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THE CONCEPT OF QUASI-TRUTH[1]

Otávio BUENO and Edelcio G. DE SOUZA

> For a long time the semantical concepts have had an evil reputation among specialists in the study of language. They have resisted all attempts to define their meaning exactly, and the property of these concepts, apparently so clear in their content, have led to paradoxes and antinomies. For that reason the tendency to reduce these concepts to structural-descriptive ones must seem quite natural and well-founded.
>
> Tarski [1933], p. 252.

1. *Introduction: truth, pragmatic truth, and quasi-truth*

Tarski's seminal study of the concept of truth (Tarski [1933]),[2] and his contributions to the development of model theory (see, for instance, Tarski [1954]), have supplied an inspiring framework in terms of which several issues intertwined with the concept of truth have been considered. From an account of semantical paradoxes to applications of the concept of truth to several philosophical domains (ranging from epistemology to the philosophy of science), Tarski's contributions found widespread and varied applications.[3] In the philosophy of science, just to take an example, the use of

[1] We are greatly indebted to Newton da Costa and Steven French for illuminating discussions and correspondence on the topics examined here, and we wish to thank Steven for his helpful comments on an earlier version of this paper. Of course, we are entirely responsible for any blunders and infelicities that still remain.

[2] For a non-technical outline of the main ideas presented in the 1933 paper, see Tarski [1944], and Tarski [1969].

[3] Despite such applications, Tarski's work, at least more recently, has not been accepted without criticism. In Etchemendy [1990], it is claimed that Tarski's account of logical truth and logical consequence is simply wrong, for it 'does not capture, or even come close to capturing, any pretheoretic conception of the logical properties' (p. 6). Two further criticisms are put forward in McGee [1992], where one claims that (1) it is only with 'heavy-handed metaphysical assumptions' that the fact that sentences are valid (according to Tarski's characterisation) is not a matter of contingent fact; moreover, according to McGee,

model-theoretic techniques in the so-called semantic approach, which has been receiving growing attention in the literature, heavily relies on such contributions.[4]

In 1986, an extension of Tarski's definition of truth was proposed by Mikenberg, da Costa and Chuaqui (see their [1986]), under the heading of pragmatic truth, and since then da Costa and French have been exploring its consequences for the philosophy of science.[5] In what respect is this an *extension* of Tarski's theory of truth? The main idea is that truth is formulated in a convenient set-theoretic context in which the epistemic 'openness' we usually find both in science and in everyday life can be accommodated. In order to do so, the authors introduce a notion of partial structure—which plays a similar role to the concept of interpretation in the standard Tarskian semantics—, and pragmatic truth is then defined in terms of such a kind of structure. The Tarskian formal definition of truth, as we shall see, can then be viewed as a particular case of pragmatic truth, when one is restricted to genuinely total structures (those that are not partial). Thus, the extension presented is concerned with a formal issue (which can, of course, receive different philosophical interpretations).

Having said that, it may seem a bit odd that an extension of Tarski's definition should be a formal notion of *pragmatic* truth. After all, isn't Tarski's view a form of *correspondence* theory? And aren't these rival interpretations of truth? In order to overcome this apparent paradox, some issues should be disentangled.

We should demarcate from the outset, as clearly as possible, *philosophical interpretations* of the notion of truth from a *formal definition* of truth.[6]

(2) it is far from being obvious that 'being true in every model is any guarantee that a sentence is true' (p. 273). Etchmendy's view is then criticised, and Tarski's account is defended, in Sher [1996]. Finally, for a critical discussion of why the notion of truth in a structure is *not* found in Tarski [1933], see Hodges [1986]. Of course, this is *not* a critique of Tarski's definition of truth, but a clarification of Tarski's 1933 account. (For related clarifications about Tarski's contribution to our understanding of truth, logical truth and logical consequence, see Etchemendy [1988].) A systematic account of some philosophical critiques to Tarski's theory of truth can be found in Kirkham [1992], pp. 175-210, which includes: (a) objections to Tarski's material adequacy condition, (b) criticisms of the fact that Tarski has not defined truth, but only truth-in-a-particular-language, and (c) claims to the effect that Tarski's theory is vacuous, or is not ontologically neutral. Of course, as Kirkham notices, several of these criticisms are simply off the mark.

