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HUME'S IS-OUGHT THESIS IN LOGICS WITH ALETHIC-DEONTIC BRIDGE PRINCIPLES

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1. *Introduction*

1.1 *A brief history of the problem.* No statement about OUGHT can be derived from statements about IS. This is Hume's famous *is-ought thesis* (Hume 1739/40, p. 469). For centuries, it was the object of a continuing metaethical debate, without leading to an agreement (cf. Hudson 1969 for the contemporary state). In the last three decades several philosophers and logicians started to examine Hume's thesis on the level of mathematical exactness provided by formal logic.[1] That this did not happen earlier is not an accident. For "ought" is not a simple predicate but a *sentential* operator having statements as its argument which express actions or (other kinds of) states of affairs. Therefore, an adequate reconstruction of Hume's is-ought thesis was not possible within standard first order logic but had to wait until the development of the framework of generalized modal logic as the logic of intensional sentence operators. But the framework of monomodal propositional deontic logic – which was the framework of the first result on Hume's thesis proved by Kutschera (1977) – is too narrow for being philosophically adequate, on two reasons. *First,* many pretended is-ought arguments in naturalistic theories of ethics refer in their premises not only to contingent but to *necessary* facts, e.g. about the nature of human life or of social collectives. Therefore, the formal modal language has to be (at least) *bimodal*, containing the alethic necessity operator $\Box$ as well as a deontic obligation operator O. We call these logic *a.d.-logics* (for "alethic-deontic"). *Second,* to have philosophically interesting applications one has to pass from modal propositional to modal predicate logic. In the light of these considerations, Kaliba (1982) and Galvan (1988) have extended their investigations to a.d.-propositional logic, and Stuhlmann-Laeisz (1983) and

[1] Cf., among others, Prior (1960), Shorter (1961), Kurtzman (1970), Kutschera (1977), Morscher (1974, 1984), Kaliba (1981, 1982), Humberstone (1982), Stuhlmann-Laeisz (1983, 1986), Galvan (1988), Pidgen (1989), Schurz (1989, 1991a, 1995a, 1997).

Schurz (1989) to *a.d.-predicate logic*.[2]

1.2. *Difficulties in the explication of Hume's thesis.* The attempts to give a general logical proof of Hume's is-ought thesis had to overcome several obstacles. One major obstacle was to find the right logical explication of Hume's thesis. Prior's prima facie most plausible explication was this: "no normative sentence is logically inferrable from a consistent set of nonnormative sentences".[3] Thereby, Prior characterized normative sentences as those which have 'normative content'. But what does this exactly mean? Two cases are clear. First: every *elementary normative* sentence – i.e., every sentence of the form OA (where $A, B\ldots$ range over sentences) – and every *purely normative* sentence – i.e., every sentence built up from elementary normative sentences by logical symbols – has 'normative content', provided that it is not already logically true (cf. Prior 1960, p. 200). Second: every *purely descriptive* sentence – i.e., every sentence which does not contain an obligation operator (or any other normative operator) – does not have 'normative content'. The problem comes with *mixed* sentences, having both descriptive and normative components. For example, do implications like $p \rightarrow Oq$ or $\Box(p \rightarrow Oq)$ have normative content, or not? Since they can be interpreted as conditional obligations, one would be inclined to answer with 'yes' (cf. Prior 1960, p. 202). But however that may be, Prior faces us with an amazing paradox: *wherever* we draw the distinction between nonnormative and normative sentences, there *must* always be inferences from nonnormative premises to normative conclusions, already due to simple laws of propositional logic. For, consider the following two inferences (1) $p \vdash p \vee Oq$ and (2) $p \vee Oq, \neg p \vdash Oq$. If the mixed sentence "$p \vee Oq$" is counted as normative, then (1) is an example of an is-ought-inference, and if it is counted as nonnormative, then (2) is an example of an is-ought-inference. So Hume's thesis in Priors explication is violated in every possible case.

Prior concluded from his paradox that Hume's is-ought thesis is simply false: "one simply *can* derive conclusions which are 'ethical' ... from premises none of which have this character" (p. 206). Prior noticed that the inferences underlying his paradox are somehow ethically irrelevant or trivial, but neither Prior nor the related papers of Kurtzman (1970) and Pidgen

[2] In a third step one might include various further intensional operators concerning human beliefs or interests (cf. Schurz 1997, ch. 8.2). For the purpose of this paper the framework of alethic-deontic predicate logic is sufficient.

[3] Prior spoke of ethical versus nonethical sentences (1960, p. 200).

(1989) succeeded in finding a suitable definition of what it means for an inference to be "irrelevant" or "trivial". Like Prior they end up with the conclusion that a general proof of Hume's is-ought thesis fails. MacIntyre (1981) turns this into a point of principle. He argues that a purely logical definition of ethical irrelevance or triviality and hence a general logical proof of Hume's theses can't be given, because what is ethically trivial is itself dependent on one's ethical view (p. 57f). It will soon turn out that this argument is wrong.

Meanwhile, other philosophers suggested to get rid off Prior's paradox by excluding mixed sentences from the range of Hume's thesis and hence by asserting this thesis in the following restricted form, which I call the *special* Hume thesis, abbreviated *SH*: no purely normative sentence which is not logically true is derivable from any consistent set of purely descriptive premises. This strategy was pursued, among others, by Harrison (1972, p.72), Kutschera (1977), Kaliba (1982) and Stuhlmann-Laeisz (1983). The concentration on *SH* led to progress insofar for the first time it was possible to find restricted but clear proofs of Hume's thesis. Ultimately the exclusion of mixed sentences from the range of Hume's thesis was not a truely satisfying solution, but only a way of 'defining away' the problem. For certain mixed sentences, namely conditional obligations, play the most important role in ethical and juridical theories – and they clearly *do* have ethical content. So what is needed is a generalization of Hume's thesis which is applicable also to mixed conclusions from purely descriptive premises.

In (1991a) I have developed such a generalization, called the *general Hume thesis*, abbreviated *GH*. It is based on the concept of an *Ought-irrelevant* conclusion, which in turn is a special application of a more general approach to relevant deduction developed for various further purposes (Schurz 1991b). *GH* claims that every conclusion following from a set of purely descriptive premises is completely ought-irrelevant. Thereby, a conclusion A of a set of premises Γ is called *completely Ought-irrelevant* if it is possible to replace every predicate in A by every *other* possibly complex predicate (of the same arity) on *exactly those* occurrences which lie in the scope of an obligation operator O, *salva validitate* of the deduction $\Gamma \vdash_L A$ (in other words, also $\Gamma \vdash_L A^*$ is valid, where A* denotes the result of this replacement). For illustration, if $\Box(p \rightarrow Oq)$ is a completely ought-irrelevant conclusion following from premises D, then also $D \vdash_L \Box(p \rightarrow OA)$ will hold for every sentence A, in particular for $A = \neg q$. Obviously, is-ought-inferences with ought-irrelevant conclusions are without any ethical use. For instance, if an ethical naturalist derives from purely descriptive premises that every person ought to be married, and this conclusion is ought-irrelevant, then the same set of descriptive premises will also imply, e.g., that every person ought to be a murderer, or ought to be unmarried, etc. Note also that the concept of ought-irrelevance employed in *GH* is a purely logical concept and hence not subject to MacIntyre's critique according to which explications of Hume's thesis based on concepts of ethically "irrelevant" or "trivial" conclusions are always relative to one's ethical viewpoint.

1.3. *Results on Hume's thesis in logics without bridge principles.* The second obstacle for the enterprise of proving Hume's thesis lies in the fact that there is not only one but an infinite class of a.d.-logics, and not all but many of them are of philosophical interest. Moreover, if *GH* or a version of *SH* has been proved for a certain logic **L**, *nothing* can immediately be concluded for the truth of *GH* or *SH* in a stronger or in a weaker logic **L***. If **L*** is stronger, then this is obvious. If **L*** is weaker, this is seen as follows. Let **D** vary over sets of purely descriptive formulas and $\vdash_L$ stand for deduction in **L**. If *GH* holds in **L**, then **D** $\vdash_L$ A implies **D** $\vdash_L$ A^* for every **D**, *A* and ought-restricted substitution result A^* of *A*. Nevertheless *GH* may fail in the weaker **L***: **D** $\vdash_{L^*}$ *A* implies **D** $\vdash_L$ *A* (for **L** is stronger than **L***) which implies **D** $\vdash_L$ A^* (because *GH* holds in **L**), but **D** $\vdash_{L^*}$ A^* may still fail for some A^* (because **L*** is weaker than **L**). Similarly, if *SH* holds in **L**, then **D** $\vdash_L$ *N* implies (**D**$\vdash_L(p \wedge \neg p)$ or $\vdash_L N$), for any **D** and purely normative formula *N*. Nevertheless *SH* may be false in the weaker **L***: **D** $\vdash_{L^*}$ *N* implies **D** $\vdash_L$ *N* which impies (**D**$\vdash_L(p \wedge \neg p)$ or $\vdash_L N$), but still either **D**$\vdash_{L^*}(p \wedge \neg p)$ or $\vdash_{L^*}$ *N* may fail. Therefore, a satisfactory logical investigation of Hume's thesis should prove its truth or falsity not only for one or some logical systems, but for a class of systems as broad as possible, with the ideal aim of giving necessary and sufficient conditions for the truth of Hume's thesis.

The first logical results were obtained for *SH*. Kutschera proved *SH* for a certain monomodal deontic logic with a dyadic norm operator (1977, p. 5f, 7f), and Kaliba (1982, pp. 21-24) proved it for the a.d.-propositional logic **a***S5***d***S5*' obtained from combining the alethic **S5** with the deontic **S5**' = *DT*'5: (cf. §2 for the explanation of our 'code' to represent various a.d.-logics). Stuhlmann-Laeisz proved *SH* for all normal a.d.-predicate logics (with constant domain and rigid designators) which are obtained from combining an alethic **S5** -logic with an arbitrary set of purely normative axiom schemata (Stuhlmann-Laeisz 1983, pp. 116-119, pp. 129-150). The gripping question is whether *SH* does indeed hold for all philosophically 'reasonable' a.d.-logics – at least for those which do not contain is-ought bridge principles, in short *bridge principles* (*BPs*), among their axiom schemata (like $\Box A \rightarrow OA$ or $A \rightarrow OA$), because the logical validity of *BPs* is exactly what is philosophically doubted. Indeed this was a conjecture of Stuhlmann-Laeisz (1983, p. 147; 1986, pp.27). In (1991a) I investigated this question for the class of all normal a.d.- predicate logics. The conjecture turned out to be wrong. The proofs of Stuhlmann-Laeisz (1983) and of Kaliba (1982) were restricted to the *S*5-character of the alethic axioms. I proved *SH* for a proper extension of the Stuhlmann-Laeisz class and at the same time gave counterexamples. For instance, *SH* is violated in the a.d.-logic obtained from combining any purely deontic logic with the alethic system *G*, where □ coincides with arithmetical provability; here, the following is-ought inference is valid: $\Box\Diamond p \vdash_L \Box Oq$. But these counterexamples are not a drawback for the validity of Hume's thesis. For I was able to prove the corresponding conjecture for *GH*: *GH* holds in an a.d.-logic exactly if it is axiomatizable (or representable) without *BP*s. Thereby, an axiom schema is called a *BP* iff it contains at least one schematic letter which occurs both in the scope of some obligation operator and outside the scope of any obligation operator.[4] So even in those a.d.-logics where *SH* is violated, all examples of is-ought inferences are ought-irrelevant. I concluded that compared with *SH*, *GH* is the more important explication of Hume's thesis.

