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BLOCKS. THE CLUE TO DYNAMIC ASPECTS OF LOGIC

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Abstract
The present paper introduces a new approach to formal logic. The block approach is especially useful to grasp dynamic aspects of reasoning, including formal reasoning, that fall beyond the reach of the usual approaches. A block language, the block analysis of proofs, and semantic systems in terms of blocks are articulated. The approach is first applied to classical logic (including proof heuristics). It is used to solve two important problems for adaptive logics (that have a dynamic proof theory). Some further applications are discussed, including meaning change.

1. *Aim of this paper*

In the present paper, I want to propagate a new approach — or perhaps some related approaches — to logic. The problems that brought me to the approach are explained in section 2. The kernel of the approach is the idea of blocks. Roughly, a block is a formula, sentence, or term, that is considered as an unanalyzed entity. The entities dealt with in a proof, a semantics, etc., are not seen as well-formed strings made up from a stock of symbols, but as blocks, possibly compounded by some logical terms.

Fascinating about the block approach is that it enables us to get a grasp on a number of dynamic aspects of reasoning. Most of these aspects belong to a dimension of logic that cannot be comprehended in terms of the abstract and static features of the usual metatheory. In the present paper, I offer a diversity of illustrations of such aspects. These include dynamic aspects of proofs in classical logic (henceforth **CL**), and dynamic aspects of proof search processes. They also include the peculiarities of dynamic proofs (that are characteristic of adaptive logics); as such proofs evolve, lines written earlier may have to be deleted in view of later added lines, and it may

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become possible to add lines that could not be added before. And they include a notion of information that is relevant to logical omniscience and to the informative character of deductive moves. Finally, I shall also show that the block approach offers an outlook on meaning change.

After section 2, I apply the block approach to a familiar domain: proofs in **CL**. My main aim is to introduce the approach, to make and justify some choices while designing the block language, to show that the approach makes sense, to show that it enables us to articulate some insights that remain implicit on other approaches, and to point to some alternatives (section 6). These sections include the formulation at the predicative level of the block language (section 4) and of the block semantics (section 5). In section 8, I briefly discuss the application of the approach to proof heuristics and to some pedagogical matters, while still concentrating on **CL**.

In the next series of sections (9 to 12), I tackle the two problems advanced in section 2. This will require that I briefly introduce the basic paraconsistent logic, and the semantics as well as the (dynamic) proof theory of the inconsistency-adaptive logic **APIL1**. I show that the block approach is revealing to understand the dynamics of the proof theory, and that it enables us to solve both problems each of which is essential to logics with a dynamic proof theory.

In sections 13 and 14, I discuss some further promises of the block approach. One of them is building formal languages that allow for meaning change in one direction, viz. by the specification of formerly primitive terms. The other suggests a way to apply the block approach to other forms of meaning change as well. This suggested mechanism is intended as a formal instrument for the problem solving approach to creativity and discovery.

In the final section 15, I list some open problems and suggest some more promises of the block approach.

That the present paper is rather long and its contents diverse has two causes of a rather different nature. The first is that the approach is new, and that I better try to push its cause. The second is that this paper was written for the Leo Apostel memorial volume of *Logique et Analyse*. Leo was my best friend and the deepest philosopher I met. I wanted to write a paper that he would have liked: concerning a new idea, programmatically interesting, and with an eye on underlying philosophical problems.

2. *Two problems with dynamic proof procedures*

Several aspects of real-life reasoning (argumentation) are dynamic. We not only drop conclusions after obtaining more information, but also after we analyzed the premises better.

Some logics have a dynamic proof theory, for example adaptive logics (see, e.g., [5], [11], and [10]) and some versions of non-monotonic logics (see, e.g., [18]).[2] This is why, it may be argued, they form a step in bridging the gap between formal logic and argumentation. The step takes off from the tougher side, by introducing dynamics without loss of precision or clarity.

The dynamics of a formal logic raises two important difficulties: Is the dynamics real? And how do we gain control over it? I shall articulate both difficulties as clearly as possible, because they formed the major motivation for articulating the block approach. But before tackling them, I need to clarify the dynamic character of the proof theory of adaptive logics.

The dynamics comes to this: it depends on the stage of the proof which steps may be added to it, *and* formulas derivable at some stage need not be derivable at a later stage (and *vice versa*). The dynamics may be expressed by deleting lines written previously, or by marking lines as in or out. Such dynamics justifiedly raises suspicion. Suppose that both John and Mary write a proof from the same set of premises. They start off in different ways, add lines and delete some in view of the dynamic character of the proof theory. After a while they end up with rather different sets of derived formulas. There is nothing special about this as exactly the same situation may arise with **CL**. However, as for **CL**, one wants to require that, if a formula is derivable from the premises, then both John and Mary should be able to derive it by extending *their* proof.

Clearly, "derive" and "derivable" are ambiguous here. In view of the dynamics, we should distinguish between *derivability at a stage* and a form of derivability that is independent of the stage of the proof. The latter is called *final derivability*. A is said to be finally derivable from Γ iff there is a stage of a proof from Γ such that A is derived *at* that stage and will not be deleted at any later stage.[3]

Final derivability provides us with stability. But we need something more, viz. *uniqueness*: what is finally derivable in John's proof need to be finally derivable in (an extension of) Mary's proof as well. And it is indeed possible to show that, for any Γ, a (decent) adaptive logic defines a unique set of formulas finally derivable from Γ. Such set may be captured by the semantics: the formulas finally derivable from Γ are the semantic consequences of Γ. The general idea here is that we allow for non-classical models of Γ and define the semantic consequences of Γ as the formulas that

[2] A basic difference is that adaptive logics do not rely on any non-logical preferences — see [9].

[3] In some cases we have to add a requirement on the extensions of the proof; see, e.g., section 6 of [5].

are true in all models of Γ that are (in a precise sense) as classical as possible. As the dynamics should be minimally clear by now, I come to the two difficulties.

First, is the dynamics real? As it is possible to define, in an absolutely static although non-monotonic way, the set of semantic consequences of Γ, what is the use of the dynamics of the proof theory? Why write lines if it is determined from the outset that they will have to be deleted later on? A dynamic proof theory seems 'realistic' in that real-life reasoning proceeds dynamically. But maybe real-life reasoning is just inefficient, or clumsy, or sloppy. If so, logicians should not worry about it.

There is a partial answer to this difficulty. It may be shown that, in the context of an adaptive logic, dynamic procedures are superior in that they guide us, often much faster than any static procedure could do, towards a situation in which final derivability is reached —see [12] and [13] for this justification of dynamic tableau methods for inconsistency-adaptive logics. Concrete matters of efficiency are often overlooked by logicians, and unjustifiedly so. But although the argument seems convincing in itself, it seems to refer dynamics outside the abstract domain to which belong derivability, semantic consequence, soundness and completeness, the Gödel and Skolem theorems, and even decidability. If we want to give dynamics a place in that domain, we should at least find a *semantic* counterpart of the proof theoretical dynamics. We should, for example, show that derivability at a stage corresponds to some notion of semantic consequence.

I now come to the second difficulty: how do we gain control over the dynamics? The most important aspect of this difficulty is related to the effectiveness of proofs. If the proofs are dynamic, how can we tell whether some formula derived at a stage of a proof from Γ is finally derivable from Γ? Remember that the aforementioned definition of final derivability was non-effective: it refers to all possible future stages of a proof. As a consequence, it does not provide us with a criterion to decide on the basis of a proof at a stage (and there are no other proofs) whether some formula is finally derived. When swimming in decidable waters, semantic considerations may provide us with such criteria. But these refer us outside of the proof itself. Up to now, no published paper even partially meets the second difficulty.

Both difficulties are extremely important. If we cannot resolve them, we can hardly claim to have an appropriate insight in the nature of the dynamics of the proof theory of adaptive logics. In the present paper, viz. in sections 9–12 I shall show that both difficulties are resolved in terms of blocks. I shall do so by applying the approach to one adaptive logic only, **APIL1**, and shall keep the applications as simple as possible.

3. *Formal-logical proofs*

To see that the inference from (1) and (2) to (3) is a correct application of Disjunctive Syllogism, it is not required to be aware of the internal structure of the wffs involved. It suffices to see that the triple ⟨(1), (2), (3)⟩ has the form $\langle A \vee B, \sim B, A \rangle$.

(1) $(p \& (\sim q \supset (r \& s))) \vee (((r \vee p) \supset q) \vee s)$
(2) $\sim (((r \vee p) \supset q) \vee s)$
(3) $p \& (\sim q \supset (r \& s))$

A different way to express this, is that one has to recognize a block and two compounds of blocks. I represent them here in a provisional way (in which I follow some obvious conventions on dropping parentheses):

(4) $[\![p \& (\sim q \supset (r \& s))]\!] \vee [\![((r \vee p) \supset q) \vee s]\!]$
(5) $\sim [\![((r \vee p) \supset q) \vee s]\!]$
(6) $[\![p \& (\sim q \supset (r \& s))]\!]$

Blocks may look like meta-variables, but they are not. They are formulas of the object-language that are considered as unanalyzed. Of course, there are other ways to analyze (1)-(3) in terms of blocks. Here is one of them:

(4') $([\![p]\!] \& ([\![\sim q]\!] \supset [\![r \& s]\!])) \vee ([\![(r \vee p) \supset q]\!] \vee [\![s]\!])$
(5') $\sim ([\![(r \vee p) \supset q]\!] \vee [\![s]\!])$
(6') $[\![p]\!] \& ([\![\sim q]\!] \supset [\![r \& s]\!])$

Suppose that the proof is annotated: a number and a justification is attached to each line, and the justification consists of the numbers of the lines from which the formula is derived and the rule by which it is derived. In such a case, we know which are the blocks that the author of the proof has *minimally* distinguished, and this gives us a unique block analysis.[4] If the justification of (1) and (2) is "Premise" and the justification of (3) is "(1), (2); Disjunctive Syllogism", then the analysis is as in (4)-(6). If a (correct) proof is not annotated, we may consider all possible annotations that make

[4] Later I shall also use the term "formula analysis" to indicate that a formula that occurs in a proof is reduced to one or more shorter formulas. For example, $A \& B$ is analyzed if both A and B occur in the proof. Similarly, $A \supset B$ is analyzed if either $\sim A$ or B occurs in the proof (and these may be obtained from $A \supset B$ by applying either *modus tollens* or *modus ponens* in the presence of $\sim B$, respectively A). Similarly for the formula analysis of goals (formulas one tries to derive from the premises).

it a correct proof, and each of these will give us a unique block analysis. In view of this, I shall suppose in the sequel that proofs are annotated.

Let every step in a proof (every addition of a line to the proof) define a *stage* of the proof. Obviously, the transition from stage n to stage $n+1$ may modify the block analysis of the formulas in the proof, even of the formulas that were present already at stage n. Here is an example:

1	$(p \supset \sim q) \supset (p \& (\sim r \vee \sim p))$	Premise
2	$p \supset \sim q$	Premise
3	$p \& (\sim r \vee \sim p)$	1, 2; Modus Ponens
4	p	3; Simplification

Let us now consider the four stages of this proof in terms of blocks. To make life easier, I (provisionally) represent each block by a number between double square brackets.

STAGE 1:

1	$[\![1]\!]$	Premise

STAGE 2:

1	$[\![1]\!]$	Premise
2	$[\![2]\!]$	Premise

STAGE 3:

1	$[\![2]\!] \supset [\![3]\!]$	Premise
2	$[\![2]\!]$	Premise
3	$[\![3]\!]$	1, 2; Modus Ponens

STAGE 4:

1	$[\![2]\!] \supset ([\![4]\!] \& [\![5]\!])$	Premise
2	$[\![2]\!]$	Premise
3	$[\![4]\!] \& [\![5]\!]$	1, 2; Modus Ponens
4	$[\![4]\!]$	3; Simplification

The identification of the blocks at the stages of this proof will be obvious. But if we generalize it, we hit upon a problem. Suppose that the proof is continued as follows:

5	$p \vee (p \supset \sim q)$	4; Addition

The block analysis of stage 5 will contain the same lines as stage 4 plus

5	$[\![4]\!] \vee [\![6]\!]$	4; Addition

For the application of Addition to be correct, the second disjunct *might* be any block, and in this concrete proof it happens to be $[\![2]\!]$. But there is no need to identify it as such because none of the applied inference rules requires that $[\![6]\!]$ is identical to $[\![2]\!]$.

Of course, the way in which blocks are represented may interfere here. If a block is represented as a formula between square brackets, blocks that have the same contents are identified. (Hence, the second disjunct in 5 is $[\![2]\!]$.) So, let us be careful and look at possible further complications before we settle the way in which blocks are represented.

