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# TEMPORALIZED DEONTIC WORLDS WITH INDIVIDUALS

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## 1. *Introduction*

In the previous article I constructed canonical models for a temporal deontic system I named R5-D5. In this system, the real world is related to alternative *histories* (*paths*), among which some may be "good" and only one (not good) can represent the real history of our world.

Another important issue in deontic logic is that of individuals playing a role in norms, mainly their *authorities* and *addressees*. For simplicity, I will discuss here only about the former. Again in previous works, I constructed logical systems which assumed that there is a complete compatibility between the norms emitted by different authorities, which is, I confess, rather an idealistic view. Thus, for example, if $x$ makes it obligatory that $p$, $y$ makes it obligatory that $q$, then any $z$ must make it permitted that $p$ and $q$. I also showed that it is almost indispensable to use *sets* of authorties. In the example, since $p$ is made obligatory (by $x$) and $q$ (by $y$), $p$ &$q$ is made obligatory. But by whom? By their set, of course. The system KDUX (vide *infra*) captures these main intuitions.

## 2. *A combination of alethic, deontic and temporal modalities*: R5-D5

Here I will only recall the readings of the three modal operators in R5-D5:

$R_t A$ = it is realized at the instant $t$ that $A$.
$\Box A$ = it is necessary that $A$.
$OA$ = it is obligatory that $A$.
and the mechanism of their evaluation:

$\vDash_{\alpha,t} R_{t'} A$ iff $\vDash_{\alpha,t'}$ A [1]

$\vDash_{\alpha,t} \Box A$ iff for every $\beta$ such that $Rt\alpha\beta$, $\vDash_{\beta,t'} A$

$\vDash_{\alpha,t} OA$ iff for every $\beta$ such that $St\alpha\beta$, $\vDash_{\beta,t'} A$

[1] In the sequel I will write only $\vDash_{\alpha,t}$ instead of $\vDash^{\mathcal{M}}_{\alpha,t}$.

I refer to the previous article for the semantic properties on $R$ and $S$, axiomatisation and adequation[2].

### 3. *A deontic system with sets of authorities*: KDUX

#### 3.1. *Axiomatics*

KDUX is as KDU, except that it adds sets of authorities ($X$, $Y$, ...) as arguments or indexes of operators and has a second rule of deduction which takes into account the compatibility between norms emanating from different authorities.

Let $E = \{x, y, \ldots\}$ be a set of authorities and let $X$, $Y$,... be various subsets of $E$. We define the operator

$O_X A$: (some subset of) $X$ makes it obligatory that $A$

Def. $P_X$ $P_X A = \neg O_X \neg A$

$P_X A$: no (some subset of) $X$ makes it obligatory that not $A$, i.e. (every subset of) $X$ permits that $A$

KDUX is the set of the following axiomatic elements :

| | |
|---|---|
| $KO_X$ | $O_X(A \rightarrow B) \rightarrow (O_X A \rightarrow O_X B)$ |
| $DO_X$ | $O_X A \rightarrow \neg O_X \neg A$ |
| $UO_X$ | $O_X(O_X A \rightarrow A)$ |
| $RNO_X$ | $A / O_X A$[3] |
| $RSrO_X$ | $X \subseteq Y \Rightarrow \vdash O_X A \rightarrow O_Y A$ |

The three axioms and the first rule are isomorphic to those of KDU, the same set of authorities being added as an index. The second rule, $RSrO_X$, states an intuitive truth: if one set of authorities is included in another set, then every obligation emanating from the former is also an obligation emanating from the latter. It is noteworthy that this rule includes a general normative compatibility between different authorities. For example, from it, one can derive the theorem:

$$O_X A \rightarrow \neg O_Y \neg A$$

i.e. $$O_X A \rightarrow P_Y A$$

2 See the Conclusion for the latter property.

3 As for $RNt$, $RN\Box$, $RNO$ in the previous paper, this rule could be written $\vdash A \Rightarrow \vdash O_X A$.

which says that if a set of authorities, $X$, makes it obligatory that $A$, then any other set of authorities, $Y$, permits $A$[4].

### 3.2. *Semantics*

When only isolated authorities are handled, not sets of authorities, models are build on a ternary relation $Sxww'$ ($S \subseteq E \times W \times W$, $E$ = a set of authorities, $W$ = a set of worlds), saying that $w'$ is permissible to $w$ according to $x$. The world $w'$ is a variant of w which complies with the norms emitted by $x$. The relation $S$ must be at least serial (we assume that a norm of every authority can be fulfilled):

— $Sx$-seriality: $\forall w \exists w' \forall x\, Sxww'$ [5]
— $Sx$-secondary-reflexivity: $\forall x \forall ww'(Sxww' \Rightarrow Sxw'w')$

Other properties are plausible, like:

— $Sxy$-second-authority-reflexivity: $\forall xy \forall ww'(Sxww' \Rightarrow Syw'w')$
— $Sxy$-transitivity: $\forall xy \forall ww'w''[(Sxww' \& Syw'w'') \Rightarrow Sxww'']$ [6]

In this first approach, however, I will only consider $Sx$-seriality and $Sx$-secondary-reflexivity.

Note that only *worlds*, but not *paths*, are used since time is not under consideration in that semantics.

