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FACTS, SEMANTICS AND INTUITIONISM

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Abstract
We outline an intuitive semantics for intuitionism in the light of van Fraassen's theory of facts. We regard an intuitionistic proof as the content of a positive fact. We establish an adequacy result for the given semantics. The connection with Kripke semantics is also investigated.

1. *Introduction*

Van Fraassen [6] proposed a theory of facts designed to advance the basic idea of Russell [12], among others. The philosophical significance of van Fraassen's theory of facts lies in its providing a way to ponder facts seriously for semantic explication. In fact, he established an alternative semantics for *tautological entailment*, a fragment of relevant logic. Unfortunately, van Fraassen's approach through facts has not been much explored for other logical systems in spite of its wide applicability to non-classical logics.

The aim of this paper is to give a new semantics for the intuitionistic propositional logic called *van Fraassen semantics* from the standpoint of theory of facts. Unlike van Fraassen's original framework, we focus on the notion of positive fact to describe intuitionistic truth. Intuitionistic negation is then defined in terms of incompatibility of positive facts following the suggestion of Demos.

The rest of the paper is structured as follows. In section 2, we give an overview of van Fraassen's theory of facts. In section 3, we propose a new semantics for intuitionism within the framework of theory of facts. In section 4, we relate van Fraassen semantics to Kripke semantics for intuitionism and also to metavaluational analysis. The facts semantics provided lies midway, so to say, between Kripkean worlds semantics and the metavaluation alternative which interprets intuitionistic syntax. It eschews worlds widely recognized as problematic, while duly avoiding syntax. This paper

[1] Richard Sylvan suddenly died in 1996. The final version of this paper was prepared by the first writer.

ends in section 5, with a brief summation and suggestions as to further connections and research.

2. *Van Fraassen on Facts*

Research in the semantics for formal logic has produced many different formalizations, e.g. classical models, Kripke models, lattice models, and so on. A principal aim of these semantical frameworks is to define the concept of truth in terms of abstract objects. It is thus natural to consider facts, prime suppliers of truth, as foundations of the semantics for various logical systems. This line of research was first done by Russell [12] and was extended by van Fraassen [6]; also see Anderson and Belnap [3]. In particular, van Fraassen's theory of facts suggested an objective of reconstruction of non-classical model theories. What is more, after progressing through the theory we shall come to see how to simplify it considerably for these purposes.

The guiding idea of van Fraassen's theory of facts is that a sentence A is true (false) iff some fact that makes A true (false) is the case. To elaborate this formalization, van Fraassen introduced the notion of *atomic fact* (or complex) to be defined as a proposition. For complex facts, conjunctive (or disjunctive) facts are also needed. Since we consider here only propositional logic, it is possible to simplify van Fraassen's model M as a pair $\langle D, h\rangle$, where D is a set of objects and h is a function from the set of atomic sentences to D. Intuitively, D can be regarded as a set of facts. *A fact* in M is any non-empty set of complexes in M. The union of facts $e_1, \ldots, e_n$, called a conjunctive fact, is expressed as $e_1 \cdot \ldots \cdot e_n$. Here, we can extend the notion of product for sets of facts. If X and Y are two sets of facts then the product $X \cdot Y$ of X and Y is the set of facts $e \cdot e'$ such that e is in X and e' is in Y. It is said that *e forces e'* in M if both e and e' are facts in M and e' is a subset of e. A complex p in M is the case in M iff $h(p) \in D$ and a fact e is the case in M iff all its members are the case in M.

To formulate truth conditions, for every sentence A we need the set $T(A)$ of facts that makes A true in M and the set $F(A)$ of facts that make A false in M. We here assume that for any atomic sentence p there is exactly one fact that makes p true in M, namely $\{P\}$ and exactly one fact that makes it false in M, namely $\{-P\}$, where $-P$ denotes the complement of P in M. Although we do not admit negative facts here, we need to assume some prior classification of the relation in the sense that for every complex p there also exists a complex that makes p false. See Russell [12] for a related discussion. We can then extend these sets by induction on the complexity of A as follows:

$$T(p) = \{P\},$$
$$F(\sim p) = \{-P\},$$
$$T(\sim A) = F(A),$$
$$F(\sim A) = T(A),$$
$$T(A\&B) = T(A) \cdot T(B),$$
$$F(A\&B) = F(A) \cup F(B).$$

Disjunction can be introduced by definition. The connection of the sets $T(A)$ and $F(A)$ is that A is true (in model M) iff some fact in $T(A)$ obtains, and false (in model M) iff some fact in $F(A)$ obtains. We denote by $T^*(A)(F^*(A))$ the set of all facts that force some fact in $T(A)(F(A))$.