Consider the following application of Addition:

6 $\quad p \vee ((p \supset \sim q) \supset (p \& (\sim r \vee \sim p)))$ $\quad$ 4; Addition

How shall this be blockwise analyzed? The second disjunct of 6 happens to be the first premise. But the block analysis does *not* reveal this. By the time the analysis arrives at line 6, we find there

6 $\quad [\![p \vee ((p \supset \sim q) \supset (p \& (\sim r \vee \sim p)))]\!]$ $\quad$ 4; Addition

We know that we have to analyze it as

6 $\quad [\![p]\!] \vee [\![(p \supset \sim q) \supset (p \& (\sim r \vee \sim p))]\!]$

But remark that the block $[\![(p \supset \sim q) \supset (p \& (\sim r \vee \sim p))]\!]$ nowhere occurs in the proof, and hence *cannot* be identified with anything. For sure, $[\![p \supset \sim q]\!] \supset ([\![p]\!] \& [\![\sim r \vee \sim p]\!])$, viz. $[\![2]\!] \supset ([\![4]\!] \& [\![5]\!])$, occurs in the proof. But nothing in the (annotated) proof reveals that the second disjunct of 6 is identical to the first premise. Only further analysis might reveal this — but no sensible further analysis ever would reveal it. So, there seems to be no good reason to write the second disjunct of 6 as $[\![2]\!] \supset ([\![4]\!] \& [\![5]\!])$.[5]

In such cases, the best policy seems to follow the strictest convention: we consider blocks as identical *only* if they *must* be identified in view of the annotation of the proof. This convention saves us further trouble. Indeed, if the second disjunct of 6 were $(p \supset \sim q) \supset (\sim r \vee \sim p)$, should we analyze it as $[\![2]\!] \supset [\![5]\!]$? And if it were $p \supset ((p \supset \sim q) \supset (p \& (\sim r \vee \sim p)))$, should we analyze it as $[\![4]\!] \supset ([\![2]\!] \supset ([\![4]\!] \& [\![5]\!]))$?

In view of the convention, I shall write a block as a number and a formula between double square brackets. The formula will be called the *contents* of the block. The numbers allow us to indicate that two blocks with the same

[5] The matter would be different if we learned from the annotation of the proof that 6 was written by application to 1 and 4 of the derivable (but redundant) inference rule $A, B \,/\, A \vee B$.

contents are not identified, as in $[\![4, p]\!]$ and $[\![5, p]\!]$ The contents is mentioned because we should be able to analyze it further in order to add steps to a proof. Especially in view of the complications required by predicate logic, I also introduce a shorthand notation and write $[\![p]\!]^4$ and $[\![p]\!]^5$ instead of $[\![4, p]\!]$ and $[\![5, p]\!]$. Let me stress again that the only function of the numbers is to express that two blocks that have the same content are or are not identified.

4. *The general block-analysis and the block language (for predicate logic)*

What is the general procedure to be used in the block analysis of a proof? We first write each formula in (the stage of) the proof as a single block. Starting from the top, we look at the justification of each step. If a block follows from one or two other blocks by some rule, the rule specifies whether some blocks have to be analyzed further and in which way they should be analyzed. An important *convention* is that, if a block is divided, then it is divided everywhere in the proof. In stage 4 of the previous example, block $[\![p\&(\sim r\vee\sim p)]\!]^3$ is replaced by $[\![p]\!]^4\&[\![\sim r\vee\sim p]\!]^5$ in both 3 and 1. If we did not do so in 1, we could not see from the block proof that 3 follows from 1 and 2.[6]

Moving to the *predicative* level involves quite a few complications.[7] At first sight, it seems natural to let quantifiers function like the unary connective "$\sim$" with respect to blocks. This gives us such blocks as $[\![2, (\forall x)(Px\vee\sim Rax)]\!]$ and such block formulas as $(\forall x)[\![17, Px\vee\sim Rax]\!]$, $(\forall x)([\![12, Px]\!]\vee[\![29, \sim Rax]\!])$ — there is no system whatsoever behind the numbers of these blocks and of those used in later examples, and I use the full notation until the complications are settled.

The main complication is related to instances of formulas. Clearly, it should be possible that both $[\![14, Pa]\!]$ and $[\![52, Pa]\!]$ occur in a proof (as unidentified blocks). But suppose we are block-analyzing a proof that contains

i	$(\forall x)(Px)$	...
i+1	Pa	from i by Universal Instantiation

or that contains

6 Some purposes do not require that we can see so. So, sometimes there is no need for the convention — see section 8.

7 I shall disregard functions, but including them does not involve any special difficulties.

j	$a=b$	...
$j+1$	$Pa \supset Qa$	...
$j+2$	$Pa \supset Qb$	from j and $j+1$ by Substitutivity of Identity

What are the recognized blocks in such cases? In the first example, we have to recognize a block that is common to i and $i+1$, we have to see that x occurs in that block in i, and that a occurs in the block in $i+1$, and we have to see that the block in i is preceded by a universal quantifier over the very same x. It seems to be a step in the good direction to analyze the first example as follows:

i	$(\forall x)[\![7, P-]\!](x)$	...
$i+1$	$[\![7, P-]\!](a)$	from i by Universal Instantiation

in which the dashes denote open places that have to be filled by the variable or constant mentioned behind the block. On the same account, the second example becomes

j	$a=b$	...
$j+1$	$[\![5, Pa \supset Q-]\!](a)$	...
$j+2$	$[\![5, Pa \supset Q-]\!](b)$	from j and $j+1$ by Substitutivity of Identity

Two things are special here. First, in view of the justification of this proof fragment, there is no need for the first occurrences of a in $j+1$ and $j+2$ to be recognized as such. The second remark concerns line j. Its formula may be seen as a block with two open spaces, in which case it would be analyzed, say, as $[\![4, - = -]\!](a, b)$. But, in view of the justification of this proof fragment, the block $[\![4, - = -]\!]$ should be recognized as an identity. Once it has been so recognized, there is no sense in which it might be not identified with any other identity.

The remark hinges on the nature of identity formulas. On the one hand, identity is a binary predicate, which shows in expressions like $[\![4, - = -]\!](a, b)$. Once the binary predicate is identified as an identity, its force becomes that of a logical term, and hence its meaning becomes fully transparent.

Clearly, this approach should be sophisticated. Variables and constants should themselves be seen as blocks, that need not to be identified even if their contents is identical. *If*, for whatever reasons, the first occurrence of a in $j+1$ and the first occurrence of a in $j+2$ have been localized, then the present analysis gives us:

j	$a=b$	...
$j+1$	$[\![5, P- \supset Q-]\!](a, a)$	...
$j+2$	$[\![5, P- \supset Q-]\!](a, b)$	from j and $j+1$ by Substitutivity of Identity

However, there is no need why the two occurrences of a in j+1 should be identified. This leads to a more sophisticated (and final) analysis:

j	$[\![11, a]\!] = [\![12, b]\!]$	...
j+1	$[\![5, P- \supset Q-]\!]([\![21, a]\!], [\![11, a]\!])$	...
j+2	$[\![5, P- \supset Q-]\!]([\![21, a]\!], [\![12, b]\!])$	from j and j+1 by Substitutivity of Identity

Applied to the first example, this analysis results in

i	$(\forall[\![35, x]\!])[\![7, P-]\!]([\![35, x]\!])$	...
i+1	$[\![7, P-]\!]([\![121, a]\!])$	from i by Universal Instantiation

We can easily increase readability by extending the shorthand notation. I shall write the number of a block as a superscript following the right bracket, forget the brackets around constants and variables (as the number refers to one symbol only), and write the constants and variables (followed by their superscripted number) instead of the dashes that indicate their place. On these conventions, we obtain a still unambiguous (and more readable) notation:

i	$(\forall x^{35})[\![Px^{35}]\!]^{7}$	...
i+1	$[\![Pa^{121}]\!]^{7}$	from i by Universal Instantiation

and

j	$a^{11}=b^{12}$	...
j+1	$[\![Pa^{21} \supset Qa^{11}]\!]^{5}$	...
j+2	$[\![Pa^{21} \supset Qb^{12}]\!]^{5}$	from j and j+1 by Substitutivity of Identity[8]

The upshot is that we have four kinds of blocks. Let us give at once names to their sets. Let sB be the set of *sentential blocks*. These contain fully unanalyzed formulas, for example $[\![14, p]\!]$, $[\![81, a{=}b]\!]$, $[\![95, (r\&Pa) \vee (\forall y)(Ry \supset Qby)]\!]$ — in shorthand: $[\![p]\!]^{14}$, $[\![a{=}b]\!]^{81}$, $[\![(r\&Pa) \vee (\forall y)(Ry \supset Qby)]\!]^{95}$. cB and v B will be the set of *constant blocks* and *variable blocks* respectively, having members as, for example, $[\![18, c]\!]$ and $[\![63, x]\!]$ — in shorthand c^{18} and x^{63}. Finally, fB will be the set of *functional blocks*. These contain a formula with, say, n ($\geq$ 1) open places, and have to be followed

[8] Remember that I supposed that the first occurrence of a in j+1 and in j+2 had been localized. If they are not, the block formulas read $[\![Pa \supset Qa^{11}]\!]^{5}$ and $[\![Pa \supset Qb^{12}]\!]^{5}$; in full notation $[\![5, Pa \supset Q-]\!]([\![11, a]\!])$ and $[\![5, Pa \supset Q-]\!]([\![12, b]\!])$.

by an *n*-tuple of members of $\mathsf{cB} \cup \mathsf{vB}$; examples: $[\![15, P\text{–}]\!]([\![99, a]\!])$, $[\![54, P\text{–}\&{\sim}(Q\text{–} \supset R\text{--})]\!]([\![39, a]\!], [\![14, a]\!], [\![38, x]\!])$ and $[\![71, (\exists x)R\text{–}cx \supset b{=}d]\!]([\![39, a]\!])$; in shorthand: $[\![Pa^{99}]\!]^{15}$, $[\![Pa^{39}\&{\sim}(Qa^{14} \supset Ra^{39}x^{38}]\!]^{54}$ and $[\![(\exists x)Ra^{39}cx \supset b{=}d]\!]^{71}$.

In the last formula, the occurrence of a^{39} is *transparent*, whereas the occurrence of the other constants and variables is not;[9] no constants or variables occur transparently in members of sB. Remark that, in the second example of a functional block, two transparent occurrences of *a* are not identified with each other.

It should be clear to the reader that, on the above conventions and in view of the first paragraph of this section, any stage of an annotated proof defines a unique block analysis. The latter displays a structure that corresponds to the minimal discriminations made by the author of the proof. As the analysis is unique, we may define *block proofs* as obtained from 'normal' proofs by the block analysis. This saves me the trouble to introduce a proof theory for blocks — it is possible to do so, but we would not gain anything for present purposes.

Any correct proof of *any* formal logic may be reconstructed in terms of blocks along the above lines. Also, each stage contains at least as much discriminations as each previous stage; in other words, a block may be turned into a block formula, but not the other way around.[10]

Some readers might object that it is impossible to find out that $[\![P \vee {\sim}Rab]\!]^{7}$ and $[\![P \vee {\sim}Rab]\!]^{7}$ are two occurrences of the same block without coming to know the formula contained in the block. Actually, they are mistaken. The mental operations required in checking that two blocks are identical need not include the mental operations required to grasp the contents of the blocks. Similarly for identifying one block as an instance of another block. Direct arguments are obtained from observation. A strong indirect argument derives from (efficient) computer programming. Any programmer knows that there is no need for a computer programme to recognize any symbol in the string "$(p \vee q)\&{\sim}r$" as a propositional letter, connective, etc., in order to find out that it is identical to "$(p \vee q)\&{\sim}r$". Similarly, a procedure that is able to recognize "$(p \vee q)\&{\sim}r$" as the concatenation of "$(p \vee q)$", "&", and "${\sim}r$" need not find out anything about the contents of the first and third elements of the concatenation. However, a programme needs to recognize the second string of the concatenation as a conjunction symbol in order to derive the first string by Simplification. Similarly, a procedure may identify $Pa \vee {\sim}Raa$ as an instance of

[9] I relaxedly use "transparent" to refer both to formulas and to block formulas.

[10] Alternatives are possible here, but they do not fit in the present programme.

$(\forall x)(Px \vee {\sim}Rax)$ as follows: after deleting the quantifier in the second string, compare each symbol in the strings; if the symbol in the second string is an x, the symbol in the first string must be an a; otherwise the symbols must be identical.[11]

I now come to the definition of the *block language* LB. I shall proceed in an indirect way. Let L be the usual language schema for **CL**; for future reference, I say that L is defined in the usual way from $\langle S, C, V, P^1, P^2, \ldots\rangle$, in which S is the set of sentential letters, C the set of (letters for) constants, V the set of variables, and P^r the set of predicates of rank r. Let F be the set of formulas and W the set of wffs of L.

sB is defined in such a way that, for any $A \in W$, sB contains a denumerable set of blocks of the form $[\![i, A]\!]$. Similarly for cB (from C) and for vB (from V). The definition of functional blocks is just a trifle more complicated. Any $A \in F{-}W$ defines a set $\sigma(A)$ of strings in which at least one occurrence of a free variable is replaced by a "–"; for example $\sigma(Pxax \supset (\forall x)Qxy) = \{P{-}ax \supset (\forall x)Qxy, Pxa{-} \supset (\forall x)Qxy, Pxax \supset (\forall x) Qx{-}, P{-}a{-} \supset (\forall x)Qxy, P{-}ax \supset (\forall x)Qx{-}, Pxa{-} \supset (\forall x)Qx{-}, P{-}a{-} \supset (\forall x)Qx{-}\}$. fB contains a denumerable set of blocks of the form $[\![i, A]\!]$. for any member of $\{A \mid \text{for some } B \in F{-}W, A \in \sigma(B)\}$. Where there are n open places in A, we shall say that $[\![i, A]\!]$ has rank n; let fB^1, fB^2, ... be the sets of functional blocks of rank 1, 2, ...

