Logique & Analyse **149** (1995), 43-53

A DECISION PROCEDURE FOR VON WRIGHT'S OBS-CALCULUS

Filip BUEKENS

1. Von Wright (1983) presents an interesting an fruitful integration of action logic into classical deontic logic.[1] The first part of this paper presents a brief overview of von Wright's OBS-calculus and discusses some alternative proposals and minor amendments to the original proposal. The second part develops a decision procedure for well-formed formulae. I follow Hintikka's approach based on a possible world-semantics for deontic operators.

2. In 'Norms, Truth and Logic' G.H. von Wright presents a logic of action which differs considerably both in notation and interpretation from action logics previously developed in, for instance, *Norm and Action* (Von Wright 1963). Starting-point for his new BS-calculus is the idea that (intentional) state of affairs like opening a door, leaving a door open or closing a door, are the result of two different types of action. Agents can produce states, i.e. change the contradictory state into the one which obtains when the action is successful, or sustain states, i.e. prevent them from ceasing to obtain. This distinction accounts for the difference between opening a door and keeping a door open. Productive actions are opposed to destructive actions. A destructive action changes a state into its contradictory state, and an agent who sustains the contradictory of a state suppresses that state, i.e. prevents it from coming to obtain. A logic of action is thus basically a logic of (causing) changes, letting changes obtain and preventing changes from coming to obtain.

Von Wright thus considers eight possible achievement actions. For both practical and theoretical reasons they are considered as a set of actions which are mutually exclusive and jointly exhausting the realm of achievement actions. The formalism of the BS-calculus contains two primitive operators: 'B' stands for the production or destruction of a state; 'Bp' can be read as 'producing the state of affair p', and '$B{\sim}p$' for producing the

[1] Part of this paper was written while I was a research assistant at the Center for Juridical Logic of the University of Leuven. I'd like to thank M. Sergant and K. Declercq for useful comments on earlier drafts of this paper.

contradictory state $\sim p$. 'S' stands for sustaining a state of affairs. 'Sp' can be read as sustaining a state of affairs p, whereas '$S\sim p$' stands for suppressing a state p. Note that the list of achievement actions makes a distinction between the propositional negator and the ommission-sign which indicates the opposite action is being performed:

1)	Bp	producing (a state of affairs) *p*
2)	$\neg Bp$	leaving the state p to continue absent
3)	Sp	sustaining (the state of affairs) *p*
4)	$\neg Sp$	letting the state p cease to obtain
5)	$B\sim p$	destroying *p*
6)	$\neg B\sim p$	leaving p to continue present
7)	$S\sim p$	suppressing *p*
8)	$\neg S\sim p$	letting the state come to obtain

The fact that these eight actions exhaust all possible achievement-actions entails that

$$\text{(T1)}\quad Bp \veebar \neg Bp \veebar Sp \veebar \neg Sp \veebar B\sim p \veebar \neg B\sim p \veebar S\sim p \veebar \neg S\sim p$$

is a tautology. Moreover, the eight primitive actions are mutually exclusive, which ensures us that, for instance,

$$\text{(T2)}\quad \sim\text{Bp} \leftrightarrow \neg Bp \veebar Sp \veebar \neg Sp \veebar B\sim p \veebar \neg B\sim p \veebar S\sim p \veebar \neg S\sim p$$

is a tautology in the BS-calculus (a BS-tautology, for short). The mutual exclusiveness of the eight possible achievement-actions is due to two different reasons: first, one cannot produce and at the same time omit to produce p. Therefore, Bp and $\neg Bp$ are incompatible. By the same token, Sp, $B\sim p$ and $S\sim p$ cannot be performed together with $\neg Sp$, $\neg B\sim p$ and $\neg S\sim p$ respectively. The second reason why different actions are exclusive is that they require different conditions of application. In order to produce p, p must be absent and remain absent unless an agent produces p. Performing $\neg S\sim p$ requires p'*s* absence and its becoming present; the action, then, consists in letting the state come to obtain. This explains why, e.g. Bp and $S\sim p$ are mutually exclusive actions.

