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# G. H. VON WRIGHT'S TRUTH-LOGICS AS PARACONSISTENT LOGICS

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## 1. *G. H. von Wright's Truth-Logics*

Georg Henrik von Wright has presented his hierarchy of truth-logics in three papers, 'Truth and Logic' (1984), 'Truth, Negation, and Contradiction' (1986) and 'Truth-Logics' (1987). However, the ideas behind these logics can be seen as early as in von Wright's 'On the Logic of Negation' (1959) and 'Time, Change and Contradiction' (1983, originally from 1968). Firstly, I shall introduce some main points of these logics calling special attention to their paraconsistent features. Secondly, I shall make some — mainly philosophical and very general — comparisons between truth-logics and other paraconsistent logics on the market and discuss some related philosophical topics.

One motivation behind von Wright's truth-logics is to establish "a link between ... *the great tradition* in logic — from Aristotle to Frege and Russell and modern mathematical or symbolic logic — and another ... tradition stemming mainly from Hegel" (1984, 131). He wants to try to do justice to some criticisms aimed at the very basic laws of classical logic. He wants to do this with a minimum of departure from the classical patterns of logic. According to von Wright "this can be done without indulging in what I would regard as 'extravagances' of many-valued and intuitionist logic" (1986, 5-6).

G. H. von Wright's truth-logics are based on the following ideas: the basic truth logic *TL* introduces the notion of truth in the object language of the calculus. This is done by employing a new symbol " *T* " (to be read "it is true that") in front of sentences.

Using this symbol " *T* ", a distinction between two kinds of negations can be made: an *external* negation $\sim T$ to be read "it is not true that" and an *internal* negation $T\sim$ to be read "it is true that not". The internal negation is an *affirmation* and signifies falsehood while the external negation is a *denial* and signifies (mere) not-truth. This idea can be found in *Analytica Priora* (51b23-24) where Aristotle says that " 'to be not-good' and 'not to be good' are not the same".

In the basic truth logic the expression $TA \,\&\, {\sim}T{\sim}A$ says that the proposition $A$ is *univocally true*. ${\sim}TA \,\&\, T{\sim}A$ says that $A$ is *univocally false*. $TA \,\&\, T{\sim}A$ means that $A$ is *both true and false* and finally ${\sim}TA \,\&\, {\sim}T{\sim}A$ that A is *neither true nor false*.[1] First two cases refer to classical truth-values true and false. In the third case we can use the term *truth-value overlap* (or *truth-value glut*) while in the last case we can speak of a *truth-value gap*. Now there are 16 different ways in which some or several of these four possibilities can be permitted or refuted and accordingly there are 16 different truth-logics. Only some of them seem to be of interest, and four of them deserve special consideration. They are classical logic (CL), the logic admitting truth-value gaps (TL), the system admitting truth-value overlaps (T'L) and finally the logic which admits both gaps and overlaps (T"L). (see, e.g., von Wright 1987, 311-314.)

Before going to the paraconsistent character of truth-logics I take an example from the very core of these systems. The Law of Excluded Middle in the form $TA \vee {\sim} TA$ is valid in all four systems mentioned above. It says that every proposition is either true or not true. von Wright calls it the *weak* form of the law. It must be distinguished from the *strong version* (or the Law of Bivalence) $TA \vee T \sim A$ saying that every proposition is either true or false. This is logically equivalent with $T(A \vee \sim A)$ and they are valid in some truth-logics but not in some others (von Wright 1987, 315).

It must be noted here that in the basic truth-logic TL the Law of Non-Contradiction in the logically equivalent forms ${\sim} T(A \,\&\, {\sim} A)$ and ${\sim}(TA \,\&\, T \sim A)$ is valid. It is not valid in the stronger form $T \sim (A \,\&\, {\sim} A)$ which is in fact logically equivalent with the Law of Bivalence (von Wright 1986, 11).

