

LOGICAL NEGATION IN ENGLISH?

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Not less than three [*sic*] eminent philosophers have recently claimed that there is no logical connective negation in English. I will conceal their identities (they are guilty enough) and refer to them collectively as "Professor Capulet". I take it that Prof. Capulet means that no word or short phrase exists in English which can be regularly prefixed to a declarative sentence to yield its contradictory denial (or one such) —at least not in the way that " \sim " when prefixed to any formula X of the propositional calculus produces a formula Y that contradicts X (e.g., " $\sim p$ " contradicts " p ", " $\sim(p \vee \sim q)$ " contradicts " $p \vee \sim q$ ", etc.). For example, sentence-initial "not" cannot be such a logical connective (operator) in English because (i) some uses of it don't work at all (e.g., "Not Socrates is wise" is ungrammatical) and (ii) some uses of it don't produce *contradictory* denials (e.g., "Not any Greek is a liar" is the contrary, not the contradictory, of "Any Greek is a liar").

Contrary to Prof. Capulet's claim, I believe there IS such a logical connective in English —viz., "not" in sentence-initial position. (Various other phrases like "it's not the case that", and "what follows is false:" are not negation connectives in the intended sense. For they are grammatical transformations of forms in which the negative is a modifier (adjective) embedded in a predicate ("... is not the case", "... is false (not true)")— very like typical infixed "not" producing contradictories of singular sentences ("Socrates is not a Persian").) Here are typical examples of logical connective "not" in English:

- (1) Not all men are Greeks; Not every/each man is a Greek.
Not (very) many men are Greeks; Not (very) much wine comes from Greece.
Not a lot of men are Greeks (wine comes from Greece).
Not a few men are Greeks; Not a little wine comes from Greece.
(denial of "Few men are Greeks", not of "A few men are Greeks")
Not one Greek is a liar (two, 300, etc. Greeks are liars); Not more than 29 are.

Of course, there are some borderline grammatical cases (not obviously acceptable to every English speaker) and there are cases where prefixing "not" produces an ungrammatical sentence (roughly no sentence at all; i.e.,

something that is not even a candidate for being a contradictory denial). (2) lists some borderline cases and (3) some of the ungrammatical cases. (Actually, (2) are okay for me, but questioned by some others.)

- (2) Not most Greeks are liars; Not three-fourths of the Greeks are liars, etc.

Not almost-all Greeks are liars; not few Greeks are liars.

- (3) Not Socrates is wise; Not not all men are Greeks.

Not (all men are liars and Socrates is wise) (=impossible parse).

Not if Socrates is wise then all Greeks are liars.

So, in disagreeing with Prof. Capulet I admit that there are lots of cases where prefixed "not" does not work to produce a contradictory denial. Further, there are borderline cases (not clearly successful cases). And worse than (2) and (3), there are clearly grammatical cases that go the wrong way; i.e., a *contrary* denial (rather than a contradictory one) is produced; i.e., "Not any Greek is a liar" is contrary to "Any Greek is a liar". And still further, there are more cases like those wherein the prefixing of "not" IS grammatical, but gives the wrong result; e.g., "Not *many* more than one-quarter of the singers are poets", which with the intoned emphasis marked entails that more than one-quarter of the singers *are* poets—something which the contradictory of "Many more than one-quarter of the singers are poets" shouldn't do (more on this in conclusion below). And all this aside from the so-far-unmentioned fact that INfixed (rather than prefixed) "not" sometimes produces contradictory denials— e.g., "*All* that glitters *is* not gold" (Shakespeareanly intoned); i.e., "not" can be used in another way to get the desired effect.

I don't deny these troubles with my position. Rather, I just reply that the rule for "not" to produce contradictories when prefixed to sentences must be a contextual rule. So, the rule must look something like:

- (4) NOT + S = contradictory-of-S

only if (i) "NOT + S" is grammatical,
& only if (ii) S begins with certain (not all) quantifier words,
& only if (iii) intonation pattern is "right",
etc.