[4] On the semantic approach, see for instance: van Fraassen [1970], van Fraassen [1980], van Fraassen [1989], Suppes [1961], Suppes [1967], Suppes [1995], and Suppe [1989].

[5] This exploration constitutes what is called the *partial structures approach*. We shall briefly consider this approach in section 4, *below*.

[6] We owe this demarcation to Newton da Costa.

Strictly speaking, Tarski's achievement lies within the latter, whereas all standard theories of truth (correspondence, coherence, pragmatic theories etc.) are to be found among the former. The idea is that formal definitions, at least in principle, are 'neutral' towards certain substantial issues, or at least as 'neutral' as mathematical formulations can possibly be. Of course, mathematical constructions may have been motivated by the most varied viewpoints, but *qua* mathematical constructions, they in general underdetermine possible interpretations. So, although Tarski may have been *motivated* by the idea of grasping the intentions contained in the correspondence theory, the definition he has produced does not necessarily depend upon that (strictly speaking, it depends much more on set theory than on any such motivation!). This is the reason why it is possible to rely on (a conveniently adapted formal version of) *his* definition in order to present a *pragmatic* notion of truth, as Mikenberg, da Costa and Chuaqui have done.

In their paper, a formal notion of pragmatic truth was put forward as a possible mathematical counterpart to certain pragmatist views of truth (such as Peirce's and James's). The authors, of course, do not claim to have represented mathematically such views, but only to have been *motivated* by them in order to formulate their own proposal. More recently, however, something stronger has been suggested. As is well known, in his paper on truth, Tarski claims that

> throughout this work I shall be concerned exclusively with grasping the *intentions* which are contained in the so-called *classical* conception of truth ('true —corresponding with reality'). (Tarski [1933], p. 153; the first italic is ours.)

Similarly, da Costa and French have persuasively argued that certain pragmatist 'intentions' underlying Peirce's and James's conception of truth can be grasped by the formal notion of pragmatic truth (see da Costa and French [1996], chapter 1). Among such 'intentions', there is the idea of convergence to the truth,[7] that finds, as we shall see, a nice formulation in the formal version of pragmatic truth.

[7] Such an idea is presented by Peirce in several ways. Consider, for instance, the following passage, in which the stability of the convergence to the truth is stressed: 'Different minds may set out with the most antagonistic views, but progress of investigation carries them by force outside themselves, to one and the same conclusion. This activity of thought by which we are carried, not where we wish, but to a foreordained goal is like a question of destiny. No modification of the point of view taken, no natural bent of mind, can enable a man to escape the predestinate opinion. This great hope is embodied in the conception of truth and reality' (Peirce [1965]). One of the main features of pragmatic truth, as presented by Mikenberg, da Costa and Chuaqui, consists in the fact that if a theory is pragmatically

So, as formulated by Mikenberg, da Costa and Chuaqui, pragmatic truth constitutes a change at the *formal* level of truth (with the introduction of partial structures), that leads to a different *interpretation* of truth (of a pragmatist stance). Nevertheless, as was soon realised, just as in Tarski's case, different *philosophical interpretations* of the formalism of pragmatic truth are possible. Due to the possibility of grasping, with pragmatic truth, certain 'intentions' underlying the pragmatist conception, a *pragmatist* interpretation can be advanced. However, one can also interpret the formalism in terms of an *epistemic possibility of truth*. According to this latter interpretation, if it is assumed that certain laws and statements are accepted in science, then every model which is compatible with such laws and statements represents an epistemic possibility of truth, and the formalism can be seen as representing such epistemic possibility (see da Costa, Chuaqui and Bueno [1997]). In order to remain neutral with regard to these alternative interpretations, the term 'quasi-truth' has then been used.

In the present note, we wish to put forward a different *definition* of quasi-truth, still following the main guidelines of da Costa's version (and to this extent, we shall be considering the problems from a *formal* level). There are two main reasons for presenting such a definition. First, it can be more straightforwardly applied to some of the fields in which quasi-truth has been considered thus far, besides supplying useful conceptual tools for the partial structures approach. Second, a distinct *philosophical* outlook, of an empiricist line, can be nicely accommodated in terms of it, in the sense that our framework advances what seems to be an appropriate notion of truth for empiricism. Thus, the present work can be seen as providing both a general, further argument for the partial structures approach, and a particular argument for an empiricist proposal. And in order to do so, we shall approach the issues from both *formal* and *philosophical* viewpoints.

