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INDEXED ADAPTIVE LOGICS*

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1. *Introduction*

A system dealing with knowledge and information, as, for example, our brain, is never complete nor fully integrated and contains seldom merely true information. It is likely that inconsistencies arise in such a system when information is retrieved, especially when new (true or false) information is added. This new information can be provided by more than one source (newspapers, books, radio, people, etc.) and, obviously, the more different the sources, the greater the chance of inconsistency.

In *standard* (as opposed to *indexed*) inconsistency-adaptive logics (SIAL) it does not make any difference from which source which bits of information were acquired; once proved that a formula p behaves inconsistently (both p and $\sim p$ can be inferred from the premises), one cannot any longer infer, for example, q from $\{p \vee q, \ \sim p\}$.

But what if $p \vee q$ and $\sim p$ were provided by some source S_1 and p by another source S_2? Presumably, source S_1 would also shoulder q, but a SIAL-system would not be able to draw this conclusion because the appearance of p, provided by S_2, forbids the inference.[1]

In the present paper, a logic is devised that is able to cope with this problem and, in general, with epistemic notions like *update* and *revision*. The major advantage of this logic is that, using indices to represent different sources, it is capable of distinguishing between sources and the information provided by them. Thus it offers a richer inference relation than standard inconsistency adaptive logics.

This logic, which I shall call IIAL from now on, is based on the Inconsistency Adaptive Logic APIL1, as presented in [Batens 199+]. As

* I wish to thank Diderik Batens and Natasha Kurtonina for valuable comments and interesting discussions.

1 Since we cannot any longer rely on the consistent behaviour of p, the inference $p \vee q, \ \sim p \vdash q$ is not justified.

we shall see, APIL1 can be seen as a special case of this indexed logic, namely the case in which only one source is involved.

2. *A general knowledge system*

In order to get a general view on the problem and to show the relevance of this logic, I shall set forth a general scheme that represents a knowledge system and shows how such a system might work, which mechanisms and processes are involved and what kind of relations are relevant.

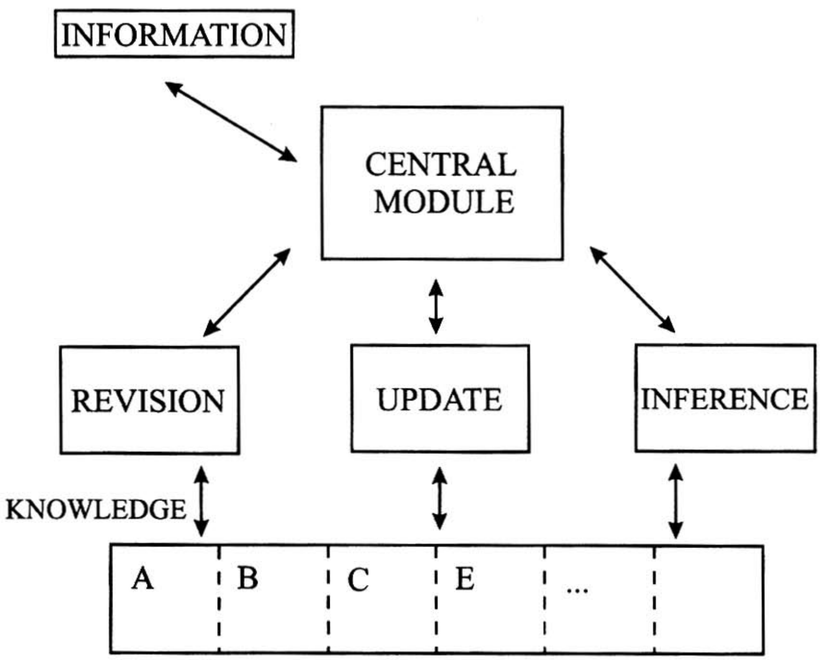


Fig. A

There are six major modules and seven arrows that represent relations between the modules.

2.1. *Information*: this is the module that provides the system with new information. It is responsible for the system's input. In a human knowledge system, new information is generated by the senses, like reading a newspaper, listening to someone, etc. The module is the bridge between external information sources and the system; it makes it possible to communicate with the world. I will not get into the details of this module right now; let us assume that the system receives information through this channel, no matter how this works.