For each action we can define an ordered triple $<a, b, c>$ in which a represents the state of affairs before the agent performs the action, b the state of affairs that would obtain if the agent does not act (intentionally) and c the state of affairs that obtains if the agent performs the action. To the eight primitive actions correspond the following triples:

1) $Bp: \langle \sim p, \sim p, p \rangle$	5) $B\sim p: \langle p, p, \sim p \rangle$
2) $\neg Bp: \langle \sim p, \sim p, \sim p \rangle$	6) $\neg B\sim p: \langle p, p, p \rangle$
3) $Sp: \langle p, \sim p, p \rangle$	7) $S\sim p: \langle \sim p, p, \sim p \rangle$
4) $\neg Sp: \langle p, \sim p, \sim p \rangle$	8) $\neg S\sim p: \langle \sim p, p, p \rangle$

The first and the second proposition in the ordered triple represent the conditions of application of an action. $\neg Bp$ and Bp can be performed only if the state of affairs in question is absent and remains absent, whereas, e.g. $S\sim p$ requires p's absence but its becoming present. Our 'ontology' thus consists of four different types of conditons of application. For each condition of application two mutually exclusive actions can be defined. When we introduce ordered couples, such as $\langle \sim p, \sim p \rangle$, we tacitly assume that they are conditions of application corresponding to the first and second element in the given triples.

3. The argument of a B- or S-operator (or an omission-sign, followed by one of these operators) may be a molecular compound of variables. It is therefore useful to formulate distribution laws in order to be able to give for every wff of the BS-calculus containing a compound of variables in the argument a normal form in which all arguments of B- or S-operators are atomic propositions. Von Wright amply discusses the various possibilities and concludes that the following distribution laws should be preferred:

$$B(p \underline{\vee} q) \leftrightarrow (Bp \,\&\, Bq) \underline{\vee} (Bp \,\&\, \neg Bq) \underline{\vee} (\neg Bp \,\&\, Bq)$$
$$\neg B(p \underline{\vee} q) \leftrightarrow (\neg Bp \,\&\, \neg Bq)$$
$$B(p \,\&\, q) \leftrightarrow ((Bp \,\&\, \neg Bq) \underline{\vee} (\neg Bq \,\&\, Bq) \underline{\vee} (\neg Bq \,\&\, \neg Bp))$$
$$S(p \underline{\vee} q) \leftrightarrow ((Sp \,\&\, Sq) \underline{\vee} (Sp \,\&\, \neg Sq) \underline{\vee} (\neg Sp \,\&\, Sq))$$
$$\neg S(p \underline{\vee} q) \leftrightarrow (\neg Sp \,\&\, \neg Sq)$$
$$S(p \,\&\, q) \leftrightarrow (Sp \,\&\, Sq)$$
$$\neg S(p \,\&\, q) \leftrightarrow ((Sp \,\&\, \neg Sq) \underline{\vee} (\neg Sp \,\&\, Sq) \underline{\vee} (\neg Sp \,\&\, Sq))$$

Gardies (1983) suggests that only some of these principles are correct. His alternative for the normal form of $S(p \,\&\, q)$ would consist in an alternation containing, inter alia, $Sp \,\&\, \neg B \sim q$. But this cannot be accepted for the obvious reason that the normal form of a complex action such as $S(p \,\&\, q)$ must consist in a disjunction of actions having the same conditions of application as the formula on the right side of the biconditional. Accepting Gardies' alternative goes against a basic principle of the BS-calculus: actions have conditions of application and re-describing actions in terms of a set of disjunctions of conjunctions must respect those conditions.

4. The eight primitive actions or their moleculary variants can be preceded by deontic operators. We adopt the following formation-rules for well-formed formulae in the OBS-calculus:

(i) well-formed formulae from the BS-calculus, preceded by deontic operators O and P
(ii) truth-functional compounds of BS-expressions preceded by an O- or P-operator
(iii) truth-functional compounds of formulae satisfying conditions (i) or (ii);
(iv) no other formulae than those which satisfy (i) - (iii) are accepted.