1.4. *Hume's thesis in a.d.-logics with the must-ought and the ought-can-principle*. The view that a *BP* can never be analytically true and hence can never be a candidate of a logical axiom is not shared by all philosophers.[5] There are in particular two *BP*s which are treated by at least almost all ethicians "as if" they were analytically true. These are the *ought-can principle* (*OC*), which requires the obligatory to be possible, and the *means-end principle* (*ME*), which says that if *A* is obligatory (i.e., an end), and *B* a necessary condition for realizing *A* (i.e., a mean for *A*), then *B* is also obligatory.

(*OC*): $OA \rightarrow \Diamond A$ equivalent: $\Box A \rightarrow PA$
(*ME*): $(OA \wedge \Box(A \rightarrow B)) \rightarrow OB$ equivalent: $\Box(A \rightarrow B) \rightarrow (OA \rightarrow OB)$

[4] Hence the class of a.d.-logics axiomatizable without BPs is greater than that of a.d.-logics axiomatizable without mixed axiom schemata. E.g. $(aT \vee dT')$: = $(\Box A \rightarrow A) \vee O(OB \rightarrow B)$ is mixed but not a *BP*; so $\mathbf{K}^{ad} + (aT \vee T')$ is axiomatizable without *BP*s, and *GH* holds in it.

The equivalent versions at the right directly reflect the character of these principles as *BP*s – implications with a purely descriptive antecedent and a purely normative consequent. In normal a.d.-logics, (*ME*) is equivalent with the (seemingly stronger) *must-ought principle*, which claims the necessary to be obligatory (cf. §3):

(MO): $\Box A \to OA$ (equivalent version: $PA \to \Diamond A$).

Also (*MO*) is treated by most ethicians "as if" it were analytically true (cf. Schurz 1997, ch. 11.4, and 1995a).

It is therefore an ethically important question whether Hume's thesis is seriously violated if *OC* or *MO* is added to the underlying logic. Galvan (1988) has undertaken the first step in this direction. He proved the following result for the propositional a.d.-logic $\mathbf{L} = \mathbf{a}S5\mathbf{d}D4{+}OC$, which is obtained from combining alethic **S5** with deontic *D*4 and adding *OC* (cf. §2): there exists no **L**-consistent set of purely descriptive formulas **D** and no **L**-falsifiable *elementary* normative formula *OB* such that $\mathbf{D} \vdash_L OB$ (1988, p. 50, theorem 1). He shows that this result is preserved if the purely normative axiom $OA \to \Box OA$ is added to **L** (1988, p.58, theorem 3).[6] These results are encouraging. But in the light of what was said in §1.3, they are not enough general, in two respects. First, Galvan's theorems apply only to elementary *obligations* but not to elementary *permissions*: the latter ones clearly *are* logically inferrable in the presence of (*OC*) from a suitable set **D**.[7] Second, Galvan's theorem 1 is restricted to the special logical system $\mathbf{L} = \mathbf{a}S5\mathbf{d}D4$ and fails if one takes another standard system. An example given by Galvan (p. 53) is the system $\mathbf{L}^* = \mathbf{a}S5\mathbf{d}D5$ [where $d5 := PA \to OPA$]: here $\Box p \vdash_{L^*} Pp \vdash_{L^*} OPp$ is an example of an is-ought inference violating Galvan's theorem 1. One might conjecture that at least if the set **D** of purely descriptive premises contains no necessity operator, then the bridge principles *MO* or *OC* will not enable the derivation of an **L**-falsifiable purely normative sentence from **D**. Even this is not true. If the

5 Cf. Schurz (1995a and 1997, ch. 11) for an extensive investigation of the question whether there exist analytically true *BP*s. On reasons explained there, I do not consider Moore's famous "open question" argument as entirely convincing.

6 Theorem 2 of Galvan (p. 54) contains a result about SH in the logic $\mathbf{a}S5\mathbf{d}D45$ without *BP*s.

7 Galvan's point that the derivation of an elementary permission is not a 'real' violation of Hume's thesis (p. 47, p. 54) does not convince me.

alethic part contains the axiom (B), we have, for example, $\neg A \vdash_L \neg\Diamond\Box A \vdash_L \Box\neg\Box A \vdash_L P\neg\Box A$ (by OC) $\vdash_L P\Diamond\neg A$.

Yet it is possible to prove a *weakened* version of Hume's thesis which applies to a very broad class of a.d.-logics containing MO or OC. The key idea is the notion of a *trivial* is-ought inference (IOI). Its primary application are inferences which have in their conclusion a categorial or conditional obligation or permission of *first degree* (i.e. without nestings of O-operators).[8] Since all normatively contentful statements of direct practical importance are categorial or conditional first degree obligations or permissions, we call them briefly the *practically normative* statements, and speak also of practical obligations or permissions, respectively. The general form of a practical obligation is $(\Box)(\forall x_{1-m})(D_1 \rightarrow (\forall x_{1-n})OD_2)$. Hereby, $(\forall x_{1-k})$ stands for a possibly empty string of universal quantifiers, $(\Box)$ stands for zero or one occurrence of $\Box$, and "$D_1 \rightarrow$" may be missing, in which case the obligation is categorial (and hence purley normative), else it is conditional (and hence mixed). Similar for a practical permission (P instead of O).[9] It is important that together with descriptive initial conditions practically normative statements imply elementary first degree obligations or permissions (OD or PD); for this reason only boxes and universal quantifiers (but not diamonds or existential quantifiers) may occur in the bracketed formula parts.[10]

For every sentence A, A^{-O} denotes the result of omitting all obligation- and permission-operators in A (inductively defined in the obvious way). Note that if A^* results from A by replacing P-occurrences by their definitions in terms of O, then A^{-O} is tautologically equivalent with $(A^*)^{-O}$ and results from it by double negation elimination: $(PA)^{-O} = A^{-O}$ and $(\neg O\neg A)^{-O} = \neg\neg(A^{-O})$. If A is a practical obligation or permission, then

8 Generally speaking, an O-occurrence in A is of first degree if it does not lie in the scope of some O, and else it is of higher degree; A is of first degree if all of its O's are of first degree; else A is of higher degree. For an inductive definiton of the modal degree of a formula cf. Fine/Schurz (1996).

9 In logics with constant domain $\Box\forall xA$ is equivalent with $\forall x\Box A$. This is not so if domains may vary from world to world, whence for this case, the general form of a conditonal obligation would be $(\Box)(\forall x_{1-m})(\Box)(D_1 \rightarrow (\forall x_{1-n})OD_2)$.

10 Our notion of "practically normative" is grammatically rather restrictive. In a more extended sense one might also call a statement practically normative if it logically implies one in our restrictive sense. We do not need this extended terminology because these cases are handled with help of our distinction of "direct" and "indirect" triviality in §3.

$A^{-O} = (\Box)(\forall x_{1-m})(D_1 \rightarrow (\forall x_{1-n})D_2)$. A practical obligation is called satisfied, and a practical permission is called realized, if the statement required or permitted by it is *true*, provided the descriptive initial condition is true. Using the "A^{-O}"-notation, we may briefly say that a practically normative statement A is satisfied or realized iff A^{-O} is true.

If **D** $\vdash_L A$ and A is practically normative, then A is said to be an O(ught)-trivial conclusion of **D** if also A^{-O} is L-derivable from **D**. This means that if the descriptive premises **D** are true, then the practical obligation or permission which they entail *must* be satisfied, or realized (resp.). Such inferences are trivial because applied ethicists are interested in norms and rights which need not be always satisfied or realized in the real world. For instance, if an ethicist derives "all men should help the poor" $(\forall x \forall y(Mx \wedge Py \rightarrow OHxy))$ from a descriptive premise set **D**, he does not want to claim that his premises already imply that all men in fact help the poor $(\forall x \forall y(Mx \wedge Py \rightarrow Hxy))$. In the opposite, if this would be the case, like in the example "all men breathe", the ethicist would not be very interested in deriving this as a norm, in our example "all men should breathe". Similar, if the ethicist derives "all men are permitted to live in freedom" $(\forall x(Mx \rightarrow PFx))$, then he does not want to claim that also "all men do live in freedom" $(\forall x(Mx \rightarrow Fx))$ is inferrable from his premise set, for again, this would make his inference trivial.

For inferences with practically normative conclusions we can prove a very broad triviality result. After presenting some logical background in §2, we will prove in §3 for all standard a.d.-logics extended by OC or MO that every practically normative statement following from a descriptive premise set is O-trivial. In §4 we discuss the question of triviality for purely normative conclusions which are not practically normative, e.g. conclusions of the form $Op \rightarrow Oq$. In §5 we strengthen our results by introducing a notion of strong triviality. We also indicate how our results can be transferred to a.d.-logics with a deontic part which is weaker than normal.

2. *Some logical background*

Concernig our *metalanguage*: D will always represent a descriptive sentence and **D** a set of such, N a normative sentence and **N** a set of such; **M** a model, M a class of such, F a frame, **F** a class of such and **L** a logic. Small greek letters $\alpha, \beta, \ldots$ denote possible worlds; $d_1, d_2, \ldots$ individuals (in possible worlds); capital A, B, ... arbitrary formulas, capital greek $\Gamma, \Delta, \ldots$ arbitrary formula sets; X, Y, ... denote axiom schemata; **X**, **Y**, ... sets of them. We introduce only one set of individual variables without an extra set of individual constants (following, e.g., Hughes/Cresswell 1984, p. 164ff). Hence our free individual variables play the same role as individual constants in languages with the distinction between individual variables and individual constants, and we may use the notions "formula" and "sentence" interchangeable.