The set of all blocks, $sB \cup cB \cup vB \cup fB$, is denumerable, and hence all blocks that have a different second element may be given a different first element (i.e. number).[12]

BF, the set of block formulas, is the smallest set such that (i) $sB \subseteq BF$, (ii) if $A \in fB^n$ and $\alpha_1, \alpha_2, \ldots, \alpha_n \in cB \cup vB$, then $A(\alpha_1, \alpha_2, \ldots, \alpha_n) \in BF$, (iii) if $\alpha_1, \alpha_2 \in cB \cup vB$, then $\alpha_1{=}\alpha_2 \in BF$, (iv) if $A \in BF$, then ${\sim}A \in BF$, (v) if A, $B \in BF$, then $(A \supset B)$, $(A \& B)$, $(A \vee B)$, $(A \equiv B) \in BF$, and (vi) if $A \in BF$ and $\alpha \in vB$, then $(\forall\alpha)A$, $(\exists\alpha)A \in BF$. The set of well-formed block formulas, BW, may simply be defined by saying that $A \in BW$ iff the result of deleting all occurrences of "⟦", "⟧", ",", and numbers in A is a member of W. Needless to say, BF and BW are denumerable.

[11] That the matter is more complicated if a universal quantifier over x occurs within the scope of the first one does not change anything to the general claim I am making.

[12] Here is an obvious way to do it. Given a list $A_1, A_2, \ldots$ of the members of F, start a new list and add, for each i (≥ 1), first a copy of all formulas already in the list (if any) and next A_i. This gives us: $A_1, A_1, A_2, A_1, A_1, A_2, A_3$, etc. This well-defined list contains each member of F a denumerable number of times. The latter list is then turned into a list of blocks: $[\![4^1\text{-}3, A_1]\!]$, $[\![4^2\text{-}3, A_1]\!]$, $[\![4^3\text{-}3, A_2]\!]$, etc. which contains all members of sB. For cB, vB, and fB we use respectively $4^i{-}2$, $4^i{-}1$, and 4^i ($i \geq 1$).

5. *Block semantics for* **CL**

The semantics for the propositional level is very simple. v assigns values from {0, 1} to members of BF and the rules for block formulas are as usual. For example, $v([\![p]\!]^7 \& [\![q]\!]^4) = 1$ iff $v([\![p]\!]^7) = v([\![q]\!]^4) = 1$. Remark, however, that nothing prevents, for example, that $v([\![p\&q]\!]^{17}) = 1$ and $v([\![p]\!]^7) = 0$; or that $v([\![p]\!]^7) \neq v([\![p]\!]^{12})$. The values of primitive blocks are independent, even if their contents are related. It is important to realize this; it is a typical feature of the block approach and essential for its force, as we shall see in subsequent sections.

I now come to the semantics for the predicative version. This semantics is very close to the usual semantics for **CL**. As one would expect, members of cB and vB are treated like constants and variables, members of sB are treated like sentential letters, and members of fB^r are treated like predicates of rank r. Here is the list of meta-variables that I shall use from now on:
– $\emptyset^r{}_1, \emptyset^r{}_2, \ldots$ for members of fB^r
– $\pi^r{}_1, \pi^r{}_2, \ldots$ for members of P^r
– $\alpha, \beta, \alpha_1, \ldots$ for members of $cB \cup vB$ as well as for members of $C \cup V$
– $A, B, \ldots$ for members of BF as well as for members of F
All ambiguities will be resolved by the context.

A **BCL** model $M = \langle D, v\rangle$ in which the domain D is a set and v is an assignment defined by

C1.1 $v : sB \rightarrow \{0, 1\}$
C1.2 $v : cB \cup vB \rightarrow D$
C1.3 $v : fB^r \rightarrow \mathcal{P}(D^r)$ (the power set of the r-th Cartesian product of D)

The valuation function v_M determined by the model M is defined as follows:

C2.1 $v_M : BW \rightarrow \{0, 1\}$
C2.2 where $A \in sB$, $v_M(A) = v(A)$
C2.3 $v_M(\emptyset^r(\alpha_1 \ldots \alpha_r)) = 1$ iff $\langle v(\alpha_1), \ldots, v(\alpha_r)\rangle \in v(\emptyset^r)$
C2.4 $v_M(\alpha = \beta) = 1$ iff $v(\alpha) = v(\beta)$
C2.5 $v_M({\sim}A) = 1$ iff $v_M(A) = 0$
C2.6 $v_M(A \supset B) = 1$ iff $v_M(A) = 0$ or $v_M(B) = 1$
C2.7 $v_M((\forall \alpha)A(\alpha)) = 1$ iff $v_{M'}(A(\alpha)) = 1$ for all models M' that differ at most from M in $v(\alpha)$.

Other logical terms are supposed to be explicitly defined. Validity and semantic consequence are defined as usual.

For **PIL**, the basic paraconsistent logic presented below, I consider ω-complete models only. In preparation, I introduce this restriction (in the same style) in the above block semantics. This leads to the following changes:

C1.2 $v : cB \cup vB \rightarrow D$ is such that $D = \{v(\alpha) | \alpha \in cB \cup vB\}$
C2.7 $v_M((\forall\alpha)A(\alpha)) = 1$ iff $v_M(A(\beta)) = 1$ for all $\beta \in cB \cup vB$

For the following lemmas and theorems, we suppose that **CL** refers to a specific inference system that defines (normal) **CL**-proofs and hence block proofs. Depending on the context, **BCL** refers to **CL**-block-proofs or to the above block semantics.

There is an obvious way to faithfully embed **BCL** in **CL**. Considering L, let $S = \{p_1, p_2, \ldots\}$, $C = \{a_1, a_2, \ldots\}$, $V = \{x_1, x_2, \ldots\}$, and, for any r, $P^r = \{P^r{}_1, P^r{}_2, \ldots\}$. Also, let all blocks with a different second element have a different first element — see the next to last paragraph of the previous section. Define a translation function tr as follows:

where $[\![i, \alpha]\!] \in cB$, $\mathtt{tr}([\![i, \alpha]\!]) = a_i$
where $[\![i, \alpha]\!] \in vB$, $\mathtt{tr}([\![i, \alpha]\!]) = x_i$
where $[\![i, A]\!] \in sB$, $\mathtt{tr}([\![i, A]\!]) = p_i$
where $[\![i, \phi^r]\!] \in vB$, $\mathtt{tr}([\![i, \phi^r]\!]) = P^r_i$
$\mathtt{tr}(\sim A) = \sim\mathtt{tr}(A)$
$\mathtt{tr}(A \supset B) = \mathtt{tr}(A) \supset \mathtt{tr}(B)$; similarly for $\vee$, &, and $\equiv$
$\mathtt{tr}(\alpha=\beta) = \mathtt{tr}(\alpha)=\mathtt{tr}(\beta)$
$\mathtt{tr}((\forall\alpha)A) = (\forall\,\mathtt{tr}(\alpha))\mathtt{tr}(A)$; similarly for $\exists$
$\mathtt{tr}(\Gamma) = \{\mathtt{tr}(A) \mid A \in \Gamma\}$.

The embedding is completed by the two following theorems.

Theorem 1. $\Gamma \vdash_{\mathbf{BCL}} A$ iff $\mathtt{tr}(\Gamma) \vdash_{\mathbf{CL}} \mathtt{tr}(A)$.
Proof. For the left-right direction, consider a block-proof (at its 'final' stage) of A from Γ. Transform it to a list of wffs obtained by replacing any block formula B in the block proof by $\mathtt{tr}(B)$. To see that the result is a proof of $\mathtt{tr}(A)$ from $\mathtt{tr}(\Gamma)$, first note that the translation of each premise of the block proof is a member of $\mathtt{tr}(\Gamma)$, and that $\mathtt{tr}(A)$ is the last step in the proof. Next, an obvious induction on the length of the proof delivers the required result. To make things concrete, suppose that the proof proceeds in terms of a system with axiom schemes. If the block formula C is added because it is an axiom, then its form should reveal so. But then it is easily seen that $\mathtt{tr}(C)$ is an axiom as well. If C is derived by Modus Ponens from the block formulas $D \supset C$ and D, then obviously $\mathtt{tr}(C)$ follows by Modus Ponens from $\mathtt{tr}(D \supset C)$, i.e. $\mathtt{tr}(D) \supset \mathtt{tr}(C)$, and $\mathtt{tr}(D)$. Similarly for predicative rules.

The right-left direction is even easier. Simply consider the block analysis of the proof of tr(A) from tr(Γ). The block analysis of the original proof is obtained by systematically replacing members of sB, cB, vB, and fB^r by members of the same sets. ■

Theorem 2. $\Gamma \vDash_{\mathbf{BCL}} A$ iff $\mathrm{tr}(\Gamma) \vDash_{\mathbf{CL}} \mathrm{tr}(A)$.
Proof. For the left-right direction, suppose that $\mathrm{tr}(\Gamma) \nvDash_{\mathbf{CL}} \mathrm{tr}(A)$ and hence that there is a **CL**-model $M = \langle D, v \rangle$ that verifies tr(Γ) and falsifies tr(A). This model is readily transformed into a **BCL**-model $M' = \langle D, v' \rangle$ by setting $v'(A) = v(\mathrm{tr}(A))$ for any block A. I leave it to the reader to verify (by induction) that M' verifies Γ and falsifies A.
For the right-left direction we suppose that $\Gamma \nvDash_{\mathbf{BCL}} A$ and proceed in the same way, this time transforming the **BCL**-model that verifies Γ and falsifies A into a **CL**-model that verifies tr(Γ) and falsifies tr(A). ■

In view of the correctness and completeness of **CL** with respect to its semantics:

Corollary 1. $\Gamma \vdash_{\mathbf{BCL}} A$ iff $\Gamma \vDash_{\mathbf{BCL}} A$.

It is important to realize that we consider a block proof as the result of the block analysis of the *final* stage of a (normal) proof of A from Γ (where $\{A\} \cup \Gamma \subseteq$ F). A is a semantic consequence of Γ in view of the blocks that have been discriminated in members of Γ and in A at the final stage of the block proof. This tells us nothing about the relation between the semantic counterpart of the different stages of the block proof. I shall comment on that in the next section.

6. *Stage by stage* vs. *continuous semantics*

It is useful to consider the semantic counterparts of the different stages of a proof. As far as the previous analysis is concerned, the models of stage n need not be a superset of the models of stage n+1. Consider again the different stages of the proof of section 3. The models of the premises of stage 2 verify $[\![(p \supset {\sim}q) \supset (p\&({\sim}r \vee {\sim}p))]\!]^1$ and $[\![p \supset {\sim}q]\!]^2$; $[\![p\&({\sim}r \vee {\sim}p)]\!]^3$ is false in some of them. The models that verify the premises in stage 3 verify $[\![p \supset {\sim}q]\!]^2$, $[\![p\&({\sim}r \vee {\sim}p)]\!]^3$, and $[\![p \supset {\sim}q]\!]^2 \supset [\![p\&({\sim}r \vee {\sim}p)]\!]^3$, but some such models falsify $[\![(p \supset {\sim}q) \supset (p\&({\sim}r \vee {\sim}p))]\!]^1$.
In other words, the block semantics proceeds stage by stage. If a proof is correct, then, at each stage, all block wffs occurring in the block analysis of the proof at that stage are semantic consequences of the block premises at that stage. But the models of the block premises at some stage need not

verify the block premises at a previous (or later) stage if the block premises at the two stages are different.

One might prefer a continuous semantics: a semantics on which the models of the premises not only verify all block wffs that occur in the proof at the stage, but also verify all block wffs that occur at previous stages. After all, if a block formula A belongs to the proof at stage n, and disappears from it at stage $n+1$, then it has been analyzed, i.e. turned into a (more complex) block formula B. There clearly is a relation between the A and B, in that the latter is an analysis of the former. This fact would be clearly revealed by a continuous semantics. A continuous semantics would moreover reveal that more and more consequences of the premises are obtained as the proof (and the block analysis of the premises) proceeds, and that, at least for a monotonic logic like **CL**, no losses occur during the journey.

Let us again consider the proof of section 3. The models of stage 2 are obviously a subset of the models of stage 1. The first problem occurs at stage 3, at which the block

(7) $[\![(p \supset \sim q) \supset (p \& (\sim r \vee \sim p))]\!]^1$

disappears from the proof and is replaced by

(8) $[\![p \supset \sim q]\!]^2 \supset [\![p \& (\sim r \vee \sim p)]\!]^3$

To obtain a continuous semantics, we want the models of each stage to be a subset of the models at the previous stage. In other words, we want to characterize the transition between stage 2 and 3 as the transition from the set of models of stage 2 to the subset containing the models in which (7) and (8) have the same truth value, viz. 1. This might even be expressed in the notation, by writing (9) rather than (8) — a line I shall not continue.

(9) $[\![[\![p \supset \sim q]\!]^2 \supset [\![p \& (\sim r \vee \sim p)]\!]^3]\!]^1$

A continuous semantics is easily obtained. On the stage by stage semantics, the models for a stage of the proof are the models verifying the premises as analyzed at that stage. On the continuous semantics, the models of a stage are those that verify the premises as analyzed at the stage and at each previous stage.

It is easily seen that, if a set of premises is consistent then each stage of the proof has a set of models on the continuous semantics, and this set becomes in general smaller as the proof proceeds. To be precise, the set becomes smaller whenever a block is analyzed as a block formula or whenever two blocks are identified. It is also easy to see that, on the continuous semantics, the set of models of a proof at a stage is the intersection of the set of models of that and each previous stage on the step by step semantics. If the premises are consistent, this intersection is never empty.