After reviewing the standard account of quasi-truth in section 2, we will suggest, in section 3, a new formulation of it, pointing out along the way in what respects this version can strengthen the empiricist case. In section 4, some philosophical considerations about this new formulation will then be considered, including a discussion of its relationship with the partial structures approach.

2. *Truth and quasi-truth*

Why do we need the concept of truth? Answers will generally indicate the centrality of this notion to our conceptual scheme. Truth can be used to

true, it will remain forever as such, and thus the stability of the convergence to the truth that Peirce touches upon can be formally represented.

characterise the validity of reasoning, to support the beliefs that inform our actions, or to present an aim of science (see Horwich [1990], p. 1). Of course, in each case, the answers presented generally rely on particular *interpretations* of truth. Some of these interpretations either are too strong to be accepted without several qualifications (this seems to be the case of a full-blooded correspondence view),[8] or have never been articulated in an acceptable form (as in the case of the coherence theory thus far).[9] In such a context, it might be opportune to introduce a weaker notion of truth, which does not face the troubles that threaten the correspondence doctrine, and which is more manageable than the extant coherence views. An appropriate concept of quasi-truth may be a promising candidate. Of course, such a concept will have to be 'sufficiently strong' in order to meet the philosophical, logical and methodological expectations surrounding a notion of truth. But how strong?

It is very difficult not to bring the realist-empiricist debate into focus while considering this issue. There are at least two kinds of answer here. On the one hand, there are those, of a more *realist* persuasion, who will argue that the notion of quasi-truth is nothing but a provisional surrogate for truth *tout court,* and as science is further developed, quasi-truth will be systematically replaced by truth (even if only in very ideal conditions). On the other hand, *empiricists* will claim that quasi-truth is everything we should strive for, given that (a) there are no means of actually establishing truth, nor (b) any need for doing so, at least as far as our interpretation of the scientific enterprise is concerned. With regard to (a), underdetermination arguments are presented pointing out that the empirical information at our disposal is compatible with several distinct theoretical representations of the phenomena. Concerning (b), particular empiricist interpretations are then articulated.

According to constructive empiricism, one of the best formulated of such interpretations, science can be understood as an activity whose aim is not the elaboration of true theories, but of empirically adequate ones.[10] The important point in this context is that the notion of empirical adequacy,

[8] Thus far nobody has been able to present an acceptable account of this mysterious notion of correspondence, and even if one accepts Popper's suggestion to the effect that Tarski's theory of truth has done precisely this job (see Popper [1972], pp. 319-340), it is far from being obvious, as Newton da Costa has pointed out, how Tarski's account can be applied to certain domains of physics, such as quantum mechanics.

[9] Rescher, for instance, whose work on the coherence theory is very detailed (see Rescher [1973]), strictly speaking has developed a coherence theory of justification, not of truth.

[10] For a detailed formulation of constructive empiricism, see van Fraassen [1980], [1985], [1989], and [1991].

which of course is weaker than truth (see van Fraassen [1980], p. 64), can then receive a clear formulation in terms of quasi-truth (see Bueno [1997], and Bueno [1996]). Moreover, underdetermination arguments also find their way within the partial structures approach, due to the formal features of the notion of quasi-truth (and this fact, of course, already points out the strong links between quasi-truth and the empiricist view). Let us see why.

Da Costa's version of quasi-truth[11] is put forward as a weaker notion of truth, appropriate for the 'partialness' and the 'openness' typically found in science and in everyday life.[12] As a matter of fact, we hardly (if ever) have at our disposal complete information about any particular domain that we happen to be investigating. And, to a certain extent, this epistemic predicament can be formally represented with the introduction of a convenient concept of partial structure. A *partial structure* $A = \langle D, R_i \rangle_{i \in I}$ is a set-theoretic construct in which D is a non-empty set, and $(R_i)_{i \in I}$ is a family of partial relations on D. A *partial relation* is then a relation that it is not necessarily defined for every n-tuple of objects of its domain.[13] This feature can be seen as a formal counterpart to the epistemic predicament just mentioned, given that partial structures can be thought of as modelling a particular empirical domain.