The(bimodal) language $\mathcal{L}$ of an alethic-deontic first order logic, in short an *a.d.-logic*, contains in its *vocabulary* (1) a denumerably infinite set $\mathcal{V}$ of *individual variables* x, y, z (possibly indexed) (2) for each $n \geq 0$, a denumerably infinite set $\mathcal{R}^n$ of *n-place predicates* F, G, H..., (possibly indexed; *propositional variables* p, q... are the 0-ary predicates), and (3) the following *logical symbols*: $\neg$ (negation), $\vee$ (disjunction), $\forall$ (universal quantifier), $\Box$ (alethic necessity operator), O (deontic obligation operator).[11] The symbols $\rightarrow$ (material implication), $\wedge$ (conjunction), $\leftrightarrow$ (equivalence), $\top$ (Verum), $\bot$ (Falsum), $\exists$ (existential quantifier), $\Diamond$ (alethic possibility operator) and P (deontic permission operator) are defined as usual in classical (modal) logics. We identify the language $\mathcal{L}$ with the set of its (well-formed) formulas, according to the usual formation rules. $\mathcal{L}^a$ stands for the purely alethic and $\mathcal{L}^d$ for the purely deontic language. The notions of free and bound variables, of alphabetic variants of a formula (obtained via renaming of bound variables) and of substitution for variables are explained as usual; $A[y_{1-n}/x_{1-n}]$ denotes the result of the simultaneous substitution of y_i for x_i in A $(1 \leq i \leq n)$ under the usual conditions prohibiting confusion of variables (cf. Bell/Machover 1977, pp. 54 - 67).

An *alethic-deontic frame*, in short an *a.d.-frame*, is a triple $F = \langle W, R, S \rangle$ where $W \neq \emptyset$ (a nonempty set of possible 'worlds'), $R \subseteq W \times W$ (the alethic accessibility relation), and $S \subseteq W \times W$ (the deontic ideality relation). The notions of a (purely) alethic frame (*a.-frame*) $\langle W, R \rangle$ and a (purely) deontic frame (*d.-frame*) $\langle W, S \rangle$ are obtained by dropping R and S, respectively. A model for the language $\mathcal{L}$, in short an *a.d.-model*, is a quintuple $M = \langle W, R, S, Dm, v \rangle$, where $F = \langle W, R, S \rangle$ is an a.d.-frame – we say that M is *based on* the frame F –, $Dm \neq \emptyset$ (a nonempty domain of individuals), and v is a value assignment (or interpretation function) which satisfies the following conditions: (1) for all $x \in \mathcal{V}: v(x) \in Dm$ (*constant domain* and *rigid designators*), (2) for all $F \in \mathcal{R}^n: v(F) \subseteq (Dm^n \times W)$ (hence for all $p \in \mathcal{P}$, $v(p) \subseteq W$').

11 Every individual variable occurring bound also counts as a logical symbol, whereas free individual variables and predicates count as nonlogical symbols.

Given a a.d.-model M and an individual d, then $M[x{:}d]$ denotes the model which is like M except it assigns d to x. The notion of "Formula $A \in \mathcal{L}$ *is true at world* α *in model* $M = <W, R, S, Dm, v>$" (where $\alpha \in W'$), in short: "$(M, \alpha) \models A$", is inductively defined as usual: (i) $(M, \alpha) \models Fx_1 \ldots x_n$ iff $< v(x_1), \ldots, v(x_n), \alpha > \in v(F)$; (ii) the cases $A = \neg B$ or $A = B \vee C$ are obvious; (iii) $(M, \alpha) \models \Box B$ iff for all $\beta \in W$ with $R\alpha\beta{:}\,(M, \beta) \models A$; (iv) $(M, \alpha) \models OB$ iff for all $\beta \in W$ with $S\alpha\beta{:}\,(M, \beta) \models A$; (v) $(M, \alpha) \models \forall xB$ iff for all $d \in Dm$, $(M[x{:}d], \alpha) \models A$. The notions of a pure a.-model and a pure d.-model are obtained by dropping S or R and the relevant clauses, respectively. $\Gamma \subseteq \mathcal{L}$ is said to be true at α in M iff all $A \in \Gamma$ are true at α in M. $A \in \mathcal{L}$ is *valid in* a model M iff A is true at *all worlds* α in M, and $A \in \mathcal{L}$ is *valid on* a frame F iff A is valid in *all models* based on F. $\Gamma \subseteq \mathcal{L}$ is (simultaneously) *satisfiable* in M iff Γ is true at some world in M; Γ is satisfiable on F iff Γ is satisfiable in some model M based on F.

A normal alethic-deontic predicate logic (for constant domains and rigid designators), in short an *a.d.-logic*, is every subset of $\mathcal{L}$-formulas which contains all instances of the following *axiom schemata*

(Taut): All tautological schemata
(*aK*): $\Box(A \to B) \to (\Box A \to \Box B)$ (*dK*): $O(A \to B) \to (OA \to OB)$
($\forall$1): $\forall A \to A[y/x]$, provided y is free for x in A (for all $x, y \in \mathcal{V}$)
($\forall$2): $\forall x(A \to B) \to (\forall xA \to \forall xB)$ (for all $x \in \mathcal{V}$)
($\forall$3): $A \to \forall xA$, provided x is not free in A. (for all $x \in \mathcal{V}$)
(*aBF*): $\forall x \Box A \to \Box \forall xA$ (*dBF*): $\forall xOA \to O \forall xA$ (for all $x \in \mathcal{V}$)

and which is closed under the following *rules:*

(*MP*) $A, A \to B \,/\, B$ ($\forall R$) $A \,/\, \forall xA$ (for all $x \in \mathcal{V}$)
(*aR*) $A \,/\, \Box A$ (*dR*) $A \,/\, OA$
(Subst) $A \,/\, \sigma A$ – the rule of substitution for predicates

The *minimal* a.d.-logic, $\mathbf{K}^{\mathbf{ad}}$, is defined as the closure of all instances of the abovementioned axioms schemata under the four rules (*MP*), ($\forall R$), (*aR*) and (*dR*)[12] The minimal alethic logic $\mathbf{K}^{\mathbf{a}}$ is obtained by restricting to formulas of the purely alethic language $\mathcal{L}^a$ and by dropping all axiom

12 As is well-known, a shorter axiomatization of the nonmodal quantifier part which is equivalent to ($\forall$1-3) + ($\forall R$) is given by ($\forall$1) + ($\forall R^*$):= "$A \to B/A \to \forall xB$ provided x is not free in A" (Hughes/Cresswell 1984, p.166). The axiomatization used by us (cf. e.g. also Fine 1978) simplifies some of our proofs.

schemata and rules containing O; simiarly for the minimal normal deontic logic $\mathbf{K^d}$. (Taut), *(aK)*, *(dK)*, *(MP)*, *(aR)*, *(dR)* represent the *propositional* part of $\mathbf{K^{ad}}$, and ($\forall$1-3), *(aBF)*, *(dBF)*, ($\forall R$) the *quantifier part* of $\mathbf{K^{ad}}$ ("*BF*" stands for "Barcan formula"). Some remarks on the rule of substitution for predicates are appropriate. This rule is just the generalization of the analogous well-known substitution rule for propositional variables. Note that the additional requirement of closure under substitution is unnecessary when a *particular* a.d.-logic **L** (e.g. $\mathbf{K^{ad}}$) is defined as the *closure* of certain axiom schemata (including the minimal ones) under the four rules *(MP)*, ($\forall R$), *(aR)* and *(dR)*, because the fact that axioms as well as rules are formulated as *schemata* will guarantee that **L** is closed under substitution. But the general definition of an a.d.-logic as *any* (normal) extension of $\mathbf{K^{ad}}$ makes no reference to a particular axiomatization with help of axiom schemata; so closure under substitution is not automatically guaranteed and must be required separately.

The notion of uniform substitution for predicates has been explicated by Kleene for nonmodal predicate logic (cf. 1971, pp.155-162). Here a brief informal description is sufficient (cf. Schurz 1991a, 1995b for details). Consider an n-ary predicate F attached with certain pairwise distinct variables $x_1, \ldots, x_n$, which figure as *name form variables* carrying the substitution for $Fx_1 \ldots x_n$. Then uniform substitution of a formula B for $Fx_1 \ldots x_n$ means replacing each occurrence of $Fz_1 \ldots z_n$ by the corresponding B-substitution-instance $B[z_{1-n}/x_{1-n}]$, provided certain restrictions are satisfied which prohibit confusion of variables. Nonmodal predicate logic and $\mathbf{K^{ad}}$ is closed under this substitution rule (cf. Kleene 1971, Schurz 1991a Appendix). It is also easily seen that for every a.d.-logic **L** defined in the above way there exists a set Θ_L of *additional axiom schemata* such that **L** is *representable* by Θ_L, i.e. it may be equivalently defined as the closure of all instances of the minimal $\mathbf{K^{ad}}$-axiom schemata and the schemata in Θ_L under the rules *(MP)*, ($\forall R$), *(aR)* and *(dR)*. For, let Δ be the set of **L**'s theorems which are not contained in $\mathbf{K^{ad}}$ and let Δ^* be the subset of all Δ-formulas which are *maximally general w.r.t.* substitution for predicate letters. If we reformulate the formulas in Δ^* as schemata (replacing atomic formulas by schematic letters) we will obtain a set Θ_L of additional axiom schemata with help of which **L** can be represented (for more details cf. Schurz 1997, ch. 2.4.4). Not every Θ_L obtained in this way will be decidable. If Θ_L is decidable, then **L** is said to be *axiomatizable* by Θ_L (which means that **L** is recursively enumerable).

Standard a.d.-logics are axiomatized by a finite additional set Θ_L of purely propositional axiom schemata (see below), which impose certain structure on the *frames* of the logic. That an axiom schema *(X)* *corresponds* to a frame condition (C_X) means that (X) is valid on a frame $<W, R>$ if and only if (C_X) holds for $<W, R>$. Recall some well-known axiom schemata of propositional monomodal logic and their corresponding semantic conditions on frames. "*aX*" ("*dX*") ["*a+dX*"] means that X is a standard axiom for alethic logics (for deontic logics) [for both kinds of logics].

