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THE SIMPLEST MEINONGIAN LOGIC

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The expression *Meinongian logic* is rarely used[1]. However, we refer quite often to Meinong's views when logical problems of ontology, existence or inconsistency (paraconsistency) are discussed[2]. In particular, since the acceptance of an inflated sphere of nonexistent objects is likely the main point of Meinong's ontology according to the most popular interpretation, usually the ideas underlying free logics are associated with the name of this Austrian philosopher. But certainly the mere toleration of nonexistent objects is not enough to call a logic *Meinongian.* As a matter of fact, Meinong's ontology and especially his theory of objects is highly elaborate what, after all, makes it susceptible to many misinterpretations and misunderstandings[3]. Yet we can point to some distinctive principles which characterise the ontology more closely. Thus, according to Meinong, every object is constituted of properties and every set (class) of properties constitutes exactly one object. The set of properties associated with an object is interpreted as the *Sosein* of the object. An object possesses a property if it is its constitutive property or it is entailed by such a property. Objects need not exist in order to possess properties (are *ausserseiend*). Identical objects are constituted by the same properties. Since there are no limitations concerning the cardinality and qualitative selection of properties, objects can be incomplete and inconsistent (roughly an object is incomplete if for some property it neither possesses this property nor the complementary property; an object is inconsistent if it possesses con-

[1] This term was not used even by philosopher who first started to build Meinongian logics, i.e. Terence Parsons. Probably the only persons who use it frequently are the author of this paper and D. Jacquette; see his *Meinongian Logic. The Semantics of Existence and Non-existence* (to appear).

[2] As a good example we can mention here the monumental work of R. Routley, see his *Exploring Meinong's Jungle and Beyond*, Department Monograph #3, Philosophy Department, Research School of Social Sciences, Australian National University, Canberra 1980.

[3] B. Russell strongly criticised Meinong's ontology accusing it of inconsistency. Meinong reply for that criticism was very unconvincing and perhaps that is why his views have been forgotten for a long time. Only recently Meinong's ontology was rediscovered and a formally consistent interpretation of it was provided.

tradictory properties). Certainly one can gain a better intuitive understanding of the contents of Meinong's ontology when one thinks of Meinongian objects as possible objects of consciousness (such an interpretation helps despite the notorious ambiguity of the phrase "object of consciousness").

Recently several philosophers and logicians have made efforts to render Meinongian views consistent. There have appeared several theories of objects formalising Meinong's ontology, or, which are inspired by it. Such theories have been created by T. Parsons, R. Routley, W. Rapaport, E. Zalta, D. Jacquette, J. Paśniczek[4]. Some of these theories may be considered as logics sensu stricto, just as *Meinongian logics*. The most developed and known are Parsons' and Zalta's logics. These logics are strong second-order intensional systems based on a very intricate language and semantics. As such they are extremely complicated. At the same time, their deductive side is rather trivial - almost nothing interesting can be deduced from the axioms what has not already been expressed explicitly in the axioms themselves. Perhaps that is why the logics are unattractive not only for philosophers but for logicians as well. There arises the question whether any logic which accommodates the basic principles of Meinong's ontology must be necessarily so complicated.

I am going to develop in this paper a quite simple logic - *M*-logic - which (despite its simplicity) might be considered as a basic Meinongian logic. I call *M*-logic "the simplest" one for the following reasons: (1) it is extensional in the sense that properties and relations are represented only by their extensions, (2) it differs slightly from classical first-order logic (sharing the same alphabet), (3) it is a first order system, (4) it is closer to the natural language syntax than classical logic. Generally, *M*-logic has smaller expressive power than Parsons' and Zalta's logics. However, due to its naturalness and simplicity, *M*-logic can be easily extended to stronger systems, e.g. intensional and second-order ones. These systems fulfil another principle of Meinong's ontology (besides those mentioned above)