It is then in terms of partial structures that quasi-truth will be defined. In order to do so, however, a further concept has to be introduced. Given a partial structure A, we say that a structure B is an *A-normal structure* if the relations in B extend the partial relations in A to total ones (and thus B contains exclusively total relations; that is, relations defined for all n-tuples of their respective domains). Of course, given a partial structure A, there may be several distinct A-normal structures B that extend A to a total structure. Necessary and sufficient conditions for the existence of A-normal

[11] As a matter of fact, the definition of quasi-truth presented in Mikenberg, da Costa and Chuaqui [1986], under the heading of pragmatic truth, has later been simplified by da Costa (see da Costa [1986], and da Costa, Bueno and French [1998]). The version we are now going to present is this second, simplified one. The two versions, however, are essentially equivalent.

[12] This aspect has been extensively explored by da Costa and French (see section 4, *below*).

[13] Formally speaking, a partial relation R on a set D can be identified with a triple $\langle R_1, R_2, R_3 \rangle$, where R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that R_1 is the set of n-tuples that belong to R; R_2 of those n-tuples that do not belong to R; and R_3 of the n-tuples for which it is not defined whether they belong to R or not. Of course, when R_3 is empty, R is a usual n-place relation that can be identified with R_1. This characterisation is put forward in da Costa and French [1990], p. 255, note 2. In section 3, *below*, we shall present it in a slightly different setting.

structures can be found in Mikenberg, da Costa and Chuaqui [1986].[14] The main idea, however, consists in requiring that the extension be done in such a way that it be consistent with certain accepted sentences P (this actually supplies a constraint for the admissible extensions).

A sentence S will then be *quasi-true* in a partial structure A if there is an A-normal structure B in which S is true (in the Tarskian sense). If a sentence S is not quasi-true in a partial structure A (according to an A-normal structure B), we say that S is quasi-false (in A according to B). As we shall see in a moment, in terms of this notion of quasi-truth, a formal framework appropriate to consider several problems in the philosophy of science can be advanced.

We can now return to the underdetermination argument mentioned above. In fact, in terms of partial structures and quasi-truth the notion of underdetermination can be forcefully presented. (And due to this feature, of course, such a framework seems to be quite compelling for an empiricist.) What happens is that, given a partial structure A, there are several distinct A-normal structures that extend A into a total structure. It is for this reason that the notion of quasi-truth is weaker than truth: the former is meant to represent only a 'partial conformity' between the structures under consideration. A theory which is quasi-true presumably grasps only certain aspects of its domain, and it leaves open various theoretical possibilities and further extensions. Indeed, the fact that such a theory is quasi-true is compatible with several distinct extensions of the relevant partial structures, and in this sense an unavoidable underdetermination is at the heart of the concept of quasi-truth. This is nothing but a different aspect of the same kind of underdetermination that the empiricist explores in his or her account of science.

Thus, quasi-truth seems appropriate, at a philosophical level, for an interpretation of science, such as the empiricist, that stresses the 'openness' and 'partialness' of our knowledge[15] (the use of partial structures is particularly relevant at this point), and avoids a commitment to certain strong proposals, such as those that claim that each particular element of our models has to find a counterpart in reality[16] (the underdetermination found in quasi-truth should make us aware of the difficulties with this claim). Moreover, given that in the version of quasi-truth to be here advanced no necessary commitment to the correspondence theory is to be found, the very idea of

[14] We shall return to this point in section 3, *below*. For a further discussion, see Bueno [1997], section 3.1.

[15] This point is considered by van Fraassen in van Frassen [1994].

[16] This, of course, is one of the main points of van Frassen [1989].

an exact counterpart of each element of our models with reality is blocked from the outset.

It goes without saying that da Costa has deliberately *not* tied his formal account of quasi-truth to such an empiricist interpretation, and even when exploring the partial structures approach with French, they have decided to remain as neutral as possible with regard to such issues (in the sense of not trying to defend a particular interpretation of the formalism, but to investigate various alternatives). Given the distinction we mentioned about philosophical interpretations of truth and formal definitions of it, this is undoubtedly a sensible strategy.