Van Fraassen gave the semantic entailment relation that A *semantically entails* B ($A \Vdash\!= B$) if whenever A is true, so is B. We can then strengthen this notion by saying that whatever makes A true, also makes B true, that is $A \Vdash B$ called a tautological entailment relation. Van Fraassen identified $A \Vdash\!= B$ with the condition that $T^*(A)$ is included in $T^*(B)$ in any model. Van Fraassen proved that A tautologically entails B iff $A \Vdash\!= B$ is valid. Thus, van Fraassen's theory of facts can be considered as one of the model theories for tautological entailment. The reader should consult van Fraassen [6] for the detailed exposition.

3. *Facts and Intuitionism*

Kripke [8] established a model-theoretic semantics for intuitionism by means of possible worlds, in terms of which the intuitionistic truth can be captured through an accessibility relation with a partial ordering, reflecting increase of information. In fact, Kripke's approach is compatible with the now familiar description of intuitionists' mental activity in their mathematical reasoning, that more information can prove more statements. Thus, if we view the actual world as a partially specified set of facts then theory of facts seems to offer a promising alternative to Kripke semantics for intuitionism.

There are essentially two strategies to develop a new semantics for intuitionism based on the theory of facts. One is to relate intuitionistic truth to positive facts and to define negation in some way. The other is to use both positive and negative facts to define truth and falsity conditions of formulas in question. The present paper pursues the former approach. However, we are faced with a serious difficulty in formulating intuitionistic truth within van Fraassen's original semantics. This is because we cannot properly account for the intuitionistic nature of logical connectives. It would thus seem necessary to revise a formal framework. Our proposal is to dualize the set of facts that makes a formula true with its content and to connect the content with the truth conditions in order to identify the notion of intuitionistic proof. In other words, this corresponds to amalgamating non-standard "fact" semantics and standard classical semantics. Surprisingly, intuitionists allow us to interpret atomic formulas *classically*. So the question is how we can describe intuitionistic features in this setting.

It is in fact possible to relate the notion of fact to intuitionistic truth in various ways within the framework of theory of facts. One attractive idea is to regard an intuitionistic proof as the content of a positive fact. By extracting the content of facts, we can formalize the concept of intuitionistic truth. The idea is not new. For example, Dummett [4] presented an interpretation of intuitionistic statements in terms of content. Dummett [4, p. 363] says:

> "If we take it as a primary function of a sentence to convey information, then it is natural to view a grasp of the meaning of a sentence as consisting in an awareness of its *content*; and this amounts to knowing the conditions under which an assertion made by it is correct."

Our idea consists of two phases. First, all atomic formulas are interpreted truth-functionally as in classical logic. Second, any formula can be assigned as the content corresponding to the intuitionistic truth. These two notions, i.e. the classical truth assignment and the content of a statement, are linked in such a way that if a formula is true then the content of the formula coincides with a maximal set of facts. The proposed formalization can thus capture intensional aspects of intuitionistic truth in terms of facts.

A van Fraassen model for the intuitionistic propositional logic I is a tuple $M = \langle FACT, v, T\rangle$, where $FACT$ is a non-empty set of van Fraassen facts (i.e. each a set of complexes), v is a valuation $v\colon At \to \{1, 0\}$, and T is a content function assigning each formula an element of $FACT$, i.e. T: wff $\to$ $FACT$, where At is a set of atomic formulas (propositional variables). There are the following two restrictions on a van Fraassen model, namely

(F1) If $v(A)=1$ then for any B, $T(B)\subseteq T(A)$,
(T0) $T(A\&\neg A)=\varnothing$.

Here, some explanation is in order. (F1) guarantees that intuitionistic truth can be connected with a set of facts being closed under the increase of new facts. (T0) ensures intuitionistic logic implicitly consistent. We also assume that the content function T satisfies the following extra conditions, i.e.