(a+dD): $\neg\Box\bot$ (or: $\Box A \rightarrow \Diamond A$)
R serial ($\forall\alpha\exists\beta R\alpha\beta$; likewise for O and S)
(aT): $\Box A \rightarrow A$
R reflexive: $\forall\alpha R\alpha\alpha$
(dT'): $O(OA \rightarrow A)$
S secondary reflexive ($\forall\alpha((\exists\beta S\beta\alpha) \rightarrow S\alpha\alpha)$)
(aB): $\Diamond\Box A \rightarrow A$
R symmetric
(dB'): $O(POA \rightarrow A)$
S secondary symmetric ($\forall\alpha,\beta((\exists\gamma S\gamma\alpha) \rightarrow (S\alpha\beta \rightarrow S\beta\alpha))$)
(a+$d4$): $\Box A \rightarrow \Box\Box A$
R transitive (likewise for O and S)
(a+$d5$): $\Diamond A \rightarrow \Box\Diamond A$
R euclidean ($\forall\alpha,\beta,\gamma(R\alpha\beta \wedge R\alpha\gamma \rightarrow R\beta\gamma)$); likewise for O and S)

Usually, **a**T is is viewed as the smallest standard alethic logic, and **d**D as the smallest standard deontic logic, at least within the realm of normal logics. There exists a variety of other well-investigated additional propositional axiom schemata.[13] However, we also shall admit the case where Θ_L contains additional *predicate logical axiom schemata* imposing additional structure on the models. One example is the axiom $\Box\exists xA \rightarrow \exists x\Box A$, which semantically corresponds to classes of models satisfying Fine's notion of local isomorphism (1978, p. 136).

Since every a.d.-logic is representable by some Θ_L, we can classifiy the class of a.d.-logics in terms of what is contained in the set Θ_L. Close to the Lemmon-code (cf. Bull/Segerberg 1984, p. 20) we write **aXdYadZ** for an a.d.-logic which is representable by a set **X** of additional alethic axiom schemata plus a set **Y** of additional deontic axiom schemata, and a set **Z** of additional bimodal axiom schemata (thus $\Theta_L = \mathbf{X} \cup \mathbf{Y} \cup \mathbf{Z}$. Logics of the type **aXdY** (containing only *monomodal* schemata) are called *a.d.-combinations*, because they combine the two monomodal logics **aX** and **dY** with-

13 Consult the standard literature; e.g. Segerberg (1971), Hughes/Cresswell (1984), Chellas (1980), Rautenberg (1979); for deontic logic cf. Aqvist (1984).

out any bimodal axiom schemata. It is easy to see that the frames of a.d.-combinations are just combinations of the respective pure frames; i.e. <*W*, *R*, *S*> is a frame for **aXdY** iff <*W*, *R*> is one for **aX** and <*W*, *S*> is one for **dY** (cf. Fine/Schurz 1996, Prop. 1). Instead of the set **X** one may also list the particular axiom schemata contained in **X** in non-bold letters. Thus, **a***T4***d***DT'* is the a.d.-combination containing (*aT*), (*a*4), (*dD*) and (*dT'*) as additional axiom schemata; **a***S5***d***D* contains the alethic **S5**-axiom schemata (*T5* = *TB*45) and the deontic axiom schema (*dD*) in its additional set. In particular, $\mathbf{a}\varnothing\mathbf{d}\varnothing := \mathbf{K}^{\mathbf{ad}}$ with empty Θ_L. **L**+**Z** denotes the extension of the logic **L** by the 'additional axioms' in **Z.**

Every $A \in \mathbf{L}$ has an **L**-*proof*; "$\vdash_L A$"denotes provability in **L**. *A* is *deducible* (or derivable) in **L** from Γ, in short $\Gamma \vdash_L A$, iff $\vdash_L \left(\bigwedge \Gamma_f \rightarrow A\right)$ for the conjunction $\bigwedge \Gamma_f$ of some finite subset $\Gamma_f \subseteq \Gamma$.[14] In particular, $\varnothing \vdash_L A$ iff $\vdash_L \top \rightarrow A$ iff $\vdash_L A$. Note that ∀-rule, *a*-rule and *d*-rule are not valid as deduction rules – they may only be applied to logical theorems, but must not be applied to nonlogical premises (e.g. $\vdash_L A \Rightarrow \vdash_L \Box A$, but not $A \vdash_L \Box A$). Only *MP* is valid as a deduction rule, since only *MP* preserves truth at a world in a model (cf. Schurz 1994). $\Gamma \subseteq \mathcal{L}$ is **L**-consistent if *not* $\Gamma \vdash_L p \wedge \neg p$.

M is a model for a given a.d.-logic **L** iff all **L**-*t*heorems are valid in *M*; similarly *F* is a frame for **L** iff all **L**-theorems are valid on F. **M** (**L**) and **F**(**L**) denote the classes of all models or frames for **L,** respectively. An a.d.-logic **L** is said to be *weakly [strongly] model-complete* iff every **L**-consistent $A \in \mathcal{L}$ $[\Delta \subseteq \mathcal{L}$, resp.$]$ is satisfiable in some model in **M**(**L**). **L** is *weakly [strongly] frame-complete* iff every **L**-consistent $A \in \mathcal{L}$ $[\Delta \subseteq \mathcal{L}$, resp.$]$ is satisfiable on some frame in **F**(**L**). (Strong completeness entails weak completeness, but not vice versa.) The standard technique to prove model-completeness of a given logic **L** is to construct the so-called *canonical model* of **L,** built up from maximally **L**-consistent and ∀-complete formula sets in $\mathcal{L}$. With this technique it is provable that every a.d.-logic is strongly model-complete. The proof is analogous to that for purely alethic logics (cf. Hughes/ Cresswell 1984, ch. 1, 9); for an exposition see Schurz (1991a–Appendix, 1997–Appendix). This method of proof does not automatically yield frame-completeness of **L,** but only if it can be shown in addition that the frame of the canonical model is a frame for **L**. This is

14 This definition of $\vdash_L$ is, e.g., choosen by Chellas (1980, p. 47), Aqvist (1984, p. 666f), Bull/Segerberg (1984, p. 19). A well-known alternative consists in using $\forall\Box O(Ax_l)$ as axioms and *MP* as the only rule (cf. Leblanc 1976, pp. 224ff; Stuhlmann-Laeisz 1983, p. 63).

trivially true for $\mathbf{K^{ad}}$ (for any a.d.-frame is a frame for $\mathbf{K^{ad}}$), so $\mathbf{K^{ad}}$ is strongly frame-complete. Whether it holds for stronger a.d.-logics depends on their additional axioms in Θ_L. If the frame of the canonical model is a frame for **L**, the logic **L** (and its additional axiom set) is said to be *canonical* (cf. Hughes/Cresswell 1984, p. 56). Canonicity implies strong frame-completeness (but not vice versa). All of the above mentioned axiom schema and hence the standard logics axiomatized by subsets of them are canonical. A well-known axiom schema which is weakly frame-complete but *not* canonical is (*aG*): $\Box(\Box A \rightarrow A) \rightarrow \Box A$; **a***G* is the modal logic of provability in arithmetics. An example of a *frame-incomplete* axiom schema is (*VB*): $\Diamond\Box A \vee \Box(\Box(\Box A \rightarrow A) \rightarrow A)$.

An a.d.-logic is called *propositionally representable* iff it has an additional set Θ_L consisting solely of purely propositional axiom schemata. Makinson (1971) has proved that every consistent propositionally representable modal logic is valid on a singleton frame – on the reflexive singleton frame $F^+ := <\{\alpha\}, \{<\alpha, \alpha>\}>$ if it is consistent with (*D*), and on the irreflexive singleton frame $F^- := <\{\alpha\}, \varnothing>$ if it contains $(Ver) := o\bot$ (o the modal operator). This result does not generalize to all a.d.-logics with additional predicate logical axiom schemata; e.g. take the mentioned example of $\mathbf{K^a} + \Box\exists xA \rightarrow \exists x\Box A$, which is invalid on every frame. However, if a modal predicate logic is valid on at least some frame, it will be certainly also valid on either F^+ or on F^-, for F^+ is a *p*-morphic image of any serial frame and F^- is a generated subframe of any non-serial frame (cf. Hughes/Cresswell 1984, p. 70ff; Schurz 1997, ch. 2.6). These facts will be of importance for theorem 3.

Canonicity and frame-completeness is well-investigated for monomodal propositional logics. Let us denote a *propositional* modal logic by adding the suffix "**–0**". Thus **a***T***–0** is propositonal **T** and its predicate logical counterpart is **a***T*. Two important questions are: (1) Does frame-completeness and canonicity always transfer from an **a.0**-logic **aX–0** to its predicate logical counterpart **aX**? (2) Does frame-completeness and canonicity always transfer from an a.-logic **aX** and a d.-logic **dY** to its combination **aXdY**? Question (1) is stated in Hughes/Cresswell (1984, p. 183f) as an open problem. Today, the question is answered, and the answer is *no*. An interesting counterexample is given by the axiom schema (*M*): $\Box\Diamond A \rightarrow \Diamond\Box A$ (due to MacKinsey). Together with **a***S4***–0** this axiom yields the well-known logic **a***S*4.1**–0** = **a***T*4*M***–0** which is canonical. Its predicate logical counterpart **a***S*4.1, however, can be shown to be frame-incomplete; only if one adds the additional predicate logical axiom schema $\Diamond\Box\exists xA \rightarrow \Diamond\exists x\Box A$, the logic becomes canonical (cf. Schurz 1997, Prop. 4). A restricted completess-transfer is proved in Schurz (1997, Prop. 5) saying that if **aX–0** *is* canonical and its frames are closed under subframes, then **aX** is canonical, too. Concerning the second question, a very general transfer result for *propositional* bimodal logics has been established in Fine/Schurz (1996). Schurz (1997, Prop. 6) proves canonicity-transfer for bimodal *predicate* logics.