Incidentally, the continuous semantics is attractive for a different reason. Every step in the analysis of the premises provides us with new information. The continuous semantics reveals this in that the set of models becomes restricted with every such step. The continuous semantics also reveals that each step in the analysis of the premises is in principle informative: if one were to skip over a number of such steps, and move directly to a deeper analysis of the premises, then one obtains a larger set of models.

7. *Information from deduction, omniscience, and uninformative moves*

It is often said that deduction does not provide one with new information. This largely derives from the view that a deductive inference is correct iff all the information contained in the conclusion is contained in the premises. This view is obviously correct, but reveals only part of the truth about the matter. There is a sense in which it is ridiculous to claim that, for example, Gödel's theorem did not provide us with new information. And for someone who is not a trained logician, to see that $\sim B$ is derivable from $\sim(A\&B)$ and A may be most informative.

The continuous block semantics offers a means to approach this notion of information. It does not allow for an exhaustive theory about it, but explicates a sensible part of the story. With every new step in the block analysis, more information is unveiled. That this is so, is expressed in terms of the most traditional and unquestioned criterion: the set of models is restricted.[13]

Omniscience is directly related to deductive information. Omniscience is partly related to the available deductive rules and to economic considerations. Even if each of these partly goes beyond the resources of the block approach, the latter allows for a sensible partial explanation of the riddle.

That certain sentences belong to one's knowledge system, does neither warrant nor require that these sentences have been analyzed to the deepest level — if there even is one, see sections 13 and 14. Only to the extent that the block analysis of these sentences is pushed deeper, they do release the information contained in them.

[13] The information meant here is quite distinct from the one that plays a role in the relevance logic tradition (to which I am most sympathetic). The notion of information that surfaces from that tradition is a rather old-fashioned one: sets of derivable disjunctions of primitive wffs.

The results offered by the block approach seem rather realistic. A person who derives A from $A\&B$, is bound to see that B is derivable as well. But this person should not realize all the consequences of either A or B, and should not necessarily identify these blocks, say $[\![A]\!]^i$ and $[\![B]\!]^j$, with all blocks that share their content with one of them. The least one should conclude is that the block approach, in contradistinction to the traditional approach to deduction, provides us with an instrument for discussing the consequences that a rational person should derive from his or her knowledge.

I now come to a problem that is completely and satisfactorily solved by the block approach. All sensible logicians realize the difference between, say, Simplification and Modus Ponens on the one hand, and Addition and Irrelevance (A / $B \supset A$) on the other. The former two are informative in a sense in which the latter two are not. And yet, the move from $A\&B$ to A involves a weakening, just as much as the move from A to $A \vee B$. On traditional approaches, the distinction is phoney. In [15], Hempel argued that the problem of partial self-explanation cannot be solved, because any explanation can be phrased as a partial self-explanation: the explanation of Pa from $(\forall x)(Qx \supset Px)$ and Qa can be reformulated as an explanation of $(Pa\&Qa) \vee (Pa\&{\sim}Qa)$ from $(\forall x)(Qx \supset Px)$ and $(Pa\&Qa) \vee ({\sim}Pa\&Qa)$, which Hempel deems a partial self-explanation.[14]

The block approach enables us to make the desired distinction. *Informative* moves such as Simplification and Modus Ponens require a deeper block analysis of the premises. *Uninformative* moves such as Addition and Irrelevance at most require the introduction of a new block. In model theoretic terms: informative moves require a restriction of the models whereas uninformative ones do not.

In a sense, the block approach discloses a truism. When hearing an utterance, some people apprehend it and are even able to retain it for a long time, but fail to see its consequences because they fail to see its logical structure. Some do not realize that "Children are singing" and "Children are inexperienced" start off with a different quantifier. As all the words sound familiar, no bell rings; they think they grasped both sentences. And in a sense they do: they can reproduce them. Logicians often conclude that such people are just unintelligent, but this is only part of the truth. As we shall see in section 14, the human ability to (consciously or more probably unconsciously) neglect deductive information contained in sentences might be essential for — that is, a prerequisite for — the progress of our knowledge

[14] Years ago, in [1], I tried to crack this nut by introducing a theory of meaning relations. An attempt to refine this theory brought me to paraconsistent logics.

in general and of our conceptual systems more in particular. Needless to say, this is not a plea against elementary logic.[15]

8. *Proof heuristics and pedagogical matters*

I refer to [6] for natural proof heuristics, and consider the type of formal system and the type of heuristics discussed there. Blocks are very intuitive in proof heuristics (and in automatic theorem proving).

A proof search contains analyzing moves and goal directed moves. Some analyzing moves apply to a single formula. In this case, one typically goes one level down in looking at the block's contents. For example, one looks for blocks that can be analyzed as $[\![\ldots]\!]\&[\![\ldots]\!]$ in order to apply Simplification, or for blocks that can be analyzed as $\sim([\![\ldots]\!]\&[\![\ldots]\!])$ in order to apply Negation of Conjunction. The matter is somewhat more complicated if the analyzing move requires the intervention of another formula. For example, in order to apply Modus Ponens, one looks for a block that can be analyzed as $[\![\ldots]\!]\supset[\![\ldots]\!]$, where the antecedent either occurs as such in the proof or may be analyzed in such a way that it is identical to a block formula that occurs in the proof.

It is often useful to *check off* the formulas that have been analyzed. This helps to direct (especially the students') attention. For example, in the presence of a block formula of the form $\sim A$ or of a block formula of the form B, a block formula of the form $A \supset B$ may be checked off. Again, A and B may be single blocks, but often they are not (at the stage at which the complex formula is checked off or at a later stage).

Goal directed moves may themselves lead to the analysis of a (sub)goal or to steps in the proof, for example the introduction of a hypothesis. I leave to the reader the exercise to translate the instructions for such moves from [6] in terms of blocks.

I also refer to section 6 and 7 where I argued that the continuous semantics reveals the informative character of the steps in the block analysis of the premises.

The present purpose (proof search) does not require that the block analysis of previous stages is retained, and does not require that a block is analyzed everywhere in the proof. For example, once a block formula has been

[15] Elsewhere, I defended a contextualist view — see [3], [7] and [8]. The same mechanisms — even those of the deductive logic sort — do not apply in all situations. 'Natural' languages should be considered as flexible means for communication and thinking, rather than as monolithic systems. The meanings of their terms get at best fixed, and usually only partially and vaguely, in a specific context. The present section seems well in agreement with these views.

checked off, it becomes 'in principle' useless and might again be considered as a single (checked off) block. I shall not pursue this line of thought here, as I need the block analysis from section 4 for the subsequent sections.

It is correct that the proof heuristics story may be told in terms of metavariables. But this is so only because they share some features with blocks. Moreover, the story is more transparent in terms of blocks. Especially beginning students should be taken away from being impressed by the complexity of strings of symbols. We want them to learn, for example, to see (1) as a disjunction of two blocks, and (2) as the negation of the first block in (1). More often than not, it is extremely helpful to convince them to disregard the contents of the blocks.

Precisely the same situation arises if one tries to teach students to formalize sentences from natural language in an *ad hoc* formal language. Especially (but not only) in the case of a propositional formal language, one tries to induce the students to see a rather complex sentence as the concatenation of a few blocks and some logical terms. I now leave the matter and turn to demonstrating the use of the block approach with respect to dynamic proofs.

9. *The paraconsistent logic* **PIL**

The inconsistency-adaptive logic **APIL1** is based on the paraconsistent logic **PIL**. The latter consists of full positive classical logic together with $A \vee {\sim}A$ (alternatively, $(A \supset {\sim}A) \supset {\sim}A$). A (perhaps unexpected) result is that replacement of identicals is restricted as follows:[16]

A=2 $\alpha{=}\beta \supset (A \supset B)$ where B is obtained by replacing in A an occurrence of α *that occurs outside the scope of a negation* by β

Semantically, **PIL** is characterized[17] as follows. Let $\mathsf{F}^{\sim}$ be the set of formulas of the form ${\sim}A$. A model is a couple $\mathsf{M} = \langle \mathsf{D}, \mathsf{v} \rangle$ in which D is a set and v is an assignment function defined by:

C1.1 $\mathsf{v} : \mathsf{S} \rightarrow \{0, 1\}$

C1.2 $\mathsf{v} : \mathsf{C} \cup \mathsf{V} \rightarrow \mathsf{D}$ is such that $\mathsf{D} = \{\mathsf{v}(\alpha) \mid \alpha \in \mathsf{C} \cup \mathsf{V}\}$

[16] See [11] and [13] for a justification of the proviso, which does not prevent that full positive **CL** (theorems and rules) is contained in **PIL**.

[17] C1.2 restricts the semantics to models with countable domain D. A more general semantics is presented in [13], but the present is closer to the usual **CL**-semantics.

C1.3 $v : P^r \rightarrow \mathcal{P}(D^r)$ (the power set of the r-th Cartesian product of D)
C1.4 $v : F^{\sim} \rightarrow \{0, 1\}$

The valuation function v_M determined by the model M is defined as follows:

C2.1 $v_M : W \rightarrow \{0, 1\}$
C2.2 where $A \in S$, $v_M(A) = v(A)$
C2.3 $v_M(\pi^r\alpha_1 \ldots \alpha_r) = 1$ iff $\langle v(\alpha_1), \ldots, v(\alpha_r)\rangle \in v(\pi^r)$
C2.4 $v_M(\alpha{=}\beta) = 1$ iff $v(\alpha) = v(\beta)$
C2.5 $v_M({\sim}A) = 1$ iff $v_M(A) = 0$ or $v({\sim}A) = 1$
C2.6 $v_M(A \supset B) = 1$ iff $v_M(A) = 0$ or $v_M(B) = 1$
C2.7 $v_M(A \& B) = 1$ iff $v_M(A) = 1$ and $v_M(B) = 1$
C2.8 $v_M(A \vee B) = 1$ iff $v_M(A) = 1$ or $v_M(B) = 1$
C2.9 $v_M(A \equiv B) = 1$ iff $v_M(A) = v_M(B)$
C2.10 $v_M((\forall \alpha)A(\alpha)) = 1$ iff $v_M(A(\beta)) = 1$ for all $\beta \in C \cup V$
C2.11 $v_M((\exists \alpha)A(\alpha)) = 1$ iff $v_M(A(\beta)) = 1$ for at least one $\beta \in C \cup V$

Truth in a model, semantic consequence and validity are defined as usual. I refer to [11] for a proof that **PIL** is sound and complete with respects to the **PIL**-semantics and for some further meta-theorems.

10. *The inconsistency-adaptive logic* APIL1

I now turn to the inconsistency-adaptive logic **APIL1**. This logic adapts itself to the specific inconsistencies that occur in a theory or set of premises. The basic sources about **APIL1** are [5] and [11]. I refer to those papers (and to [10]) for motivational matters and for many technical details, but mention enough technical stuff to give logicians a feel of what is specific about **APIL1**. Let me start with an example of a very simple **APIL1**-proof.

1	$p \vee {\sim}q$	Premise	
2	${\sim}p \vee r$	Premise	
3	q	Premise	
4	${\sim}s \vee t$	Premise	
5	$s \& {\sim}p$	Premise	
[6	p	1, 3; $A \vee {\sim}B, B \,/\, A$	$\{q\}$] deleted at stage 11
[7	r	2, 6; ${\sim}A \vee B, A \,/\, B$	$\{q, p\}$] deleted at stage 11
8	s	5; $A \& B \,/\, A$	$\emptyset$
9	${\sim}p$	5; $A \& B \,/\, B$	$\emptyset$
10	$(p \& {\sim}p) \vee {\sim}q$	1, 9; $A, B \vee C \,/\, (B \& A) \vee C$	$\emptyset$

11 $(p\&{\sim}p)\vee(q\&{\sim}q)$	10, 3; $A\vee B, C / A\vee(C\&B)$	$\emptyset$
12 t	4, 8; ${\sim}A\vee B, A / B$	$\{s\}$

Lines 6 and 7 are typical conditional derivations. The rule $A\vee{\sim}B, B / A$ is not valid in **PIL**. Nevertheless, **APIL1** enables us to apply the rule provided B is *reliable*. At the moment one may take this to mean that one may rely on the consistent behaviour of B. In this specific case, B corresponds to q, and for this reason we list $\{q\}$ as the fifth element of line 6. The reasoning for line 7 is similar, with a small exception. To derive line 7 from 2 and 6, p should be reliable. But line 6 depended itself on the reliability of q. Hence line 7 depends on the reliability of both p and q, as is indicated in its fifth element. For lines 8–10, no formula needs to be reliable. At line 11, we discover that the consistent behaviour of p is connected to that of q: one of them behaves inconsistently. The strategy followed by **APIL1** is to consider both as unreliable.[18] As a result, lines 6 and 7 have to be deleted as soon as line 11 is added to the proof. Line 12 contains again a conditional derivation. As the proof is propositional, it is easy to see that, on the present premises, s is consistent and that its consistent behaviour is not connected to the consistent behaviour of any other formulas. Whence line 12 will not be deleted at any later stage of the proof. Below I offer the precise definitions and rules that govern this proof.