## 2. *Paraconsistent Truth-Logics*

von Wright calls the truth-logics allowing truth-value gaps *paracomplete* and those allowing truth-value overlaps *paraconsistent*. In the paraconsistent logic T'L a new "truth-operator" $T'$ is introduced. It can be defined, however, in terms of $T$ as follows: $T'A =_{df} {\sim} T \sim A$.[2] According to von Wright the symbol $T$ refers to a *strict* sense of "true" while $T'$ is the symbol for the *liberal* or *more lax* notion of truth.

[1] This approach can be compared with, e.g., relevant systems which approve as truth-values not only members from the set $\{0,1\}$ but from its power set $\{\varnothing, \{0\}, \{1\}, \{0,1\}\}$.

[2] The relation between these two truth-operators has obvious connections with relations between various intensional operators but they cannot be discussed here.

In T'L the Law of Non-Contradiction in the form $\sim(TA \,\&\sim TA)$ is valid but in the form $\sim(TA \,\&\, T\sim A)$ it is not valid. Neither is the so called Duns Scotus Law or Principle in the form $(TA \,\&\, T\sim A)\rightarrow TB$ valid (while the corresponding principle is valid in TL). Therefore von Wright's paraconsistent system is not explosive and is immune to trivialization. Since $\sim(TA \,\&\sim TA)$ is valid in T'L it follows that a contradiction in the form $TA \,\& \sim TA$ (the one and the same proposition both is and is not true) trivializes the system. (von Wright 1987, 320-321.)

Maybe this is enough about the formal side and I shall give a clarifying example below. Truth-logics have not been commented very much in the paraconsistent literature. I shall only mention Katalin Havas (1986), Joseph Wayne Smith (1990) and Hristo Smolenov (1986). Without hesitation I can say that, as logics in general, truth logics have been constructed in such a simple and elegant way that can seldom be found. But in the same breath somebody could say that as paraconsistent logics they are not suitable. They are not suitable, it could be said, because they do not take contradictions seriously. But I shall return to this question a little later.

## 3. *On paraconsistent logics in general*

Meanwhile, a few words about paraconsistent logics in general. A paraconsistent logic could be defined, as Ayda I. Arruda (1980, 2), by saying that a logic that "can be employed as underlying for inconsistent but non-trivial theories" is a paraconsistent logic. But as I shall discuss later this definition sounds somewhat dissatisfactory to me. Therefore I prefer an alternative definition given by Graham Priest and Richard Routley (1984, 2) in the following way:

> Let $\Vdash$ be a relation of logical consequence. $\Vdash$ may be defined either semantically ... or proof theoretically ..., or in some other way. $\Vdash$ is *explosive* (or in Batens' (1980) terminology *destructive*) iff for all $A$ and $B$ $\{A, \sim A\} \Vdash B$. It is *paraconsistent* iff it is not explosive. A logic is *paraconsistent* iff its logical consequence relation is.

I use the term *Duns Scotus Law* or *Principle* to refer generally to various formulations of the principle that accepts the explosive nature of logic.

Since the 1950's dozens of different formal paraconsistent systems have been developed. The four main approaches to paraconsistency are the following: 1) da Costa's systems or systems with a weakened negation, 2) relevant paraconsistent logics, 3) non-adjunctive systems based on Jaqkowski's ideas, and 4) G. H. von Wright's hierarchy of Truth logics. The first three categories form the standard classification but I have added von Wright's Truth logics as a category of its own.[3]

By *paraconsistency* I refer to a view that there are important paraconsistent theories. We must distinguish from paraconsistency a position that there are certain true contradictions. This position is called *dialetheism* (or strong paraconsistency) in the literature. It must be noted that paraconsistency does not imply dialetheism: we can believe that even though there are no true contradictions there can be interesting and important paraconsistent theories because they sometimes can be approximately true or because they can have some other useful properties.

In the light of this distinction it can be seen that it is at least slightly inaccurate when e.g. Joseph Wayne Smith writes (1986, 106) that "we agree that the burden of proof is upon the paraconsistentist to establish that there are *true contradictions*".

