2^2) = 16$ rows. The table for $m = 3$ (i,j,k) values is therefore $4(3^2) = 36$ rows. The conjunction table then has this form:

A_1	A_2	$A_1 \& A_2$	$O_i(A_1 \& A_2)$	$O_j(A_1 \& A_2)$	$O_k(A_1 \& A_2)$
P_i	P_i	P_i	T	–	–
P_i	P_j	P_k	–	–	T
P_i	N_i	N_i	F	–	–
P_i	N_j	N_k	–	–	F
P_j	P_i	P_k	–	–	T
P_j	P_j	P_j	–	T	–
P_j	N_i	N_k	–	–	F
P_j	N_j	N_j	–	F	–
N_i	P_i	N_i	F	–	–
N_i	P_j	N_k	–	–	F
N_i	N_i	N_i	F	–	–
N_i	N_j	N_k	–	–	F
N_j	P_i	N_k	–	–	F
N_j	P_j	N_j	–	F	–
N_j	N_i	N_k	–	–	F
N_j	N_j	N_j	–	F	–

The assignments given here are offered only by way of illustration. Indefinitely many other interpretations might also be given. There is nevertheless an intuitive presumption in favor of this application. It is governed by the idea that P values are dominated by N values (as is truth by falsehood in the semantics for propositional conjunction). Source indices are conserved only when identical, and otherwise their conjunction introduces another, conjunctive, obligation source. As expected, true source-indexed many-valued deontic conjunctions are vastly outnumbered by false and undetermined conjunctions.

In the limiting case, where by Castañeda's suggestion, index '1' denotes an *overriding obligation source* (such as the principle of utility, categorical imperative, divine command, or the like), it is unproblematic by principle (D) to conclude on the basis of the equivalence $O_i(A)$ iff A is prescribed$_1$, that $O_i(A)$ is T (F) iff A is (unqualifiedly) $P(N)$. That is, the point of qualifying or relativizing the prescription value of act or practition A is irrelevant in the limiting case.[7] Within the proposed semantic framework, Castañeda's idea is represented by defining function $f(i,j) = min(i,j)$. Then $min(i,i) = i$. The minimal function induces a linear ordering on the O_i such

[7] Castañeda (1975), p. 142; (1981), pp. 77-78. (See Axiom A11a: $O_1(A) \supset A$).

that $O_i(A_1)$ & $O_j(A_2)$ is interpreted as $O_{min(i,j)}(A_1$ & $A_2)$. The order converges on $min(i,j) = 1$, which defines Castañeda's overriding obligation for the conjunction as $O_1(A_1$ & $A_2)$. In all other imaginable applications, and hence theoretically in general, particularly for nonabsolutist axiologies in which no overriding obligation sources are admitted, a full indeterminate expansion of multiple prescription values is required.[8]

APPENDIX

Pair-Wise Evaluation of Many-Valued Deontic Predications

There is another method of managing compound obligation sources, by *pair-wise evaluation*.[9] Pair-wise evaluations of many-valued deontic predications by contrast with the general proposal examined above involves consideration of the values of just two acts or practitions at a time.

This is possible because propositional connectives can be limited to negation and conjunction. The conjunction of any number of distinctly indexed deontic predications can therefore be evaluated by considering first two conjuncts, determining the evaluation of their conjunction, and then evaluating the conjunction of this with a third conjunct, and so on. Thus, there is never a need to bring more than two values into comparison in a single application of the appropriate semantic definition.

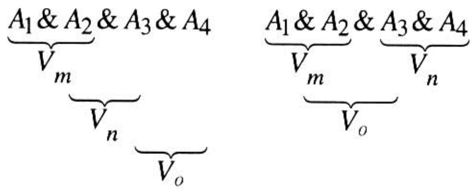
The method guarantees a kind of isomorphism with classical propositional logic, in the sense that no more than two prescription values ever come into play in determining the value of a compound deontic predication. This in turn raises an interesting philosophical question, since it might be understood as reducing all supposedly multiple-valued logics to classical bivalence. If a semantic isomorphism with bivalent logic is sufficient to qualify a formalism as bivalent, then, paradoxically, if pair-wise semantic evaluation is sound and effective, there simply are no nonstandard many-valued logics. This need not be an intolerable implication, and it might be thought that the reduction of many-valued to bivalent semantics, if not desirable, is at least conceivable, perhaps even

8 An argument for many-valued deontic logics is already implicit in considerations about the need for values representing what is morally good, bad, and indifferent. See Chisholm (1982); (1986), pp. 69-75.