Suppose that, in an extension of the proof, we were able to derive $p\&{\sim}p$ without relying on any other formula. At that moment, line 11 would not any more provide a reason to consider q as unreliable. Hence, a duplicate of line 6 could be added again to the proof. In general, the addition of new lines may render underivable formulas that were derived earlier (because new formulas turn out to be unreliable), but may also render formulas derivable that were not derivable earlier (because formulas that appeared unreliable earlier turn out to be reliable). In other words, we have to distinguish between derivability at a stage, and final derivability. As mentioned in section 2, a formula is finally derivable if it is derived at some stage and will not be deleted in any (possibly qualified) extension of the proof.

The example should give the reader an idea of both the dynamic character of **APIL1**-proofs, and also of the adaptive character of this logic. The logic reacts to the specific (disjunctions of) inconsistencies that occur in the proof. By its reference to reliability, it allows us to interpret a set of premises 'as consistently as possible'.

Let me now turn to the precise formulation of **APIL1**. As this is a predicative logic, several definitions are somewhat tedious. At the propositional

[18] Other inconsistency-adaptive logics follow a different strategy. See [11] for the **APIL2**-strategy, which is less cautious than that of **APIL1**.

level, all is much simpler. I trust that the above example will make it easier for the reader to get a grasp on the definitions.

Where $(A\&{\sim}A)$ is a formula in which the variables $\alpha_1, \ldots, \alpha_k$ $(k \geq 0)$ occur free (in that order), let $\exists(A\&{\sim}A)$ be $(\exists\alpha_1)\ldots(\exists\alpha_k)(A\&{\sim}A)$. Let $\mathrm{DEK}(A_1, \ldots, A_n)$ refer to $\exists(A_1\&{\sim}A_1) \vee \ldots \vee \exists(A_n\&{\sim}A_n)$. I shall say that $A_1, \ldots, A_n$ are the *factors* of $\mathrm{DEK}(A_1, \ldots, A_n)$. As permutations of the factors and of the quantifiers in "$\exists$" result in equivalent formulas, I shall also use sets to refer to any such permutation, as in $\mathrm{DEK}(\Sigma)$.

It is handy to write **APIL1**-proofs in a specific format (as in the above example) according to which each line of a proof consists of five elements:

(i) a line number,
(ii) the formula derived,
(iii) the line numbers of the wffs from which (ii) is derived,
(iv) the rule[19] that justifies the derivation, and
(v) the set of formulas on the consistent behaviour of which we rely in order for (ii) to be derivable by (iv) from the formulas of the lines enumerated in (iii).

The formula in (ii) is a wff, but (v) may contain open formulas. For example, Px occurs in (v) if (ii) is derived relying on the falsehood of $(\exists x)(Px\&{\sim}Px)$.

We need some definitions. *A occurs unconditionally* at some line of a proof iff the fifth element of that line is empty. *A behaves consistently at a stage of a proof* iff $\exists(A\&{\sim}A)$ does not occur unconditionally in the proof at that stage. The consistent behaviour of A_1 is *connected to* the consistent behaviour of the members of Δ iff $\mathrm{DEK}(\{A\} \cup \Delta)$ occurs unconditionally in the proof and $\mathrm{DEK}(\Theta)$ does not occur unconditionally[20] in the proof for any $\Theta \subset \{A\} \cup \Delta$. A is *reliable* at a stage of a proof iff A behaves consistently at that stage and its consistent behaviour is not connected to the consistent behaviour of any other formula.

Apart from the premise rule, proofs in **APIL1** are governed by an unconditional (meta-)rule, a conditional (meta-)rule and a deletion rule (that causes the dynamics of the proofs).

RU If $\vdash_{\mathbf{PIL}} (A_1\&\ldots\&A_n) \supset B$, and $A_1, \ldots, A_n$ occur in the proof, then add B to it. The fifth element of the new line is the union of the fifth elements of the lines mentioned in its third element.

19 As appears from the examples, I mean an application of the meta-rules RU or RC (see below), rather than one of these rules themselves.

20 See sections 4 and 5 of [4] for the rationale of this requirement.

RC If $\vdash_{\mathbf{PIL}}$ DEK$\{C_1, \ldots, C_m\} \vee ((A_1 \& \ldots \& A_n) \supset B)$, and $A_1, \ldots, A_n$ occur in the proof, then add B to it *provided* that each factor of DEK$\{C_1, \ldots, C_m\}$ is reliable (at that stage). The fifth element of the new line is the union of $\{C_1, \ldots, C_m\}$ and of the fifth elements of the lines mentioned in its third element.

RD If C is not (any more) reliable, then delete from the proof all lines the fifth element of which contains C.

Wffs that occur unconditionally are **PIL**-derivable from the premises (and cannot possibly be deleted later). The unconditional occurrence of DEK-formulas at a stage determines which formulas are reliable at that stage. Wffs that occur in the proof at a stage are derivable at that stage. But we clearly want a more stable notion, called *final* derivability, that does not depend on the stage of the proof.

Definition. An extension of an **APIL1**-proof from Γ is *intelligent* iff: if both DEK(Σ) and DEK$(\Sigma \cup \Phi)$ occur unconditionally in the extension, then the former precedes the latter.[21]

Definition. A is *finally derived* at some line in an **APIL1**-proof iff it is the second element of that line and the line will not be deleted in any intelligent extension of the proof.

Definition. $\Gamma \vdash_{\mathbf{APIL1}} A$ (A is *finally derivable* from Γ) iff A is *finally derived* at some line in an **APIL1**-proof from Γ.

Like **CL**, **PIL** is not decidable. Hence, unlike for its propositional fragment, there is no algorithm for writing intelligent proofs. Yet, $Cn_{\mathbf{APIL1}}(\Gamma)$, the set of **APIL1**-consequences of Γ, may be characterized without referring to the dynamics of the proofs (but by referring to **PIL** only).

In order to characterize **APIL1** semantically, we need some more definitions. $\mathrm{EK(M)} = \{A \mid v_{\mathrm{M}}(\exists(A \& {\sim}A))=1\}$. DEK$(\Delta)$ is a minimal DEK-consequence of Γ iff $\Gamma \vDash_{\mathbf{PIL}}$ DEK(Δ) and, for no $\Theta \subset \Delta$, $\Gamma \vDash_{\mathbf{PIL}}$ DEK(Θ). U(Γ), the set of unreliable formulas, is the set of the factors of all minimal DEK-consequences of Γ. A **PIL**-model M of Γ is an **APIL1**-model of Γ iff $\mathrm{EK(M)} \subseteq \mathrm{U}(\Gamma)$.

[21] Intuitively: if a formula is reliable on the premises (in an absolute sense), then it is not unreliable at any stage in an intelligent extension of the proof.

Definition. $\Gamma \models_{\mathbf{APIL1}} A$ iff A is true in all **APIL1**-models of Γ.[22]

That the proof theory of **APIL1** is sound and complete with respect to its semantics is proved in [11]. I now turn to the problems raised in section 2.

11. *Block semantics for* **APIL1** — *a dynamic semantics*

In section 2, I argued that we need a semantic counterpart for the dynamics of the proof theory of inconsistency-adaptive logics. The block approach (and nothing else that I can imagine) enables us to do so. Let **BPIL**, respectively **BAPIL1**, refer to the block proof theory as well as to the block semantics of **PIL**, respectively **APIL1**.

The **BPIL**-semantics is easily obtained. We consider the block language LB and define $\mathsf{BF}^{\sim}$ as the set of all (primitive and other) block-formulas of the form $\sim A$ — "$\sim$" followed by a block formula. For this language we define a **BPIL**-model as a couple $\mathrm{M} = \langle \mathrm{D}, \mathrm{v} \rangle$ in which D is a set and v is an assignment function defined by:

C1.1 $\mathrm{v} : \mathsf{sB} \rightarrow \{0, 1\}$
C1.2 $\mathrm{v} : \mathsf{cB} \cup \mathsf{vB} \rightarrow \mathrm{D}$ is such that $\mathrm{D} = \{\mathrm{v}(\alpha) \mid \alpha \in \mathsf{cB} \cup \mathsf{vB}\}$
C1.3 $\mathrm{v} : \mathsf{fB}^r \rightarrow \mathcal{P}(\mathrm{D}^r)$ (the power set of the r-th Cartesian product of D)
C1.4 $\mathrm{v} : \mathsf{BF}^{\sim} \rightarrow \{0, 1\}$

The valuation function $\mathrm{v_M}$ determined by the model M is defined as follows:

C2.1 $\mathrm{v_M} : \mathsf{BW} \rightarrow \{0, 1\}$
C2.2 where $A \in \mathsf{sB}$, $\mathrm{v_M}(A) = \mathrm{v}(A)$
C2.3 $\mathrm{v_M}(\phi^r \alpha_1 \ldots \alpha_r) = 1$ iff $\langle \mathrm{v}(\alpha_1), \ldots, \mathrm{v}(\alpha_r) \rangle \in \mathrm{v}(\phi^r)$
C2.4 $\mathrm{v_M}(\alpha = \beta) = 1$ iff $\mathrm{v}(\alpha) = \mathrm{v}(\beta)$
C2.5 $\mathrm{v_M}(\sim A) = 1$ iff $\mathrm{v_M}(A) = 0$ or $\mathrm{v}(\sim A) = 1$
C2.6 $\mathrm{v_M}(A \supset B) = 1$ iff $\mathrm{v_M}(A) = 0$ or $\mathrm{v_M}(B) = 1$
C2.7 $\mathrm{v_M}(A \& B) = 1$ iff $\mathrm{v_M}(A) = 1$ and $\mathrm{v_M}(B) = 1$
C2.8 $\mathrm{v_M}(A \vee B) = 1$ iff $\mathrm{v_M}(A) = 1$ or $\mathrm{v_M}(B) = 1$
C2.9 $\mathrm{v_M}(A \equiv B) = 1$ iff $\mathrm{v_M}(A) = \mathrm{v_M}(B)$
C2.10 $\mathrm{v_M}((\forall \alpha) A(\alpha)) = 1$ iff $\mathrm{v_M}(A(\beta)) = 1$ for all $\beta \in \mathsf{cB} \cup \mathsf{vB}$
C2.11 $\mathrm{v_M}((\exists \alpha) A(\alpha)) = 1$ iff $\mathrm{v_M}(A(\beta)) = 1$ for at least one $\beta \in \mathsf{cB} \cup \mathsf{vB}$

[22] The inconsistency-adaptive logic **APIL2**, which is also based on **PIL**, has a much simpler semantics: the **APIL2**-models of Γ are the minimally abnormal **PIL**-models of Γ. However, the proof theory of **APIL2** is more complex than that of **APIL1**.

Truth in a **BPIL**-block-model, semantic consequence and validity are defined as usual. That the **BPIL**-semantics is sound and complete with respect to **BPIL**-proofs may be shown, as for **CL**, by an embedding of **BPIL** in **PIL**. Actually the translation function is the same as for **CL** and so proceeds the proof of

Theorem 3. $\Gamma \vdash_{\mathbf{BPIL}} A$ iff $\mathtt{tr}(\Gamma) \vdash_{\mathbf{PIL}} \mathtt{tr}(A)$.

The proof theory of **BAPIL1** is defined (again) as the result of the block analysis of an **APIL1**-proof.

The semantics of **BAPIL1** is obtained from the semantics of **BPIL** in the same way as the semantics of **APIL1** is obtained from the semantics of **PIL**. So, I repeat, metavariables now referring to block formulas:

Definition. Where $\mathtt{M}$ is a **BPIL**-model, $\mathtt{EK(M)} = \{A \mid v_{\mathtt{M}}(\exists(A\&{\sim}A))=1\}$.

Definition. $\mathtt{U}(\Gamma)$ is the set of the factors of minimal $\mathtt{DEK}$-consequences of Γ.

Definition. $\mathtt{M}$ is an **BAPIL1**-*model* of Γ iff it is a **BPIL**-model of Γ and $\mathtt{EK(M)} \subseteq \mathtt{U}(\Gamma)$.

Definition. $\Gamma \models_{\mathbf{BAPIL1}} A$ iff A is true in all **BAPIL1**-models of Γ.

If you think that I reached a trivial result, you could not be more mistaken. The result is *natural* in that the proof theory and semantics for **BAPIL1** are obtained exactly as the proof theory and semantics for **BCL** were obtained before. But the result is *far from trivial.* Indeed, the **BAPIL1**-semantics is a dynamic semantics. It corresponds to the *stages* of a **BAPIL1**-proof. In other words, $\Gamma \models_{\mathbf{BAPIL1}} A$ captures **APIL1**-derivability *at a stage*, a form of derivability that may be overruled at a later stage.

To illustrate the result, let us consider again the example from the previous section. Here is the block analysis for stage 7:

1	$[\![p]\!]^1 \vee {\sim}[\![q]\!]^2$	Premise	
2	${\sim}[\![p]\!]^1 \vee [\![r]\!]^3$	Premise	
3	$[\![q]\!]^2$	Premise	
4	$[\![{\sim}s \vee t]\!]^4$	Premise	
5	$[\![s\&{\sim}p]\!]^5$	Premise	
6	$[\![p]\!]^1$	1, 3; $A \vee {\sim}B, B \,/\, A$	$\{[\![q]\!]^2\}$
7	$[\![r]\!]^3$	2, 6; ${\sim}A \vee B, A \,/\, B$	$\{[\![q]\!]^2, [\![p]\!]^1\}$

What is significant about this is that, with respect to the present block analysis, 6 and 7 are *finally* derived from the premises 1–5. This is easily verified by the block semantics. However, for some readers it will be easier to see what happens if one replaces each block by a propositional letter. That leads to:

1'	$p \vee {\sim}q$	Premise	
2'	${\sim}p \vee r$	Premise	
3'	q	Premise	
4'	s	Premise	
5'	t	Premise	
6'	p	1, 3; $A \vee {\sim}B, B / A$	$\{q\}$
7'	r	2, 6; ${\sim}A \vee B, A / B$	$\{q, p\}$

And it is easily seen indeed that, in **APIL1**, 6' and 7' are semantic consequences of 1'–5'.