In 'Norms, Truth and Logic' deontic operators are given satisfaction conditions: an obligation Op is satisfied if and only if p is always the case on occasions that offer the agent an opportunity to perform p. Pp is satisfied if p is at least one time the case on an occasion that allows us to perform p. It is easy to see that this is a simple re-formulation of a possible-worlds semantics for standard deontic logic. Note, however, that the formula $O(p \veebar \sim p)$, rejected in von Wright's 'old system' (von Wright 1951) is accepted here as a theorem. (See section 6 for a model-theoretical formulation.)

A central point in von Wright's OBS-calculus is that the satisfiability of a norm, say, OBp, $O(Bp \veebar Bq)$ or $PB(p \veebar q)$ may depend on whether the conditions of application of the norm actually obtain. If the circumstances are such that it is impossible to produce p, then it is natural to say that the obligation to produce p is 'irrelevant' in those circumstances. OBp can be satisfied if and only if the circumstances allow the agent to produce p, that is, if condition of application $<\sim p, \sim p>$ obtains. The conditions of application of $PB(p \veebar q)$ can be derived as follows: first, we give the normal form of $B(p \veebar q)$ according to the distribution principles presented in section 3. The result is $P((Bp \,\&\, Bq) \veebar (Bp \,\&\, \neg Bq) \veebar (\neg Bp \,\&\, Bq))$. $PB(p \veebar q)$ can be satisfied on occasions where both p and q can be produced (although the agent actually has a choice as to which of the alternatives he shall perform). The set of conditions of application of $PB(p \veebar q)$ is $\{<\sim p, \sim p>, <\sim q, \sim q>\}$.

$P(Bp \veebar Bq)$ has a wider range of conditions of application. First we rewrite the given formula in its disjunctive normal form: $P((Bp \,\&\, Bq) \veebar (\sim Bp \,\&\, Bq) \veebar (Bp \,\&\, \sim Bq))$. If we replace the negated actions by a seven-termed unnegated expression (based on pinciple (T2)) and distribute the result, we get a 15-termed disjunction. The disjuncts have different conditions of application (for instance, $(Bp \,\&\, Bq)$ can be performed if the conditions of application $<\sim p, \sim p>$ and $<\sim q, \sim q>$ obtain, whereas $Bp \,\&\, \neg S \sim q$ requires that $<\sim p, \sim p>$ and $<\sim q; q>$ obtain). The result of a relativization to different conditions of application of the 15-termed disjunction is a 7-termed conjunction $P\big((Bp \,\&\, Bq) \veebar (Bp \,\&\, \neg Bq) \veebar (\neg Bp \,\&\, Bq)\big) \,\&\, P\big((Bp \,\&\, Sq) \veebar (Bp \,\&\, \neg Sq)\big) \,\&\, P\big((Bp \,\&\, B \sim q) \veebar (Bp \,\&\, \neg B \sim q)\big) \,\&\, P\big((B \sim p \,\&\, Bq) \veebar (\neg B \sim p \,\&\, Bq)\big) \,\&\, P(\neg Bp \,\&\, Bq) \,\&\, P\big((S \sim p \,\&\, Bq) \veebar (\neg S \sim p \,\&\, Bq)\big) \,\&\, P\big((Sp \,\&\, Bq) \veebar (\neg Sp \,\&\, Bq)\big)$. To the seven conjuncts the following sets of conditions of application correspond: $\{<\sim p, \sim p>, <\sim q, \sim q>\}$; $\{<\sim p, \sim p>, <q, q>\}$; $\{<\sim p, \sim p>, <q, \sim q>\}$; $\{<\sim p, \sim p>, <\sim q, \sim q>\}$; $\{<p, \sim p>, <\sim q, \sim q>\}$; $\{<p, p>, <\sim q, \sim q>\}$; $\{<\sim p, p>, <\sim q, \sim q>\}$.