[4] Let $X = \{x\}$, $Y = \{y\}$, $Z = \{x,y\}$. $X \subseteq Z$ is true, so we have $O_X A \rightarrow O_Z A$. By virtue of $DO_X$, $O_Z A \rightarrow \neg O_Z \neg A$ holds, i.e. $O_Z A \rightarrow P_Z A$ and therefore, by syllogism, $O_X A \rightarrow P_Z A$. Now, $Y \subseteq Z$ is true, too, so that $O_Y A \rightarrow O_Z A$ holds, too. Therefore, by contraposition and substitution of $A$ by $\neg A$, $\neg O_Z A \rightarrow \neg O_Y A$, i.e. $P_Z A \rightarrow P_Y A$. Finally, by syllogism: $O_X A \rightarrow P_Y A$, where $X$ and $Y$ are two singletons completely independent.

[5] The order of quantifiers are essential. The fact that the universal quantification on $x$ follows the existential quantification on $w'$ means that a permissible world can always be found which realizes the norms emitted by all the authorities together. Only this order preserves the complete normative compatibility between authorities.

[6] As easily checked, Sxy-second-authority-reflexivity corresponds to the axiom $O_X(O_Y A \rightarrow A)$ and therefore implies the thesis $O_X O_Y A \rightarrow O_X A$; $Sxy$-transitivity corresponds to $O_X A \rightarrow P_X O_Y A$. The former thesis expresses that when $x$ makes it obligatory for $y$ to make something obligatory, then he ($x$) makes it obligatory this thing, which is plausible; the latter says that when $x$ makes it obligatory something, then he permits for any $y$ to make this thing obligatory, which is very plausible, too. Cf. [Bailhache 1991], p. 167ff.
Of course, $O_X(O_X A \rightarrow A) (= UO_X)$ is a special instance of $O_X(O_Y A \rightarrow A)$, as Sx-secondary-reflexivity is of Sxy-second-authority-reflexivity.

Now, in order to introduce sets of authorities, we *define* the ternary relation $SXww'$ ($S \subseteq P(E) \times W \times W$, $E=$ a set of authorities, $W$ = a set of worlds), saying that $w'$ is permissible to $w$ according to $X$, in this manner[7]:

$$SXww' = \&_i \; Sx_i ww'$$

where $X = \{x_1, x_2, \ldots\}$ and $\&_i$ represents the (meta-)conjunction on all the $x_i$ in $X$. Thus, when a world is permissible according to a set of authorities, it means that it is permissible according to each authority of this set.

Obviously, $Sx$-seriality implies $SX$-seriality and $Sx$-secondary-reflexivity implies $SX$-secondary-reflexivity:

— $SX$-seriality: $\forall w \exists w' \forall X \; SXww'$
— $SX$-secondary-reflexivity: $\forall X \forall ww' (SXww' \Rightarrow SXw'w')$

Obviously again, if $X \subseteq Y$, then the conjunction represented by $SY\,ww'$ contains all the conjuncts of $SX\,ww'$, so that the former implies the latter:

— $SX$-set-regularity[8]: $\forall XY \forall ww' [X \subseteq Y \Rightarrow (SYww' \Rightarrow SXww')]$

I will not take care here of other possible properties based upon other plausible properties of $Sx\,ww'$.

3.3. *Adequation*
Again the proof of soundness and completeness is a routine task; the latter does not even require using of canonical models. I shall not dwell here on the proofs. The correspondence between semantic properties and axiomatic elements can be scheduled as follows:

| | |
|---|---|
| $KO_X$ | standardness of the model |
| $DO_X$ | $SX$-seriality |
| $UO_X$ | $SX$-secondary-reflexivity |
| $RNO_X$ | standardness of the model |
| $RSrO_X$ | $SX$-set-regularity |

[7] Cf. [Bailhache 1991], p. 100.

[8] I choose this bad denomination, waiting for a better proposal.

## 4. *Mixing authorities into modalities*: R5-DX5

### 4.1. *Building a system with the help of axiomatics and semantics together*

The simplest way to start the job is probably to add sets of authorities to the temporal deontic $St$-relation (the alethic relation, $R$, remains unchanged). So $St\alpha\beta$[9] and $SXww'$ merge into a relation with four arguments, $StX\alpha\beta$, which can be read "From time $t$, the path $\beta$ is a variant of the path $\alpha$, good according to the set of authorities $X$". Obviously, the previous properties, $St$-seriality, $St$-secondary-reflexivity, $St$-secondary-ramification, and $SX$-seriality, $SX$-set-regularity, should be merged into the following ones:

— $StX$-seriality: $\forall t\, \forall\alpha\, \exists\beta\, \forall X StX\alpha\beta$[10]
— $StX$-secondary-reflexivity: $\forall t\, \forall\alpha\, \forall\beta\, \forall X(StX\alpha\beta \Rightarrow StX\beta\beta)$[11]
— $StX$-secondary-ramification:
$\forall t\, \forall t'\, \forall\alpha\, \forall\beta\, \forall\gamma\, \forall X\, [(t'<t\ \&\ St'X\alpha\beta \& StX\beta\gamma) \Rightarrow St'X\beta\gamma]$
— $StX$-set-regularity: $\forall t\, \forall XY\, \forall\alpha\beta\, [X \subseteq Y \Rightarrow (StY\alpha\beta \Rightarrow StX\alpha\beta)]$