Of course, this is a *very* schematic rule—only a promise for one. Fine. That is all I would claim, that there *exists* a rule of English whereby certain uses of "not" produce contradictory denials just like the logical connective for negation in the propositional calculus does. The objections to this strat-

egy are several. First, it will be objected that the English rule *must* be contextual, but then it can't parallel the logical connective (operator). For the absoluteness of the logical connective for negation rests simply in the fact that no such necessary conditions must hold; i.e., "~" prefixed to any propositional logic formula produces contradictories of the formula, *period*. (Not "only under certain conditions".) So, any extra conditions at all (to make such a rule as (4) work) just confirm the claim that there is no logical connective of negation in English.

In addition to objecting to the very existence of conditions (that rule (4) *is* contextual at all), each particular condition can be objected to. For example, the first condition might be objected to. You might claim that the rule has to be universal in that every prefixation of "not" to a declarative sentence has to *be* grammatical (i.e., the rule has to resemble a genuine grammatical transformation in this regard). Further, you might object (*vis à vis* the third condition) that ambiguity cannot be produced through prefixing "not", since negation in propositional logic isn't ambiguous.

I will examine these in turn. First, why can't any rule which reveals that logical connective for negation occurs in English *be* contextual? Consider logical conjunction for a parallel. Prof. Capulet might say that logical conjunction (from the proposition calculus) *does* occur in English in that "and" when placed between two declarative sentences joins them into one compound English sentence just like the typical signs for logical conjunction do (producing a true compound if and only if each conjunct is true). And he may add that this rule is *non-contextual*. For every such use of "and" that way produces English logical conjunctions. However, I have some objections to this proposal. First, it is simply *not* true that every such use of "and" produces the exact correlate of logical conjunction. For one thing, many such uses involve a tacit ordering in time (and so violate the commutativity of *logical* conjunction); e.g., "John swallowed the pill and he put it in his mouth" certainly does not follow logically (as it would have to if this "and" were nothing other than logical conjunction) from "John put the pill in his mouth and he swallowed it". So, it is only *some* uses of "and" between declarative sentences that functions exactly like logical conjunction. Secondly, the supposedly non-contextuality of the rule can be challenged by noting that it is not every use of "and" itself, but only its use between full sentence clauses that produces an English instance of logical conjunction. That is, looked at more broadly (which I claim is the appropriate way to look at it), "and" produces logical conjunctions when flanked by expressions that are sentence clauses but does not do so when flanked by expressions that are noun phrases (or as, I will show, *sometimes* does not with noun phrases). So, this is a contextual rule, since "and" is logical conjunction in English only *under certain conditions*.

For example, consider

(5) John and the stranger arrived late.

Here "and" joins two noun phrases to produce a compound subject, not a logical conjunction. Of course, Prof. Capulet may reply that there is a tacit logical conjunction expressed; i.e., (5) in English presupposes or implies the truth of (6), to which it is grammatically related (by some grammatical transformation such as so-called "conjunction reduction"):

(6) John arrived late and the stranger arrived late.

Now if you were to take this tack to defend the hypothesis that "and" in (5) was really no counter-example to an absolute rule for logical conjunction "and" in English, then I will do the same for sentences like "*All that glitters is not gold*" (Shakespeareanly pronounced). That is, I will claim that

(7) *All who are politicians are not liars*

doesn't really give a case of contradictory denial via *infix* "not" because there is a tacit understanding that the "not" goes in front logico-semantically speaking (perhaps derived by grammatical transformation from "Not all who are politicians are liars"). So, leaning on the possibility of grammatical transformations to support the absoluteness of "and" for logical conjunction is a ploy that will backfire. In any case, there are other cases like (8) wherein the use of "and" to conjoin noun phrases *cannot* be correctly hypothesized to relate to some other form with ordinary logical conjunction.

(8) John and the stranger lifted the piano.