[4] Cf. T. Parsons, *Nonexistent Objects*, Yale University Press, 1982; R, Routley, op. cit.; William Rapaport, Meinongian Theories and Russellian Paradox, *Noūs* 12, 1978; Nonexistent Objects and Epistemological Ontology, *Grazer Philosophische Studien* Vol. 25/26 1985; E. Zalta, *Abstract Objects: An Introduction to Axiomatic Metaphysics*, Dordrecht: Reidel 1983; *Intensional Logic and the Metaphysics of Intentionality*, MIT Press, 1988; D, Jacquette, op. cit.; K. Perszyk, *Nonexistent Objects. Meinong and Contemporary Philosophy* Kluwer, Nijhoff 1993; J. Paśniczek, *Meinongowska wersja logiki klasycznej. Jej zwiazki z filozofia jezyka, poznania, bytu i fikcji* (in Polish: The Meinongian Version of Classical Logic. Its Relevance for the Philosophy of Language, Knowledge, Being, and Fiction), M. Curie-Sklodowska University Press 1988.

which is particularly troublesome for logical treatment - we call it the *dual predication principle*. In the second part of my paper I will mention ways of carrying out such extensions. Curiously enough, second-order versions of *M*-logic look almost the same as *M*-logic itself and the essential difference lies in relaxing the grammar rules of the formal language.

I

The main problem which we must face when starting to build a Meinongian logic is the problem of a proper choice of formal language. It is commonly shared opinion that the logic should be bivalent[5]. Thus we cannot express predication in the classical first-order language since in the case of incomplete and inconsistent objects it would lead to contradiction. So, how is it possible to render consistently sentences like *a is squared, a is not squared* on the ground of Meinongian logic? This can be carried out in the simplest way by introducing complex predicates correlated semantically with complex properties; in particular the negation of the predicate *is squared* will be the predicate *is not squared* (or better: *is non-squared*). Obviously, the predication involving complex predicates cannot be generally reducible to the predication involving only simple predicates together with respective sentential connectives. For the sake of generality, we should introduce more complex predicates than negations, at least all those that can be expressed in the language of first-order logic. Usually we do that by means of the awkward lambda notation[6]. However the same can be done by appealing exclusively to the primitive symbols of classical logic. In particular, the negation of *P* we express simply by $x\neg Px$ (instead of expressing it by $[\lambda x\neg Px]$). Generally, any expression of the form xA, where A is a formula, will be a predicate expression. Subject-predicate sentences with the subject a will be represented by formulas of the form axA. For example the following sentences: *John is not silly, John is silly and lazy, John loves himself, John loves Mary, Mary is loved by John, John loves somebody, Somebody is loved by John* will have the following forms respectively: $ax\neg Px$, $ax(Px \wedge Qx)$, $axRxx$, $axbyRxy$, $byaxRxy$, $ax\exists yRxy$, $\exists yaxRxy$. We see that in this new syntax constants occupy the same places as quantifiers in the classical syntax. And this is the very idea of the formal language for our Meinongian logic, call it *M*-language. So in *M*-language constants and quantifiers are of the same category - the *term* category. Informally, we consider terms to be semantically correlated with objects. It

[5] There is one exception - D.Jacquette claims that such a logic must be three-valued and he applies Łukasiewicz logic to construct Meinongian logic, cf. op. cit.

[6] Cf. Parsons and Zalta, op.cit.

means that not only constants but quantifiers as well are categorematic expressions. Perhaps the idea is not very astonishing. Meinong himself ascribed quantifier objects, i.e. objects such as *every dog, some dog*, to his ontology[7]. Also Frege interpreted quantifiers as objects of special kind: second-order concepts. Besides, it is commonplace in the contemporary philosophy of language to treat names and quantifiers as expressions of the same category. Here we have in mind *R*. Montague's works and current semantical analyses of noun phrases by means of the notion of generalised quantifiers[8].

Now let us describe *M*-logic in some detail.