3. MO, OC and practically trivial is-ought-inferences

First let us record some basic facts about the *BP*s *MO* and *OC*.
(1) *MO* is logically equivalent with the *means-end principle* (*ME*): $(OA \wedge \Box(A \rightarrow B)) \rightarrow OB$ in all normal logics. [*Proof*: *MO*⇒*ME*: $O(A \rightarrow B)$ follows from $\Box(A \rightarrow B)$ by *MO*, and OB from $O(A \rightarrow B)$ and OA by the axiom (*dK*) and *MP*. *ME*⇒*MO*: Assume $\Box A$. $\Box A$ is $\mathbf{K}^{\mathbf{a}}$-equivalent with $\Box(\top \rightarrow A)$, and $O\top$ is a $\mathbf{K}^{\mathbf{d}}$-theorem, so OA follows by *ME*.] If the deontic part of the a.d.-logic is only *regular* (i.e. weaker than normal) and hence does not contain $O\top$ (cf. §5, last paragraph), then *MO* implies, but is not implied by *ME*.
(2) A weak analogue of *OC*, where "$\Box$" is replaced by "logical necessity", is already contained in the axiom (*dD*), which implies $OA \rightarrow \neg(A \leftrightarrow \bot)$, and a corresponding weak analogue of *MO* is already contained in the *d*-rule: $\vdash_L A \Rightarrow \vdash_L OA$. Of course, *OC* and *MO* are much stronger since the interpretation of $\Box$ is not restricted to logical necessity.
(3) *OC* implies the axioms (*aD*) and (*dD*) in $\mathbf{K}^{\mathbf{ad}}$ [because $\neg\Diamond\bot \rightarrow \neg O\bot$ is the contraposition of an *OC*-instance, and $\neg\Diamond\bot \in \mathbf{K}^{\mathbf{ad}}$; similar $O\top \rightarrow \Diamond\top$ is an *OC*-instance, and $O\top \in \mathbf{K}^{\mathbf{ad}}$].
(4) *MO* and (*dD*) together imply *OC* in $\mathbf{K}^{\mathbf{ad}}$ [$\Box\neg A \rightarrow O\neg A$ by (*MO*), $O\neg A \rightarrow \neg OA$ by (*dD*), which gives $\Box\neg A \rightarrow \neg OA$, which is equivalent with $OA \rightarrow \Diamond A$].
(5) The semantic frame conditions which correspond to *OC* and *MO* and canonically characterize these *BP*s are:

$$(C_{OC}): \forall\alpha\exists\beta: R\alpha\beta \wedge S\alpha\beta \qquad (C_{MO}): \forall\alpha,\beta: S\alpha\beta \rightarrow R\alpha\beta$$

The proof of correspondence is straightforward. Canonicity is proved in the standard way (cf. Schurz 1997, ch. 6.2); the proof establishes that if **L** is canonical and $\mathbf{B} \subseteq \{MO, OC\}$, then **L+B** is canonically characterized by $\mathbf{F}(\mathbf{L}) \cap \mathbf{F}(\mathbf{B})$ [where $\mathbf{F}(\mathbf{B})$ = the class of a.d.-frames satisfying **B**].

Theorem 1.1 tells us that in all a.d.-combinations **aXdY** enriched with *OC* and *MO* the rule "A/A^{-o}" is admissible, provided $(aT) \in \mathbf{X}$ and **dY**+(*dD*) is valid on at least one frame (this is true, e.g., if **dY**+(*dD*) is consistent and propositionally representable; recall §2). So, this result applies to all standard a.d.-combinations in a very weak sense of "standard", enriched with *OC* or *MO*. It immediately entails, via lemma 1, that all these logics are *practically I(s)-O(ought)-trivial* in the sense that every practically normative conclusion derivable in them from purely descriptive premises is *O*-trivial (def. 1).

Definition 1: An a.d.-logic **L** is *practically I(s)-O(ught)-trivial* if for every purely descriptive $\mathbf{D} \subseteq \mathcal{L}$ and practically normative $A \in \mathcal{L}$ with $\mathbf{D} \vdash_L A$, A is an *O(ught)-trivial* conclusion of **D**, i.e. also $\mathbf{D} \vdash_L A^{-O}$ holds.

Lemma 1: If the rule "A/A^{-O}" is admissible in an a.d.-logic, then it is practically *I-O*-trivial.

Proof: Assume $\mathbf{D} \vdash_L A$, then $\vdash_L D \rightarrow A$, where $D := \bigwedge \Delta$ for some finite $\Delta \subseteq D$. Hence $\vdash_L (D \rightarrow A)^{-O}$ (by assumption), and $(D \rightarrow A)^{-O} = D \rightarrow A^{-O}$, because D is purely descriptive. So $\mathbf{D} \vdash_L A^{-O}$, whence **L** is practically *I-O*-trivial. Q.E.D.

Theorem 1: For every a.d.-combination **aXdY** where **X** contains *(aT)* and **dY**+*(dD)* is valid on at least one frame, and for every (possibly empty) set of *BP*s $\mathbf{B} \subseteq \{MO, OC\}$:

(1.1) The rule "A/A^{-O}" is admissible in **aXdY+B**.

(1.2) **aXdY+B** is practically *I-O*-trivial.

Proof: (1.1): By the arguments of §2, **dY**+*(dD)* and hence **dY** is valid on the reflexive singleton frame. So, **dY** is consistent with *(dTriv)*: $OA \leftrightarrow A$. Note that in **dY**+*(dTriv)*, every sentence A is logically equivalent with A^{-O} [by replacement of logical equivalents]. We abbreviate **aXdY+B** as **L**. - Let $< A_1, \ldots, A_n := A >$ be an **L**-proof of A, and consider the sequence $< A_1{}^{-O}, \ldots, A_n{}^{-O} := A^{-O} >$. We show that it is an **L**-proof (of A^{-O}), by induction on its length:

For the axioms: (i) Assume A_i is an instance of a nonmodal or a monomodal alethic axiom schema X (either a basic one or one in **X**). So $A_i = \sigma X$ where σ is a substitution function sending schematic letters to $\mathcal{L}$-formulas. By defining $\sigma^* L =(\sigma L)^{-O}$ for each schematic letter L in X, it is easy to see that $A_i{}^{-O} = \sigma^* X$, i.e. that $A_i{}^{-O}$ is itself an instance of X, because X contains no *O*-operator (the inductive proof is trivial and hence omitted). Hence $A_i{}^{-O}$ is itself an **L**-theorem. - (ii) If A_i is an instance of a monomodal deontic axiom schema Y (either a basic one or one in *Y*), then by similar considerations as in (i.), $A_i{}^{-O}$ will be instance of Y^{-O}. Now, Y^{-O} is a formula schema of nonmodal (1st order) predicate logic. Since in **dY**+*(dTriv)*, Y^{-O} is logically equivalent with Y, Y^{-O} must be a theorem of

dY+(*dTriv*), whence (all instances of) Y^{-o} must be valid on the reflexive singleton frame. So Y^{-o} must be a theorem of nonmodal predicate logic (otherwise it could be falsified on this frame, by imposing a falsifying model $<Dm, v_\alpha>$ on it). If follows that A_i^{-o} is an **L**-theorem. - (iii) Assume $A_i = \Box B \rightarrow OB$ is an instance of $(M\ O)$. Then $A_i^{-o} = \Box(B^{-o}) \rightarrow (B^{-o})$, which is an **L**-*a*xiom because $(aT) \in \mathbf{X}$. Similarly, assume $A = OB \rightarrow \Diamond B$ is an instance of (OC). Then $A_i^{-o} = (B^{-o}) \rightarrow \Diamond(B^{-o})$, which is again an **L**-*a*xiom because $(aT) \in \mathbf{X}$.

For the rules: (iv) Assume $A_i = \Box A_j$ follows from A_j by the rule (aR). Then $A_i^{-o} = \Box(A_j^{-o})$ follows from A_j^{-o} by (aR), and A_j^{-o} is **L**-provable by ind. hyp. – (v) If $A_i = O A_j$ and follows from A_j by the rule (dR), then $A_i^{-o} = A_j^{-o}$ and A_j^{-o} is **L**-*p*rovable by ind. hyp. - (vi) Assume $A_i = \forall u A_j$ follows from A_j by $(\forall R)$. Then $A_i^{-o} = \forall u(A_j^{-o})$ follows from A_j^{-o} by $(\forall R)$, and A_j^{-o} is **L**-provable by ind. hyp. - (vii) If A_i follows from A_j and $A_j \rightarrow A_i$ by (MP), then $A_i^{-o}=$ follows from A_j^{-o} and $(A_j \rightarrow A_i)^{-o} = (A_j^{-o} \rightarrow A_i^{-o})$ by (MP), where both A_j^{-o} and $(A_j \rightarrow A_i)^{-o}$ are **L**-provable by ind. hyp.

(1.2): Follows from theorem 1.1 and lemma 1. Q.E.D.

A necessary condition for an a.d.-combination **aXdY** to be "standard" is (besides $D \in \mathbf{Y}$ and $T \in \mathbf{X}$) that the deontic part is *unmixed*, i.e. the axiom schemata are purely normative. Theorem 1 applies not only to all standard a.d.-combinations, but even to those which contain in **dY** some *monomodal deontic bridge principles*, like $A \rightarrow OA$ or $A \rightarrow PA$, etc. Also the addition of these *BP*s to standard a.d.-combinations (plus any of *MO* or *OC*) will keep them practically *I-O*-trivial. The reason for this is clear from step (ii) in the proof of theorem 1: a monomodal deontic *BP Y* can be consistent (in **dY**+*D*) only if Y^{-o} is a nonmodal first order theorem.

Theorem 1 can be generalized in two respects. Corollary 1.1 tells us that if we replace "$(aT) \in \mathbf{X}$" by "$(aT) \in \mathbf{aXdY+B}$" and require **B** to be nonempty, we obtain not only a sufficient but also a *necessary* condition for the admissibility of the "A/A^{-o}"-rule. (Note: it might be that although $(aT) \notin \mathbf{X}$, (aT) is derivable from other **L**-axioms, e.g. when **Y** contains the *BP* $OA \rightarrow A$ and *B* contains *MO*.) In corollary 1.2 we generalize theorem 1 and corollary 1.1 to arbitrary sets of *BP*s **B** by replacing "$(aT) \in \mathbf{aXdY+B}$" by "$\mathbf{B}^{-o} \subseteq \mathbf{aXdY+B}$" (where $\mathbf{B}^{-o} := \{X^{-o} | X \in \mathbf{B}\}$). A second information contained in corollary 1.2 (a) (and in the second half of corollary 1.1) concerns the admissibility of the "A/A^{-o}"-rule: though generally stronger than practical *I-O*-triviality, it is *equivalent* with the latter in all logics **aXdY+B** where **dY**+(*dD*) is valid on at least one frame and the *BPs* in **B** are practically normative schemata.