Having spelled out our preferred interpretation (the empiricist one), we shall now consider a different way of presenting the formalism. In proceeding this way, someone may complain that we are putting the cart before the horse, given that strictly speaking the formalism should come first, instead of the interpretation. Fair enough. But we hope that, by making our bias explicit from the outset, the reader may see more clearly in what respects the formalism offered here supplies some support for the empiricist case.

3. *Quasi-truth, expanding models and degrees of quasi-truth*

In this section, we shall put forward our account of quasi-truth, and in order to do so, we shall first make some general considerations about the language we shall be using.[17] The language of our quantification theory is standard. Its vocabulary has the following symbols:

(1) logical connectives: $\neg$, $\vee$, $\wedge$, $\rightarrow$;
(2) quantifier symbols: $\forall$, $\exists$;
(3) parentheses: (,);
(4) an infinite list of individual variables: $x_0, x_1, x_2, \ldots$;
(5) an infinite list of individual constants: $c_0, c_1, c_2, \ldots$;
(6) for each $n \geq 0$, an infinite list of n-place predicate symbols: $F^n_0, F^n_1, F^n_2, \ldots$

The concepts of term, formula, subformula, sentence, free and bound variable etc. are the usual ones. Moreover, we use the following notation:

(1) $A, B, C, A_1, B_1, C_1, \ldots$ as metalinguistic variables for formulas;
(2) $v, v_1, v_2, \ldots$ as metalinguistic variables for object language variables;
(3) $t, t_1, t_2, \ldots$ as metalinguistic variables for terms;

[17] In what follows, we shall adopt (and wherever necessary, adapt) the notation and some basic, standard definitions about quantification language presented in Grandy [1977].

(4) A_t^s to indicate the formula which results from substituting t for s in A, provided that (i) if s is a variable, t is substituted only for free occurrences of s, and (ii) if t is a variable, every new occurrence of t is free; if these conditions are not met, A_t^s is simply A.

Just as in da Costa's version of quasi-truth, partial structures also play a fundamental role here. Their main function is to supply an interpretation for our quantification language, and due to this role, from now on we are going to call them *partial models*.

Definition 1. (Partial models) A *partial model for quantification language* is an ordered pair $\langle D, I\rangle$, where D is an non-empty set, and I is a function such that:

(1.1) for each constant c, $I(c) \in D$;

(1.2) for each predicate symbol F_i^n, $I(F_i^n) = \langle I_T(F_i^n), I_F(F_i^n), I_U(F_i^n)\rangle$, where

(i) $I_T(F_i^n), I_F(F_i^n), I_U(F_i^n) \subseteq D^n$ are pairwise disjoints;

(ii) $I_T(F_i^n) \cup I_F(F_i^n) \cup I_U(F_i^n) = D^n$. ($D^n$ is the set of n-tuples of objects in D.)

One of the points of this definition is to make explicit that the notion of a 'partial relation' is formulated in the metalanguage, and it is meant to reflect a lack of information we have about whether certain relations between the objects of D hold or not. To a certain extent, this reflects our epistemic condition, in which the partiality is not (claimed to be) something 'out there' in the world, as it were, but concerns *our information* about the world. This is precisely the role of the I_U-component in the definition of a partial model: to supply a formal tool in order to represent such epistemic 'openness'. Roughly speaking, the I_U-component will include all the n-place relations for which, according to the interpretation supplied, we still do not know whether they hold (in which case they will belong to I_T) or not (in which case they will belong to I_F). Thus, in this characterisation, a 'partial relation' could be viewed as something 'overdetermined': not having enough information to discriminate whether a certain relation holds or not for certain objects, we simply add all the unknown possibilities to the I_U-component.

Just as in the standard Tarskian account, the notion of quasi-truth is not to be directly defined, but we shall need a detour through a convenient notion of quasi-satisfaction. In order to do so, we shall first introduce some terminology and notation.