M-language

As we hinted earlier, the alphabet for *M*-language consists of the same symbols as the alphabet of classical logic, i.e.:

(1) sentential connectives: $\neg$, $\supset$ (the other sentential connectives are introduced by means of the usual definitions).
(2) the universal quantifier symbol: $\forall$.
(3) the identity symbol: =.
(4) individual variables: $x_1, x_2, \ldots$
(5) constants: $a_1, a_2, \ldots$
(6) predicate symbols: $P_1, P_2, \ldots$
(7) brackets: (,).

Let us assume that metavariables *s,t* range over terms, i.e. symbols listed in (2) and (5); $x, y, y_1, y_2, \ldots$ over variables.

The grammar of *M*-language is defined as follows: (a) every expression of the form $Py_1 \ldots y_n$ and $x = y$ is a formula; (b) if *A, B* are formulas, then

[7] I rely here on Grossmann's interpretation of Meinong, Cf. Reinhardt Grossmann, *Meinong*, Routledge & Kegan Paul 1974.

[8] Cf. Richard Montague, The Proper Treatment of Quantification in Ordinary English, in: *Formal Philosophy*, ed. Richmond Thomason, Yale University Press, 1974. From the bulk of current literature concerning generalised quantifiers we may recommend two papers: J. Barwise & R. Cooper, Generalized Quantifiers and Natural Language, *Linguistics and Philosophy*, 4 1981; D. Westerståhl, Quantifiers in Formal and Natural Languages, in: D. Gabbay and F. Guenthner, *Handbook of Philosophical Logic*, Vol. IV, 1989.

$\neg A$ and $(A \supset B)$ are formulas; (c) if A is a formula then xA is a *predicate*; (d) if Π is a predicate then $t\Pi$ is a formula[9].

M-system

A M-system is defined by the following axiom-schemata and rules of inference:

$M1$ Classical truth-functional tautologies.
$M2$ $\forall x(A \supset B) \supset (txA \supset txB)$
$M3$ $A \supset \forall xA$, provided x is not free in A.
$M4$ $\forall xA \supset A(y \mid x)$
$M5$ $txA \supset tyA(y \mid x)$, where $A(y \mid x)$ is obtained from A by freely substituting every occurence of x by y.
$M6$ $x=x$
$M7$ $x = y \supset (A \supset A(y \| x))$, where $A(y \| x)$ results from A by freely substituting every or some occurence of x by y.
MP if $\vdash_M A \supset B$ and $\vdash_M A$ then $\vdash_M B$
MG if $\vdash_M A$ then $\vdash_M txA$ and $\vdash_M \neg tx\neg A$

Let us list now some selected theorems of M-system.

$M8$ $tx(A \supset B) \equiv (A \supset txB)$, provided x is not free in A
$M9$ $\neg tx\neg(A \supset B) \equiv (txA \supset B)$, provided x is not free in B
$M10$ $\forall xA \supset txA$
$M11$ $txA \supset \exists xA$
$M12$ $tx(txA \supset A)$
$M13$ $tx(A \wedge B) \supset (txA \wedge txB)$
$M14$ $\exists x\neg ty(x \neq y) \supset (tx\neg A \supset \neg txA)$
$M15$ $\exists xty(x = y) \supset (\neg txA \supset tx\neg A)$
$M16$ $tx\neg ty(x \neq y) \supset (txA \wedge txA \supset tx(A \wedge B))$
$M17$ $\neg tx\neg ty(x = y) \supset (tx(A \vee B) \supset txA \vee txB)$
$M18$ $\neg sx\neg ty(x = y) \supset (sxA \supset txB)$

[9] We can define the set of formulas in the simpler way replacing conditions (c) and (d) by the single condition: if A is a formula then txA is a formula.

$M19 \quad tx(x = y) \wedge \neg tx(x \neq y) \supset (txA \equiv A(y \mid x))$[10]

M-semantics

By a model of a M-language we mean a pair $M = [D, I]$ where D is a non-empty set called the domain of interpretation, I is a function defined on terms and predicate symbols called the interpretation:

(a) $I(t) \subset \mathcal{P}(D)$, where $\mathcal{P}(D)$ is the power set of D, $I(t) \neq \varnothing$ and $I(t) \neq \{\varnothing\}$; in particular $\mathrm{I}(\forall) = \{D\}$
(b) $I(P) \subset D^n$, for a n-argument predicate symbol P

[10] Proofs of these theorems usually proceed along the same lines as those of classical first-order logic, although sometimes are quite elaborate. Let us prove for example the theorem $M8$. Let x be not free in A.