According to the previous adequations, these properties must respectively correspond to the following axioms and rules:

| | |
|---|---|
| $DO_X$ | $O_XA \rightarrow \neg O_X\neg A$ |
| $UO_X$ | $O_X(O_XA \rightarrow A)$ |
| $O_Xt2$ | $t'<t \Rightarrow R_{t'}O_X(R_{t'}O_XR_tA \rightarrow R_tO_XR_tA)$ |
| $RSrO_X$ | $X \subseteq Y \Rightarrow \vdash O_XA \rightarrow O_YA$ |

On the other hand, $KO_X$ and $RNO_X$ are preserved, like $DO_X$, $UO_X$ and $RSrO_X$, since the introduction of time into $KDUX$ makes no change for these axiomatic elements (as was the case for such an introduction into $KDU$ for $KO$, $UO$ and $RNO$, which are elements of R5-D5). For the time being, the sole problem will be to check that $O_Xt2$ and its corresponding semantic property, $StX$-secondary-ramification, are really plausible.

Let us examine, however, what the other properties and axiomatic elements of R5-D5 become when set of authorities are added. The remaining

[9] Because I am not explicitely working with canonical models in the present paper, I prefer $Rt\alpha\beta$ and $St\alpha\beta$ formulation to $\alpha_tS\beta_t$ and $\alpha_tS\beta_t$, used in the previous article.

[10] Observe the situation of quantifiers.

[11] Of course, do not confuse $StX$-secondary-reflexivity with the property alluded above, $Sxy$-second-authority-reflexivity. In the former the (set of) authority does not change; on the contrary, in the latter the secondarity means only that a world is already permissible and the authority changes.

properties are those which mix alethic and deontic modalities, *SRt*-implication, *RSt*-transitivity, *RSt*-post-implication, corresponding to the axioms $\Box O$, $\Box O4$, $\Box Ot$. In addition, R5-D5 contains the axiom $Ot1$, which will have also to be extended with sets of authorities.

I shall now study the five elements, $\Box O$, $\Box O4$, $Ot1$, $Ot2$, $\Box Ot$, with their corresponding semantic properties, in this order.

4.1.1. $\Box O$ *with SRt-implication*
Obviously, the axiom

$$\Box O \qquad \Box A \to OA$$

must be converted into

$$\Box O_X \qquad \Box A \to O_X A$$

Semantically, the property

— *SRt*-implication: $\forall t\, \forall \alpha\, \forall \beta\, (St\alpha\beta \Rightarrow Rt\alpha\beta)$

must be converted into

— *SRtX*-implication: $\forall t\, \forall \alpha\, \forall \beta\, \forall X\, (StX\alpha\beta \Rightarrow Rt\alpha\beta)$

Everything which is necessary belongs to the things good according to any *X*.

4.1.2. $\Box O4$ *with RSt-transitivity*
Similarly, the axiom

$$\Box O4 \qquad OA \to \Box OA$$

must be converted into

$$\Box O_X 4 \quad O_X A \to \Box O_X A$$

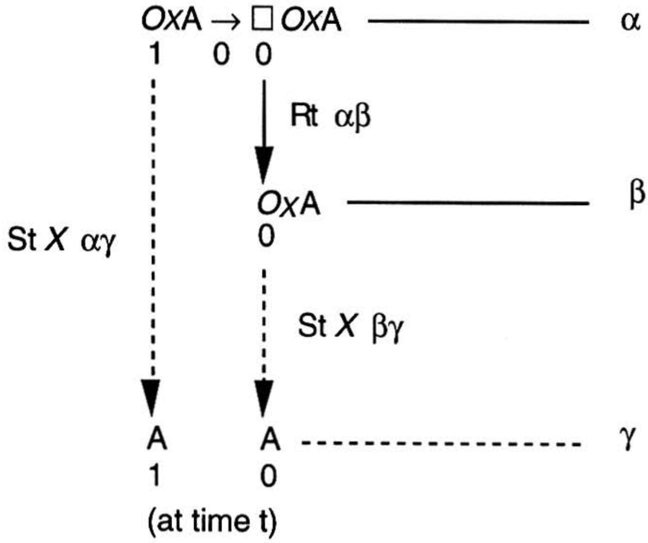
and the property

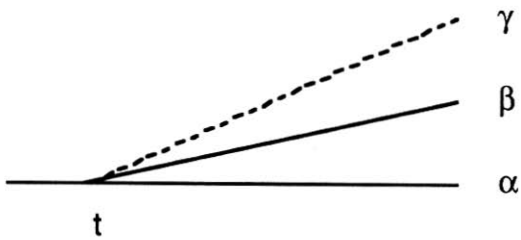
— *RSt*-transitivity: $\forall t\, \forall \alpha\, \forall \beta\, \forall \gamma\, [(Rt\alpha\beta\ \&\ St\beta\gamma) \Rightarrow St\alpha\gamma]$

into

— *RStX*-transitivity: $\forall t\, \forall \alpha\, \forall \beta\, \forall \gamma\, [(Rt\alpha\beta\ \&\ StX\beta\gamma) \Rightarrow StX\alpha\gamma]$

Semantic evaluation of $\Box O_X 4$ makes us to understand the reason of its truth:




$\gamma$ is a path, good according to $X$ with respect to $\beta$ at time $t$. Since $\beta$ is an alternative to $\alpha$ at time $t$, $\gamma$ is good according to $X$ with respect to $\alpha$, too, at the same time.