In order to keep things simple, we assume that information is presented as logical well formed first order propositions, which can be represented by variables like

p = "It is raining."
$\sim q$ = "It is not the case that he is ill".

2.2. *Central Module*: this is the module that is responsible for all kinds of decisions to be made in the knowledge system; it *rules the roast*, so to speak. The central module e.g. decides to activate the Revision module when a contradiction occurs in the system, it determines which logic is best suited for which job, it evaluates the importance and relevance of situations with respect to the system etc. In a way we could say that the *CM* is the *rational motor* of the system.

2.3. *Update*: this is the module I am concerned most about in the present paper; its role is to extend the knowledge. As is well known, updating a knowledge system involves more than simply adding new information to the present system; the new information can be inconsistent, it can contradict the present knowledge, etc. The update module is responsible for examining the new information and the possible acceptance of it. I will concentrate on only one (and perhaps the most important) problem an update mechanism can come to face : contradiction.

2.4. *Inferences*: this part of the system deals with questions like "What can I infer from my present state of knowledge?", "Which premises do I have to accept in order to validate this conclusion?" etc. This module is *intern*, that is: it works on a given, as one could say, knowledge base. It is not directly connected with the external world, but of course, during an inference process, it is very well possible that, e.g., contradictions or other kinds of problems arise, on which the central module can decide to activate an update or revision process.

2.5. *Revision*: As already stated, inconsistencies are a major problem in a knowledge system. The difficulty is not just the contradiction itself, which can be easily eliminated by rejecting one of the two parts, say "p" or "~p", because most of the times, a contradiction does not come out of the blue, but *arises*; that is to say, it is inferred from the system. Thus, in order to get rid of it, one has to revise all the relevant information (or knowledge) out of which the contradiction was inferred.

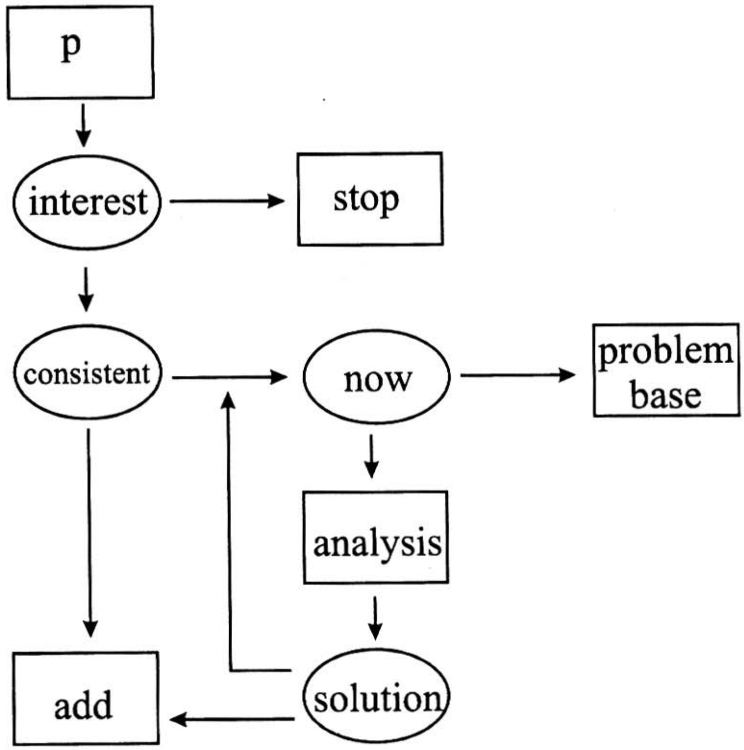
2.6. *Knowledge*: this module contains the bare data, information, structured in several *contexts*. This part of the scheme is of course very misleading; it suggests that there is a well defined and distinct part of the system which contains the data, while other parts contain inference rules, contradiction solving mechanisms and so on, while, from a contextual point of view, this is of course far from realistic. But this scheme is only a way of making certain epistemic notions and processes clear, it is not an attempt to give a realistic look upon a, for instance, human knowledge system. It could, however, be a pretty good description of a classic computer expert system, in which there is indeed a sharp distinction between data and algorithms.

3. *Update*

As said before, the update process copes with new information, provided by the *information* module. When we receive new information, we certainly do not automatically accept it. A lot of factors are involved. A famous rule that is frequently used in non-monotonic contexts is:

When it is consistent to accept "*p*", accept "*p*",
otherwise, reject "*p*".