1. $A \supset ((A \supset B) \supset B)$ $(M1)$
2. $\forall x(A \supset ((A \supset B) \supset B))$ (MG)
3. $\forall xA \supset \forall x((A \supset B) \supset B)$ $(M2)$
4. $A \supset \forall x((A \supset B) \supset B)$ $(3, M3)$
5. $A \supset (tx(A \supset B) \supset txB)$ $(4, M2)$
6. $tx(A \supset B) \supset (A \supset txB)$ (5)
7. $B \supset (A \supset B)$ $(M1)$
8. $\forall x(B \supset (A \supset B))$ (MG)
9. $txB \supset tx(A \supset B)$ $(M2)$
10. $\neg A \supset (A \supset B)$ $(M1)$
11. $tx(\neg A \supset (A \supset B))$ (MG)
12. $\neg A \supset tx(A \supset B)$ $(11, 6)$
13. $(A \supset txB) \supset tx(A \supset B)$ $(9, 12)$
14. $tx(A \supset B) \equiv (A \supset txB)$ $(6, 13)$

From M8 we easily get $M10$ and $M12$ (starting respectively from $M4$ and $txA \supset txA$).

An important property of M-system is duality: if we replace in any theorem-scheme the metavariable 't' by the expression '$\neg t \neg$', in all places where 't' appeares, then the resulting formula will be a theorem-schema as well. The proof of the duality follows straightforwardly from the structure of axiomatics for M-system. The property of M-system allows to obtain theorems dual to theorems already proved. E.g. from $M10$ we get: $\forall xA \supset \neg tx\neg A$, and consequently $M11$. $M9$ is "almost" dual to $M8$, $M14$ to $M15$, $M16$ to $M17$.

An assignment in D is a function V which assigns to every variable an element of D. Given V, by V^x_d we mean the function which is just like V except possibly $V^x_d(x)=d$.

In M-semantics truth conditions for atomic formulas, for negation and implication are the same as in classical semantics. What is new in M-semantics is the truth condition for predication:

Formula txA is true in M with respect to the assignment V iff there exist
(*) $X \in I(t)$ such that:
$X \subset \left[d \in D\text{: } A \text{ is true in } M \text{ with respect to } V^x_d\right]$.

In particular, the formula $txPx$ is true in M iff there exists $X \in I(t)$ such that $X \subset I(P)$. Notice also that the condition retains the meaning of the universal quantifier with respect to the given interpretation.

A formula of M-language is M-valid iff it is true in every M-model with respect to any assignment. For M-logic completeness, compactness, and Skolem-Löwenheim theorems hold[11].

It is easily seen that M-logic closely resembles the classical first-order logic. Axioms $M1$, $M3$, $M4$, $M6$, M7 and MP rule are usually adopted for the classical system. The only additional axiom and rule of inference needed for the complete axiomatisation of classical logic are special cases of $M2$ and MG respectively (we mean here the distribution of universal quantifier and the ordinary generalisation rule). Thus, M-logic turns out to be an extension of classical logic and an essential extension indeed. Its

[11] It is worth emphasising that the completeness of M-logic easily follows from the completeness of the classical first-order logic. Compactness and Skolem-Löwenheim theorems hold for the same reasons as they hold for the classical logic.