4.1.3. *Ot*1
Obviously,

$Ot1 \qquad R_t OA \leftrightarrow R_t OR_t A$
must be converted into

$O_X t1 \qquad R_t O_X A \leftrightarrow R_t O_X R_t A$

(for the corresponding semantic property, see "Canonical Models for Temporal Deontic Logic", § 3.3).

4.1.4. *Ot2 with St-secondary-ramification*
As said above,

$$O_X t2 \qquad t'<t \Rightarrow R_{t'} O_X (R_{t'} O_X R_t A \rightarrow R_t O_X R_t A)$$

with *StX*-secondary-ramification:

$$\forall t\, \forall t'\, \forall \alpha\, \forall \beta\, \forall \gamma\, \forall X\, [(t'<t\ \&\ St'X\alpha\beta \& StX\beta\gamma) \Rightarrow St'X\beta\gamma]$$

are apparently good candidates for extending the elements of R5-D5. We have now to check that. To this end, let us draw the diagram of $O_X t2$.

$R_{t'} O_X (R_{t'} O_X R_t \mathrm{A} \rightarrow R_t O_X R_t \mathrm{A})$ ———— α
0 0

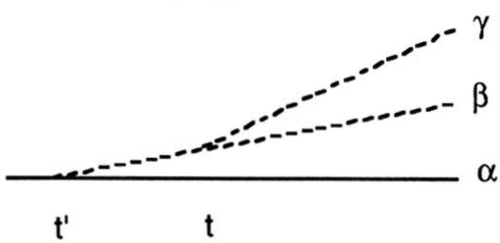
↓ $St'X$ αβ

$R_{t'} O_X R_t \mathrm{A} \rightarrow R_t O_X R_t \mathrm{A}$ - - - - - - β
1 1 0 0 0

↓ $St'X$ βγ ↓ $St\ X$ βγ

$R_t$A - - - - - A $R_t$A - - - - - - γ
1 1 0 0

time t' time t



$\beta$ is a path good according to $X$ at time $t'$. The absurdity concluding the proof ($A$=1 and $A$=0 on $\gamma$ at time $t$) stems from the fact that the conjunction of $St'X\alpha\beta$ and $StX\beta\gamma$ implies $St'X\beta\gamma$, i.e. the $StX$-secondary-ramification. A new problem is to know whether the variation of sets of authorities into the formula is plausible or not. For example, what about $R_{t'}O_X(R_{t'}O_YR_tA \rightarrow R_tO_YR_tA)$? In this case, the path $\gamma$ should be understood as good, from time $t$, for *Y in the goodness of X* (in the path $\beta$). In addition, to obtain conclusive absurdity, the section of $\gamma$ between $t'$ and $t$, identical to the section of $\beta$, must be conceived as good according to $Y$. This is rather problematic, but not completely improbable since we can admit that the goodness of $Y$ viewed from that of $X$ amounts to the mere goodness of $X$. I shall return to this problem in a while with the examination of $\Box Ot$.

4.1.5. *$\Box Ot$ with RSt-post-implication*

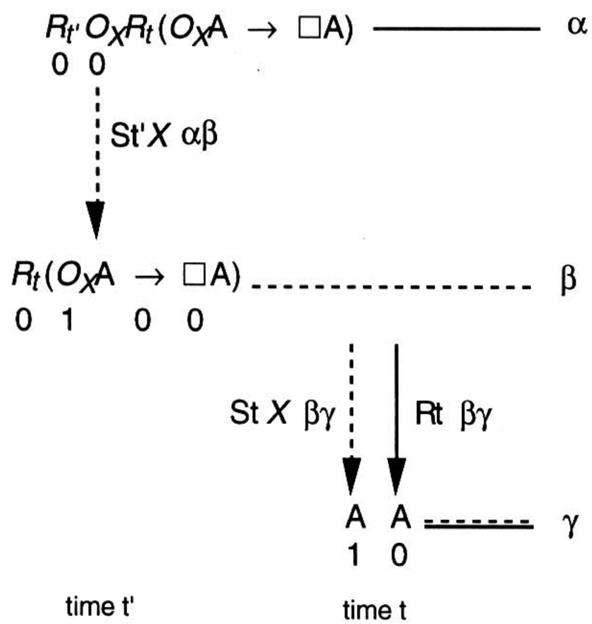
The simplest way to extend

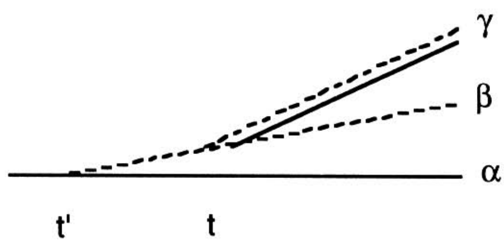
$$\Box Ot \qquad t'<t \Rightarrow R_{t'}OR_t(OA \rightarrow \Box A)$$

is to keep the same set of authorities $X$ in the whole formula:

$$\Box O_Xt \qquad t'<t \Rightarrow R_{t'}O_XR_t(O_XA \rightarrow \Box A)$$

As usual, however, complete light on the matter, if possible, can only be thrown by semantic diagrams. So let us draw the diagram of $\Box O_Xt$.