However, I think that it is not too daring to claim that paraconsistent logics have never been very popular. I think that most philosophers and logicians consider paraconsistent logics absurd and think that the burden of proof is upon the paraconsistentist to establish that there are good reasons to develop and employ paraconsistent logics. If we are dealing with *true* contradictions the burden of their proof can be upon a dialetheist. Even von Wright asks if "true contradiction" is a "*contradictio in adjecto*" (1984, 36). And all classically educated logicians have learned that it is: the linguistic expressions "contradiction" and "inconsistency" refer to something that can never be true - that cannot be true in any models or in any possible worlds or in any situations etc. Even such an extraordinarily good new textbook of philosophical logic as Pascal Engel's *The Norm of Truth* (*La norme du vrai*) shares the received view by saying that "[t]he least that one can expect from a logical system seems to be *consistency* or *non-contradiction*" (1991, 222).[4] But in the light of the following example the situation does not seem absurd at all. That is: paraconsistent logics do not seem absurd if we accept von Wright's system T'L a real paraconsistent logic.

[3] On paraconsistent logics, see, e.g. Arruda 1980, Batens 1980, da Costa 1974. 1982, da Costa and Marconi 1989, D'Ottaviano 1990, Jaqkowski 1969, Priest 1987, Priest and Routley 1984, Priest, Routley and Norman 1989 and Rescher and Brandom 1980.

[4] However, later in his book Engel discusses also paraconsistent logics (pp. 286-287).

## 4. *Truth logics as paraconsistent logics*

Let us have a look at von Wright's own example (see, e.g., 1987, 318-319). There he considers a process such as rainfall which goes on and on and then gradually stops. Now we can say that during some stretch of time it is definitely raining ($A$) and later during some stretch of time it is definitely not raining ($\sim A$) (Fig. 1). Between these two states there is a "zone of transition": we cannot *say for certain* if it is raining or not. It is neither true nor false to say that it is definitely raining $(\sim TA \,\&\, \sim T \sim A)$ However, as long as some drops are still falling we could say that it is still raining. But we can as well say that it is no longer raining because only some drops are falling. Now, according to von Wright, we could instead of saying that it is neither raining nor not-raining as well say that it is both raining and not-raining $(T'A \;\&\; T' \sim A)$. G. H. von Wright now suggests that something like this happens in a so called Dialectical Synthesis (1984, 36-39). But we must notice that in the picture (Fig. 1) we employ the strict sense of "true" above the line while below the line the liberal sense of "true" discussed above is used. According to von Wright this shift in the concept of truth can be understood especially in situations of *becoming* or *change* or *process* and also in cases of *vagueness*. (We can naturally interpret the line representing, e.g., a person's age where he is first definitely young ($A$) and then definitely not-young ($\sim$A).)

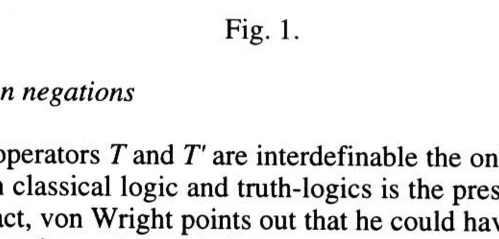


Fig. 1.

## 5. *An example on negations*

Since the truth operators $T$ and $T'$ are interdefinable the only notational difference between classical logic and truth-logics is the presence of the truth operator $T$. In fact, von Wright points out that he could have used two symbols for negation instead for similar purposes. Generally speaking the question concerning the meaning of logical constants are in a central position when dealing with any paraconsistent logic. Particularly negations play an important role in many paraconsistent logics, especially in the systems with a weakened negation developed by e.g. da Costa and Batens. In some systems there is only a weak negation (*strictly* paraconsistent logics in Batens' (1980) terminology) while in some other systems classical negation and weak negation are interdefinable (*materially* paraconsistent in Batens' (1980) terminology) as different truth-operators in von Wright's systems. G. H. von Wright himself considers systems with *only* a non-classical negation "crippled". He wants to develop a system where the *one* notion of negation can "behave" both classically and non-classically (1987, 312). But my question is the following: Must we in this light speak about conceptual shift in other paraconsistent logics as well? Some formulas of the form *A* & ~*A* are true in, say, one of da Costa's systems. But are they in fact "true" in a more liberal sense than, e.g., in classical logic because here the symbol ~ refers to a weak negation? In the light of an approach like this the claim that contradictions are not "taken seriously" in paraconsistent logics seems very understandable. Let us suppose that we want to defend a strong paraconsistent or dialetheistic position admitting contradictions in the actual world. Should we now demand (or develop) better logics than the existing paraconsistent logics: we do not want any mild or weak or liberal notions, we want to be strict in the good old Aristotelian way! But on the other hand the meaning of some logical constant(s) or some other principles have to be changed if we want to create a new kind of logic. If nothing is changed the result is - the classical logic!!!