Let us outline the completeness proof. The soundness of M-system can be proved in an obvious way. Suppose $\models_M A$ i.e. that A is M-valid. For M-logic there holds a theorem which is a generalisation of classical theorem on *prenex normal form*. The theorem says that every formula is logically equivalent to a formula in *prenex normal form*. In particular the formula A will be equivalent to a formula $t_1y_1, \ldots, t_ny_nB$ such that (1) for every term t occurring in A, t itself or the term dual to t, i.e. the term $s = \neg t \neg$ is among $t_1, \ldots, t_n$; (2) B is a term-free formula (the theorem on the prenex normal form for M-logic follows from the duality of M-system and from some of its characteristic theorems such as $M8$ and $M9$). Thus $\models_M t_1y_1, \ldots, t_ny_nB$. It means that also the formula $Q_1y_1, \ldots, Q_ny_nB$ where $Q_i = \exists$ if $t_i = \exists$ and $Q_i = \forall$ otherwise, is M-valid (for if a formula is valid for a constant term t it is valid for the universal quantifier put in the place of t as well). But $Q_1y_1, \ldots, Q_ny_nB$ is a classical formula. Since for such formulas the classical semantics is equivalent to M-semantics then $\models_C Q_1y_1, \ldots, Q_ny_nB$. And by the completeness of classical logic: $\vdash_C Q_1y_1, \ldots, Q_ny_nB$. As we have mentioned, M-system contains the classical system. So $\vdash_M Q_1y_1, \ldots, Q_ny_nB$. From M 2 and M 10 it follows that $\vdash_M Q_1y_1, \ldots, Q_ny_nB \supset t_1y_1, \ldots, t_ny_nB$ and thus $\vdash_M t_1y_1, \ldots, t_ny_nB$. Applying once again the theorem on the prenex normal form we get $\models_M A$.

expressive power, especially that concerning ontological matters, far exceeds everything that can be said on the ground of classical logic. Yet *M*-logic remains a first-order logic[12]. Perhaps it is worth mentioning here that *M*-logic can be considered as a logic of unary monotone increasing quantifiers[13].

Let us comment briefly on the Meinongian character of *M*-logic. According to *M*-semantics, Meinongian objects are represented in the logic by sets of sets of individuals what is tantamount to extensionally treating objects as sets of properties (see the definition of interpretation). A given object possesses a property iff the extension of the property is equal to or broader than the extension of some property constituting the object (see the condition (*)). It is easily seen that inconsistent and incomplete objects can be available in *M*-semantics. For example, *the round square* is such an object when it is interpreted by the set: {*set of circles, set of squares*}. The object in question possesses properties: being round, being a square, being non-round, being a geometrical figure, but does not possess the property of having 1 m^2 area and the property of not having 1 m^2 area. a represents an inconsistent object if for some predicate P, both formulas $axPx$ and $ax\neg Px$ are true; a represents an incomplete object if for some predicate P, both formulas $axPx$ and $ax\neg Px$ are false[14]. Generally, for any set of properties

[12] As it will be noticed below, in *M*-logic only individuals are quantified. Besides, *M*-logic with respect to its metalogical properties listed earlier fulfils the Lindström definition of first-order logic.

[13] The terminology is according to Westerståhl, *op. cit.*

[14] Notice, however, that according to *M*-logic, no object is strongly inconsistent in the sense of possessing some inconsistent property; the formula $ax(Px \wedge \neg Px)$ is always false. Conversely, every object is weakly complete in the sense of possessing all universal properties - the formula $ax(Px \vee \neg Px)$ is always true. I think that this perfectly agrees with our intuitions concerning objects of consciousness. If we think of objects, even if fictional characters, we do not ascribe them plain contradictions because this would probably mean that they were devoid of sense. Rather we tend to think of them as being consistent (the formula $ax(\neg(Px \wedge \neg Px))$ is true of all of them) although they might turn out not to be so ($axPx$ and $ax\neg Px$ might be true). While creating fictional characters and ascribing them consciously some mutually inconsistent properties, we treat intentionally the characters as consistent (i.e. as possible). The situation of completeness is even more clear. Objects of consciousness *qua* objects of consciousness are always incomplete. Because of the fintitude of our minds, we cannot grasp objects in all their properties. And, of course, we are not able to create complete fictions. However objects are given to us as complete entities. Despite the fact that Sherlock Holmes, being a fictional character, is incomplete and probably inconsistent, yet he is intentionally created as a real person, i.e. as a consistent and complete entity.

there will be an object represented in the M-semantics by the set of extensions of these properties.