One can immediately see how the validation of $\Box O_X t$ depends on *RStX*-post-implication under the form:

$$(t' < t \;\&\; St' X\alpha\beta \;\&\; Rt\beta\gamma) \Rightarrow StX\beta\gamma$$

This property is very similar to its version without sets of authorities. It only says that every alternative, at $t > t'$, of the path $\beta$, which is good according to $X$ at $t'$, should be good according to $X$ at $t$. Apparently, we can admit this property without restriction.

4.2. *The problem with* $\Box O_X t$

Things are not so easy, however. In § 4.1.1. I explained that the axiom

$$\Box O_X \qquad \Box A \rightarrow O_X A$$

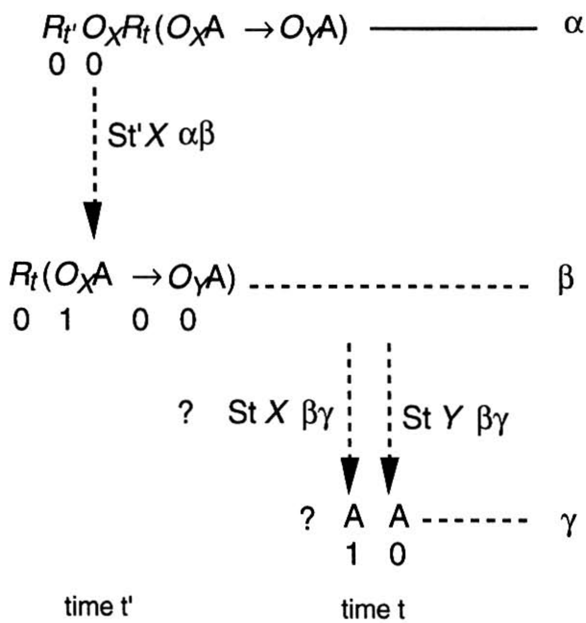
should be laid down, with the same (great) degree of verisimilitude as $\Box O$. Then we derive

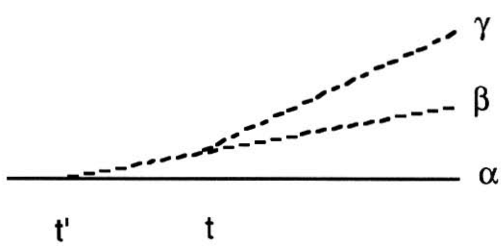
$$\Box A \rightarrow O_Y A$$

and, by $\Box O_X t$:

$$t' < t \Rightarrow R_{t'} O_X R_t (O_X A \rightarrow O_Y A)$$

To understand this sentence, let us use our best tool, viz. a diagram.



The proof by absurdum will achieve only if[12]

$$(St'X\alpha\beta \,\&\, StY\beta\gamma) \Rightarrow StX\beta\gamma \qquad (*)$$

Now, this property holds in virtue of *RStX*-post-implication

$$(St'X\alpha\beta \,\&\, Rt\beta\gamma) \Rightarrow StX\beta\gamma$$

jointly with *SRtX*-implication (used with *Y* instead of *X*)

$$StX\alpha\beta \Rightarrow Rt\alpha\beta$$

Since *RStX*-post-implication and *SRtX*-implication appeared plausible, the property (*) should be admitted.

Under these circumstances, one can now re-examine the status of *StX*-secondary-ramification. We have accepted its *weak* version without problem (cf. last paragraph, at the end):

$$(St'X\alpha\beta \,\&\, StX\beta\gamma) \Rightarrow St'X\beta\gamma$$

And we projected to study its *strong* version

$$(St'X\alpha\beta \,\&\, StY\beta\gamma) \Rightarrow St'Y\beta\gamma$$

which would validate the formula

$$R_{t'}O_X(R_{t'}O_YR_tA \rightarrow R_tO_YR_tA).$$

But, as readily checked, the property (*), jointed to the weak *StX*-secondary-ramification, implies a *StX*-secondary-ramification only intermediate between the weak and the strong ones:

$$(St'X\alpha\beta \,\&\, StY\beta\gamma) \Rightarrow St'X\beta\gamma \qquad (**)$$

The meaning of this is straightforward. The goodness according to $Y$ at time $t$ viewed by $X$ from time $t'$ is goodness according to $X$ at time $t$ (cf. (*) ) or even $t'$ (cf. (**) ), but not goodness according to $Y$ from time $t'$. In other words, deontic feedback of $Y$ on $X$ is not conceivable. Isn't it a wise solution?

[12] From now on, I omit $t' < t$, which will be always supposed to be true.

4.3. R5-DX5: *resulting construction*

Let us now summarise the result of our research about a temporal deontic system with sets of authorities.

4.3.1. *Semantic properties*

$R$ is a ramified equivalence relation, as in R5-D5: $Rt$-reflexivity, $Rt$-symmetry, $Rt$-transitivity, $Rt$-ramification.