It is now easy to see why the corpus of norms $OBp \,\&\, O\neg Bp$ is contradictory, whereas $OBp \,\&\, \mathrm{OB} \sim p$ form a consistent corpus of norms: in the first case, both norms have the same conditions of application, whereas those conditions differ in the second case. This point reminds us of the fact that norms often exist only in the context of a corpus. Corpora or sets of norms are consistent or inconsistent; a corpus is consistent if and only if all the norms that consitute it are satisfiable. There is, of course, a close link between the (in)consistency of a corpus of norms and the validity of, say, an implication: an implication is valid iff the negation of the consequens, conjoined to the antecedens, results in an inconsistent formula. This entails that an implication is contingent iff a conjunction of the negation norm of the consequens with the antecedens is consistent, i.e. results in a consistent corpus.

These ideas are developed or tacitly assumed in 'Norms, Truth and Logic', but no formal decision procedure is given for wff of the OBS-calculus. Hintikka (1971) develops a model-theoretical approach to deontic logic based on the notion of consistency of a set of norms. I shall now adapt his ideas to Von Wright's OBS-calculus.

5. Let us first solve an important preliminary problem. Suppose that a given formula of the form $P \rightarrow Q$ is a tautology in the OBS-calculus. This entails that $P \,\&\, \sim Q$ is an inconsistent corpus of norms. $\sim Q$ is the negation-norm of Q. The question we have to answer first is how to construe the negation-norm of a given OBS-expression. We distinguish three cases:

(case a) *Q has unique conditions of application (example: OBp)*
Given a BS-expression with unique conditions of appication, preceded by an O- or P-operator. We construe the normal form of the BS expressions (according to the principles in 2.). The result is an m-termed subclass of 2^n possible disjuncts (where n = the number of propositional variables in the BS-expression). The negation norm is obtained by changing the O-operator

into a P-operator (or vice versa) and adding to it an argument containing the $2^n - m$ disjuncts that do not occur in the original normal form. The negation norm of OBp and $PB(p \veebar q)$ are $P \neg Bp$ and $O(\neg Bp \,\&\, \neg Bq)$ respectively. This procedure respects the principle that the conjunction of a given norm with its negation norm results in an contradiction and that the negation norm of Q should have the same conditions of application as Q itself (This follows, in fact, from the former principle.)

(case b) *Q is a molecular compound of norms with unique conditions of application (example: $OBp \veebar PSq$)*
We put the negation-sign before the compound and distribute it. Next, we substitute the norms preceded by a negation sign by their negation norms (cf. (case a)). If the norm preceded by the negation sign has several conditions of application, then follow the procedure sub (c). An example: the negation norm of $OBp \veebar PSq$ is $P \neg Bp \,\&\, O \neg Sq$. Once again, the conjunction of Q and its negation norm is a contradiction.

(case c) *Q is a norm with different conditions of application (example: $P(Bp \veebar Bq)$)*
Von Wright (1983:191) suggests the following procedure: (1) derive the normal form of $Bp \veebar Bq$ (a 15-termed disjunction of conjunctions - cf. sub 2.) Each of the disjuncts can be seen as a complex action to which three cases of omission correspond. (2) Replace each conjunct by the corresponding cases of omission and change the P-operator into an O-operator. The result is a 45-termed obligation. (In fact, some disjuncts are identical and can be eliminated, which results in a 28-termed disjunction.) This procedure cannot be accepted: the conjunction of $P(Bp \veebar Bq)$ with its negation norm (construed along the lines suggested by Von Wright) is not a contradiction. In fact, the negation norm Von Wright derives is a tautology.

An alternative method is a combination of cases a and b. We first relativize the given norm to its conditions of application (see section 4). The result is a conjunction of norms, each consisting of m of 2^n possible disjuncts. Each conjunct has unique conditions of application. The result is a formula of the form described in case b. Thus the negation-norm of $P(Bp \veebar Bq)$ is the seven-termed disjunction of obligations $O(\neg Bp \,\&\, Bq) \veebar O((\neg Bp \,\&\, Sq) \veebar (\neg Bp \,\&\, \neg Sq)) \veebar O((\neg Bp \,\&\, B{\sim}q) \veebar (Bp \,\&\, \neg B{\sim}q)) \veebar O((\neg Bp \,\&\, S{\sim}q) \veebar (\neg Bp \,\&\, \neg S{\sim}q)) \veebar O((Sp \,\&\, \neg Bq) \vee (\neg Sp \,\&\, \neg Bq)) \veebar O((\neg B{\sim}p \,\&\, \neg Bq) \veebar (B{\sim}p \,\&\, \neg Bq)) \veebar O((S{\sim}p \,\&\, \neg Bq) \veebar (\neg S{\sim}p \,\&\, \neg Bq))$. What remains to be given is a decision procedure to show that a given set of OBS-expressions is (in)consistent.