Corollary 1: For every **L**:= **aXdY,** where **dY**+(dD) is valid on at least one frame:
(1.1) For nonempty $\mathbf{B} \subseteq \{MO, OC\}$: $(aT) \in \mathbf{L}+\mathbf{B}$ $\Leftrightarrow$ the rule "A/A^{-O}" is **L**+**B**-admissible $\Leftrightarrow$ **L**+**B** is practically *I-O*-trivial.
(1.2) (a) For every set of practically normative *BP*s **B**: $\mathbf{B}^{-O} \subseteq \mathbf{L}+\mathbf{B}$ $\Leftrightarrow$ the rule "A/A^{-O}" is **L**+**B**-admissible $\Leftrightarrow$ **L**+**B** is practically *I-O*-trivial. – (b) For every set of *BP*s **B**: $\mathbf{B}^{-O} \subseteq \mathbf{L}+\mathbf{B}$ $\Leftrightarrow$ the rule "A/A^{-O}" is **L**+**B**-admissible $\Rightarrow$ **L**+**B** *i*s practically *I-O*-trivial.

*Proof: (*1.1*):* We prove a cycle of implications. The two $\Rightarrow$-steps are proved as in the proof of theorem 1.1+2. For the $\Leftarrow$-step, assume **L**+**B** is practically *I-O*-trivial. Then for every **L**+**B**-theorem *A* which is practically normative, A^{-O} will be an **L**+**B**-theorem (for note that A is **L**-inferrable from the empty premise set). So either $(\Box p \rightarrow Op)^{-O}$ or $(Op \rightarrow \Diamond p)^{-O}$ or both will be in **L**, and both are tautologically equivalent with $\Box p \rightarrow p$; moreover every substitution instance of $\Box p \rightarrow p$ will be in **L** (because **L** is closed under substitution), whence $(aT) \in \mathbf{L}$. (1.2)(*a*)*:* Again a cycle of implications is proved. For the first $\Rightarrow$-step we argue as in the proof of theorem 1.1; by our assumption step (iii) of this proof goes through for all schemata in **B**. The second $\Rightarrow$-step follows from lemma 1. Now assume **L**+**B** is practically *I-O*-trivial and $A = \sigma Z$ is an instance of a *BP* *Z* in **B**. Since *Z* is practically normative, Z^{-O} and all substitution instances thereof will be in **L**+**B** Because A^{-O} is a substitution instance of Z^{-O} (for the modified substitution function $\sigma^{*}L = (\sigma L)^{-O}$), also A^{-O} will be in **L**+**B**. (1.2)(*b*)*:* The $\Rightarrow$-direction of the equivalence follows as in (1.2)(a), the $\Leftarrow$-direction of it is obvious; the implication follows from lemma 1. Q.E.D.

A *BP* is called practically *I-O*-(non)trivial if its addition to a standard a.d.-combination yields a practically *I-O*-(non)trivial logic. Corollary 1.2 implies the following *necessary condition for practically nontrivial BPs*: For every a.d.-combination **L** = **aXdY** where **dY**+(dD) is valid on at least one frame (and $\Box$ may be interpreted as any *descriptive* sentence operator), the addition of a bridge principle *BP* to **L** will enable pratically nontrivial is-ought inferences *only if* (1.) *BP* is bimodal, i.e. it states an $\Box$-*O-interaction* and (2.) **L** does not contain BP^{-O}. *BP*s which are practically nontrivial are, e.g., those which connect subjective propositional attitudes like belief or desire with obligations, because the logics of such attitudes do not contain (*T*).

Besides monomodal (alethic or deontic) axiom schemata an a.d.-logic may contain certain bimodal but unmixed and hence purely normative axiom schemata like $(N1) := OA \rightarrow \Box OA$, $(N2) := \Box OA \rightarrow O\Box A$ or $(N3) := O\Box A \rightarrow \Box OA$. Does theorem 1 hold if one of them is added to the logic enriched with $\{MO, OC\}$? Not in general. For example, in $\mathbf{L} = \mathbf{a}T\mathbf{d}D{+}OC{+}N1$ we have the inference $\Diamond\Box p \vdash_L \Diamond Pp$ [via OC] $\vdash_L Pp$ [via $N1$], although p is not derivable from $\Diamond\Box p$ in **L**. It would only be so if **L** would contain the axiom aB. At least one can show that theorem 1 is preserved under the addition of ($N2$) or ($N3$).

Corollary 2: Assume $L := \mathbf{aXdY{+}B}$ is as in theorem 1, and $\mathbf{N} \subseteq \{N2, N3\}$. Then the rule "$A/A^{-O}$" is **L+N**-admissible, whence **L+N** is practically *I*-*O*-trivial.

Proof: There is just one additional step to the proof of theorem 1.1 (step iii): assume $A_i = \Box OB \rightarrow O\Box B$, or $A_i = O\Box B \rightarrow \Box OB$, is an instance of $N2$ or $N3$, resp. Then $A^{-O} = \Box B \rightarrow \Box B$ in both cases, which is an **L**-theorem. Q.E.D.

4. Direct and indirect O-(non)triviality

The logical core of theorem 1 is the admissibility of the rule "A/A^{-O}". So far we have evaluated the consequences of this rule only with respect to practically normative conclusions. What does it imply for arbitrary conclusions? To answer this question we must extend the notion of triviality to arbitrary conclusions. It would be tempting just to extend our previous definition to arbitrary conclusions, calling any conclusion following from **D** trivial if also $\mathbf{D} \vdash_L A^{-O}$ holds. This extended definition would immediately imply that the logics of theorem 1 are not only practically but generally *I*-*O*-trivial, i.e. trivial w.r.t. all conclusions following from descriptive premises. But such an extended definition would be inadequate. Let us explain why.

Ethicists usually emphasize that the means-end-principle has not the power to admit the derivation of norms from purely descriptive premises alone; only if in addition certain *fundamental* norms are *given*, (*ME*) allows the derivation of various *derived* norms. For example, from the fundamental norm Op and the neccessary implication $\Box(p \rightarrow q)$ we may infer the derived norm Oq [via the steps $\Box(p \rightarrow q) \vdash_L O(\mathrm{p} \rightarrow \mathrm{q}) \vdash_L Op \rightarrow Oq$]. In a similar way we may infer derived permissions from fundamental obligations with help of OC: $\Box(p \rightarrow q), Op \vdash_L Pp$ [via the steps $\Box(p \rightarrow q) \vdash_L P(p \rightarrow q)$ and $P(p \rightarrow q), Op \vdash_L Pq$]. However, $\Box(p \rightarrow q), Op \vdash_L Oq$ implies $\Box(p \rightarrow q) \vdash_L Op \rightarrow Oq$ by deduction theorem, which is an inference with a purely descriptive premise and a purely normative conclusion. It would be counterintuitive to call the conclusion $Op \rightarrow Oq$ trivial, in spite of the fact that also $\Box(p\to q) \vdash_L p\to q$ holds. Though $Op\to Oq$ is not *directly* relevant for the derivation of practical norms, it is *indirectly* relevant, since given we agree upon Op, we obtain a further practical norm, namely Oq, which is nontrivial because $\Box(p\to q)$, $Op \nvdash_L q$. Intuitively we would consider the conclusion $Op\to Oq$ as trivial only if also $Op\to q$ would follow from its descriptive premise set, as in the example $\Box q \vdash_L Op\to Oq$.

This tells us is two important things. *First,* that the extension of our definition of triviality to arbitrary conclusions is inadequate, because there exist purely normative statements N following from descriptive premises $\mathbf{D}$ which are nontrivial in spite of the fact that $\mathbf{D} \vdash_L N^{-O}$ holds. *Second,* that the above intuitive claim – that *ME* only allows to infer derived norms from fundamental norms plus descriptive premises, but not to infer fundamental norms from descriptive premises alone – is not exactly true, on two reasons. On the one hand, there exist nontrivial and purely normative conclusions following from descriptive premises, as in the example $\Box(p\to q) \vdash_L Op\to Oq$, but they are not *practically* normative. On the other hand, there exist practically normative conclusions following from purely descriptive premises with help of *ME*, as in the example $\Box(p\to q)$, $\Box p \vdash_L Oq$ (following from the previous example by the additional *MO*-step $\Box p \vdash_L Op$), but they are *trivial.* Hence the exact reformulation of the above intuitive claim is nothing but our theorem 1.2: *ME* ($\equiv$*MO*) and *OC* do not allow the derivation of nontrivial and practically normative conclusions from descriptive premises alone, but only from descriptive premises plus certain fundamental practical norms.

The attempt to develope a definition of triviality in terms of the omission of O-operators for conclusions of arbitrary complexity leads into various difficulties. Often clear intuitions are missing. On these reasons I suggest to evaluate the triviality of arbitrary conclusions in an indirect but simpler way, namely with respect to the practically normative statements implied by them together with other statements.

Definition 2: Assume $\Gamma \vdash_L A$. Then:
(2.1) A is a *O-nontrivial* conclusion of Γ iff there exists a set Δ containing *descriptive* or *practically normative* statements[15] and a practically normative statement B such that $\{A\}\cup\Delta \vdash_L B$, $\Delta \nvdash_L B$, and B is a O-nontrivial conclusion of $\Gamma\cup\Delta$ (i.e. $\Gamma\cup\Delta \nvdash_L B^{-O}$). Else A is a (directly as well as indirectly) O-trivial conclusion of Γ.

15 We do not allow any statements in Δ, for then every conclusion would come out as indirectly nontrivial.

(2.2) If a Δ exists satisfying (2.1) and containing only descriptive statements, then A is a *directly O-nontrivial* conclusion of Γ (else A is directly O-trivial).
(2.3) If some Δ satisfying (2.1) but no Δ satisfying (2.2) exists, then A is called an *indirectly O-nontrivial* conclusion of Γ.

In other words, an O-nontrivial conclusion of Γ serves as a relevant *intermediate step* in the derivation of a nontrivial practically normative statement from Γ plus additional premises Δ. It is directly O-nontrivial if the additonal premise set is descriptive; else indirectly O-nontrivial. Only directly nontrivial conclusions can *stop* the ethical justification *regress* in the search for practically normative statements. Indirectly nontrivial conclusions cannot do this, but they become indirectly relevant in the derivation of new and nontrivial practically normative statements from others which are given. For example, $Op \rightarrow Oq$ is a indirectly nontrivial conclusion from $\Box(p \rightarrow q)$, because $Op \rightarrow Oq$, $Op \vdash_L Oq$ and Oq follows nontrivially from $\{\Box(p \rightarrow q), Op\}$. But it is a directly trivial conclusion of $\Box(p \rightarrow q)$: there exists an additional descriptive premise, namely $\Box p$, such that $Op \rightarrow Oq$, $\Box$p $\vdash_L Oq$, but Oq is a trivial conclusion of $\{\Box(p \rightarrow q), \Box p\}$, since $\Box(p \rightarrow q), \Box p \vdash_L q$.