(T1) $T(A\& B)=T(A)\cap T(B)$,
(T2) $T(A\vee B)=T(A)\cup T(B)$,
(T3) $T((A\vee B)\rightarrow C)\subseteq T(A\rightarrow C)\cap T(B\rightarrow C)$,
(T4) $T(A\rightarrow B)\cap T(B\rightarrow C)\subseteq T(A\rightarrow C)$,
(T5) $T(A\rightarrow B)\cap T(A\rightarrow\neg B)=T(\neg A)$,
(T6) $T(\neg A)\cup T(B)\subseteq T(A\rightarrow B)$,
(T7) $T(A)\cap T(A\rightarrow B)\subseteq T(B)$.

Then, we can specify the valuation v generally as follows:

$v(A\& B)=1$ iff $v(A)=1$ and $v(B)=1$,
$v(A\vee B)=1$ iff $v(A)=1$ or $v(B)=1$,
$v(\neg A)=1$ iff $v(A)=0$ and $T(A)=\varnothing$,
$v(A\rightarrow B)=1$ iff $(v(A)=0$ or $v(B)=1)$ and $T(A)\subseteq T(B)$.

Of course in every case, $v(\neg A)=0$ and $v(A*B)=0$ otherwise (where $*\in\{\&,\vee,\rightarrow\}$). Obviously, both conjunction and disjunction are here interpreted in the same way as in classical logic. However, it is not the case for negation and implication due to their intuitionistic flavor. For instance, intuitionistic implication cannot be identified with material implication. This is because $A\rightarrow B$ supposedly embodies the transformation of a proof of A into that of B. Accordingly, the above truth condition emerges. Since intuitionistic negation can be seen as a special case (i.e. $A\rightarrow false$) of implication, we can describe its interpretation similarly. Namely, intuitionistic negation $\neg A$ can be introduced as the incompatibility of a positive fact of A, in the sense A is false and has no content. For $v(\neg A)=1$ iff $v(A\rightarrow false)=1$, with $false=A\&\neg A$ for arbitrary A, i.e. iff $v(A)=1$ and $T(A)=\varnothing$ using(T0). This reading is in fact in accordance with the basic interpretation of negation as proposed by Gabbay [7].

To round off van Fraassen semantics we introduce a standard truth definitions, namely $\Gamma\models A$ for any set of formulas Γ and a formula A when for every van Fraassen model M if for all $B\in\Gamma$, $v(B)=1$ then $v(A)=1$. A is valid iff it is true in all models. We first establish the soundness:

Theorem 1 (Soundness theorem)
If $\Gamma\vdash A$ then $\Gamma\models A$,
where $\vdash$ is the provability relation in I.

Proof: It suffices to check that every axiom is valid in van Fraassen models and that modus ponens preserves validity. Before establishing the requirement, we need to give a Hilbert presentation of *I*. That below is due to Dummett [4].

(A1) $A \to (B \to A)$
(A2) $A \to (B \to A \& B)$
(A3) $A \& B \to A$
(A4) $A \& B \to B$
(A5) $A \to (A \vee B)$
(A6) $B \to (A \vee B)$
(A7) $(A \vee B) \to ((A \to C) \to ((B \to C) \to C))$
(A8) $(A \to B) \to ((A \to (B \to C)) \to (A \to C))$
(A9) $(A \to B) \to ((A \to \neg B) \to \neg A)$
(A10) $A \to (\neg A \to B)$

The inferences are closed under *modus ponens* (MP). As before, $\to$ and $\neg$ denote intuitionistic implication and negation, respectively. Now, (A1) is true by (F1) and (T6). By (T6), $\mathrm{T}\,T(A) \subseteq T(\neg B) \cup T(A) \subseteq T(B \to A)$. Also, as $v(A)=0$ so $v(A \to (B \to A))=1$ follows, as when $v(A)=1$, $T(\mathrm{B}) \subseteq \mathrm{T}(\mathrm{A})$ by (F0) and $v(B)=0$ or $v(A)=1$. (A2), (A3) and (A4) are the immediate from (T1). (A5) and (A6) follows from (T2). (A7) is shown to be true by (T3) and (T1). (A8) is immediate from (T1) and (T4). From (T5) and (T0), we establish (A9). (A10) is true by (T6). It is also not difficult to show that *modus ponens* preserves truth. This completes the soundness proof.