Let us now turn to the block analysis of stage 11 of the proof:

1	$[\![p]\!]^1 \vee {\sim}[\![q]\!]^2$	Premise	
2	${\sim}[\![p]\!]^1 \vee [\![r]\!]^3$	Premise	
3	$[\![q]\!]^2$	Premise	
4	$[\![{\sim}s \vee t]\!]^4$	Premise	
5	$[\![s]\!]^6 \& [\![{\sim}p]\!]^7$	Premise	
6	$[\![p]\!]^1$	1, 3; $A \vee {\sim}B, B / A$	$\{[\![q]\!]^2\}$
7	$[\![r]\!]^3$	2, 6; ${\sim}A \vee B, A / B$	$\{[\![q]\!]^2, [\![p]\!]^1\}$
8	$[\![s]\!]^6$	5; $A\&B / A$	$\emptyset$
9	$[\![{\sim}p]\!]^7$	5; $A\&B / B$	$\emptyset$
10	$([\![p]\!]^1 \& [\![{\sim}p]\!]^7) \vee {\sim}[\![q]\!]^2$	1, 9; $A, B \vee C / (B\&A) \vee C$	$\emptyset$
11	$([\![p]\!]^1 \& [\![{\sim}p]\!]^7) \vee ([\![q]\!]^2 \& {\sim}[\![q]\!]^2)$	10, 3; $A \vee B, C / A \vee (C\&B)$	$\emptyset$

On the block analysis, 6–11 are indeed semantic consequences of 1–5 and as no DEK-formula occurs in 1–11, there is still no reason whatsoever to delete any line from this proof. Such a reason is only obtained when 11 is *recognized* as a DEK-formula, in other words, if $[\![{\sim}p]\!]^7$ is identified with ${\sim}[\![p]\!]^1$. Once that is done, lines 5, 9, 10 and 11 become:

5	$[\![s]\!]^6 \& {\sim}[\![p]\!]^1$	Premise	
9	${\sim}[\![p]\!]^1$	5; $A\&B / B$	$\emptyset$
10	$([\![p]\!]^1 \& {\sim}[\![p]\!]^1) \vee {\sim}[\![q]\!]^2$	1, 9; $A, B \vee C / (B\&A) \vee C$	$\emptyset$
11	$([\![p]\!]^1 \& {\sim}[\![p]\!]^1) \vee ([\![q]\!]^2 \& {\sim}[\![q]\!]^2)$	10, 3; $A \vee B, C / A \vee (C\&B)$	$\emptyset$

At this point, and only at this point, lines 6 and 7 are deleted.

The feature is worth being stressed. Just as one may fail to recognize $s\&\sim p$ as a conjunction of two wffs, and hence fail to apply Simplification to it, one might fail to recognize 11 as a disjunction of contradictions, and hence fail to delete 6 and 7. In other words, the mental operations for deriving $([\![p]\!]^1\&[\![\sim p]\!]^7) \vee ([\![q]\!]^2\&\sim[\![q]\!]^2)$ are different from the mental operations required to recognize this as $([\![p]\!]^1\&\sim[\![p]\!]^1) \vee ([\![q]\!]^2\&\sim[\![q]\!]^2)$.

However, the rules RC and RD refer to the occurrence of DEK-formulas in the proof. If one were to fail to see that 11 is a DEK-formula, and hence would fail to delete lines 6 and 7 at stage 11, then the proof would not any more be correct. To put it differently, RC and RD *require* that DEK-formulas *be transparent*. It follows that *only* the last block analysis is the correct block analysis of stage 11 of the proof.

It is easily seen that $[\![p]\!]^1$ and $[\![r]\!]^3$ are not semantic consequences of the premises according to the correct block analysis of stage 11: there are **BAPIL1**-models of the premises in which $[\![p]\!]^1$ and $[\![r]\!]^3$ are false.

What exactly happened from a semantic point of view? On the step by step analysis, the **BAPIL1**-models of the premises at stage 10 and the **BAPIL1**-models of the premises at stage 11 *do not overlap*. Indeed, $[\![p]\!]^1$ and $[\![r]\!]^3$ are true in all the former models, but not in some of the latter. What even more strikes the eye is that the former models are consistent whereas none of the latter is.

On the continuous analysis, we obtain ... a discontinuity. The apparent riddle is readily removed if we look at the **PIL**-models. There, continuity is obviously preserved. The transition from stage 10 to stage 11 requires that we identify, in analyzing 5, the value of $[\![\sim p]\!]^7$ with the value of $\sim[\![p]\!]^1$. As a result, all consistent **PIL**-models of the premises at stage 10 are eliminated, as appears from line 11. So, all **BAPIL1**-models of the premises at stage 10 — all of these are consistent — are eliminated from the **PIL**-models of stage 11. Hence, the **BAPIL1**-models of the premises at stage 11 cannot possibly be **BAPIL1**-models of the premises at stage 10.

Two comments seem useful. The first is that, in a much deeper sense than was the case for **CL**, the block analysis of the premises proves informative. It even induces drastic changes in insight concerning the consequences of the premises. The second comment is that the dynamic character of the proof theory of **BAPIL1** provides a deep understanding of the non-monotonicity of the logic. The dynamics does not derive from adding premises, but from realizing (step by step) what the premises mean.[23] To put it more generally, there is a different dimension to logic than the abstract and static features expressed by the usual notions of derivability and semantic conse-

[23] The relation with logical omniscience is apparent. If we were logically omniscient, we would, even in the case of non-monotonic logics, have no need for any dynamics.

quence. We experience this in searching for a proof, even in **CL**, but more clearly where non-monotonicity is involved. The block semantics reveals this further dimension, and it does so in a perfectly stringent and formal way.[24]

Let us now turn to an extremely perverse **APIL1**-proof: the only premise seems to lead to the clearcut derivability of a consequence, but the further analysis of this consequence leads to its rejection.

1	$(p\&{\sim}p) \vee (q\&(p\&{\sim}p))$	Premise	
2	$q\&(p\&{\sim}p)$	1; $(A\&{\sim}A) \vee B \,/\, B$	$\{p\}$

Here 2 is derivable from 1 on the condition that p is reliable at this stage of the proof from the premises. But obviously, $p\&{\sim}p$ is derivable from 2 (and hence from 1). Only, nothing in the present proof enables us to see so: no DEK-formula occurs in the proof. So, the above proof is a correct **APIL1**-proof (at stage 2) of 2 from 1.

Applying the block analysis to stage 2 of the proof, we obtain:

1	$([\![p]\!]^1\&{\sim}[\![p]\!]^1) \vee [\![q\&(p\&{\sim}p)]\!]^2$	Premise	
2	$[\![q\&(p\&{\sim}p)]\!]^2$	1; $(A\&{\sim}A) \vee B \,/\, B$	$\{[\![p]\!]^1\}$

Remark that 2 is only derivable from 1 iff it is realized that the first disjunct of 1 has the structure $[\![p]\!]^1\&{\sim}[\![p]\!]^1$. If the inconsistency were not recognized, 2 would not be derivable.

What about the semantics? Only two blocks matter: $[\![p]\!]^1$ and $[\![q\&(p\&{\sim}p)]\!]^2$. The premise is true in three sorts of **BPIL**-models: those in which $v([\![p]\!]^1) = v([\![q\&(p\&{\sim}p)]\!]^2) = 1$ and $v({\sim}[\![p]\!]^1) = 0$, those in which $v([\![p]\!]^1) = 0$ and $v({\sim}[\![p]\!]^1) = v([\![q\&(p\&{\sim}p)]\!]^2) = 1$, and, finally, those in which $v([\![p]\!]^1) = v({\sim}[\![p]\!]^1) = 1$ and $v([\![q\&(p\&{\sim}p)]\!]^2)$ is either 1 or 0. So, 1 has consistent models, and in all of these $v([\![q\&(p\&{\sim}p)]\!]^2) = 1$. Whence $[\![q\&(p\&{\sim}p)]\!]^2$ is a **BAPIL1**-consequence of 1.

Extending the proof, we sooner or later shall find (perhaps with different line numbers):

3	q	2; $A\&B \,/\, A$	$\{p\}$
4	$p\&{\sim}p$	2; $A\&B \,/\, B$	$\{p\}$

[24] Leo would have liked this: the endeavour to develop dynamic proof theory and block semantics belongs to the same philosophical enterprise that takes ideological problems serious *and* approaches them in scientific terms.

It is easy to see that, whenever A is **APIL1**-derivable from some premises on the condition that the members of Σ are reliable, then $\text{DEK}(\Sigma \cup \{A\})$ is unconditionally derivable from the premises. Whence any sensible person shall continue the proof by

5	$(p\&{\sim}p) \vee (p\&{\sim}p)$	1; $A \vee (B\&C)$ / $A \vee C$	$\emptyset$

As 5 is a DEK-formula and hence is transparent, the block analysis gives us:

1	$(\llbracket p \rrbracket^1 \& {\sim}\llbracket p \rrbracket^1) \vee (\llbracket q \rrbracket^3 \& (\llbracket p \rrbracket^4 \& {\sim}\llbracket p \rrbracket^4))$	Premise	
[2	$\llbracket q \rrbracket^3 \& (\llbracket p \rrbracket^4 \& {\sim}\llbracket p \rrbracket^4)$	1; $(A\&{\sim}A) \vee B$ / B	$\{\llbracket p \rrbracket^1\}$] del. at 5
[3	$\llbracket q \rrbracket^3$	2; $A\&B$ / A	$\{\llbracket p \rrbracket^1\}$] del. at 5
[4	$\llbracket p \rrbracket^4 \& {\sim}\llbracket p \rrbracket^4$	2; $A\&B$ / B	$\{\llbracket p \rrbracket^1\}$] del. at 5
5	$(\llbracket p \rrbracket^1 \& {\sim}\llbracket p \rrbracket^1) \vee (\llbracket p \rrbracket^4 \& {\sim}\llbracket p \rrbracket^4)$	1; $A \vee B\&C)$ / $A \vee C$	$\emptyset$

Remark that 2–4 have to be deleted at stage 5, even if $\llbracket p \rrbracket^1$ and $\llbracket p \rrbracket^4$ are not identified at that stage. But these blocks may be identified, and that will provide us with the information that $p\&{\sim}p$ is (finally and unconditionally) derivable from 1. If this wff is derived at line 6, the block analysis of stage 6 is as that for stage 5 except in that $\llbracket p \rrbracket^4$ is replaced everywhere by $\llbracket p \rrbracket^1$ and that the following line is added

6	$\llbracket p \rrbracket^1 \& {\sim}\llbracket p \rrbracket^1$	1; $A \vee A$ / A	$\emptyset$

Actually, nothing about q is derivable from 1: 1 is **PIL**-equivalent to $p\&{\sim}p$, and it can be demonstrated that, if Γ and Δ are **PIL**-equivalent, then they have the same **APIL1**-consequences.

Two further comments seem useful. The first is that the deleted lines have taken care of the (formula) analysis of 1. In this sense, they are far from useless (except for a trained logician who sees at once that $p\&{\sim}p$ is derivable from 1). Remark that 1 does not have any consistent models, whereas its block analysis (at stages 1–3) does have consistent block models. This supports my claim that the block analysis and the dynamic proofs reveal an aspect of reasoning that surpasses the usual semantics.

The second comment is that the dynamic character of the proof does not merely derive from the non-monotonic character of the logic. It is *not* the case that lines 2–4 are derivable from 1 but have to be deleted in view of the presence of further premises. The dynamics derives from the (block analysis and formula) analysis of 1, and the block semantics captures this analysis nicely.

Let me now return to the first problem I mentioned in section 2: to find a *semantic* counterpart of the proof theoretical dynamics. This problem is clearly solved (even if I did not offer any proofs). But exactly how is it solved? There is no distinct semantic counterpart for each stage of a proof. But there is one for each set of stages that share the same block analysis. Put in other terms, if stage n+1 does not introduce a new block premise and does not require that a block is replaced by a block formula, then the formula derived at that stage is a semantic consequence of the premises as analyzed at stage n (where "semantic consequence" refers to the block semantics).[25] If stage n+1 introduces a new block premise or requires that a block is replaced by a block formula, then the formula of line n+1 need not be a semantic consequence of the premises as analyzed at stage n — in the continuous semantics, the models are restricted.

The block approach offers new insights in the proof dynamics. In devising the proof theory, I was constantly haunted by the question which formulas should count as an indication that some lines of the proof should be deleted. Is line 4 sufficient to delete 2–3? If so, why not also line 4 itself. And if we unconditionally derive A, and later unconditionally derive ~A, should this be taken as a symptom of the unreliability of A, or should we wait until A&~A has been unconditionally derived? The block approach offers the guideline to answer such queries. It provides the insight that is necessary to see the effects and viability of the conventions we might introduce at such points. I shall not continue the matter here, but the block approach indeed suggests a slight modification to the rule RC and to the implicit rule concerning the transparency of DEK-formulas.