Let α be a function which assigns, to each individual variable, an element of D, and to each constant c, $I(c)$. Such a function is said to be a *sequence in* $\langle D, I\rangle$. We use α, β, γ, α_1, β_1, γ_1, ... as metalinguistic variables that

range over sequences. Moreover, we use the notation $\alpha \approx_v \beta$ to express that the sequences α and β agree on all variables, except possibly v, i.e., for all $v' \neq v$, $\alpha(v') = \beta(v')$.

Definition 2. (Quasi-satisfaction) The relation α *quasi-satisfies* A *in* $\langle D, I\rangle$ is defined recursively:

(2.1) α quasi-satisfies $F_i^n t_1 ... t_n$ in $\langle D, I\rangle$ iff $\langle \alpha(t_1), ..., \alpha(t_n)\rangle \in I_T(F_i^n) \cup I_U(F_i^n)$;

(2.2) α quasi-satisfies $\neg A$ in $\langle D, I\rangle$ iff α does not quasi-satisfy A in $\langle D, I\rangle$;

(2.3) α quasi-satisfies $A \vee B$ in $\langle D, I\rangle$ iff α quasi-satisfies A in $\langle D, I\rangle$ or α quasi-satisfies B in $\langle D, I\rangle$;

(2.4) α quasi-satisfies $A \wedge B$ in $\langle D, I\rangle$ iff α quasi-satisfies A in $\langle D, I\rangle$ and α quasi-satisfies B in $\langle D, I\rangle$;

(2.5) α quasi-satisfies $A \rightarrow B$ in $\langle D, I\rangle$ iff α does not quasi-satisfy A in $\langle D, I\rangle$ or α quasi-satisfies B in $\langle D, I\rangle$;

(2.6) α quasi-satisfies $\exists v A$ in $\langle D, I\rangle$ iff for some β, $\alpha \approx_v \beta$, and β quasi-satisfies A in $\langle D, I\rangle$;

(2.7) α quasi-satisfies $\forall v A$ in $\langle D, I\rangle$ iff for all β, $\alpha \approx_v \beta$, and β quasi-satisfies A in $\langle D, I\rangle$.

The main point of this definition consists, of course, in condition (2.1), in which the I_U-component of the partial model enters. One may wonder why this is a definition of *quasi*-satisfaction. The answer is clear: because we are explicitly taking into account those relations (to be found in the I_U-component) about whose 'epistemic status' we are still uncertain. Depending upon how such relations are later dealt with (with the growth of our knowledge about the domain under consideration, they may become elements of I_T or I_F), our claims of quasi-satisfaction may change. For instance, if such relations actually become elements of I_F, a sequence that once quasi-satisfied a formula $F_i^n t_1 ... t_n$ will no longer quasi-satisfy it.

This is, therefore, a rather liberal notion of satisfaction. This fact reflects a similar feature in da Costa's formulation of quasi-truth, something we have already noted: given a partial structure A, there are several distinct A-normal structures that extend A to a total structure. In order to use the standard Tarskian semantics (which was articulated only for genuinely total structures) while using partial structures, da Costa's strategy was to make a detour through A-normal structures. Given that these are total structures, all that we know about Tarski's definition of truth could then be naturally imported to the definition of quasi-truth. The problem, however, was to guarantee the existence of such A-normal structures. In order to do so, as we have already mentioned, a set P of certain accepted sentences has to be in-

troduced,[18] and we have to perform a particular construction. The idea is that, for each partial relation R_i^n (in a given partial structure A), we construct a set R_i of atomic sentences and negations of atomic sentences such that the n-tuples that satisfy R_i^n correspond to atomic sentences, and the n-tuples that do not satisfy R_i^n to negations of atomic sentences. Let then $R = \cup_{i \in I} R_i$. A simple pragmatic structure A admits an A-normal structure if, and only if, the set $R \cup P$ is consistent.

It should be noted, however, that the strategy of considering quasi-satisfaction directly allows one to circumvent this construction and the associated set P. This, of course, is by no means a problem for da Costa's characterisation, but just points out the different strategies involved in the two definitions of quasi-truth.

Nonetheless, in both strategies we find a similar 'underdetermination': the plurality of A-normal structures (given a partial structure A) in da Costa's characterisation, and a liberal view of satisfaction here. This feature points out the similar role such concepts have in the definition of quasi-truth; indeed, quasi-truth is to be defined either in terms of quasi-satisfaction, or in terms of A-normal structures (depending on the strategy adopted). Moreover, the 'underdetermination' is also basic for quasi-truth —in fact, it makes quasi-truth weaker than truth—, and is extensively explored by the empiricist.