Let us go back to truth-logics. What is this new sense of, e.g., "true" in the truth-logic T'L? von Wright himself compares some truth-logics with intuitionistic logic. In my view the situation can be interpreted in the following way: both intuitionistic logic and T'L seem to give up the so called "absolute notion" of truth. They both seem to replace it with a new notion of truth, and that is a notion that maybe at least implicitly contains some epistemic elements. What happens in these two logics is that they move in opposite directions: the main difference seems to be that when intuitionistic logic defines "true" in a stricter way than classical logic, T'L defines it in a more liberal way. Roughly speaking, in intuitionistic logic "true" is defined in the sense of "proved". But it seems to me that in T'L "true" can be interpreted as "not proved or shown to be false". In other words, if we cannot say whether *A* or ~*A* is false, let us accept both of them, at least for the time being until one of them has been shown to be false.

But can this interpretation be extended to other paraconsistent logics as well? It is said in connection with systems with a weakened negation that some sentences of the form *A* & ~*A* can be true. Can some or all of these systems be interpreted in a way that sentences of the form *A* & ~*A* are "true" in a more liberal sense than, e.g., in classical logic - as the situation is in T'L? Does this mean that, in these paraconsistent logics, contradictions are not "taken seriously" either? Is the weak negation in the systems described above a "real" negation at all? Or is it some kind of strange, positive, intensional operator? I think that most paraconsistent logicians themselves would not approve this interpretation.

Now we can go back to Arruda's definition according to which a logic that "can be employed as underlying for inconsistent but non-trivial theories" is a paraconsistent logic (1980, 2). A paraconsistent logician may be of the opinion that naturally he takes contradictions seriously - on the level of the theory. He may also say that he does not believe in true contradictions - on the level of the theory. But on the level of the underlying logic, the situation is different - there we can find both *A* and ~*A* true because "~" is a symbol for a weak negation (Fig. 2). A clear difference has been made between these two levels.

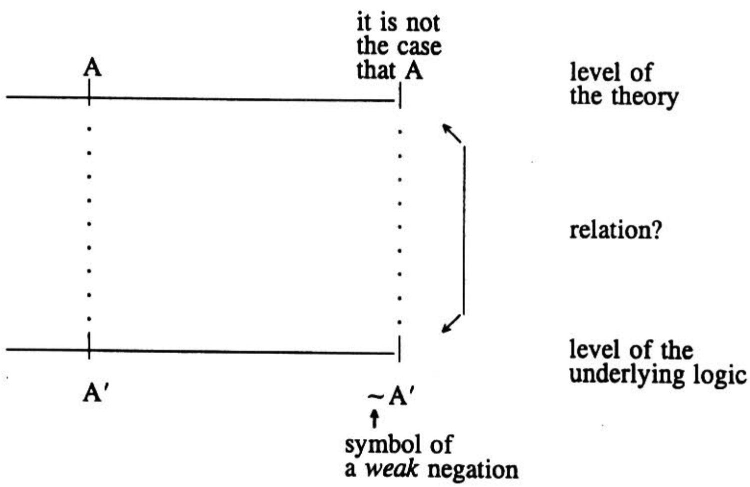


Fig. 2.

But if so, what is the relation or connection between these two levels? What are the criteria that allow us to say that this is a right or good logic to be employed as underlying for inconsistent but non-trivial theories? Is the only criterion that it must not be explosive? That does not sound acceptable. This question is highly interesting and important but has not been dealt with very much in the literature.