12. *A proof theoretical criterion for final derivability*

I now come to the second problem mentioned in section 2: devise a proof theoretic criterion for final derivability in **APIL1**. I shall restrict the discussion to the propositional level — the predicate level requires numerous complications, but offers no fundamentally new insights.

Let me first point out that I do not promise a 'positive test' for final derivability. If we have a positive test for $\Gamma \vdash A$, we may start a Turing Machine, with A (as its goal) and (the possibly infinite set) Γ (as its premises) on its tape, and we have the guarantee that, if A is finally **APIL1**-derivable from Γ, the Machine will stop with a positive answer after a finite time. There is a positive test for **PIL**-derivability,[26] there also is a decision

[25] Remark that this holds even if a new block is introduced by Addition, Irrelevance, or a similarly uninformative move.

method for final derivability for finite (and most plausibly also for infinite) Γ in the propositional fragment of **APIL1**, but whether there is a positive test for final derivability in **APIL1** is still an open question — the criterion presented below may be turned into a positive test for finite sets of propositional formulas, and there is a good chance that it can be generalized. But for now, I promise a criterion: some **APIL1**-proofs allow us to decide that A is finally derivable from Γ.

All unconditionally derived formulas are finally derived from the premises. So, let us consider a proof from the premises $A_1, \ldots, A_m$:

1	B_1	...	Δ_1
2	B_2	...	Δ_2
...			
n	B_n	...	Δ_n

in which $\Delta_n \neq \emptyset$. Our problem will be to decide whether B_n is finally derived from the premises at line n. In order to do so, we have to find out whether all members of Δ_n are reliable on the premises $A_1, \ldots, A_m$. Let us first consider a special case. Suppose that the last line of the proof reads

n	$\sim(p\&\sim p)$	$- / \sim(A\&\sim A)$	$\{p\}$

where the rule "$- / \sim(A\&\sim A)$" indicates that any wff of the form $\sim(A\&\sim A)$ may be derived at any stage of a proof (on the supposition that A is reliable)[27]. The block analysis of this proof will deliver the following last step (the block number is arbitrary):

n	$\sim([\![p]\!]^7\&\sim[\![p]\!]^7)$	$- / \sim(A\&\sim A)$	$\{[\![p]\!]^7\}$

If p does not occur in any premise, then n is obviously finally derived. It is even possible to show — and I shall soon do so — that n is finally derived if p is a subformula of some premise but $\sim p$ is not, or if $\sim p$ is a subformula of some premise, but p does not occur in any premise outside the scope of a negation. If both p and $\sim p$ occur in some premise outside the scope of a negation, then p might be unreliable on the premises and hence we should take action.

26 The next edition of Boolos and Jeffrey's [14] should absolutely offer some examples on paraconsistent logics. Turing Machines just love non-standard logics (even if they don't apply them).

27 By an application of RC in view of $\vdash_{\mathbf{PIL}} (p\&\sim p) \vee \sim(p\&\sim p)$.

The first type of action concerns the block analysis. If both p and $\sim p$ occur in some premise outside the scope of a negation, then all such occurrences of p should be made transparent. In other words, if p does not occur within the scope of a negation in a formula $\ldots p \ldots$ of the proof, the block analysis should be pushed further until we obtain $\ldots [\![p]\!]^7 \ldots$; similarly, if $\sim p$ does not occur within the scope of a negation in a formula $\ldots \sim p \ldots$ of the proof, the block analysis should be pushed further until we obtain $\ldots \sim [\![p]\!]^7 \ldots$ The second type of action concerns the *formula* analysis. This means that certain derivations *should* be made in the proof in order to split formulas into their components, as when A and B are derived by Simplification from $A \& B$. After these preparatory remarks, let us move on to the general formulation of the criterion.

The criterion refers to a set (of block formulas) and to two requirements (on the proof). First, I shall specify a set $\prod$ of block formulas. The set contains the formulas of which we want to find out whether they are reliable. The first requirement concerns the block analysis: it should be pushed further until all members of $\prod$ occur transparently (with some exceptions that I shall specify). The second requirement concerns the formula analysis: it should warrant that any member of $\prod$ is unreliable at the final stage of the proof if it is unreliable on the premises — see footnote 21.

As the central difficulty concerns the derivability of DEK-formulas, it is useful to recall that the occurrence of a line

i	B_i	$\ldots$	Δ_i

warrants that one may add the line

j	$B_i \vee \text{DEK}(\Delta_i)$	$\ldots$	$\emptyset$

(The converse does not hold as line i can only be written iff all members of Δ_i are reliable at the stage.) An immediate consequence of this feature is that the occurrence of a line

i	$C \& \sim C$	$\ldots$	Δ_i

warrants that $\text{DEK}(\Delta_i \cup \{C\})$ is a DEK-consequence (but not necessarily a minimal DEK-consequence) of the premises.

Which formulas should be included in $\prod$? B_n is finally derived at line n of the proof iff all members of Δ_n are reliable on the premises. So $\Delta_n \subseteq \prod$. Remember that Δ_n contains (blocks and) block formulas.

Suppose that $[\![p \& q]\!]^k \in \prod$. Even if $p \& q$ is made transparent everywhere in the proof, we might fail to see that p and q are derivable (conditionally or unconditionally) from the premises. Hence, we shall require: (i) all blocks that occur in a member of $\prod$ are added to $\prod$, and (ii) if the formula of a block in $\prod$ contains $A\$B$, where "\$" is any binary connective, then both A and B are made transparent.[28]

There is a further complication. Suppose that $\Delta_i \cap \Delta_n \neq \emptyset$ and that line i reads

$$i \quad C\&D \qquad \ldots \qquad \Delta_i$$

whereas the proof also contains

$$j \quad {\sim}C\&E \qquad \ldots \qquad \Delta_j$$

The proof may be extended as follows

i'	C	...	Δ_i
j'	${\sim}C$	...	Δ_j
j''	$C\&{\sim}C$	...	$\Delta_i \cup \Delta_j$
j'''	$\text{DEK}(\Delta_i \cup \Delta_j \cup \{C\})$	...	$\emptyset$

Hence, some members of Δ_n are unreliable on the premises, unless there is a $\Theta \subset \Delta_i \cup \Delta_j \cup \{C\}$ such that $\text{DEK}(\Theta)$ is unconditionally derivable from the premises and $\Theta \cap \Delta_n = \emptyset$. This means that we should require that, whenever $\Delta_i \cap \Delta_n \neq \emptyset$, then $\Delta_i \subseteq \prod$ and $B_i \in \prod$.

Remark that this requirement also applies to line n itself. Indeed, if line n reads

$$n \quad (C\&D)\&{\sim}C \qquad \ldots \qquad \Delta_n$$

then only by pushing the block analysis down to the level of C shall we be able to find out that the proof may be extended as follows

n'	$C\&{\sim}C$	...	Δ_n
n''	$\text{DEK}(\Delta_n \cup \{C\})$	...	$\emptyset$

and hence that some members of Δ_n behave unreliably on the premises unless $\text{DEK}(C)$ is itself unconditionally derivable from the premises.

[28] Requirement (ii) entails that the block analysis is pushed further until all blocks contain either a sentential letter or a formula of the form ${\sim}A$. The proof that the latter need not be analyzed further is similar to the proof of 'the difficult bit' at the end of the present section.

Let me summarize. We need to know whether all members of Δ_n behave reliably on the premises, that is: whether some member of Δ_n occurs in a minimal DEK-consequence of the premises. The obvious way to find this out is by first going after the DEK-consequences of the premises of which some member of Δ_n is a factor, and then checking whether, for any such DEK-consequence DEK(Θ), there is a DEK-consequence of the premises DEK(Λ) such that $\Lambda \subset \Theta$ and $\Lambda \cap \Delta_n = \emptyset$. This lead to the following definition of Π: the smallest set such that (i) $\Delta_n \subseteq \Pi$, (ii) if $\Delta_i \cap \Pi \neq \emptyset$, then $\Delta_i \subseteq \Pi$ and $B_i \in \Pi$, (iii) if block A occurs in a member of Π, then $A \in \Pi$, and (iv) the block analysis of members of Π should be pushed further until each block that belongs to Π or occurs in a member of Π is either a primitive formula or of the form $\sim A$.

Pushing the block analysis deeper down. Two remarks are at hand. The first is this: making A transparent in all formulas of the proof requires (i) that A is itself (a block or) a block formula and (ii) that all occurrences of A are identified with each other. The second remark is that the conditions on the block analysis are exactly as in the earlier example: if, for some $A \in \Pi$, both A and $\sim A$ occur in some premise outside of the scope of a negation, then, if A, respectively $\sim A$, do not occur in the scope of a negation in $\ldots A \ldots$, respectively $\ldots\sim A\ldots$, the block analysis of this formula should be continued until we reach $\ldots[\![A]\!]^k\ldots$, respectively $\ldots\sim[\![A]\!]^k\ldots$

The analysis of block formulas. Let an atom be either a primitive block formula, a block formula of the form $\sim B$, or a block formula of the form $B\&\sim B$. I define a (special) conjunctive normal form CNF° for block formulas: A is in CNF° iff A is a conjunction of one or more block formulas of the form $[A \supset]B$ in which A is a conjunction of (zero or more) atoms and B is a disjunction of (one or more) atoms. For each formula, there are several equivalent formulas that are in CNF°.

Three steps are to follow. First step: a block formula that is not in CNF° and occurs in the proof on the condition Δ is analyzed iff an equivalent block formula in CNF° occurs in the proof on some condition $\Theta \subseteq \Delta$. Second step: a block formula $A\&B$ occurring in the proof on the condition Δ is analyzed iff A occurs in the proof on some condition $\Theta \subseteq \Delta$ and B occurs in the proof on some condition $\Lambda \subseteq \Delta$. It is easily seen that, if all formulas are analyzed in as far as the first two steps allow us to do so, then the only non-analyzed formulas in the proof have the form $[A \supset]B$ in which A is a conjunction of (zero or more) atoms and B is a disjunction of (one or more) atoms.

The third and final step is to decide whether the members of Δ_n are reliable in the proof. There is an obvious *semantic* criterion to do so: check whether the set of non-analyzed formulas in the proof has a model in which all members of Δ_n are consistent. Of course it is possible to offer a proof theoretic criterion, but I skip this because it is complicated and there obviously is one anyway.

Is the criterion adequate? I have to show that, if all members of Δ_n turn out to be reliable, then continuing the proof, and hence pushing the block analysis further, cannot result in the unreliability of some member of Δ_n. I shall show that this holds, not only for Δ_n, but for Π as well. There is one difficult bit: if $C \in \Pi$, then it is not necessary to locate C in a formula in which it only occurs as a subformula of a negated formula. Apart from this, the matter is obvious. Suppose that we are able to derive (in the usual proof)

i	B_i	...	Δ_i
j	B_j	...	Δ_j

where (i) $\Delta_i \cap \Pi \neq \emptyset$ and (ii) $\Delta_j \cap \Pi = \emptyset$ and no member of Π is a subformula of B_j.

Suppose that, from i and j, we are able to derive unconditionally $\text{DEK}(\Theta \cup \Lambda)$, where $\Theta \subseteq \Pi$ and $\Lambda \cap \Pi = \emptyset$. As $B_i \vee \text{DEK}(\Delta_i)$ and $B_j \vee \text{DEK}(\Delta_j)$ do not have any common subformula,[29] it is obvious that either $\text{DEK}(\Theta)$ is derivable from line i or $\text{DEK}(\Lambda)$ is derivable from line j. So, if some member of Π is reliable on the present block analysis, then continuing the proof (and pushing the block analysis further beyond what is required by the test) cannot possibly make it unreliable.

Similarly, if some member of Π is unreliable on the present block analysis, then continuing the proof (and the block analysis) cannot possibly make it reliable. This follows from the reasoning in the previous paragraph if we set $\Lambda = \emptyset$. In other words, the problem is solved provided I prove the difficult bit.

I have to show that, if $C \in \Pi$, then it is not necessary to locate C in a formula in which it only occurs as a subformula of a negated formula. Let me first spell out the problem that might occur, and then show that it does not.

Consider a proof that contains the following lines:

i	----~(... C ...)---	...	$\{\ldots\}$
j	A	...	$\{C, \ldots\}$

In line i, the context outside the parentheses may consist of two nil strings, but the context within the parentheses should not be nil — occurrences of

[29] This follows from the definition of Π in view of (i) and (ii).

$\sim C$ that do not themselves occur within the scope of a negation should be made transparent.

In preparation of the demonstration, consider the conditions under which **APIL1** enables one to 'open up' negated complex formulas. I list the relevant consequences and the conditions:

from:	to derive:	should be reliable:
$\sim\sim A$	A	$\sim A$
$\sim(A\&B)$	$\sim A \vee \sim B$	$A\&B$
$\sim(A \vee B)$	$\sim A$ (resp. $\sim B$)	$A \vee B$
$\sim(A \supset B)$	A (resp. $\sim B$)	$A \supset B$

Suppose now that A and some negated complex wff of which A is a subformula occur (conditionally or unconditionally) in some **APIL1**-proof. I now show that, if A behaves reliably on the present block analysis, then so it does in an extension of the proof in which the negated complex formula is 'opened'. I first consider the basic cases. I shall list all details for the first case only.

Case 1. Consider a proof containing the following lines:

i	A	...	Δ_i
j	$\sim\sim\sim A$	...	Δ_j

According to the requirement on the block analysis, the formula on line j may just be a single unanalyzed block (or a negation followed by a block, etc.). I have to show that, if the block $[\![A]\!]^i$ is reliable on this analysis, then it will remain reliable if the block on line j is further analyzed, viz. as $\sim\sim\sim[\![A]\!]^i$.