Notice that if the I_U-component is empty, we have the standard definition of satisfaction. The following proposition is then immediate.

Proposition 1. If α satisfies A in $\langle D, I \rangle$, then α quasi-satisfies A in $\langle D, I \rangle$.

Having presented this characterisation of quasi-satisfaction, we can now state the notion of quasi-truth, following the Tarskian strategy.

Definition 3. (Quasi-truth) A formula A is *quasi-true* (respectively *true*) in $\langle D, I \rangle$ iff A is quasi-satisfied (respectively satisfied) in $\langle D, I \rangle$ by all sequences in $\langle D, I \rangle$.

[18] The introduction of this set P can of course be motivated by considerations from scientific practice, where in general there is a basic core of laws and statements that we wish to preserve while articulating and extending our theories. Moreover, as da Costa and French have pointed out, this set plays a role in interpreting quasi-truth as a pragmatist conception of truth, given that, according to the latter, certain statements are to be taken as true in the correspondence sense. In the formalism presented by da Costa and French, obvious candidates for such statements are, of course, the elements of P (see da Costa and French [1996], chapter 1).

Definition 4. (Quasi-validity) A formula is *quasi-valid* (respectively *valid*) iff A is quasi-true (respectively true) in all partial models.

The next proposition, that relates truth and quasi-truth, validity and quasi-validity, is then immediate.

Proposition 2.
(1) If A is true in $\langle D, I\rangle$, then A is quasi-true in $\langle D, I\rangle$.
(2) If A is valid, then A is quasi-valid.

One of the points of the present account of quasi-truth is to supply a framework in order to compare partial models. We shall present here two proposals. The first is obtained by the following definition.

Definition 5. (Expanding models) Let $M_1 = \langle D_1, I_1\rangle$ and $M_2 = \langle D_2, I_2\rangle$ be distinct partial models for quantification language. We say that M_2 *expands* M_1 if
(5.1) $D_1 = D_2$;
(5.2) for each individual constant c, $I_1(c) = I_2(c)$;
(5.3) for each predicate symbol F_i^n, we have that:
(i) $I_{1T}(F_i^n) \subseteq I_{2T}(F_i^n)$;
(ii) $I_{1F}(F_i^n) \subseteq I_{2F}(F_i^n)$;
(iii) $I_{1U}(F_i^n) \subseteq I_{2T}(F_i^n) \cup I_{2F}(F_i^n) \cup I_{2U}(F_i^n)$.

The main idea is that a partial model M_1 *expands* M_2 if 'more' relations are taken into account by the former than by the latter. So, despite being partial, it still possible to compare them. The next proposition shows the kind of order that is introduced by the expanding relation.

Proposition 3. The expanding relation between partial models for quantification language is reflexive and antisymmetric.

In terms of the expanding relation, it is then possible to introduce a different kind of comparison between partial models, with explicit reference to the notion of truth.

Definition 6. (Approximation to the truth) Let $M_1 = \langle D_1, I_1\rangle$ and $M_2 = \langle D_2, I_2\rangle$ be distinct partial models for quantification language, and A a formula.
(a) We say that M_2 *approximates the truth of* A *in* M_1 if
(6.1) M_2 expands M_1;
(6.2) A is quasi-true in M_1;
(6.3) A is true in M_2 (in the Tarskian sense).

(b) We say that A is *approximately true in* M_1 if there is a partial model for quantification language M_2 such that M_2 approximates the truth of A in M_1.