$S$ is serial, secondary-reflexive, secondary-ramified and set-regular:

- $StX$-seriality: $\forall t \forall \alpha \exists \beta \forall X\ StX\alpha\beta$
- $StX$-secondary-reflexivity: $\forall t \forall \alpha \forall \beta \forall X\ (StX\alpha\beta \Rightarrow StX\beta\beta)$
- (weak) StX-secondary-ramification:
  $\forall t \forall t'\ \forall \alpha \forall \beta \forall \gamma \forall X\ [(t' < t\ \&\ St'\ X\alpha\beta\ \&\ StX\beta\gamma) \Rightarrow St'\ X\beta\gamma]$
- $StX$-set-regularity: $\forall t \forall XY \forall \alpha\beta\ [X \subseteq Y \Rightarrow (StY\alpha\beta \Rightarrow StX\alpha\beta)]$

In addition, there are three properties common to $R$ and $S$ (I omit the $\forall s$):

- $SRtX$-implication: $StX\alpha\beta \Rightarrow Rt\alpha\beta$
- $RStX$-transitivity: $(Rt\alpha\beta\ \&\ StX\beta\gamma) \Rightarrow StX\alpha\gamma$
- $RStX$-post-implication: $(t' < t\ \&\ St'\ X\alpha\beta\ \&\ Rt\beta\gamma) \Rightarrow StX\beta\gamma$

Remarks

- A not weak $StX$-secondary-ramification can be *derived* from weak $StX$-secondary-ramification, $RStX$-post-implication and $SRtX$-implication: $(St'\ X\alpha\beta\ \&\ StY\beta\gamma) \Rightarrow St'\ X\beta\gamma$
- The strong $StX$-secondary-ramification does not hold:
  $(St'\ X\alpha\beta\ \&\ StY\beta\gamma) \Rightarrow St'\ Y\beta\gamma$

4.3.2. *Axiomatic*

Subsystem $\mathbb{R}$ for $R_t$ and subsystem S5 for $\Box$

Subsystem KDUX for $O_X$

| | |
|---|---|
| $KO_X$ | $O_X(A \to B) \to (O_XA \to O_XB)$ |
| $DO_X$ | $O_XA \to \neg O_X \neg A$ |
| $UO_X$ | $O_X(O_XA \to A)$ |
| $O_Xt2$ | $t' < t \Rightarrow R_{t'}O_X(R_{t'}O_XR_tA \to R_tO_XR_tA)$ |
| $RSrO_X$ | $X \subseteq Y \Rightarrow \vdash O_XA \to O_YA$ |

Mixed axioms

- $R_t$ and $\Box$:

| | |
|---|---|
| $\Box t1$ | $R_t\Box A \leftrightarrow R_t\Box R_tA$ |
| $\Box t2$ | $t' \le t \Rightarrow R_{t'}\Box R_tA \to R_t\Box R_tA$ |

— $R_t$ and $O_X$:

$O_X t1$ $\quad R_t O_X A \leftrightarrow R_t O_X R_t A$

$O_X t2$ $\quad t' < t \Rightarrow R_{t'} O_X (R_{t'} O_X R_t A \rightarrow R_t O_X R_t A)$

— $\Box$ and $O_X$

$\Box O_X$ $\quad \Box A \rightarrow O_X A$

$\Box O_X 4$ $\quad O_X A \rightarrow \Box O_X A$

— $R_t$, $\Box$ and $O_X$

$\Box O_X t$ $\quad t' < t \Rightarrow R_{t'} O_X R_t (O_X A \rightarrow \Box A)$

Remark

The formula $t' < t \Rightarrow R_{t'} O_X (R_{t'} O_Y R_t A \rightarrow R_t O_Y R_t A)$ stronger than $O_X t2$ must be rejected.

4.3.3. *Adequation*

| | |
|---|---|
| $Kt$ of $\{\mathbb{P}.\}$ | standardness of the model |
| $D!t$ of $\{\mathbb{P}.\}$ | functionality of $T_t$ |
| $Rtt'$ of $\{\mathbb{P}.\}$ | uniqueness of path for evaluation of $R_t$ |
| $RNt$ of $\{\mathbb{P}.\}$ | standardness of the model |
| | |
| $K\Box$ of $S5$ | standardness of the model |
| $T\Box$ of $S5$ | $Rt$-reflexivity |
| $5\Box$ of $S5$ | $Rt$-transitivity & $Rt$-symmetry |
| $RN\Box$ of $S5$ | standardness of the model |
| | |
| $KO_X$ of $KDUX$ | standardness of the model |
| $DO_X$ of $KDUX$ | $StX$-seriality |
| $UO_X$ of $KDUX$ | $StX$-secondary-reflexivity |
| $RNO_X$ of $KDUX$ | standardness of the model |
| $RSr\,O_X$ | $StX$-set-regularity |
| | |
| $\Box t1$ | $t' = t$ in $\forall \beta_{t'}\ st\ \alpha_t R \beta_{t'}$ |
| $\Box t2$ | Rt-ramification |
| $O_X t1$ | $t' = t$ in $\forall \beta_{t'}\ st\ \alpha_t S \beta_{t'}$ |
| $O_X t2$ | $StX$-secondary-ramification |
| $\Box O_X$ | $SRtX$-implication |
| $\Box O_X 4$ | $RStX$-transitivity |
| $\Box O_X t$ | $RStX$-post-implication |

## 5. *DARB and KDUX: DXARB*?

### 5.1. *Principle of correspondence*

In [Bailhache 1993] we compared the temporal structure of R5-D5 with that of a system due to Åqvist and Hoepelman, called DARB. For details the reader will refer to these papers, here I shall only give essential features on the matter.