There is a lot of sense in this rule, but an update process is not as easy as that, for the rule presupposes that the knowledge already in the system is better or more reliable than the new information presented. Also, the rule doesnt tell us what to do when confronted with inconsistent new information, e.g., some source *A* says "*p*" and another source *B* tells us "~*p*" (except that we cannot accept the information from both *A* and *B* at the same time). Let's examine an update process a bit closer.



"flow chart" scheme representing an update mechanism.

1. The new information, e.g., $\{p\}$, or $\{p, q, \sim s\}$. It is important to note that we do not only receive the information itself (e.g., "It is raining in Saint-Andrews") but that we also know where we get this information from (like: Stephen told me that "it is raining in Saint-Andrews"). As we shall see, the origin of information is crucial in update processes.

2. Interest: is the system interested in this information, is the information relevant for the system? E.g., if I read in a newspaper that astronomers have discovered a new star in some galaxy 5F214aG, which is more than 5000 lightyears from Earth, then I shall probably soon forget it and I certainly will not consider whether this information is consistent with my (little) knowledge about astronomy (like "Is it possible that astronomers can observe stars over such a distance?"). On the other hand, if I were an astronomer, this information could be very interesting to me and I would probably want to examine it.

3. Consistency: is this new information consistent with what I already know? If so, then I can simply add it to my present knowledge system (this is the nonmonotonic rule we saw earlier), if not, I will have to investigate the matter (this is what is missing in the rule).

4. Examine: but it is very well possible that I don't have the time or the ability right now to examine or solve this problem, in that case I put the problem in a so called problem base; I know I've got a problem here (a contradiction) but I send it to Coventry, until I have got the time, energy, extra information or whatever necessary for solving it.

5. Analysis: this is where the logic shows up; exactly what information is inconsistent?, which premises do I have to accept/reject in order to get rid of the contradiction?, Is it possible to eliminate the contradiction without having to revise my whole knowledge system? etc. As mentioned before, an important factor in this process is the source from which the information was achieved. It can be helpful to know that e.g. "p" comes from hearing say and "$\sim p$" from a highly reliable eye-witness, or that source A is contradicting itself, while source B seems to provide consistent information etc.

4. *An indexed adaptive logic*

A logic that has to deal with such an update process has to be able to deal with contradictions (paraconsistency), it has to be non-monotonic (new information can defeat older information) and, of course, it has to be able to keep track of all the different sources we get our information from. A major upshot of using indices representing different sources, is that the logic becomes, as we shall see, even richer than the standard adaptive logics, and, after all, the purpose of an update is to enrich the knowledge system.

The core logic is Batens' inconsistency adaptive logic as presented in (Batens, 199+). In 4.1 I briefly describe APIL1. In section 4.2 an indexed version of APIL1, IIAL, is introduced.

4.1 *The Inconsistency Adaptive Logic APIL1*

First I make some preliminary remarks. By a theory $T = \langle \Gamma, \mathrm{L} \rangle$ I shall mean $Cn_L(\Gamma)$: the deductive closure defined by a logic L, of a set of axioms Γ. We shall say that Γ has *abnormal properties* (with respect to L) if Γ violates some of the presuppositions of L. An adaptive logic La localizes the abnormal properties of Γ. It behaves exactly like L for all the normal consequences of Γ but prevents specific rules of L from being applied to the abnormal consequences of Γ. In the proof theory, we will assume that a formula behaves normally unless proved otherwise. A straightforward example : La would allow us to infer p from $\{p \vee q, \ \sim q\}$ unless q is proved to behave abnormally ('q behaves inconsistently', 'q is unreliable').