Corollary 3.1 tells us that for the special case where A is practically normative, the definition of direct triviality coincides with our previous definition 1 for logics in which the "A/A^{-O}" rule is admissible. Corollary 3.2 states what is to be expected, namely that every conclusion A following from a descriptive premise set **D** in the logics of theorem 1 will be directly nontrivial. However, it might be indirectly O-nontrivial, as is added in Corollary 3.3.

Corollary 3: Assume **L** = **aXdY+B** is an a.d.-logic satisfying the conditions of theorem 1 or corollary 1 or 2. Then, for any Γ and A with $\Gamma \vdash_L A$:
(3.1) *If A is practically normative*, then $\Gamma \vdash_L A^{-O}$ iff A is a directly O-trivial conclusion of Γ.
(3.2) *If* Γ *is descriptive*, then A is a directly O-trivial conclusion of Γ.
(3.3) For some Δ and C, C is an indirectly O-nontrivial conclusion of Δ.

Proof: (3.1): $\Leftarrow$: If A is directly O-trivial, then for for every $\mathbf{D}^*$ and practically normative B such that $\{A\} \cup \mathbf{D}^* \vdash_L B$, $\Gamma \cup \mathbf{D}^* \vdash_L B^{-O}$; in particular for $\mathbf{D}^* := \varnothing$ and $B := A$, so $\Gamma \vdash_L A^{-O}$. $\Rightarrow$: Assume $\{\mathrm{A}\} \cup \mathbf{D}^* \vdash_L B$ for practically normative B. Hence $\mathbf{D}^* \vdash_L (A \rightarrow B)$ and thus $\mathbf{D}^* \vdash_L A^{-O} \rightarrow B^{-O}$ because "A/A^{-O}"–rule is **L**-admissible by theorem 1 or corollary 1 or 2.

Since $\Gamma \vdash_L A^{-o}$ by the assumption that A is practically normative, $\Gamma \cup \mathbf{D}^* \vdash_L B^{-o}$ follows, whence A is a directly O-trivial conclusion of Γ. (3.2): Assume again $\{A\} \cup \mathbf{D}^* \vdash_L B$ for practically normative B. Hence $\Gamma \cup \mathbf{D}^* \vdash_L B$, where $\Gamma \cup \mathbf{D}^*$ is descriptive by assumption. So by theorem 1 or corollary 1 or 2, $\Gamma \cup \mathbf{D}^* \vdash_L B^{-o}$. Thus A is a directly O-trivial conclusion of Γ. (3.3): $\Box(p \rightarrow q) \vdash_L Op \rightarrow Oq$ is an example, for $Op \rightarrow Oq$, $Op \vdash_L Oq$ but $\Box(p \rightarrow q), Op \nvdash_L q$. Q.E.D.

Note also that there are various cases of conclusions following from nondescriptive premises which are directly nontrivial but not practically normative, e.g. conjunctions of practical obligations $\Box(p \rightarrow Oq) \wedge \Box(r \rightarrow Os)$ or iterated necessary implications $\Box(p \rightarrow \Box(q \rightarrow Or))$, provided the premises do not entail their O-omitted versions. In this way our restricted notion of a nontrivial practically normative conclusion is complemented by the much more liberal notion of a directly nontrivial conclusion.

5. *Strongly practically trivial Is-Ought-Inferences*

A practical obligation $D_1 \rightarrow OD_2$ following from $\mathbf{D}$ is called *strongly O-trivial* if not only $D_1 \rightarrow D_2$ but even $D_1 \rightarrow \Box D_2$ is $\mathbf{L}$-inferrable from $\mathbf{D}$; likewise for the boxified and universally quantified cases. As for practical triviality we may establish a result for strong practical triviality by proving that the rule "$A/A[\Box/O]$" is admissible, where $A[\Box/O]$ denotes the result of replacing every occurrence of O by $\Box$ in A. This notion is only adequate if applied to an obligation OD (or more generally, to a *positive* O-occurrence), but not if applied to a permission PD (or more generally, to a *negative* O-occurrence).[16] For practical permissions, strong I-O-triviality implies that whenever $\mathbf{D}$ implies that D is permitted, then $\mathbf{D}$ implies also that D is possible – but this is not a reason for calling $\mathbf{D} \vdash_L PD$ trivial (in some sense). Therefore, we will speak more appropriately of strong triviality *w.r.t. obligations*.

Definition 3: An a.d.-logic $\mathbf{L}$ is *strongly practically I-O-trivial w.r.t. obligations* iff for every $\mathbf{D} \subseteq \mathcal{L}^a$ and practical obligation A with $\mathbf{D} \vdash_L A$, $\mathbf{D} \vdash_L A[\Box/O]$ holds.

16 According to a well-known definition, an occurrence of a symbol ψ in a formula A is called positive [or negative] iff it lies in the scope of an even [or odd, resp.] number of negations or implication-antecedents (cf. Kleene 1967, p. 124).

We may prove strong practical *I-O*-triviality w.r.t. obligations for every a.d.-combination **aXdY**+MO under the weak condition that **aX** is stronger than **dY** in the sense that **dY**$[\Box/O] \subseteq$ **aX**, i.e. every theorem of **dY** becomes a theorem of **aX** when O is replaced by $\Box$. The result is preserved if ($N2$) or ($N3$) – and under certain conditions if (OC) or ($N1$) – are added to the logic; moreover it is generalizable similar to corollary 1.

Theorem 2: For any a.d.-combination **L**:= **aXdY+Z** with **dY**$[\Box/O] \subseteq$ **aX**:
(2.1) If **Z** $\subseteq \{MO, OC, N1, N2, N3\}$, and $aD \in$ **X** if $OC \in$ **Z**, a4 $\in$ **X** if $N1 \in$ **Z**, then the rule "$A/A[\Box/O]$" is **L**-admissible, whence **L** is strongly practically *I-O*-trivial w.r.t. obligations.
(2.2) **Z**$[\Box/\mathrm{O}] \subseteq$ **L** $\Leftrightarrow$ the rule "$A/A[\Box/O]$" is **L**-admissible $\Rightarrow$ **L** is strongly practically *I-O*-trivial w.r.t. obligations.

Proof: By similar arguments as in lemma 1 it is seen that a logic **L** is strongly practically *I-O*-trivial w.r.t. obligations if the rule "$A/A[\Box/O]$" is admissible in **L** (for note that $D = D[\Box/O]$ for any purely descriptive D). To prove the admissibility of this rule, we assume that $< A_1, \ldots, A_n := A >$ is an **L**-proof of A, and show that there exists an **aX**-proof (and thus an **L**-proof) of $A[\Box/O]$. If $A_i = \sigma X$ is an instance of axiom schema X of **aX,** then $A_i\,[\Box/\mathrm{O}] = \sigma^*X$ is an instance of X, too, by putting $\sigma^*L = \sigma L[\Box/O]$. If $A_i = Y$ is an instance of an axiom schema Y of **dY**, then $A_i[\Box/O] = \sigma^*Y[\Box/O]$ is an instance of $Y[\Box/O]$ (with σ^* defined as above), and $Y[\Box/O]$ is an **aX**-theorem by assumption. If A_i is an instance of MO, then $A_i[\Box/O] = \Box(A[\Box/O]) \rightarrow \Box(A[\Box/O])$, which is a tautology. If A_i is an instance of $N2$ or $N3$, then $A_i[\Box/O] = \Box\Box(A[\Box/O]) \rightarrow \Box\Box(A[\Box/O])$, again a tautology. If A_i is an instance of OC, then $aD \in$ **aX** by assumption, and $A_i[\Box/O] = \Box A \rightarrow \Diamond A$, which is an instance of aD. If A_i is an instance of $N1$, then ($a4$) $\in$ **aX** by assumption, and $(A_i)[\Box/O] = \Box A \rightarrow \Box\Box A$, an instance of ($a4$). If $\Box B$ is derived from B by (aR), then $(\Box B)[\Box/O] = \Box(B[\Box/O])$ is derivable from $B[\Box/O]$ by (aR), and $B[\Box/O]$ is **L**-provable by ind. hyp. Similar for ($\forall R$) and (MP). Finally if OB is derived from B by (dR), then $(OB)[\Box/O] = \Box(B[\Box/O])$ is derivable from $B[\Box/O]$ by (aR), and the latter formula is **L**-provable by ind. hyp. This establishes (2.1). For (2.2) we argue as for corollary (1.2)(b). Q.E.D.

Under more restricted conditions a similar result is provable for permissions, saying that if A is a practical permission and $D \vdash_L A$, then also $D \vdash_L A[\Box/P]$ holds, where $A[\Box/P]$ is the result of replacing P in A by $\Box$. We speak here of *strong practical I-O-triviality w.r.t. permissions.* The proof is now semantical, and the first degree restriction is essential for it, i.e. the rule "$A/A[P/\Box]$" is not generally admissible. For instance, if $\mathbf{L}$ = $\mathbf{a}T\mathbf{d}\{T',.1\}+MO$ [where (d.1) = $OPA \rightarrow POA$], then $\Box p \vdash_L Op$ [by MO] $\vdash_L$ OPp [by dT'] $\vdash_L POp$ [by d.1]; but $\Box Op$ is not $\mathbf{L}$-derivable from $\Box p$. By a frame construction we may prove strong practical *I-O*-triviality w.r.t. permissions for all logics of theorem 1 where **aX** is weakly frame-complete and **dY** is unmixed.

Definition 4: $\mathbf{L}$ is *strongly practically I-O-triviality w.r.t. permissions* iff for every $\mathbf{D} \subseteq \mathcal{L}^a$ and practical permission A with $\mathbf{D} \vdash_L A$, $\mathbf{D} \vdash_L A[\Box/P]$ holds.