From the semantic point of view, DARB handles on sets of paths. Let $X, Y \in W = \{\alpha, \beta, \ldots\}$. The deontic dimension is introduced by means of the "preference" function:

opt($X$) = the set of the "good" paths of $X$

To capture the meaning of preference, this function must fulfil the following Preference Conditions:

(*PC a*) $\text{opt}(X) \subseteq X$
(*PC b*) $\text{opt}(X) = \varnothing \Rightarrow X = \varnothing$
(*PC c*) $\text{opt}(X) \cap Y \subseteq \text{opt}(X \cap Y)$
(*PC d*) $\text{opt}(X) \cap Y \neq \varnothing \Rightarrow \text{opt}(X \cap Y) \subseteq \text{opt}(X) \cap Y$

Alethic and deontic modalities are then evaluated according to the rules:

$$\vDash_{\alpha,t} \Box A \Leftrightarrow \forall \beta \in W(\alpha \approx_t \beta \Rightarrow \vDash_{\beta,t} A)$$

or

$$\boxed{\vDash_{\alpha,t} \Box A \Leftrightarrow [\alpha]_{\approx_t} \subseteq \|A\|_{\alpha,t}}$$

where
$$[\alpha]_{\approx_t} = \{\beta \in W : \alpha \approx_t \beta\}$$
$$\|A\|_{\alpha,t} = \{\beta \in W : \alpha \approx_t \beta \;\&\; \vDash_{\beta,t} A\}$$

which can be read "*A* is necessary on $\alpha$ at *t* iff all paths accessible to $\alpha$ at *t* validate *A* at *t*".

(Absolute) obligation is derived from the conditional one:

$$\vDash_{\alpha,t} O(A / B) \Leftrightarrow \text{opt}(\|B\|_{\alpha,t}) \subseteq \|A\|_{\alpha,t}$$

that is "It is obligatory that $A$ under the condition $B$ on $\alpha$ at $t$ iff all the good paths accessible to $\alpha$ at $t$ which validate $B$ validate $A$, too". Then absolute obligation is obligation under the tautological condition:

$$\boxed{\models_{\alpha,t} OA \Leftrightarrow opt([\alpha]\approx_t) \subseteq \|A\|_{\alpha,t}}$$

which yields the reading "It is obligatory that $A$ on $\alpha$ at $t$ iff all the good paths accessible to $\alpha$ at $t$ validate $A$ at $t$".

There is an intuitive similarity between this semantic structure and that of R5-D5. More precisely, to implement a formal comparison, I proposed the following *principle of correspondence*:

$$Rt\alpha\beta \Leftrightarrow \alpha \approx_t \beta (\Leftrightarrow \beta \in [\alpha]\approx_t)$$
$$St\alpha\beta \Leftrightarrow \beta \in opt([\alpha]\approx_t)$$

We should expect, however, some differences in results because of a fundamental disagreement between the ramified structures of R5-D5 and DARB. The former requires that every good path remains good in future, while the latter does not (see the figures in [Bailhache 1993]).

5.2. *Semantic correspondence*

From the semantic point of view, applying these principles allows us to establish a total agreement for alethic modalities: every $Rt$-property (reflexivity, etc.) is validated by the correspondence for $R$. As for deontic modalities, the remaining properties, for $S$ and $R$, $S$ together, are in the following correspondence:

| | |
|---|---|
| (*PC a*) | *SRt*-implication |
| (*PC b*) | *St*-seriality |
| (*PC c*) | ? |
| (*PC d*) | *St*-secondary-ramification |
| no condition | *St*-secondary-reflexivity |
| no condition | *RSt*-transitivity |
| | ~~*RSt*-post-implication~~ |

Only *RSt*-post-implication is not validated[13]. This is not surprising since it was already the most controversial property of R5-D5 (and axiomatically

13 See [Bailhache 1993] for a formal proof that *RSt*-post-implication is not validated in DARB.

$\Box Ot$ the most controversial axiom), when expanded with sets of authorities. This constitutes some advantages for DARB versus R5-D5.

## 5.3. *Some ingredients for a draft of* DXARB

Accurate definitions of a frame, a model, etc. would be a matter of routine. Here I will restrict myself to give some semantic properties, axioms and rules, and their table of correspondence. The completeness of DARB is not proved, *a fortiori* that of DXARB.

### 5.3.1. *Semantic properties*

With

$$\text{opt}_X(E) = \text{the set of the paths of } E, \text{“good” according to } X$$

the preference conditions with authorities can be formulated:

| | |
|---|---|
| (*PCX a*) | $\text{opt}_X(E) \subseteq E$ |
| (*PCX b*) | $\text{opt}_X(E) = \varnothing \Rightarrow E = \varnothing$ |
| (*PCX c*) | $\text{opt}_X(E) \cap F \subseteq \text{opt}_X(E \cap F)$ |
| (*PCX d*) | $\text{opt}_X(E) \cap F \neq \varnothing \Rightarrow \text{opt}_X(E \cap F) \subseteq \text{opt}_X(E) \cap F$ |
| (*PCX-Sr*) | $X \subseteq Y \Rightarrow \text{opt}_Y(E) \subseteq \text{opt}_X(E)$ |

The evaluation of an obligation according to a set of authorities is very similar to that for absolute obligation:

$$\models_{\alpha,t} O_X A \Leftrightarrow opt_X([\alpha]_{\underset{t}{\approx}}) \subseteq \|A\|_{\alpha,t}$$

which should be read "(some subset of) $X$ makes it obligatory that $A$ on $\alpha$ at $t$ iff all the paths accessible to $\alpha$ at $t$, good for $X$, validate $A$ at $t$".