6. A corpus of norms can be considered as a model set, i.e. a set of formulae. To say that a corpus of norms is consistent is to say that the set of norms that corresponds to it, is satisfiable, i.e. capable of being true (under given circumstances). A set of formulae is satisfiable if it is embedded (or can be embedded) in a set μ which satisfies the following conditions (Hintikka 1971):

(C. ~) If $p \in \mu$ then not $\sim p \in \mu$
(C. &) If $(p \,\&\, q) \in \mu$ then $p \in \mu$ and $q \in \mu$
(C. $\veebar$) If $(p \veebar q) \in \mu$ then $p \in \mu$ and $q \in \mu$

The main problem now is: how to give analogous closure conditions for formulae that contain deontic operators? It intuitively makes sense to say that, if a certain state of affairs is permitted, it must be conceivable to think of a world in which an agent satisfies the given permission. Formally:

(C.P*) If $P\,p \in \mu$ then for at least one deontic alternative world $\mu*$ to μ we must have $p \in \mu*$

We assume that a deontic alternative $\mu*$ to μ is also a model set and thus satisfies the conditions (C. ~) - (C. $\veebar$). Whatever ought to be the case must be the case in any deontic alternative world to μ (This is why it makes sense to think of deontic alternatives as more or less 'perfect' worlds: deontic alternatives are model sets in which all obligations are satisfied.) We adopt the following condition:

(C.O$^+$) If $O\,p \in \mu$ and $\mu*$ is a deontic alternative to μ, then $p \in \mu*$.

All obligations that obtain in the actual world or model set also obtain in a perfect deontic alternative to μ:

(C.OO$^+$) If $O\,p \in \mu$ and if $\mu*$ is a deontic alternative to μ, then $O\,p \in \mu*$.

If *Op* obtains in a perfect deontic alternative, then *p* too is the case in the perfect alternative:

(C.O)$_{rest}$ If $O\,p \in \mu*$ and if $\mu*$ is a deontic alternative to some model set μ, then $p \in \mu*$.

These principles can be used to construct rules of analysis, i.e. rules that allow us to construct a model system Ω for a set of formulae by starting from the set λ of formulae and adjoining new formulae in order to approach Ω.

How can we expand these rules of analysis for the OBS-calculus? Let us think of μ as the actual world, containing a set or corpus of OBS-expressions. Each expression has unique conditions of application. First we split the set of formulae in subsets according to the conditions of application of the formulae in μ. This allows us to split μ into sub-worlds corresponding to various conditions of application. An example: if we have

$\mu = \{OBp, OBq, O(Bp \& Bq)\}$

Then $CA(\mu_1) = \{(\sim p, \sim p)\}, CA(\mu_2) = \{(\sim q, \sim q)\}$ and $CA(\mu_3) = \{(\sim p, \sim p), (\sim q, \sim q)\}$. The subsets of μ are

$\mu_1 = \{OBp\}$
$\mu_2 = \{OBp\}$
$\mu_3 = \{O(Bp \& Bq)\}$

Any deontic alternative model set to μ will have one of the conditions of application of μ_1, μ_2 or μ_3; a deontic alternative to μ_1 has conditions of applications identical with those of μ_1. μ^*_1, μ^*_2 are *partial deontic alternatives* to μ.