The present development is by no means the only formulation of a model theory for intuitionism based on the theory of facts. Another version of the present development begins from an unanalyzed set of facts, *LFACT*, and construes *T* as a function from wff to subsets of *LFACT*. (In the completeness arguments this amounts to interpreting *LFACT* more linguistically, as a set of true statements rather than as a set of *LFACT*.) Also of course the modelling conditions can be varied and rendered more compact. As well, further afield, there are at least three other ways to seek a van Fraassen semantics for *I*. The first is to define the consequence relation and show that it is a smallest relation satisfying the full deduction theorem. In this setting, our semantics could be substantiated through Meyer [9]. The second is to explicitly integrate a fact semantics with the concept of incompatibility in order to interpret intuitionistic negation. But, the approach seems more complicated than the semantics just given. In fact, our semantics implicitly uses the concept of incompatibility. The third alternative expands the notion of fact with an explicit reference to a possible world as suggested by van Fraassen [6]. It is, however, far from our understanding of facts as the basis for semantics, and it is somewhat contrived.

The van Fraassen semantics can alternatively be seen as a content elaboration of metavaluational theory, as sponsored by Meyer (for details see [3]). With negation defined, metavaluational assignments for intuitionism differ from those given only as regards the evaluation clause for implication $\rightarrow$. For where v is a metavaluational assignment from wff to truth-values $\{1, 0\}$, $v(A \& B) = 1$ iff $v(A) = v(B) = 1$, $v(A \vee B) = 1$ iff $v(A) = 1$ or $v(B) = 1$, $v(false) = 0$. The exception is this: $v(A \rightarrow B) = 1$ iff $v(A) = 0$ or $v(B) = 1$ and $\vdash_I A \rightarrow B$. We step out of syntactical confines altogether by simply supplanting implicational provability by —what is conventionally equated with it— content inclusion. The rule thus arrived at should already be familiar from other content semantics, those for Parry logics and for relational and relevant logics.

4. *Van Fraassen Semantics Connected with Kripke Semantics*

In this section, we prove the completeness of the van Fraassen semantics. Our strategy is to employ a correspondence between a van Fraassen model and a Kripke model, and to use the completeness of the Kripke semantics for I (see e.g. Kripke [8] or Dummett [4]) to establish completeness with respect to the van Fraassen semantics. Although we could directly prove the completeness theorem, we stick to the strategy capable of comparing both semantical frameworks.

Before doing so, we recall the basics of a Kripke semantics for intuitionism. A *Kripke model* is a triple $\langle W, \leq, V \rangle$ where $\langle W, \leq \rangle$ is a poset and V: $PV \rightarrow$ subset(W) satisfying:

$$w \in V(p) \text{ and } w' \geq w \text{ imply } w' \in V(p)$$

hen, the forcing relation $\models$ between a world and a formula is defined as

$$w \models p \text{ iff } w \in V(p).$$

Here, $w \models p$ reads "p is true at w". The forcing relation $\models$ extends generally as follows:

$w \nvDash false$,
$w \vDash A \& B$ iff $w \vDash A$ and $w \vDash B$,
$w \vDash A \vee B$ iff $w \vDash A$ or $w \vDash B$,
$w \vDash A \rightarrow B$ iff for all $w' \geq w$, $w' \vDash A$ implies $w' \vDash B$,
$w \vDash \neg A$ iff for all $w' \geq w$, $w' \nvDash A$.

A formula A is *valid* (written $\vDash_K A$) iff for all $w \in W$, $w \vDash A$. We also use the notation $\Gamma \vDash_K A$ to mean that for all $B \in \Gamma$, if $\vDash_K B$ then $\vDash_K A$.