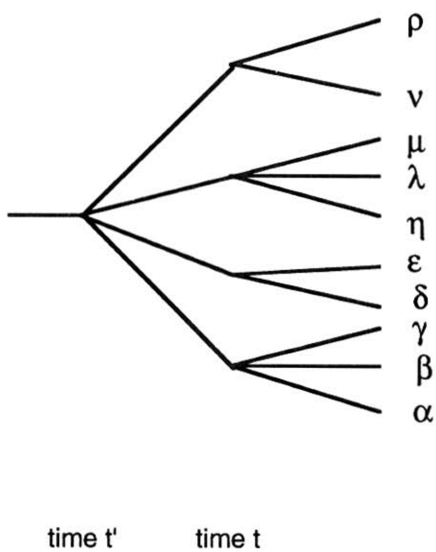
### 5.3.2. *Axiomatic elements and semantic correspondence*

An complete axiomatics for DARB is not known[14]. However, the correspondence between some axioms and properties are obvious:

[14] See [Bailhache 1993], sections 4 & 5.

| | |
|---|---|
| $\Box O_X$ | *(PCX a)* |
| $D\,O_X$ | *(PCX b)* |
| ? | *(PCX c)* |
| $O_X t2$ | *(PCX d)* |
| $RSr\,O_X$ | *(PCX-Sr)* (*StX*-set-regularity) |
| $U\,O_X$ | no condition (*StX*-secondary-reflexivity) |
| $\Box O_X 4$ | no condition (*RStX*-transitivity) |
| | ~~*RStX*-post-implication~~ |

The sole non trivial item of this table concerns the translation of *StX*-set-regularity in terms of a semantic condition of DXARB. This is performed by the *(PCX-Sr)* condition. This condition can easily be proved to be sufficient for making $RSrO_X$ valid. It is also necessary, as can be established by using canonical models. Let us take an example to illustrate how the condition works. Consider the universe:




with the following equations

$\text{opt}_{\{x\}}([\alpha]\approx_{t'}) = \{\mu,\ \upsilon\}$ $\text{opt}_{\{x,\,y\}}([\alpha]\approx_{t'}) = \{\eta\}$

$\text{opt}_{\{y\}}([\alpha]\approx_{t'}) = \{\varepsilon,\ \eta\}$ $\text{opt}_{\{x,\,y\}}([\alpha]\approx_{t}) = \{\beta\}$

$\text{opt}_{\{x\}}([\alpha]\approx_{t}) = \{\alpha,\ \beta\}$

$\text{opt}_{\{y\}}([\alpha]\approx_{t}) = \{\beta,\ \gamma\}$

Obviously, we have

$$\mathrm{opt}_{\cup_i \{x_i\}}([\alpha]\approx_t) = \cap_i \mathrm{opt}_{\{x_i\}}([\alpha]\approx_t)$$

which is in total accordance with (*PCX-Sr*)

$$X \subseteq Y \Rightarrow \mathrm{opt}_Y(E) \subseteq \mathrm{opt}_X(E)$$

Thus, there is no difficulty to express the *StX*-set-regularity in the style of DARB.

## 6. *Conclusion*

There are many reasons not to be completely satisfied with the present work.

First the intended research is not achieved. Authorities are not the unique individuals playing a role in deontic logic. Addressees, that is people whom norms are emitted towards, people who ought to fulfil obligations or who are permitted to do something, etc., play an important role, too. For the sake of brevity this paper neglected addressees.

Secondly, as explained in [Bailhache 1993], the notion of *conditional* norms is probably vital in deontic logic. Here I ignore it, this time for the sake of simplicity.

However the latter and former gaps could be readily filled up.

Thirdly, more dramatically, we can notice that whenever a new modality is added to others in a system, this system is enriched with new features (axioms or rules, corresponding to semantic properties). Thus mixing alethic and pure deontic modalities gives rise to both axioms $\Box O$ ($\Box A \rightarrow OA$), $\Box O4$ ($OA \rightarrow \Box OA$) i.e. *S R*-implication and *RS*-transitivity from the semantic point of view. Similarly for mixing *separately* $R_t$ and $\Box$, $R_t$ and $O$. Finally when the three modalities $R_t$, $\Box$ and $O$ appear together in the same system, we need $\Box Ot$ ($t' < t \Rightarrow R_{t'} O R_t (OA \rightarrow \Box A)$), i.e. *RSt*-post-implication. On the other hand mixing sets of authorities into deontic logic yields the rule $RSrO_X (X \subseteq Y \Rightarrow \vdash O_X A \rightarrow O_Y A)$. Now my question is: what new element is needed when we make the complete composition of alethic, deontic, temporal modalities with sets of authorities? I must confess that no new element should be apparently introduced. It does not augur well.

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