For on the most obvious use or reading of (8), it is not at all equivalent to (nor imply or presuppose)

(9) John lifted the piano and the stranger lifted the piano

(since pianos are heavy to lift and it is unlikely that (8) be used to assert something equivalent to (9), but rather likely that the assertion of (8) would mean that the two of them lifted the piano *together*).

Also, though many noun-phrases conjoined with "and" such as in the subject of (8) do not express logical conjunction, still other conjoining noun-phrases do express logical conjunction (especially in a way that does not promote global grammatical transformations such as relating (6) to (5)). In (9) each noun-phrase flanking "and" does (I maintain) refer to or express

a proposition, where the whole compound proposition is the subject of the predicate "is unlikely":

- (9) John's throwing the piano and the stranger's catching it is unlikely.

I hold this via "Fact-Proposition-Event Theory", by which the subject of (9) refers to a proposition because "unlikely" is a propositional predicate — distinct from, but parallel to, factive predicates like "surprising". Cf. my "Anaphoric Reference to Facts, Propositions, and Events", *Linguistics & Philosophy* 1982, or "Complex Events", *Pacific Philosophical Quarterly* 1989.

(Sometimes it is even said that "and" expresses embedded logical disjunction; e.g., "Apples and oranges are nutritious" expressing " $(x)((Ax \vee Ox) \supset Nx)$ ". I myself think this example is a little misleading. The *direct* formalizations should rather be " $(x)(Ax \supset Nx) \& (y)(Oy \supset Ny)$ ", which is merely logically equivalent to, not identical to, the prior formalization.)

I conclude, then, that if any rule for finding a logical connective in English must be an absolute rule (not-contextual, or not with necessary conditions involved), then there is no occurrence of the logical connective for *conjunction* in English. So, logical connective "not" in English is not worse off than logical connective "and" in English. *Both* are contextual, non-absolute rules. I hypothesize on the basis of these two cases that *no* logical component of a logistic system (such as disjunction, universal quantification, etc.) will be properly said to occur in English *unless* it is describable by a contextual, non-absolute rule. Prof. Capulet may conclude, then, that conjunction, quantification, and other such logical components don't occur in English for the very same kinds of reasons that sentence connective for negation doesn't. I myself conclude the opposite. Most or all such components or aspects *do* occur in English, but specifying the exact rule for each is difficult due to the complex contexts that need to be specified. (Of course, the reason I think all such logical components *do* or *can* occur in English is because that is where they came from; i.e., they are all abstractions from certain syntactic and semantic features of English and/or other natural languages. I do not think such components of logical systems and logical languages were the inventions or discoveries of philosophers who were just thinking of pure logic, pure reasoning, or even pure mathematics divorced from considerations about natural languages. Does anyone believe they were?)

Prof. Capulet has said that one reason he didn't think logical connective for negation (such as via prefixed "not") did not occur in English was because of *non*-cases like those in (3) —e.g., "Not Socrates is wise" does not express the contradictory denial of "Socrates is wise". Well, that is clearly

true. But to notice this seems to me to be sufficient for also proposing that grammaticality is the point. A prefixing of "not" to a declarative sentence must be grammatical to begin with. If it's not, then it's *not a candidate* for being the contradictory denial of the sentence. More interesting to me is the iteration of prefixed "not"s. Even if prefixing one "not" is grammatical and *does* produce a contradictory denial, still prefixing a second "not" does *not* do what it should (logically speaking). That is, "Not not all Greeks are liars" not only fails to express the contradictory denial of a contradictory denial (as it certainly can in *logic-class* English — which, of course, is not the issue herein), but is even (I would contend) *ungrammatical*. This shows that the full power of the logical connective for negation certainly does *not* occur in English. For successive iterations of negation logically speaking certainly *does* occur in propositional logic. Analogous shortcomings with "and" expressing conjunction in English also occur, however. For example, logical conjunction is a two-argument operation. Thus, successive embeddings of conjunctions within conjunctions are, logically speaking, structured pair-wise. A formula like " $p \ \& \ q \ \& \ r \ \& \ s$ " is, strictly speaking, ill-formed. The *exact* concept of logical conjunction requires further structure — e.g., " $(p \ \& \ q) \ \& \ (r \ \& \ s)$ " (among other alternatives). This fact is *not* carried over into English. We *do* find multiple conjunctions in English, such as "Al left and Harry left and Mary arrived and John disappeared". But we do not find any exact devices in English for expressing the parsing which *must* be supplied in logical notation (propositional-logic language). You might claim that in written English commas can do the job; e.g., "Al left and Harry left, and Mary arrived and John disappeared" expressing the embeddings " $(p \ \& \ q) \ \& \ (r \ \& \ s)$ ". Well, maybe. But how about some use of commas to express the parsing " $p \ \& \ (q \ \& \ (r \ \& \ s))$ "? Should it be "Al left,, and Harry left, and Mary arrived and John disappeared"?