Kripke [8] proved completeness of I relative to Kripke models:

Theorem 2 (Completeness of Kripke Models)
Let Γ be a set of formulas and A a formula. Then, we have:

$$\Gamma \vdash A \text{ iff } \Gamma \vDash_K A.$$

We are now in a position to give a completeness proof of the van Fraassen semantics. This can be achieved by converting a van Fraassen model into a Kripke model. The point is that in Kripke models we can equate propositions with sets of worlds. In fact, the observation is famous in the Kripke semantics for modal logics. It would thus be possible to regard the content of a formula as a set of worlds making the formula true. In addition, the valuation in a van Fraassen model can be a forcing relation in a Kripke model. By these two steps, we can eliminate the notion of world and provide an alternate semantics (i.e. van Fraassen semantics) incorporating the concept of the content of a formula. The next lemma is a key result for proving completeness:

Lemma (Completeness Lemma)
(1) Where A is not provable then there is a van Fraassen countermodel to A.
(2) Where $\Gamma \nvdash A$ then $\Gamma \nvDash A$.

Proof: (2) is a straightforward standard generalization of (1). As to (1), suppose A is not provable. Then by Theorem 2 there is a Kripke countermodel K with base G such that $G \nvDash A$. In fact, as Kripke also showed, there is a connected tree countermodel, with all worlds of W in K emanating from base G. Let K be such a tree countermodel.

Now define a putative van Fraassen countermodel M in terms of K as follows: $M = \langle FACT, v, T \rangle$, with $T(A) = \{w \in W: w \vDash A\}$, i.e. the range of A, or "proposition that" A, for each wff A; $v(p) = 1$ iff $G \vDash p$, for every atomic wff p;

$FACT = \{T(B)\colon G\vDash B\}$, i.e. in effect the class of true propositions.

Thus both T and v are functions of the types required.
It is shown jointly that

(M1) M is a van Fraassen model,
(M2) $v(A) = 1$ iff $G\vDash A$, for every wff A.

As to (M1), conditions on content function derived from features of ranges within Kripke intuitionistic semantics. Some examples illustrate how.

ad(T1). $T(A\&B) = \{w\colon w\vDash A\&B\} = \{w\colon w\vDash A \text{ and } w\colon w\vDash B\}$
$\{w\colon w\vDash A\} \cap \{w\colon w\vDash B\} = T(A) \cap T(B).$
ad(T3). As $\vdash ((A \to C)\&(B \to C)) \to ((A \vee B)) \to C$, for every w, when $w\vDash(A \to C)\&(B \to C)$ then $w\vDash(A \vee B) \to C$.
So $T((A \vee B) \to C) \subseteq T((A \to C)\&(B \to C))$
$= T((A \to C) \cap T(B \to C))$ by (T1)

For the rest, (T2) is like (T1) and (T4)-(T7) similar to (T3). (T5) uses the biconditional $(A \to B)\&(A \to \neg B) \leftrightarrow \neg A$, deriving from (A9) and (A10) with permutation. Finally, for (T0) similarly use $A\&\neg A \to$ *false* together with $T(\mathit{false}) = \varnothing$.

In establishing (M2), by induction from the given basis for atomic wff of *At*, it is taken, what is admissible, that negation has been definitionally eliminated, through the definition: $\neg A = A \to$ *false* with *false* in *At*. Then, the step for negation follows from other cases. The induction steps for conjunction and disjunction are immediate, as the clauses in respective models are similar. As to implication: for every wffs B and C,

$v(B \to C) = 1$ iff $v(B) = 0$ or $v(C) = 1$ and $T(B) \subseteq T(C)$
iff $v\nvDash B$ or $G\vDash C$ and,
for every w, $w\vDash B$ ensures $w\vDash C$,

by induction hypothesis,

i.e. iff, for every w (including G), when $w\vDash B$ then $w\vDash C$.
Now $G\vDash B \to C$ iff for every w such that $w \geq G$, when $w\vDash B$ then $w\vDash C$. So, when $v(B \to C) = 1$, $G\vDash B \to C$. For the other half, appeal is made to the connected tree model, where, for every $w \geq G$. So when $G\vDash B \to C$, $v(B \to C) = 1$..

Now first to conclude (M1):

ad(F1) Given $v(A)=1, G\models A$. So $T(A)\in FACT$. As $w\geq G$, $w\models A$, whence $T(A)\subseteq T(B)$

Secondly, as $G\not\models A$, $v(A)\neq 1$, concluding demonstration of a van Fraassen countermodel to A.

Theorem 3 (Adequacy of van Fraassen Modelling)

$$\Gamma\vdash A \text{ iff } \Gamma\models A$$

Proof: The soundness was proved in Theorem 1. For the completeness, we combine the completeness lemma with Theorem 2. This completes our adequacy result for this van Fraassen semantics.