The logic APIL1 is based on the monotonic paraconsistent logic PIL which is derived from classical logic (CL) by dropping the consistency presupposition: if $v_M(A)=1$ then $v_M(\sim A)=0$. Let $\exists(A\&\sim A)$ be $(\exists\alpha_1)\ldots(\exists\alpha_k)(A\&\sim A)$ where $(A\&\sim A)$ is a formula in which the variables $\alpha_1, \ldots, \alpha_k$ $(k \geq 0)$ occur free. Let $DEK\{A_1, \ldots, A_n\}$ refer to $\exists(A_1\&\sim A_1)\vee\ldots\vee\exists(A_n\&\sim A_n)$. $DEK\{A_1, \ldots, A_n\}$ is a disjunction of (where necessary) existentially quantified contradictions. $A_1, \ldots, A_n$ are the *factors* of $DEK\{A_1, \ldots, A_n\}$. The following theorem is proved in [Batens 199+]

Theorem 1. If $\vdash_{CL} A$, then, for some $C_1, \ldots, C_n (n \geq 0)$, $\vdash_{PIL} DEK\{C_1, \ldots, C_n\} \vee A$

An illustration of this theorem is the PIL-theorem $(\exists x)(Qx\&\sim Qx)\vee (((\forall x)(Px \supset Qx)\&(\exists x)\sim Qx)\supset(\exists x)\sim Px)$. Intuitively, this suggests that we derive $(\exists x)\sim Px$ from $(\forall x)(Px \supset Qx)\&(\exists x)\sim Qx$ provided Q behaves consistently.

The proof theory of the inconsistency adaptive logic APIL1 is constructed in the following way: we apply all rules of PIL unconditionally whereas other rules of classical logic (like, e.g., disjunctive syllogism) are applied on the condition that certain formulas are *reliable* with respect to their consistent behaviour. In this sense, the logic APIL1 is adaptive.

A line in an APIL1 proof consists of five elements; a line number, the derived formula, the line numbers of wffs from which the formula is derived, the inference rule that justifies the derivation and the set of formulas on which consistent behaviour we rely in order to make the derivation.

We say that a formula A *occurs unconditionally* at some line of the proof iff the fifth element of that line is empty. A formula *A behaves consistently* at a stage in the proof iff $\exists(A\&\sim A)$ does not occur unconditionally in that proof (at that stage). There is a complication: formulae can behave consistently without being reliable, for their consistent behaviour may be *connected* to the consistent behaviour of other formulae. Suppose for example that $(p\&\sim p)\vee(q\&\sim q)$ occurs unconditionally at a stage in a proof. Both p and q are unreliable although none of them behave inconsistently at that stage. The occurrence of, e.g., $(p\&\sim p)$ in that same proof would make q reliable again. In general, the consistent behaviour of A_1 is *connected to* the consistent behaviour of $A_2, ..., A_n$ *at a stage of a proof* if $DEK\{A_1, ..., A_n\}$ occurs unconditionally in the proof at that stage whereas $DEK\{A_2, ..., A_n\}$ does not occur unconditionally in it. A formula A is *reliable*, at a stage in the proof, iff it behaves consistently and its consistent behaviour is not connected to the consistent behaviour of any other formula.

Proofs in APIL1 are governed by three rules. The first is an *unconditional rule* (UR) which states that whenever $\vdash_{PIL}(A_1 \& ... \& A_n) \supset B$, and $A_1, ..., A_n$ occur in a proof, add B to the proof. The second rule is a *conditional rule* (CR): If $\vdash_{PIL} DEK\{C_1, ..., C_m\} \vee ((A_1 \& ... \& A_n) \supset B)$ and $A_1, ..., A_n$ occur in the proof, then add B to the proof *provided* that each factor of $DEK\{C_1, ..., C_m\}$ is reliable at that stage. The third rule is the *deletion rule*, which is obligatory applied at any stage of the proof, and states that if C is not (any more) reliable, all lines that contain C in their fifth element must be deleted.

As APIL1 is a dynamic logic, in which a formula derived at some stage can be deleted at a later stage, it is necessary to distinguish between *provisional* and *final* consequences. A formula is *finally derived* when it will not be deleted in any intelligent extension of the proof[2] and [199+]. APIL1 consequences from Γ are those consequences that are finally derived from Γ.

The semantics for APIL1 is obtained by defining for each Γ a subset of the PIL-models of Γ. The APIL1-models of Γ are those PIL-models in which only unreliable formulae behave inconsistently. To define this semantically, we first define: *a minimal DEK-consequence of* Γ is a formula DEK(Δ) such that (i) DEK(Δ) is true in all PIL-models of Γ and (ii) for all $\Theta \subset \Delta$, DEK(Θ) is false in some PIL-models of Γ. Let U(Γ) be the set of factors of minimal DEK-consequences of Γ. Furthermore, where M is PIL-model, let $EK(M) = \{A | v_M(\exists(A \& \sim A)) = 1\}$ Then, a PIL-model M is *maximally APIL1-normal* with respect to Γ iff (i) M is a PIL-model of Γ and (ii) $EK(M) \subseteq U(\Gamma)$. Finally, M is an *APIL1-model* of Γ iff M is maximally APIL1-normal with respect to Γ.