Now let us consider some aspects of ontological content of M-logic. First let us notice that the logic involves two categories of entities: (existing) individuals referred to by variables and Meinongian objects referred to by terms (constants). Only individuals are genuinely quantified (in objectual way). The quantification of Meinongian objects is merely simulated in the logic - see theorems: $M10$, $M11$ which mimic the classical universal instantiation and the existential generalisation. But we have at our disposal the means of identifying individuals with some Meinongian objects. The formula which is the antecedent of the theorem $M19$, i.e. $ax(x=y)\wedge\neg ax(x\neq y)$, says that the individual x is identical with the Meinongian object a, consequently the two objects possess exactly the same properties (although with respect to two different modes of predication)[15]. Thus the formula $\exists y(ax(x=y)\wedge\neg ax(x\neq y))$ may be taken as expressing the (singular) existence in M-logic. The notions of completeness (incompleteness) and consistency (inconsistency) of objects in their ordinary formulations are second-order notions; however in M-logic we can define, using only first-order means, even stronger notions. According to the theorems $M14$ and $M15$ the property expressed by $\exists x\neg ay(x\neq y)$, entails consistency of a and the property expressed by $\exists xay(x=y)$ entails completeness of a. The notions which appear in the next two theorems are even stronger. These are: $ax\neg ay(x\neq y)$ - being a *general object* and $\neg ax\neg ay(x=y)$ - being a *particular object.* For example, since $\forall x\exists y(x=y)$ is a theorem then *the universal quantifier object* is a general object whereas *the existential quantifier object* is a particular object (conversely, since $\exists x\forall y(x=y)$ is not a theorem then the first object is not particular and the second is not general one, except the case of one element domain of course). If we consider the theorem $M18$ we see that the formula $\neg ax\neg by(x=y)$ expresses the relation of being a subobject (in the sense of possessing less properties).

II

There are many directions to modify M-logic. First let us mention some weakenings of M-logic.

The simplest one consists in replacing the rule MG by the ordinary universal generalisation rule: if $\vdash A$ then $\vdash\forall xA$. The new logic, say M'-logic, differs from M-logic in that it remains true in empty domains (it is an inclusive logic) and it approves two bizarre objects: the object which

[15] From the semantical side it holds whenever $I(a)=\{\{V(x)\}\}$.

possesses all properties including inconsistent ones, and the object which possesses no properties, even tautologous ones (they are represented by $\{\varnothing\}$ and $\varnothing$ respectively - M'-semantics differs from M-semantics only in that in the former every subset of $\mathcal{P}(D)$, without exception, can be an interpretation of a term). In particular theorems $M10$, $M11$ cease to hold in M'-system[16].

A more important way of changing M-logic leads us to an intensional version of it. Notice that the logic is not only extensional but also imposes on objects strong deductive closure conditions - the axiom $M2$ from one side and the condition (*) from the other say that an objects possesses every property which is extensionally equal to or broader than some property constituting the object. Meinongians frequently insist on a strictly intensional interpretation of Meinong's theory of objects[17] and, at the same time, they identify properties constituting objects with properties possessed by the objects (although sometimes admitting that there might be an entailment between the former and the latter[18]). For phenomenological reasons I am deeply convinced that Meinongian objects understood as objects of consciousness must be in a way deductively closed although this need not be the sort of the closure embodied in M-logic. So, we may weaken $M2$ replacing it, for example, by the rule: if $\vdash_M A \supset B$ then $\vdash_M txA \supset txB$ which means that predication is closed only on logical entailment of properties. Intensional objects, contrary to the extensional ones, would be built of genuine properties not of their extensions. We might treat extensional objects as *extensions* of intensional ones but for the sake of generality, it is more appropriate to reckon both types to one category of Meinongian entities. Not underestimating the importance of building an intensional M-logic it is worth stressing that such a logic must be equipped with highly complicated (algebraic style) semantics and as such can hardly be counted as a simple Meinongian logic[19].