Before we prove our theorem we state an important semantical lemma for unmixed a.d.-combinations. For any given deontic frame $F = \langle W, S\rangle$, $\alpha \notin W$ and $\beta \in W$, $F^{\alpha+\beta} = \langle W^{\alpha+\beta}, S^{\alpha+\beta}\rangle$ is the result of adding α to F at β, and is defined as follows: $W^{\alpha+\beta} = W \cup \{\alpha\}$, and $S^{\alpha+\beta} = S \cup \{\langle\alpha,\delta\rangle \mid \langle\beta,\delta\rangle \in S\}$. For an a.d.-frame $F = \langle W, R, S\rangle$, $\alpha \notin W$ and $\beta \in W$, $F^{\alpha+\beta} := \langle W^{\alpha+\beta}, R, S^{\alpha+\beta}\rangle$. The operation $^{\alpha+\beta}$ has an important preservation property, concerning the truth of *unboxed* purely normative sentences; it follows from this property that the frames of unmixed deontic logics are closed under the operation $^{\alpha+\beta}$. Thereby a purely normative sentence is called unboxed if every $\Box$ (or $\Diamond$) contained in it lies in the scope of some O (or P).

Lemma 2: (2.1) For any a.d.-frame $F = \langle W, R, S\rangle$, $\alpha \notin W$, $\beta \in W$, model $M^{\alpha+\beta} = \langle W^{\alpha+\beta}, R, S^{\alpha+\beta}, Dm, v^{\alpha+\beta}\rangle$ based on $F^{\alpha+\beta}$: if $M = \langle W, R, S, Dm, v\rangle$ is the restriction of $M^{\alpha+\beta}$ to W (i.e., $v^{\alpha+\beta}(x) = v(x)$, $v(F) = v^{\alpha+\beta}(F)\uparrow W$), then for every unboxed purely normative N: $(M^{\alpha+\beta}, \alpha) \models N$ iff $(M, \beta) \models N$.
(2.2) For any unmixed deontic logic **dY**, **F(dY)** is closed under the operation $^{\alpha+\beta}$.

Proof: (2.1): By induction on the complexity of N, which is built up of formulas of the form OA by $\neg$, $\vee$ and $\forall$. (i) $N = OA$: $(M^{\alpha+\beta}, \alpha) \models OA$ iff for all δ with $S^{\alpha+\beta}\alpha\delta$, $(M^{\alpha+\beta}, \delta) \models A$ iff for all δ with $S\beta\delta$, $(M^{\alpha+\beta}, \delta) \models A$ (by def. of $S^{\alpha+\beta}$) iff for all δ with $S\beta\delta$ $(M, \delta) \models A$ [because: all δ with $S\beta\delta$ are in W, whence the δ-R-$S^{\alpha+\beta}$–generated submodel of $M^{\alpha+\beta}$ is identical with the δ-R-S-generated submodel of M; by the def. of $S^{\alpha+\beta}$] iff $(M, \beta) \models OA$. (ii) The induction steps for $N = \neg A$ and $N = A \vee B$ are straightforward. (iii) $N = \forall z B$: $(M^{\alpha+\beta}, \alpha) \models \forall z B$ iff for all $d \in Dm$, $(M^{\alpha+\beta}[z{:}d], \alpha) \models B$. The restriction of $M^{\alpha+\beta}[z{:}d]$ to W is $M[z{:}d]$. So we proceed:... iff for all $d \in Dm$, $(M[z{:}d], \beta) \models B$ (ind. hyp.) iff $(M, \beta) \models \forall z B$.

(2.2): Assume, for reductio, $F \in \mathbf{F}(\mathbf{dY})$, but $F^{\alpha+\beta} \notin \mathbf{F}(\mathbf{dY})$. Then there exists an instance A of an axiom schema $Y \in \mathbf{Y}$, a model $M^{\alpha+\beta} = \langle W^{\alpha+\beta}, S^{\alpha+\beta}, Dm, v^{\alpha+\beta} \rangle$ based on $F^{\alpha+\beta}$ and a world $\gamma \in W^{\alpha+\beta}$[= $W \cup \{\alpha\}$] such that A is false at γ in the model $M^{\alpha+\beta}$. A is purely normative and unboxed, because Y is purely deontic and unmixed. It follows from (2.1) that $(M^{\alpha+\beta}, \alpha) \models A$ iff $(M, \beta) \models A$, where M is the restriction of $M^{\alpha+\beta}$ to W, which is based on F. For $\gamma \neq \alpha$, $(M^{\alpha+\beta}, \gamma) \models A$ iff $(M, \gamma) \models A$ holds trivially (by def. of $M^{\alpha+\beta}$). Hence, also F cannot be a frame for **dY**, contradicting our assumption. Q.E.D.

Lemma 2 plays also a crucial role in the proof of the following special Hume thesis restricted to unboxed purely normative conclusions, which applies to a very broad class of a.d.-logics: *If* **L** *is a weakly frame-complete and unmixed a.d.-combination, N is* **L**-*falsifiable, unboxed and purely normative and* **D** *is* **L**-*consistent, then* $D \nvdash_L N$ (cf. Schurz 1997, ch. 5.4). – We turn to our theorem.

Theorem 3: Let $\mathbf{L} = \mathbf{aXdY{+}B} \subseteq \{MO, OC\}$, where **aX** is weakly frame-complete, $(aT) \in \mathbf{X}$, **dY** is unmixed and **dY**+(dD) is valid on at least one frame. Then **L** is strongly practically *I-O*-trivial w.r.t. permissions.

Proof: By the remark in §2, **dY** is valid on the singleton frame $F^+_\delta := \langle\{\delta\}, \{\langle\delta,\delta\rangle\}\rangle$. Lemma 2 implies that it is also valid on the two point frame $\langle\{\gamma,\delta\},\{\langle\gamma,\delta\rangle,\langle\delta,\delta\rangle\}\rangle$, because **dY** is unmixed and the two point frame results from F^+_δ by the operation ${}^{\gamma+\delta}$. Assume (i) $\mathbf{D} \nvdash_L \Box\forall x_{1\text{-}n}(D_1 \rightarrow \forall y_{1\text{-}m}\Box D_2)$. We must show that (ii) $\mathbf{D} \nvdash_L \Box\forall x_{1\text{-}n}(D_1 \rightarrow \forall y_{1\text{-}m} P\, D_2)$. By frame-completeness of **aX** there exists a frame $\langle W, R\rangle$ for **aX**, a model $M = \langle W, R, Dm, v\rangle$ on it and world α in M such that $(M, \alpha) \models \mathbf{D}$, but $(M, \alpha) \models \neg\Box\forall x_{1\text{-}n}(D_1 \rightarrow \forall y_{1\text{-}m}\Box D_2)$, i.e. $(M, \alpha) \models \Diamond\exists x_{1\text{-}n}(D_1 \wedge \exists y_{1\text{-}m}\Diamond\neg D_2)$. Hence for the model $M^+ = \langle W, R, Dm, v^+\rangle$ with a valuation function v^+ differing from v only in what it assigns to the variables $x_{1\text{-}n}$ and $y_{1\text{-}m}$, $(M^+, \alpha) \models \Diamond(D_1 \wedge \Diamond\neg D_2)$ (here we assume that the variables $y_{1\text{-}m}$ do not occur free in D_1.). So there exist worlds β, γ in W such that $\alpha R\beta R\gamma$ and $(M^+,\beta) \models D_1$, $(M^+,\gamma) \models \neg D_2$, whence $(M, \beta) \models D_1 \wedge \Diamond\neg D_2$. We add to $\langle W, R\rangle$ the following relation $S := \{\langle\beta,\gamma\rangle,\langle\gamma,\gamma\rangle\} \cup \{\langle\delta,\delta\rangle \mid \delta \in W-\{\beta,\gamma\}\}$. $\langle W,S\rangle$ is the disjoint union of the above two point frame and of F^+ – singleton frames; thus **dY** is valid on $\langle W, S\rangle$ (by the remarks above and lemma 2), whence **aXdY** is valid on $\langle W, R, S\rangle$ (by the remarks in §2). The axiom schemata MO and OC are valid on $\langle W, R, S\rangle$ because R is reflexive (since $aT \in \mathbf{X}$) and S is serial. In the model $M^{+*} = \langle W, R, S, Dm, v^+\rangle$, **D** is true at α, D_1 is true at β and $O\neg D_2$ is true at β (since β's only ideal world is γ at which D_2 is false), whence $(M^{+}*, \beta) \models (D_1 \wedge O\neg D_2)$, thus $(M^{+}*, \alpha) \models \Diamond(D_1 \wedge O\neg D_2)$ and hence $(M*, \alpha) \models \Diamond\exists x_{1-n}(D_1 \wedge \exists y_{1-m} O\neg D_2)$, i.e. $(M*, \alpha) \models \neg\Box\forall x_{1-n}(D_1 \rightarrow \forall y_{1-m} P D_2)$, where $M* = <W, R, S, Dm, v^{+}>$. Thus $\mathbf{D} \nvdash_L \Box\forall x_{1-n}(D_1 \rightarrow \forall y_{1-m} \Box P D_2)$. The cases where some boxes or quantifiers are missing are proved similarly; the case of a categorial permission is covered by letting D_1 be a tautology. Q.E.D.

For the strong triviality of purely normative conclusions which are not practically normative the considerations of §4 apply in the same way: if we call conclusions which are not strongly *O*-trivial weakly *O*-nontrivial, then there exist *IOI*'s with weakly *O*-nontrivial conclusions, but these conclusions are never practically normative nor directly weakly non-trivial, but only indirectly weakly nontrivial.

It is sometimes argued that normal deontic logics are too strong because they contain the axiom *O*, which is not intuitively obvious. Deontic logics which are "one step weaker" are the so-called regular logics. The minimal regular deontic logic is axiomatized by the rule (*dER*): $A \leftrightarrow B / OA \leftrightarrow OB$ (replacing equivalents) and the axioms (*dM*): $O(A \wedge B) \rightarrow (OA \wedge OB)$ (monotonicity) and (*dC*): $(OA \wedge OB) \rightarrow O(A \wedge B)$ (conjunction) (cf. Chellas 1980, part III). It is easy to see that theorems 1 and 2 go through also if the deontic part of the a.d.-logic is only regular; they will go through even if it is only classical, which means that only the rule (*dER*) is assumed. The only modification is that we must now explicitly assume that **dY**+(*dTriv*) is **L**-consistent.[17] If a deontic logic is regular and contains the Barcan formula, it can be based on Kripke frames which include, besides normal worlds, an additional set of queer worlds making every sentence of the form *OA* false (including $O\top$). Using these kinds of frames, also theorem 3 will go through for a.d.-combinations with a regular *d*-part; we just have to assume in lemma 2 and theorem 3 that the worlds are deontically normal.

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17 This is because the validity of **dY**+(*dD*) on some frame is only defined if **dY** is regular, and even in this case it will guarantee **L**-consistency of **dY**+(*dTriv*) only if the frame contains some (so-called) normal worlds.

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