4.2 *The indexed inconsistency adaptive logic IIAL*

Given the proof theory and semantics of APIL1, we can now proceed with the construction of the indexed inconsistency adaptive logic. For the sake of simplicity, I shall use propositional formulae in the examples.

2 for the definition of "intelligent extension of a proof" see Batens [1989]

We introduce indices in the following way: we write p^A to express that p provides from source A. For example, p^1, $(p \supset q)^2 / q^{1 \cup 2}$. More generally, A^Ψ mean that A was derived from the information provided from the set of sources Ψ.

I shall get to the proof theory and semantics right away, but first, in order to show how introducing indices and thus distinguishing between different sources, makes the logic APIL1 richer, let's take a look at some examples.

Suppose two sources S and T provide some information.

$S = \{\sim p \& r,\ q \supset p,\ s \supset \sim r\}$

$T = \{q \vee \sim r\}$

It is clear that the union of this information is inconsistent. An APIL1 proof could proceed as follows (pay no attention to the indices yet):

1	$\sim p \& r^S$	-	prem	$\{\}$
2	$q \supset p^S$	-	prem	$\{\}$
3	$s \supset \sim r^S$	-	prem	$\{\}$
4	$q \vee \sim r^T$	-	prem	$\{\}$
5	$\sim p^S$	1	$A \& B / A$	$\{\}$
6	$\sim q^S$	2,5	$A \supset B,\ \sim B / \sim A$	$\{p\}$
7	$\sim r^{S \cup T}$	4,6	$A \vee B,\ \sim A / B$	$\{p,q\}$
8	r^S	1	$A \& B / A$	$\{\}$
9	$r \& \sim r^{S \cup T}$	7,8	$A,\ B / A \& B$	$\{p,q\}$

At this stage we cannot any longer rely on the consistency of r, consequently the following step is not allowed:

10	$\sim s^S$	3,8	$A \supset B,\ \sim B / \sim A$	$\{r\}$

But the inconsistency of r was yielded by the unification of S and T, so if we would restrict to S only, in which, at least at this stage of the proof, r is consistent, we should be able to derive $\sim s^S$ Thus, restricting to S, it is possible to draw the conclusion that $\sim s$ follows directly from the information provided by S.

Let us take a look at a second example:

$S = \{\sim p,\ p \vee q\}$

$T = \{p\}$

	APIL1		IIAL
1	$\sim p$	1	$\sim p^S$
2	$p \vee q$	2	$p \vee q^S$
3	p	3	p^T
		4	q^S

Thus, the indexed adaptive logic, makes it possible to infer that source S *implicitly* tells us that q is true.

IIAL is a generalization of APIL1, the latter corresponding to the case in which only one source is considered. It does not make any difference whether inconsistencies occur as result of combining different sources or whether a source itself contains contradictory information.

Note that not only sources can be introduced in the inference process; the indices may also represent contexts, periods of time, etc. E.g.: the civil engineer who assumes that $F = m.a$ when calculating the strength of a bridge (context 1), but knows that $F \neq m.a$ when he is studying modern physics (context 2). The engineer's two beliefs are not problematic, as long as he does not mix up the two contexts. This knowledge structure can be formalized using IIAL.

4.3 *Proof theory for IIAL*

Definition A^Ψ *occurs unconditionally* at some line of proof iff the fifth element of that line is empty.

Definition A behaves consistently with respect to Φ *at a stage of a proof* iff there is no $\Psi \subseteq \Phi$ such that $\exists(A \& \sim A)^\Psi$ occurs unconditionally in the proof at that stage.

Definition The consistent behaviour of A_1 *is connected to the consistent behaviour of* $A_2, \ldots, A_n$ *with respect to* Φ *at a stage of a proof* if there is a $\Psi \subseteq \Phi$ such that $\mathrm{DEK}\{A_1, \ldots, A_n\}^\Psi$ occurs unconditionally at that stage whereas $\mathrm{DEK}\{A_2, \ldots, A_n\}^\Psi$ does not.