16 There hold only weaker theorems in M'-logic:
$txB \supset (\forall xA \supset txA)$
$\neg txB \supset (txA \supset \exists xA)$

17 Perhaps with one exception. M. Luise Schubert-Kalsi's who is a great expert in Meionog's thought supports the extensional reading of his ontology; personal communication.

18 E.g. Zalta. Op. cit.

M-logic can be equipped with some definition schemata. These schemata enable us to introduce new terms on the basis of terms already present (correspondingly: new objects on the basis of objects already present).

*DM*1 $\exists xA \supset ([xA]xB \equiv \forall x(A \supset B))$ (for *M*-logic)
*DM'*1 $[xA]xB \equiv \forall x(A \supset B)$ (for *M'*-logic),
where *A* contains at most the free variable *x*
*DM*2 $\hat{t}xA \equiv \neg tx\neg A$ (the inversion of term)
*DM*3 $(s \sqcap t)xA \equiv sxA \wedge txA$ (the conjunction of terms)
*DM*4 $(s \sqcup t)xA \equiv sxA \vee txA$ (the disjunction of terms)

New terms $[xA]$, $\hat{t}$, $(s \sqcap t)$, $(s \sqcup t)$ receive appropriate interpretations in *M*-semantics[20]. Now, for instance the object *the round square* can be explicitly defined of the ground of *M*-language: [*roundness*] $\sqcup$ [*squareness*]. Also the existential quantifier can be defined as the inversion of universal quantifier: $\hat{\forall}$ [21].

To describe further extensions of *M*-logic let us first tell something about *the dual predication principle* which is the most intriguing and, at the same time, one of the most controversial principles of Meinong's ontology. It can be phrased as follows: for every object, or at least for every non-existent object, we can distinguish two groups of its properties: internal and external ones. Internal properties or qualities are those through which an object appears to consciousness. External properties are those which the object exemplifies in the status of intentional object *qua* intentional. It is hard to explain more closely the exact sense of this distinction since it relies on some complicated phenomenological descriptions of objects of consciousness (intentional objects). We hope it will be enough to give

19 When I say about an intensional logic I would like to distinguish it from a mere "strongly extensional logic" which is usually associated with the logic based on possible world semantics. In the former, two properties may differ even if their extensions are the same in all possible worlds. Obviously, *M*-logic can be extended to modal *M*-logic (based on possible world semantics) in several standard ways. Perhaps it is worth noticing that, because of relative richness of *M*-language, the *de re - de dicto* distinction is applicable to all Meinongian objects and not only quantifiers, e.g.: $tx\Box A$ and $\Box txA$. What is more, the transworld identity can be expressed in such an logic.

20 $I([xA]) = [d \in D: A \text{ is true in } M \text{ with respect to } V_d^x]$, $I(\hat{t}) = \{X \subseteq D: X \cap Y \neq \emptyset$ for some $Y \in I(t)\}$, $I((s \sqcap t)) = \{X \cup Y: X \in I(s) \text{ and } Y \in I(t)\}$, $I((s \sqcup t)) = I(s) \cup I(t)$.

21 It is important here that the definitions have the status of axioms, Although, as they stand, they are not creative with respect to the original axiomatics but they could be so. For example, in the presence of *D M*2 it suffices to assume a weaker rule *GM*: if $\vdash_M A$ then $\vdash_M txA$. Also, these definition axioms are indispensable when we start to build higher-order versions of *M*-logic (see below).

some paradigmatic examples. Let us mention two famous nonexistent objects and list some of their internal and external properties. *The round square*'s internal properties are: being round, being a square, being a geometrical figure; its external properties are: being impossible, being inconsistent, being nonexistent, being thought of by J. Paśniczek, etc *Sherlock Holmes*' internal properties are: being a detective, being Dr Watson's friend, and all other properties ascribed to the hero by A.C. Doyle; its external properties are: being incomplete, being nonexistent, being a fictional character etc.