It is easily seen that $CA(\mu_1)$ and $CA(\mu_2)$ are subsets of $CA(\mu_3)$, which means that, whenever O (Bp & Bq) applies, OBp and OBq apply too, but not vice versa. Or, to put the matter more intuitively, every occasion to satisfy $O(Bp \& Bq)$ is also an occasion to satisfy *OBp* or *OBq*. This captures the idea von Wright expresses when he claims that a norm *OBp* has a wider *range of application* than $O(Bp \& Bq)$. Occasions on which we are obliged to perform *Bp* are less restricted than occasions on which we are forced to oblige to $O(Bp \& Bq)$ (see von Wright 1983:190). A natural consequence of this idea is that norms belonging to μ_1 or μ_2 also belong to μ_3. We can thus extend μ_3 with *OBp* and *OBq*: $\mu_3 = \{OBp, OBq, O(Bp \& Bq)\}$. The same remarks hold, of course, for permissions.

Let us now formulate rules of analysis that allow us to approach a model system Ω for a given set of formulae: (βp stands for any *BS*-expression and $CA(\mu)$ stands for the conditions of application of μ)

($A.\&_1$) If $\mu \in \Omega$ and $(\beta p \& \beta q) \in \mu$ but not $\beta p \in \mu$ then we may add βp to μ

($A.\&_2$) If $\mu \in \Omega$ and $(\beta p \& \beta q) \in \mu$ but not $\beta q \in \mu$ then we may add βq to μ

$(A.\underline{\vee})$ If $\mu \in \Omega$ and $\beta p \underline{\vee} \beta q \in \mu$ but neither $\beta p \in \mu$ or $\beta q \in \mu$ then we may add βp or βq to μ

$(A.\beta P^{*})$ If $\mu_a \in \Omega$ and $P\beta p \in \mu_a$ but not $\beta p \in \mu^{*}_n$ where μ^{*}_n is any partial deontic alternative to μ and $CA(\mu_a) \subseteq CA(\mu^{*}_n)$, then we may add βp to μ^{*}_n

$(A.\beta O^{+})$ If $\mu_a \in \Omega$ and $O\beta p \in \mu_a$ but not $\beta p \in \mu^{*}_n$ where μ^{*}_n is a partial deontic alternative to μ and $CA(\mu_a) = CA(\mu^{*}_n)$, then $\beta p \in \mu^{*}_n$.

$(A.\beta OO^{+})$ If $\mu_a \in \Omega$ and $O\beta p \in \mu_a$ but not $O\beta p \in \mu^{*}_a$ where μ^{*}_a is a partial deontic alternative to μ_a and $CA(\mu_a) = CA(\mu^{*}_a)$, then $O\beta p \in \mu^{*}_a$.

$(A.\beta O)$rest If $\mu^{*}_n \in \Omega$, $O\beta p \in \mu^{*}_n$ but not $\beta p \in \mu^{*}_n$, then $\beta p \in \mu^{*}_n$

$(A.\beta o^{*})$ If $\mu_a \in \Omega$ and $O\beta p \in \mu_a$ but not $\beta p \in \mu^{*}_n$ where $CA(\mu_a) \subseteq CA(\mu^{*}_n)$ then $\beta p \in \mu^{*}_n$

We also adopt the following closure conditions for the omission operator:

$(C.\neg\beta)_1$ If $\beta p \in \mu$ then not $\neg\beta p \in \mu$

$(C.\neg\beta)_2$ If $\neg\beta p \in \mu$ then not $\beta p \in \mu$

Both conditions are based on principle (T1) which says that the eight achievement actions are mutually exclusive (cf. section 2). Note that $(C.\neg\beta)_1$ and $C(C.\neg\beta)_2$ are the analogues of $(C.\sim)$ for the OBS calculus. It is assumed that all relations of partial deontic alternativeness between members of Ω are originally due to an application of the rules $(A.P^{*})$ and/or $(A.O^{*})$. The union set of all partial deontic alternatives to μ is consistent (i.e. all the formulae contained in it are satisfiable) if each partial deontic alternative is consistent (i.e. the formulae contained in the partial alternative are satifiable). By contraposition, if at least one deontic alternative to μ is inconsistent, then the union set of all deontic alternatives to μ is inconsistent too.