9 Castañeda in personal communication suggested the terminology 'pair-wise' evaluation, proposing the method as a way of preserving the isomorphism between what I have called multiple prescription values and classically bivalent truth values.

preferable to countenancing an indeterminately many-valued deontic predication semantics.[10]

Unfortunately, the pair-wise reduction of multiple deontic prescription values is excessively cumbersome. It is so unwieldy as to be impracticable, and, ultimately, theoretically unsatisfactory. Now consider the magnitude of task involved in applying the pair-wise algorithm for evaluation of act or practition conjunctions. The pair-wise treatment works something like this for conjunctions, in the example below, with four distinct obligation source-indexed act or practition conjuncts. Let 'V_i' represent indifferently prescription value P_i or N_i. The precise order of pair-wise evaluation is irrelevant to the number of values needed for complete semantic evaluation.



To compute the combined prescription value of the conjunction, the pair-wise approach requires, where n is the number of conjuncts, an evaluation depth of $4(n+(n-1))^2$ lines. This is because each conjunct might obtain prescription value from a distinct obligation source, and the pair-wise algorithm introduces an additional number less one of these values. If $n = 4$ as above, pair-wise evaluation requires $4(7^2) = 196$ lines; if $n = 5$, in a conjunction of 5 acts or practitions, pair-wise evaluation requires $4(9^2) = 324$ (where, by comparison, an ordinary truth table for 5 propositional variables contains only 32) lines!

Nor is this merely a practical inconvenience. Pair-wise evaluation depth also has theoretical implications for semantics of many-valued deontic predications. Because there are indeterminately many distinct sources of obligation needed to disambiguate the potentially conflicting obligations impinging on possible agents, a combinatorial explosion places the resolution of imaginable moral dilemmas beyond the computational capacity of any finite calculator. An agent's moral status is thereby rendered literally incomprehensible under pair-wise deontic evaluation, despite its abstract bivalent 'isomorphism' with classical propositional logic. The alternative way of handling multiple prescription values in many-valued deontic predications according to rule (C) above is so simple

[10] See Rescher (1969), pp. 14-15, for hints about controversies surrounding many-valued propositional logics.

as to be practical even for mental or pencil and paper arithmetic. It restricts the number of distinct prescription values needed to evaluate any finite conjunction of n acts or practitions to a maximum of $2(n+1)$, and the total number of combinations of prescription values needed for its semantic matrix-type analysis to no more than 2^{n+1}. The conjunctive proposal for dealing with many-valued deontic predications by rule (C) is not only more economical, requiring fewer distinct prescription values, but it is also less complicated by several factors. The basic idea is to provide a shortcut recognition of the parallel with truth value semantics for propositional conjunction, by which even one falsehood always makes an entire conjunction false, falsehood being analogous in the expected way to negative prescription.

There may nevertheless be a tradeoff in the method. By rejecting pair-wise evaluation, the proposal gives up even the pretense of bivalence or indirect semantic isomorphism with classical propositional truth value semantics. To repudiate pair-wise evaluation is to repudiate bivalence, even in the abstract semantic isomorphism sense, and with it the classical bivalent semantics that is often presupposed as underlying standard deontic logics. Yet if moral dilemmas are avoided or resolved in the standard way, by circumstantial relativization or obligation source-indexing, and if the truth values of circumstantially relativized or obligation source-indexed obligation statements are semantically related as by principle (D) to the deontic prescription values of acts or practitions, then deontic logic cannot have a classically bivalent prescription value semantics. At its deepest semantic level, for the prescription values of acts or practitions, deontic logic must be regarded as many-valued.[11]

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[11] The argument for many-valued deontic semantics was developed during my participation in Hector-Neri Castañeda's National Endowment for the Humanities (NEH) Seminar on 'Human Action: Self, Thinking, and Reality', Indiana University, 18 June - 10 August 1984. This essay is dedicated to Castañeda's memory.

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