My conclusion is that "not" when prefixed to *some* English declarative sentences *does* express logical connective negation, but only in a rather limited way. That is, there are further uses of logical connective negation in appropriate logical languages (notations) which do *not* occur in English (such as iterations — not to mention the singular sentences mentioned above). But a similar thing happens with logical conjunction "and"; i.e., every sort of variety of logical conjunction (especially with respect to embeddings) does not occur exactly in English. Only some varieties do. I understand that Prof. Capulet may conclude, then, that just as logical connective negation does not occur in English, so now also logical conjunction does not either. For only limited varieties of each occur in English. I think that the limited varieties are enough.

The final kind of objection I mentioned above concerned ambiguities. Prof. Capulet or someone else might claim that prefixing "not" to declarative sentences in English cannot be an instance of logical connective sort of

negation *unless* each such is *unambiguously* so. In brief, logical connective negation is unambiguous, so any occurrence of it in English must be too. This is an extreme demand up with which I myself will not put. A pervasive feature of every natural language on practically every occasion of use is actual or potential ambiguity. I say actual *or potential*, because ambiguities that exist often do get *disambiguated*. One might say, then, for such cases that they were not real cases of ambiguity (since now disambiguated). Saying this would be (to my way of thinking) to miss the full nature of an important aspect (ambiguity) of natural language syntax and semantics. In all natural languages, nearly every word is ambiguous. (If you don't think so, find in any ordinary dictionary an entry for a word that is single and simple. They're hard to find.) In addition, very many (I am inclined to say most, or almost-all) grammatical strings are syntactically or semantically (structure-wise) ambiguous —i.e., they admit more than one syntactic (or semantic-structure) analysis. So, I believe that many (perhaps most) strings of English words that are grammatical sentences are like the following strings

- (10) John is a tired, old athlete; The shooting of the hunters was abominable; The men who were tired slept soundly; Flying planes can be dangerous; etc.

And each possible syntactic disambiguation of a grammatical string will bring up new possibilities for interpreting negations. For example,

- (11) John is *not* a tired old athlete

may mean (at least) either that for an old athlete, John is not tired, *or* that for an athlete, John is not tired and old. These disambiguations can have different truth conditions.

Further, some strings which are semantically ambiguous may not be syntactically ambiguous.

- (12) All that glitters is not gold

when simply written can certainly appear ambiguous. For should we read or understand it intoned Shakespeareanly as (12.1) or regularly as (12.2)?

- (12.1) *ALL* that glitters *is* not gold

- (12.2) All that glitters is *not* gold

What this shows is that contrasting intonations (that are sometimes available) help to *disambiguate* logico-semantically-speaking. In sum, (12.1)

will be used to express wide-scope (contradictory denial) negation, whereas (12.2) will be used to express the opposite (which is contrariety). That is, in (12.2) the “not” does not express wide-scope negation, but internal to the predicate (or of the predicate) negation. The interesting thing here (unsettled, as I understand it, in contemporary grammatical theory) is that (12) and (12.1) and (12.2) might all have the *same syntactic* structure—i.e., (12) might not be syntactically ambiguous like the examples in (10), even though it is *potentially* semantically ambiguous. (I claim that the merely potential ambiguity of (12) is cleared up in speech. It is said and understood either one way or the other via intonation expressed or heard.)