However, this question can be naturally asked in connection with other logics as well. But I think that it is even more difficult in connection with paraconsistent logics because it seems that in inconsistent connections we cannot usually rely on, e.g., "common sense" or "intuitions".

## 6. *Global and local features of logic*

Sometimes it is claimed that there are two rival tendencies about the way how logic or logics should be developed in general. According to one tendency our ultimate goal should be one *global* or *universal* logic. Another opinion is that we should develop several *local* logics — each applicable in its own context.

But what is this distinction between one global or universal logic and several local logics? I think that to many logicians this distinction is not clear at all — or at least they do not explicitly express it. They seem to think that they are creating a new global logic but what they in fact are doing is the opposite — they develop local logics for reasoning in certain contexts, inconsistent contexts. It seems to me that sometimes also paraconsistent logicians forget that *not* all contexts are inconsistent. A paraconsistent logician can dream about a global logic when he or she starts to create a new system. But often the result can be something different. Maybe it must be admitted that as long as a global logic cannot be developed we must attribute to each logic a particular set of domains in which it is adequate.

But what could a global logic be like? E.g., Richard Routley seems to have an explicit ideal of a universal logic in mind:

> A universal logic, in the intended sense, is one which is applicable in every situation whether realised or not, possible or not. Thus a universal logic is like a universal key, which opens, if rightly operated, all locks. It provides a canon for reasoning in every situation, including illogical, inconsistent and paradoxical ones. Few prevailing logics stand up to such a test. Certainly neither classical logic, nor the main alternatives to it offered, such as intuitionistic logic, are so universal. (Routley 1980, 893.)

It seems that if we want to create a global logic which is also paraconsistent the best possibility would be to have one negation which — slightly metaphorically speaking — automatically coincides with the classical one when the degree of inconsistency in the domain is zero but differs from it when the degree of inconsistency deviates from zero — analogically with some well-known cases in physics. But whether this can be done in a logical system is a different and a difficult question.[5]

[5] The anonymous referee of this paper pointed out that a global logic in this sense is not as impossible as I suggest. Diderik Batens' dynamic dialectical logics fulfill my conditions

The problem concerning one (global) logic and several (local) logics (or logical *monism* and *pluralism*) can be considered one of the crucial questions that should be answered in connection with logic in general and particularly when we get into closer contact with non-classical logics. But von Wright's truth-logics show us among other things that there need not necessarily be any conflict between these views. His logics show it because they consist of a variety of logics which, however, form a systematic unity.

In the light of all this it is clear that we cannot ask if one of the truth-logics is the right logic or if one of them is the best one. Instead we should or could ask, e.g., which one is the best for reasoning in a given context. But as von Wright says this question "cannot be answered *in logic*, but must be answered so to speak from the platform of the *contexts*" (1987, 333). In any case von Wright has shown us that very different logics can be combined into a uniform theory. When we survey the enormous variety of existing logics on the market it may seem clear, however, that it is impossible to gather all of them under the umbrella of truth logics. G. H. von Wright says that his truth-logics represent "a *unification* of logical thinking" (1987, 334). And I can agree with him in saying that they represent at least one step to that direction.

## 7. *Conclusion*

In conclusion I shall summarize my line of thought in the following way: As logics von Wright's truth-logics are extremely elegant and worth examining and developing further. As paraconsistent logics their value can be seen among other things from the fact that they once and for all make it clear that the conceptual change has taken place. But because of this, the following fundamental question remains in the air: Can they be considered "real" paraconsistent logics at all? Truth-logics can also show us the way in which we can combine the opinions concerning local and global characters of logic: our ultimate aim can be to develop one global hierarchy of logics which is composed of several beautifully compatible subsystems. But maybe it is because of the explicit conceptual change that the jungle of truth-logics is not so inaccessible as the jungle of other paraconsistent logics.

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(see Batens 1986; 1989). I am grateful to the referee for this and many other valuable remarks.

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