Definition A is *reliable with respect to* Φ *at a stage of a proof* iff A behaves consistently with respect to Φ at that stage and its consistent behaviour is not connected to the consistent behaviour of any other formula with respect to Φ.

Unconditional Rule. If $\vdash_{\text{PIL}}(A_1\&...\&A_n)\supset B$, and $A_1^{X_1}\&...\&A_n^{X_k}$ occur in the proof, then add $B^{X1\cup...\cup Xk}$ to the proof; the fifth element of the new line is the union of the fifth elements of the lines on which $A_1^{X_1}\&...\&A_n^{X_k}$ occur.

Conditional Rule. If $\vdash_{\text{PIL}}\text{DEK}\{C_1, ..., C_m\}\vee((A_1\&...\&A_n)\supset B)$, and $A_1^{X_1}\&...\&A_n^{X_m}$ occur in the proof, then add $B^{X1\cup...\cup Xk}$ to the proof, *provided* that each of C_1, ..., C_m is reliable with respect to $\{x_1, ..., x_n\}$ at that stage. The fifth element of the new line is the union of C_1, ..., C_m and of the fifth elements the lines on which $A_1^{X_1}\&...\&A_n^{X_m}$ occur.

Deletion Rule. If C is no longer reliable with respect to Φ delete from the proof all the lines the fifth element of which contains C and the second element of which is of the form D^{Φ}.

At any stage in the proof it is obligatory to apply the Deletion Rule.

4.4 *Semantics for IIAL*

In IIAL, the notions *minimal DEK-consequence*, *unreliable formula* and U(Γ) are defined as in APIL1 (see section 4.1).

Definition Let $\Gamma^{\{x_1, ..., x_n\}}=\Gamma^{x_1}\cup...\cup\Gamma^{x_n}$ A formula A^{Φ} is *finally derivable* in an IIAL-proof iff $\Gamma^{\Phi}\vdash_{\text{APIL1}}A$

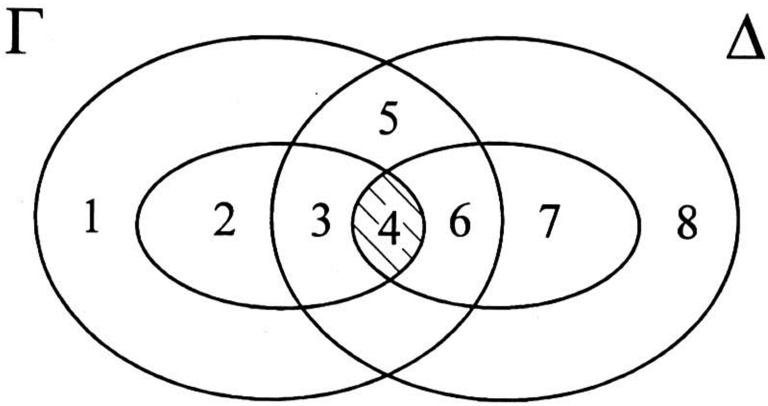
Definition A formula A is a *semantical consequence* of Γ^{Φ} iff A is true in all APIL1-models of Γ^{Φ}.

It is interesting to examine some example cases in which the models of different sources and the models of their union is compared. For simplicty's sake, let us restrict to those cases in which only two sources, Γ and Δ, are involved. Let me remind the reader that U(Γ) is the set of factors of minimal DEK-consequences of Γ.

Case 1 : $\Gamma=\{p,\ q\&s\},\ \Delta=\{p\vee\sim s,\ r\}$

In this case, Γ, Δ and $\Gamma\cup\Delta$ are consistent; U(Γ) = U(Δ) = U($\Gamma\cup\Delta$) = $\varnothing$.