The adequacy of metavaluational theory for intuitionism follows as a corollary. For it is now immediate that $\vdash_I A \rightarrow B$ iff canonically $T(A)\subseteq T(B)$. Conversely, adequacy of van Fraassen modelling can be established by expanding upon metavaluational theory.

Finally, we should evaluate the significance of the van Fraassen semantics for intuitionism. Our technical results reveal that the content of a fact confirming a formula can be viewed as the intuitionistic proof of the formula. Since the intuitionistic proof can be modelled by a set of worlds in an intuitionistic Kripke model, content in van Fraassen semantics can be naturally defined in terms of Kripke models. Nevertheless, we can dispense with the concept of world in a van Fraassen model.

It is surprising that all intuitionistic connectives can be interpreted *classically* under the restrictions on the content of a formula. This feature in the new semantics suggests that the notion of fact is flexible enough to deal with non-classical semantics. We can therefore *intensionalize* facts to serve as a semantic foundation for intuitionistic logic. In other words, there is an interesting way to link the intuitionistic truth with the mix of the (classical) truth and the content. It will be evident that our theory is slightly different from van Fraassen's original theory. But, we can enhance his idea so that theory of facts constitutes the basis for non-classical semantics.

Our proposed semantics is closely related to R.L. Epstein's [5] *set-assignment semantics* for intuitionism. Epstein also relies heavily on Dummett's [4] interpretation for intuitionistic logic. Epstein's central idea in the set-assignment semantics is that the truth-value of a compound proposition is determined by its form and the truth-values and contents of its parts. As stated above, the content of an intuitionistic proposition can be identified with the condition under which it is correct to assert the proposition a la Dummett. Our starting point is slightly different from Epstein's. We interpret the content of a proposition as its intuitionistic proof. In this sense, our semantics is more intuitive than Epstein's semantics in that intuitionistic truth directly corresponds to a fact. However, from the intuitionistic point of view there may be no differences between Epstein's and our semantics. This is because the intuitionist gives the proposition its truth by recognizing a proof of the proposition. We also note that instead of (F1) in a van Fraassen model Epstein uses the condition such that if $v(A) = 1$ then $T(A) = S$ where S is a non-empty collection representing the least bits of content. From a technical viewpoint, this idea is equivalent to (F1).

5. *Concluding Remarks*

We have presented a novel semantics for intuitionism based on van Fraassen's theory of facts. We have modified van Fraassen's theory so that we can interpret the intuitioistic truth as the content of a positive fact. It has also been shown that there are simple connections between our semantics and Kripke semantics for intuitionism. We have proved the completeness of the new semantics.

One might observe that the connection between van Fraassen's ideas and our interpretation is rather weak. In van Fraassen's fact semantics, an atomic proposition p gets assigned an atomic fact P, i.e. $T(p)=\{P\}$, as described above. However, in the intuitionistic interpretation, $T(p)$ is a set of worlds. This may be odd, but there is no reason to assume the existence of the exactly one atomic fact that makes the corresponding atomic proposition in intuitionism. This is because we may be able to have several proofs of a proposition. In this regard, van Fraassen's original theory of facts cannot be directly extended for intuitionistic logic. Another objection to our semantics is about (F1). According to (F1), if $v(p) = 1$, then for any B, $T(B) \subseteq T(p)$. Here, we may doubt whether $T(p)$ can be still called a *fact*. In our modelling, $T(p)$ is viewed as a set of atomic facts, which is related to a *canonical* proof, in that any set of facts can be constructed from a set of atomic facts. This agrees with the intuitionistic intepretation that every proof can be constructed in terms of canonical proofs.

Our investigations lead naturally to further developments. Adding quantifiers is an obvious next step. Similar fact-based semantics can no doubt be devised for a range of other non-classical logics and connected with content semantics already in circulation. Among such developments of the semantics is one briefly alluded to section 3 concerning negative facts; there we were thinking of Nelson's [10] logic of *constructible falsity*; see Akama [1, 2] and Routley [10]. This alternative appears capable of providing a promising framework for interpreting negation in non-classical logics. We hope to elaborate the idea elsewhere.

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