Now to insist that the logical connective of negation cannot be said to be expressed by any uses of “not” (prefixed or otherwise) *if* ambiguity arises will, of course, bar our saying that logical connective negation occurs in English. For actual or potential ambiguity (both syntactically and semantically) is often (usually?) present. But, as I urged before, if absence of ambiguity is to be insisted on, then no logical component or aspect of propositional or predicate logic (conjunction, quantification, etc.) will be said to occur in English. I conclude that to raise the mere presence of ambiguity in English as an objection is to *uninterestingly prevent* application of any logical concepts (of and from formal logistic systems and artificial languages) to natural language phenomena.

Ambiguity in language and speech (and its frequent disambiguation) is a central intriguing feature of natural languages. It is important to avoid ambiguity and equivocation in formal systems and languages, but just as important to admit (give in to) its centrality in natural languages. In fact, a very interesting feature of “not” as a prefix expressing logical connective negation arises with intonations disambiguating negation scope in parallel to its disambiguation via Shakespearean pronunciation. First, recall that (12.1) *cannot* be produced or understood to be an instance of the universal negative categorical. Rules of English pronunciation prevent it. (That is, say (12.1) and try to “mean” something equivalent to “No glittering thing is gold”. You can’t. The way English is prevents you. Of course, clear thinking while pronouncing is required. One must get beyond the confusions that arise on *first* discussing this in logic class. Similarly, say (12.2) and try to “mean” something equivalent to “Some glittering things are not gold”. Again, you can’t.) For English, however, this Shakespearean pronunciation choice is not a widely generalized phenomenon. (The rule is not “productive”.) For notice that (13) pronounced either as (13.1) or (13.2) does NOT produce the same alternatives (wide- vs. narrow-scope negation) as (14.1) and (14.2) do for (14):

- (13) Many singers are not poets
- (13.1) *MANY* singers *ARE* not poets

- (13.2) Many singers are *NOT* poets
- (14) All singers are not poets
- (14.1) *ALL* singers *ARE* not poets
- (14.2) All singers are *NOT* poets

That is, (as I see it) either (13.1) or (13.2) is appropriate for saying something contrary (actually sub-contrary) to "Many singers are poets". (13.1) does not mean something equivalent to "Not many singers are poets" (= "Few singers are poets") —the latter being the contradictory denial. (If it can for you, at least the phenomenon is not as clearcut as (14) alternatives.)

Something analogous (not closely similar, just analogous) to the alternatives with (14) —the Shakespearean alternatives— occurs for prefixed "not" with directly following *complex* quantifiers. Consider

- (15) Many more than 1/4 of the S are M

(where, say, "S" = "singers" and "M" = "musicians"). If we place "not" before this sentence, we get an ambiguity (I contend) which can be *disambiguated* by contrastive stressings, to wit:

- (16) *Not* many more than 1/4 of the S are M
- (17) Not *many* more than 1/4 of the S are M

(17) is the clearest side, I believe. That is, (17) seems clearly to imply or mean that "many more", as vs. just "more", is being denied for the quantity of singers that are musicians that is greater than 1/4 of the singers. One symptom of this interpretation is that when (17) is asserted, it is implied that *more* than 1/4 of the singers *are* musicians (just not *many* more). In short, the prefixed "not" in (17) is aimed at the quantitative modifier "many" of the embedded (complex) quantifier "more than 1/4". So, (17) entails (18):

- (18) More than 1/4 of the S are M.