An object may be incomplete or inconsistent only with respect to its internal properties. However with respect to its external properties the object is complete and consistent like any other existent individual. This is because objects just as object of consciousness, are in a sense real whether considered by philosophers as abstract, ideal or merely psychic entities.

Everything that has been said earlier concerns only internal predication and the subject-predicate formula axA expresses only that predication. However there is a straightforward way of extending M-language so as to render in it the external predication as well. Since, as we hinted, the external predication is always complete and consistent we may simply adopt for it the classical grammar. Thus we let constants, representing names of objects, occupy argument places in formulas just like in first-order language. For example, formulas Pa, $\exists xRxa$ express the external predication with respect to a; analogical internal predication takes the form: $axPx$, $ay\exists xRxy$

M_1-language results from M-language by assuming that constants may occupy argument places in atomic formulas on a par with variables just like in the classical language (now metavariables in the definition of formulas range over variables and constants). M_1-system is based on M-system axiom schemata extended in its application to M_1-language (axioms $M4$,$M6$,$M7$). For instance, as a particular case of $M4$ we now have the axiom: $\forall xPx \supset Pa$. In M_1-logic formulas $axPx$ and Pa are independent of each other (neither entails the other or its negation). In the semantics for M_1-logic, constants are interpreted in two different ways corresponding to the internal and the external predication respectively: one interpretation is the same as in M-semantics, i.e. $I(a) \subset \mathcal{P}(D)$ and the second interpretation is the same as in classical semantics for first order logic, i.e. $\mathbb{I}(a) \in D$. Also, the truth condition for Pa is that of classical semantics.

M_1-logic is still first-order logic and as such it is relatively poor, although now object are quantified, but only as subjects of external predication. In comparison to M_1-logic, in Parsons' and Zalta's logics, we quantify objects as subjects of internal predication as well. Besides, in these logics like in ordinary second order logics, properties and relations are also quantified.

Anyhow there is an extremely easy way to extend the quantification in M_1-logic and get a stronger logic, M_2-logic based on M_2-language. As before we need not change essentially the axiomatics but only the grammar of M_2-language. It suffices to let variables occupy term places as well: for any variables *x*, *y*, the expression *yxA* will be a formula (i.e. now *s* and *t* range not only over constants and quantifiers but also over variables). Doing so we allow objects, taken as subjects of internal predication, to be quantified. But what about the quantification of properties? Surprisingly enough, the quantification of objects may implicitly comprise the quantification of properties. Roughly the idea is that in M_2-logic *general* objects - as subjects of internal predication - can play the role of properties. For example, the object *a square* represented in *M*-language by [*squareness*] (more exactly: [*x(x is a square)*]) and in *M*-semantics by {*set of squares*} plays the role of property *being a square* (see *DM*1 and *DM*'1). The following theorem says that for every property, simple or complex, there exists an object which represents the property:

$$\exists xA \supset \exists y(yxB \equiv \forall x(A \supset B)) \quad (\text{or: } \exists y(y = [xA]))$$

(the theorem is an existential generalisation of *DM*1). Now the following can also be proved in M-logic:

$$txA \equiv tx\neg[xA]y(x \neq y)$$

This theorem has a stronger counterpart in M_2-logic:

$$\exists u \forall w(wxA \equiv wx\neg uy(x \neq y))$$

It means that every (closed) formula *txA* is replaceable by a formula containing only terms and logical symbols (the negation and identity): $tx\neg sy(x \neq y)$. Since the structure of the latter formula is fixed we may write it in short: $t \epsilon s$ where ϵ will be a kind of ontological relation in Leśniewski's spirit[22]. And this relation can simulate the relation of

[22] Actually this relation is stronger than the original Leśniewski's relation, cf. J.Paśniczek, op. cit.

internal predication. That is why we need in M_2-logic only definition axiom for objects whereas in Parsons' and Zalta's logics there are two separate definition axioms: one for objects and one for relations[23].

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[23] These axioms make the logics very powerful but they cannot be adopted in their full generality for it leads to the inconsistency of the logics. The same problem pertain M_2-logic but we won't discuss it here.