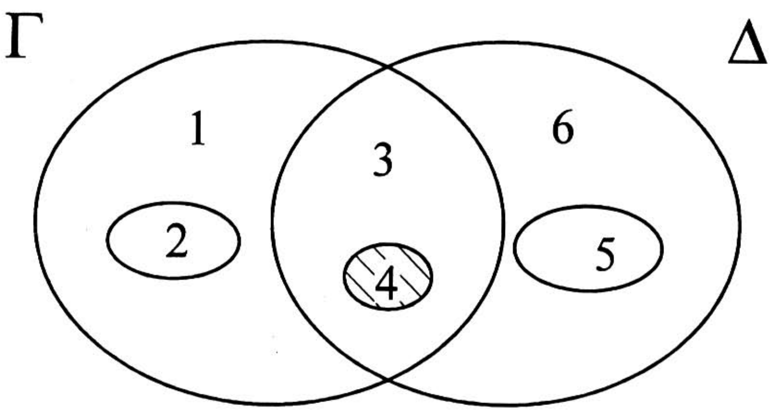
In the diagrams below, the larger ellipses represent the PIL-models of Γ and Δ respectively. The smaller ellipses represent the APIL1-models. In this case, all the APIL1-models of Γ (the models in sections 2-4), Δ (sections 4-7) and $\Gamma\cup\Delta$ (section 4) are consistent.



Case 1

Case 2 : $\Gamma = \{\sim p,\ p \vee q\},\ \Delta = \{p\}$

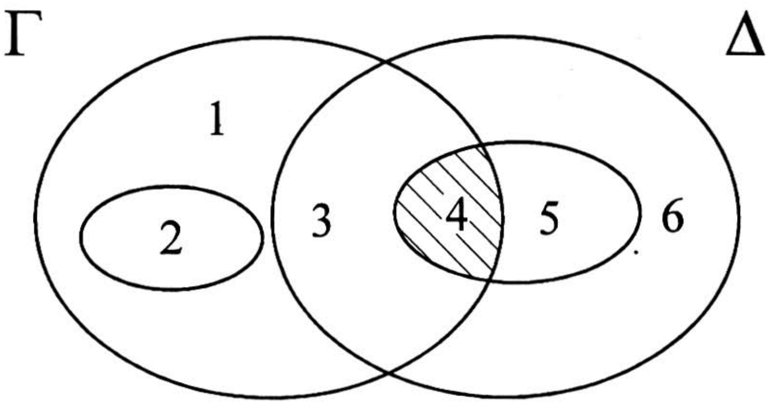
In this case, the union of the two sources yields a *new* inconsistency for both Γ and Δ; U(Γ) = U(Δ) = ∅ and U(Γ ∪ Δ) = $\{p\}$. It is clear that, e.g., $\Gamma \cup \Delta \nvdash_{\text{APIL1}} q$ and $\Gamma \cup \Delta \nvdash_{\text{APIL1}} (p \& \sim p) \supset r$. Since both the set of APIL1-models for Γ (section 2) and the set of APIL1-models of Δ (section 5) contain only consistent models, there are no common APIL1-models, because the union of Γ and Δ is inconsistent which implies that all the APIL1-models of Γ ∪ Δ (section 4) are inconsistent, too.



Case 2

Case 3 : $\Gamma=\{\sim p,\ p\vee q,\ r\},\ \Delta=\{\mathrm{DEK}(p)\}$

We have that Γ is consistent, Δ contains unreliable formula's and the union of Γ and Δ yields no new unreliable formula's; $U(\Gamma)=\varnothing$, $U(\Delta)=\{p\}$ and $U(\Gamma\cup\Delta)\subseteq U(\Delta)$. All the APIL1-models of Γ are consistent (section 2) while all the APIL1-models of Δ are inconsistent (sections 4-5). The APIL1-models of $\Gamma\cup\Delta$ are in the intersection of the PIL-models of Γ and the APIL1-models of Δ (section 4). In section 5 are models that contain *p*&~*p* and ~*r*. In the case that $\Gamma\cup\Delta$, there would be no models in sections 5 and 6.



Case 3

Case 4 : $\Gamma=\{\sim p,\ p\vee q\},\ \Delta=\{p,\ \mathrm{DEK}(r)\}$

In this case, Γ is consistent, Δ is inconsistent and $\Gamma\cup\Delta$ contains unreliable formulas that are reliable in Δ; $U(\Gamma)=\varnothing$, $U(\Delta)=\{r\}$ and $U(\Gamma\cup\Delta)=\{p,r\}$. The diagram is the same as the one for case 2.