Two comments should be made about this definition. First, despite the apparent strangeness of the following remark, such a definition supplies a *non-realist* notion of approximation to the truth, due to the use of quasi-truth as the underlying truth notion. Indeed, the concept of truth in (6.3) is simply semantic, and by no means 'substantive', given that it its nothing but a formal notion of truth, articulated in set-theoretic terms. Incidentally, it was for this reason that Tarski once remarked that he would not be bothered if someone wished to take his definition of truth as a definition of 'frue' (see Tarski [1944]). To a certain extent, the more 'metaphysical' aspects of the concept of truth (which would definitely disturb the empiricist) are deliberately disregarded in the Tarskian account. And it is in this respect that, according to the demarcation presented in section 1, this account supplies a *formal definition* of truth, not a *philosophical interpretation* of it. Moreover, given that several logical notions are articulated, from a semantic point of view, in terms of this formal notion of truth, if an empiricist had anything against *this* notion of truth, he or she would not be able to use some of the most important theoretical resources for his or her own view. (This is especially so for constructive empiricism, given the use of the semantic approach, according to which to present a scientific theory is to present a family of *models*.)

Second, the definition of approximation to the truth was presented in order to put forward a notion of *degree of quasi-truth*. In fact, instead of asking, in (6.3), that A be true in M_2, we can request that it be *quasi-true* in M_2. Now, given that, by (6.1), M_2 expands M_1, and that, by hypothesis, $M_2 \neq M_1$, A is 'more' quasi-true in M_2 than in M_1, in the sense that more information about the domain of the structures under consideration is taken into account in M_2 than in M_1 (this is, after all, one of the points of the expanding relation). So, it is possible to claim that one aspect of the development of science is the increase in the degree of quasi-truth of its theories. And given the first comment, such a move is of course entirely compatible with an empiricist view.

This, however, is by no means an accidental fact. Indeed, the present characterisation of quasi-truth seems to be particularly appropriate for the empiricist, given that besides allowing the introduction of degrees of quasi-truth, in contrast with the standard version, the notion of a genuinely total structure (the A-normal structures) is *not* required. This supplies, thus, a nice framework for representing the radical 'openness' of our knowledge, an 'openness' with which the empiricist is particularly concerned.

4. *What use is quasi-truth? The partial structures approach*

Having presented this account of quasi-truth, we wish to close the present note pointing out in what respects it reinforces the partial structures approach. Thus our proposal should be seen as a *complementary* formulation of quasi-truth (perhaps more appropriate for those of an empiricist persuasion), *vis-à-vis* the one already put forward by da Costa, and explored by he and French.

In fact, da Costa and French have argued in detail for the richness of the partial structures approach, and how it supplies a different perspective to examine several problems in the philosophy of science. For instance, in da Costa and French [1989], the logic of induction is considered from this perspective; the notion of model in science is then investigated in da Costa and French [1990]; in da Costa and French [1993*a*], theory acceptance in terms of quasi-truth is discussed; and problems related to the modelling of 'natural reasoning' are then examined in da Costa and French [1993*b*]; finally, in da Costa and French [1995], a partial structures study of inconsistent belief sets is advanced. (For a systematic presentation of da Costa's and French's view, see da Costa and French [1996].)

Our proposal, because it spells out a notion of degree of quasi-truth, yields alternative resources to examine problems related to theory change in science, and the dynamics of scientific knowledge (see also Bueno [1996]). In particular, the notion of an expanding model can be useful in this context as a tool to make comparisons between the information brought by two distinct partial models. In future works, we shall explore these possibilities.

Underlying all these proposals, we find of course the concept of quasi-truth. This is in fact the main pillar of the partial structures approach. And by now, given everything that da Costa and French have already obtained with its adoption, and due to the fact that it can receive a clear formulation, it seems natural to claim that it has been overwhelmingly 'confirmed' by the 'evidence' supplied by all the extant applications. This should at least supply a hint, if not something stronger, about its usefulness.

In his paper on truth, Tarski mentioned the 'evil reputation' of semantical concepts. It is striking how, largely due to his own work, things could have changed so dramatically during the last sixty four years. And if Tarski helped us so much in 'rehabilitating' such concepts (even if by showing

how to get rid of them), we can now explore new domains that some of these concepts, such as quasi-truth, invite us to investigate.

Bueno: Division of History and Philosophy of Science,
Department of Philosophy, University of Leeds, Leeds, LS2 9JT, UK
e-mail: phloab@leeds.ac.uk

de Souza: Department of Philosophy,
Pontifical Catholic University of São Paulo,
Rua Monte Alegre, 984, São Paulo-SP, 05014-000, Brazil
e-mail: edelcio@exatas.pucsp.br

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