Suppose that the *original* proof is *extended* in that the block at line j is further analyzed, and that the two following lines are added:

j'	$\sim A$	j; $\sim\sim B$ / B	$\Delta_j \cup \{\sim\sim A\}$
j''	$(A\&\sim A) \vee \text{DEK}(\Delta_i \cup \Delta_j \cup \{\sim\sim A\})$	i, j'; ...	$\emptyset$

in other words, $\text{DEK}(\{A, \sim A\} \cup \Delta_i \cup \Delta_j)$. If this formula is a minimal DEK-consequence of the premises, then A is unreliable. Remark that, in the block analysis of the extended proof, the block $[\![A]\!]^i$ is transparent everywhere (and occurs whenever A occurs in the usual proof).

As $\sim A \vee \sim\sim A$ is a theorem of **PIL**, two more lines can be added, in view of the unconditional rule RU, to the original proof on the *original* block analysis. I write at once the block formulation in order to clarify the point:

$k \quad \sim[\![A]\!]^i \vee \sim\sim[\![A]\!]^i \qquad \ldots \qquad \emptyset$
$k' \quad ([\![A]\!]^i \& \sim[\![A]\!]^i) \vee (\sim\sim[\![A]\!]^i \& \sim\sim\sim A]\!]^i) \qquad \ldots \qquad \mathrm{b}(\Delta_i \cup \Delta_j) \cup \{\sim\sim[\![A]\!]^i\}$

where $b(\Delta_i \cup \Delta_j)$ indicates that the set has to be replaced by the corresponding set of blocks (on the analysis of the original proof). By a fact mentioned earlier, k' immediately gives us

$k+1 \quad \mathrm{DEK}(\{[\![A]\!]^i, \sim\sim[\![A]\!]^i\} \cup \mathrm{b}(\Delta_i \cup \Delta_j) \qquad \ldots \qquad \emptyset$

This line too is derivable on the block analysis of the original proof.

Now (as the final step) consider the three following possibilities. (i) For some $\Phi \subseteq \Delta_i \cup \Delta_j$, either DEK($\Phi$), DEK($\Phi \cup \{A\}$), DEK($\Phi \cup \{\sim\sim A\}$), or DEK($\Phi \cup \{A, \sim\sim A\}$) is a minimal DEK-consequence of the premises. Then line i or line j of the original proof have to be deleted, and hence the problem evaporates. (ii) $A \& \sim A$ is a minimal DEK-consequence of the premises (on the original block analysis). Then A is already unreliable on the block analysis of the original proof. (iii) $\sim\sim A \& \sim\sim\sim A$ is a minimal DEK-consequence of the premises (on the original block analysis). Then, line j' has to be deleted (or cannot be added to the proof) and line j'' is not a minimal DEK-formula. So, if A is not unreliable on the original proof, then it is not unreliable in any continuation of it.

Case 2. Consider a proof containing A and $\sim(A \vee B)$. The reasoning proceeds as for case 1, except that we now rely on

$$A, \sim(A \vee B) \vdash_{\mathbf{PIL}} ((A \vee B) \& \sim(A \vee B))$$

Similarly for a proof containing B and $\sim(A \vee B)$.

Case 3. Consider a proof containing $\sim A$ and $\sim(A \supset B)$, respectively B and $\sim(A \supset B)$. Here, we rely upon one of

$$\sim A, \sim(A \supset B) \vdash_{\mathbf{PIL}} (A \& \sim A) \vee ((A \supset B) \& \sim(A \supset B))$$
$$B, \sim(A \supset B) \vdash_{\mathbf{PIL}} (A \supset B) \& \sim(A \supset B)$$

Case 4. Consider a proof containing A and $\sim(A \& B)$, respectively B and $\sim(A \& B)$. This case requires a slightly different approach. Suppose that A is unreliable on the block analysis of the extended proof. Hence, (10) holds (for some $\emptyset$), and hence, by properties of positive logic, also (11):

(10) $\Gamma, A, \sim(A \& B), \sim A \vee \sim B \vdash_{\mathbf{PIL}} (A \& \sim A) \vee \mathrm{DE}(\emptyset)$
(11) $\Gamma, A, \sim(A \& B) \vdash_{\mathbf{PIL}} \sim B \supset ((A \& \sim A) \vee ((A \& B) \& \sim(A \& B)) \vee \mathrm{DEK}(\emptyset))$

But (12) and (13) hold generally:

(12) $\Gamma, A, \sim(A\&B) \vdash_{\mathbf{PIL}} B \supset ((A\&\sim A) \vee ((A\&B)\&\sim(A\&B)) \vee \mathrm{DEK}(\emptyset))$
(13) $\vdash_{\mathbf{PIL}} B \vee \sim B$

Hence

(14) $\Gamma, A, \sim(A\&B) \vdash_{\mathbf{PIL}} (A\&\sim A) \vee ((A\&B)\&\sim(A\&B)) \vee \mathrm{DEK}(\emptyset)$

which means that A is unreliable on the block analysis of the original proof.

This finishes our basic cases (I skip equivalence). The general result follows by the usual induction on the complexity of the negated formulas.

13. *The direct block approach — languages without basic meaning elements.*

In the previous sections, I considered the block approach as a way of looking at usual formal logic proofs, and devised a semantics that corresponds to the result. But the block approach may very well be viewed as an alternative to the traditional approach to formal logic. I shall merely offer some sketchy comments to this alternative.

According to the traditional approach to logic, there are primitive expressions, apparently corresponding to elementary entities (elementary facts, or, more often, elementary observational statements like 'Protokolsätze'). The primitive expressions are the basic meaning elements. Moreover, the primitive expressions are usually seen as independent of each other. This appears not only from the semantics — the assignment treats the symbols as independent — but also from traditional applications such as Carnap's state descriptions. Both suppositions, however, are (known to be) unrealistic. The primitive expressions of natural languages and even of the languages used in the sciences cannot be seen as corresponding to elementary facts (etc.); and, partly for precisely this reason, they need not be independent of each other. A further (obvious) discrepancy between traditional formal languages and real life languages is that the latter are subject to an evolution which is determined in part by our changing views on the world.

In all three respects, a block approach to languages seems promising. The idea is very simple. The primitive symbols of a language are not seen as entities that have an elementary, independent, and fixed meaning, but as blocks, which here signifies: primitive carriers of meaning. To put it in other words: when the block analysis comes to an end, this now will be viewed as provisional. A conceptual analysis of the language might reveal that, e.g., some primitive predicate may be identified with a complex expression, like when Aristotle defined "human" as "rational animal". Also, a theoretical change may have as an effect that some term is 'understood' in some specific way, and hence is identified with some complex expression, like when "heat" was interpreted as an internal movement.[30] Primitive expressions that are not identified with complex ones will still be treated as independent — the block approach *as such* does not offer a way out of this — but if they are identified with more complex expressions, dependencies may appear. In allowing for a 'downward dynamics', the block approach is able to clarify at least in part both conceptual analysis and the change (in the sense of specification) of a language.[31]

The direct block approach does not require any change to the usual formalism. The change resides in the *interpretation* of the formalism. This is why the embedding from section 5 is illuminating. If sentential letters are interpreted as sentential blocks, n-ary predicates as blocks of rank n, etc., then the usual formalism will stand as is. Soundness, completeness and other properties will have to be proved in a direct way (not by relying on the usual system, as in section 5). Let me just mention a definition and a theorem for the block approach to **CL**:

Definition. A block proof (at a stage) is *correct* iff all **BCL**-models of the block premises verify all block formulas in the proof.

Theorem 4. For any **CL**-proof, if the block analysis of formulas 1 to n at stage n is semantically correct, then so is the block analysis of these formulas at stage $n+1$. (MONOTONICITY)

14. *More on meaning change*

Meaning change in scientific disciplines often occurs in the course of a problem solving process with inconsistent constraints. For example, there may be a clash between different theories, each of which have their merits, or between a theory and some principle, or between a theory and some empirical laws. In trying to solve the problem, a scientist has to analyze the situation, to locate the inconsistencies and their possible dependencies, to order the different sides of the inconsistencies in terms of preferences, and to build a theory that resolves the inconsistencies in agreement with those preferences. During this process, the meaning of some terms will have

[30] In both cases, blocks with a different contents (in the sense of section 3) are identified and hence may be analyzed in terms of the more complex contents. An obvious variation is where different complex expressions are identified. This situation belongs rather in the next section, but limitations of space prevent me from discussing it there.

[31] This obviously does not cover all forms of meaning change. Yet, as we shall see in section 14, the block approach allows more than is spelled out here.

changed. Such a change is radically different from the 'downward dynamics' mentioned in the previous section.

Still, it seems to me that the block approach enables us to understand such a change. The following suggestion is largely the result of thinking about [19] and discussing the case with Joke Meheus. The suggestions are mine; if they do not make sense, I alone am to be blamed.

In the case discussed in [19], the clash is between Sadi Carnot's theory and Joules principle, one half of which was based on a set of experimental results (but that did not form a theory). Carnot's theory was heavily dependent on a specific meaning of "heat": the identification of heat with a substance, caloric. The theory contained a conservation law for heat, and viewed the production of work as a result of a movement, viz. a fall, of heat (from a warm to a cold source). All this conflicted with Joule's principle according to which heat is transformed into work and vice versa. (Needless to say, the matter was actually more complicated — I just try to give the reader a feel of the type of problem.) At the end of a long creative process, Clausius succeeds in formulating a theory that is in several respects similar to Carnot's — most importantly, it basically explains the Carnot cycle — but that agrees with Joule's principle. As Joke Meheus argues, his reasoning can only be reconstructed in terms of an adaptive logic.

In Clausius' theory, heat obviously cannot be a substance. In this sense Clausius' interpretation of the Carnot cycle is radically different from Carnot's interpretation. But how did the meaning of "heat" change?

My suggestion is that "heat" functioned like a block in Clausius' (inconsistency-adaptive) reasoning — actually several forms of heat are involved, but that does not require any change to the suggestion. Where this block was analyzed for Carnot and (differently) for Joule, Clausius considered "heat" as an unanalyzed block — one might say as "heat, whatever that is". And he did so for a number of other terms as well. In this sense he was able to concentrate on theoretical statements and empirical laws and on cooking a consistent theory out of them. Once the theory was ready, conceptual analysis was sufficient to reveal what actually was the meaning of "heat" and of other terms — that is, the meaning of the blocks that related in a specific way according to the new theory.

My suggestion is by no means outrageous. The phenomenon described, considering a formerly analyzed block as unanalyzed, occurs frequently in everyday situations. Where words have a specific meaning for us, we have often to forget about this meaning in talking to other people. The meaning they assign to these words may be different ones, unknown to us. And we have to consider those words as unanalyzed blocks in order to follow the reasoning and slowly grasp what the meaning of those words might be for them.

It is worth pointing out that the block approach enables us to understand not only changes to a language (such as English or the language of physics), but also local and contextual changes (such as the ones we have to perform to understand others). This is important from my philosophical point of view, as I argued in [3] that languages do not exist as monolithic entities, but are fictions constructed on the basis of local and contextual verbal behaviour.

When it comes to meaning and meaning change, some people have the tendency to refer to deep-structural phenomena such as neural networks or (Freudian or other) subconscious structures. My suggestion goes rather in the opposite direction. I see meaning change as the outcome of manipulating unanalyzed blocks — rather similar to Hilbert's uninterpreted symbols — and I see the conceptual analysis that later reveals the meaning of these blocks as parasitic.

15. *In conclusion*

I hope to have clarified the block approach and to have shown that it is promising. I have discussed a reinterpretation of proofs, a clarification of proof search processes, the solution of technical problems of adaptive logics, the advantages of the direct block approach, and some outlooks on meaning change.

The promising character of the block approach is clearly connected to the *dynamic* aspects of logic. It not only enables us to devise a semantics that corresponds to dynamic proofs of adaptive logics. It also enables us to get a grasp on the dynamic aspects of proofs and proof search procedures in logics as monotonic and traditional as **CL**. And it enables us to understand some linguistic changes as well. The dynamic aspects of logic concern concrete procedures and these have been shamelessly neglected by the traditional approach (that seems merely interested in the paradise of abstraction). For this reason too, the block approach seems promising.

Many open problems remain. The direct block approach has to be worked out, and one should look for direct proofs that are independent of the traditional approach. The two problems for adaptive logics are solved only in a heuristic fashion; they are in need of proofs (for all adaptive logics) and presumably will require some reformulation of the dynamic proof theory itself. The claims on meaning change too need be worked out.

But there are promises and problems beyond the ones mentioned. In [16] and [17], Samuel Issman proposed further restrictions to Anderson and Belnap's approach to relevant logic.[32] In his understanding, the relevance of inferences depends on the relations between steps in a derivation. I do not want to take a stand here with respect to Issman's proposal. I merely refer to it to illustrate that concrete aspects of proofs may play a role in the definition of derivability and hence of semantic consequence. A realistic approach to those concrete aspects involves dynamics — were it only because of the undecidability of predicate logics. And the block approach is able to handle this type of dynamics: it is typically a dynamics that depends on the block analysis.

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32 One way of putting his idea is that the premises or hypotheses should not only be used to derive the conclusion, but should also be indispensable for it. To capture the idea, Issman introduces conditions on the relation between the steps in the derivation.

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