Case 5 : $\Gamma=\{\sim p\},\ \Delta=\{\mathrm{DEK}(p,\ q),\ p\}$

In Δ, DEK(*p*, *q*) is a minimal DEK-consequence (no result of dropping one of the disjuncts is a minimal DEK-consequence of Δ) while in $\Gamma\cup\Delta$ it is not. In other words, the union of Γ and Δ makes *q* reliable. The diagram is the same as case 3. Section 5 contains the models in which *q*&~*q* is true and $\sim p$ is false. In section 4 there are models in which $p\&\sim p$ and, for some, $q\&\sim q$ are true. The APIL1-models of $\Gamma \cup \Delta$ are those models in section 4 in which $q\&\sim q$ is false.

Case 6 : $\Gamma = \{DEK(p)\},\ \Delta = \{\mathrm{DEK}(q)\}$

In this case we have that $\mathrm{U}(\Gamma) \neq \mathrm{U}(\Delta)$ and $\mathrm{U}(\Gamma \cup \Delta) = \mathrm{U}(\Gamma) \cup \mathrm{U}(\Delta)$. The diagram is the same as for case 2. In all the APIL1-models of Γ (section 2) we have $p\&\sim p$ (no other contradictions), in all APIL1-models of Δ (section 5) we have $q\&\sim q$ (and no other contradictions) so there cannot be a common model. In the APIL1-models of $\Gamma \cup \Delta$ (section 4) both $p\&\sim p$ and $q\&\sim q$ are true.

Case 7 : $\Gamma = \{DEK(p),\ r\},\ \Delta = \{\mathrm{DEK}(p),\ \mathrm{DEK}(q)\}$

Since $\mathrm{U}(\Gamma)$ is a subset of $\mathrm{U}(\Delta)$, and no new inconsistencies arise in $\Gamma \cup \Delta$, $\mathrm{U}(\Gamma \cup \Delta) = U(\Delta)$. The minimal DEK-consequences of $\Gamma \cup \Delta$ are $\{\mathrm{DEK}(p),\ \mathrm{DEK}(q)\}$. This case is similar to case 3, except that the APIL1-models of Γ are inconsistent too.

Case 8 : $\Gamma = \{DEK(p,\ q),\ r\},\ \Delta = \{\mathrm{DEK}(p,\ q),\ s\}$

$\mathrm{U}(\Gamma) = \mathrm{U}(\Delta)$ and $\mathrm{U}(\Gamma \cup \Delta) = \mathrm{U}(\Gamma) \cup \mathrm{U}(\Delta)$.
This case is similar to case 1, except that all APIL1-models are inconsistent.

Case 9 : $\Gamma = \{p,\ DEK(q)\},\ \Delta = \{\sim p,\ \mathrm{DEK}(r)\}$.

Here we have that $\mathrm{U}(\Gamma) \neq \mathrm{U}(\Delta)$ and $\mathrm{U}(\Gamma) \cup \mathrm{U}(\Delta) \subset \mathrm{U}(\Gamma \cup \Delta)$. The minimal DEK-consequences of $\Gamma \cup \Delta$ are DEK(p), DEK(q), DEK(r). The diagram is similar to the one for case 2. The APIL1-models of $\Gamma \cup \Delta$ (section 4) have $p\&\sim p$, $q\&\sim q$ and $r\&\sim r$, while the only inconsistencies in the APIL1-models of Γ is $q\&\sim q$ and the APIL1-models of Δ $r\&\sim r$.

Case 10 : $\Gamma = \{DEK(p),\ r\},\ \Delta = \{\mathrm{DEK}(p),\ \sim r\}$

In this case, $U(\Gamma) = U(\Delta)$ and $U(\Gamma) \cup U(\Delta) \subset U(\Gamma \cup \Delta)$. This case is similar to case 2, except that all APIL1-models are inconsistent. The APIL1-models are in section 5.

Case 11 : $\Gamma = \{DEK(q), \ r\}, \ \Delta = \{p \vee s, \ \sim s, \ \mathrm{DEK}(q, \ s)\}$

The diagram is similar to the one in case 1. The only minimal DEK-consequences of $\Gamma \cup \Delta$ is DEK(q). The APIL1-models of $\Gamma \cup \Delta$ (section 4) are those in which the only contradiction is $p\&{\sim}p$.

6. *Conclusion*

The refinement of a knowledge base by keeping track of the sources from which information is retrieved, makes it possible to set up a richer consequence relation. IIAL provides such a consequence relation. Furthermore, since inconsistency adaptive logics handle inconsistent sets of premises in a very flexible and accurate way, IIAL offers an excellent basis for an epistemic logic.

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