We shall now examine whether the following conjunction of OBS-expressions is satisfiable:

(1) $OBp \,\&\, OBq \,\&\, O(Bp \,\&\, Bq)$

μ contains the subsets $\mu_1 = \{OBp\}$, $\mu_2 = \{OBq\}$ and $\mu_3 = \{O(Bp \,\&\, Bq), OBp, OBq\}$. By $(A.\ \beta o^{*})$ we derive (3) and (4). (5) - (7) are obvious:

(2) $\beta p \in \mu^{*}_1$

(3) $\beta q \in \mu^{*}_2$

(4) $Bp \,\&\, \beta q \in \mu^{*}_3$

(5) $Bp \in \mu^{*}_3$

(6) $Bq \in \mu^*_3$

All partial deontic alternatives to μ are consistent, and that suffices to show that our original conjunction of formulae (1) is satisfiable too.

Contrary to the validity of $O(p \,\&\, q) \rightarrow Oq$ in ordinary deontic logic we are well advised not to accept

(1) $O(Bp \,\&\, Bq) \rightarrow OBp$

as an OBS-tautology. An occasion that allows one to perform $Bp \,\&\, Bq$ may not allow one to perform *Bp simpliciter*. We first rewrite the implication as a conjunction of $O(Bp \,\&\, Bq)$ with the negation-norm of OBp. μ thus consists of two subsets $\mu_1 = \{O(Bp \,\&\, Bq)\}$ and $\mu_2 = \{P\neg Bp\}$. $CA(\mu_2) \subseteq CA(\mu_1)$, which entails that μ_1 can be expanded with $P\neg Bp$ (rule $(A.\beta P^*)$).

Applying the Rules of Analysis, we obtain

(2) $\neg\beta p \in \mu^*_2$
(3) $(Bp \,\&\, Bq) \in \mu^*_1$
(4) $Bp \in \mu^*_1$
(5) $Bq \in \mu^*_1$

Rule $(A.\,\beta P^*)$ does not require that $\neg Bp$ is added to μ^*_1: what matters here is that we have a choice as to whether we apply $(A.\beta P^*)$ to μ_1 or μ_2. Applying $(A.\beta P^*)$ to μ_1 results in an inconsistency. But we can apply $(A.\beta P^*)$ to μ_2, which does not result in an inconsistent set of actions. We conclude that μ^*_1 and μ^*_2 are consistent and that $O(Bp \,\&\, Bq) \rightarrow OBp$ is, as expected, a contingent statement.

To conclude, let us show that

(1) $P(Bp \veebar Bq) \rightarrow (PBp \veebar PBq)$

is an OBS-tautology. We relativize $P(Bp \veebar Bq)$ to its conditions of application and add to the result the negation-norm of $PBp \veebar PBq$. The result is a 9-termed conjunction. The model set of formulae can be divided into 9 different subsets. One subset is

(2) $\mu_1 = \{P((Bp \,\&\, Bq) \veebar (Bp \,\&\, \neg Bq) \veebar (\neg Bp \,\&\, Bq))\}$

The conditions of application of the subset which contains *OBp* are a subset of the conditions of application of μ_1. The same holds for *OBq*. It is now easy to see that the expanded set

(3) $\{P((Bp \,\&\, Bq) \veebar (Bp \,\&\, \neg Bq) \veebar (\neg Bp \,\&\, Bq)),\ OBp,\ OBq\}$

is inconsistent: principles $(C.\neg\beta)_1$ and $(C.\neg\beta)_2$ rule out the consistency of μ^*_1. Now, if one subset of μ^* is inconsistent, then μ^* is inconsistent and the formula given an OBS-tautology.

University of Tlburg

REFERENCES

von Wright, G.H. (1951), 'Deontic Logic', *Mind* 60 (1951), 1-15

von Wright, G.H. (1963), *Norm and Action*, London 1963

von Wright, G.H. (1983), 'Norms, Truth and Logic', in ibid., *Practical Reason*, Oxford 1983, 131-209

Gardies, J.L. (1983), 'Logique d'action (ou du changement) et logique déontique", in *Logique et Analyse* 1983, 71-89

Hintikka, Jaakko (1971), 'Some Main Problems in Deontic Logic', in R. Hilpinen (ed.), Deontic Logic: *Introductory and Systematic Readings*, Dordrecht 1971, 59-104