Now (16) is different. I take it that there is another way of denying (15) via prefixed "not" which (i) does not amount just to denying the "many" component and (ii) expresses the contradictory denial of (15). I think (16) does that. (This is, I believe, more arguable than my description of the nature of (17). However, if (16) does not quite do what I believe it does, then we would have to provide a longer expression for the contradictory sort of denial of (15) — say, "It's not the case that many more than 1/4 of the S are M".) In other words, (16) expresses wide-scope denial (propositional logic negation) of (15). And perhaps even the special stress noted in (16) is op-

tional; i.e., the real contrast is between (17) and the same string with the absence of special stress on "many". (Appropriate background for this feature of (17) occurs in my discussion of so-called "quantifier negation" in "On the Logic of 'Few', 'Many', and 'Most'", *Notre Dame Journal of Formal Logic*, 1979, 20(1), 155-177.)

So, it seems clear (to me anyway) that (16) is equivalent to the denial of (15); i.e., I will assume that "(16) = (15)" is true. Further, a little thought will confirm that

(16) = (15) = Almost 3/4 of the S are not-M

For further help with *why* this is true see my "Complexly Fractionated Syllogistic Quantifiers" (in *Journal of Philosophical Logic* 1991), especially square of opposition (5). (5) is appended below!

To nail down the potential ambiguity of (15) which I claim is DISambiguated via intonation contrasts —(17) being one way it is disambiguated and (16) the other— it is necessary to show that (16) and (17) are logically distinct. This is the same thing as showing that (15) and (17) are distinct. In short, wide-scope negation of (15) is logically inequivalent to denial of the "many" component. (This strikes me as analogous to the Shakespearean intonation phenomena in that a special intonation shows where the negation goes.) To show this turns out to be more lengthy than I would like (though, of course, it is possible I have missed an obvious economy). Here is the *idea* behind the longer demonstration. First (to be brief about it), assume that 3/4 of the actual singers really are *non-musicians* —i.e.:

(19) 3/4 the S are non-M.

(That is possible —even likely I think. So, pretend that that possible world is the real one.) Now the question here is whether (17) and the contradictory denial of (15) are really inequivalent. One way to proceed is via *reductio* argument; i.e., hypothesize that (17) and $\sim(15)$ are equivalent and see if it leads to a contradiction. It does (at least it quickly does in this world wherein 3/4 of the actual singers are non-musicians).

In outline, the argument is:

1. (17) $\equiv \sim(15)$ RAA hyp (in world where (19) is true)
2. (17) \supset (18) English semantic data (see above)
3. $\sim(15) \supset$ (18) Sub. in 2 via 1
4. (19) $\supset \sim(15)$ From entailment down columns in (5) below
5. (19) $\equiv \sim(18)$ From squares in (5) below
6. (19) \supset (18) From lines 3 and 4 (H.S.)

7. $\sim(18) \supset (18)$ Sub. in 6 via 5
8. (18) From line 7

So, given the equivalence (a supposed *reductio* hypothesis) we have deduced that (18) is true. But (18) *contradicts* (19) and (19) is true in the world we are in (or so we have pretended anyway). So, for *this* particular world, assuming that (17) is equivalent to (16) or to $\sim(15)$ leads to a contradiction. So, logically speaking we have to give up something. I myself do *not* think we have proved that it is impossible that 3/4 of the actual singers are non-musicians. (Anyone who has known as many bad singers as I have will agree that singers are *not* by definition musicians. I would be very suspicious of anyone who said that $\sim(19)$ could be proved as a matter of logic.) I think we have to *give up* the other side of the matter — viz., that it is possible that (17) is equivalent to $\sim(15)$. From this I am willing to generalize that for any such sentences (with various related rational fractions between 0 and 1), wide-scope denial of a "Many more than m/n ..." sentence is *inequivalent* to a sentence of the form "Not *many* more than m/n ..."

(In the short argument just given, I mentioned entailment down columns in squares of opposition — as a reason in step 4. I have discussed this in my early article "On the Logic of 'Few', 'Many', and 'Most'", *op. cit.* and in my most recent one (among other places) — "Complexly Fractionated Syllogistic Quantifiers", *op. cit.*)

In the more complete *reductio* of the hypothesis that (17) is equivalent to $\sim(15)$, some additional sentences are required — viz.,

- (20) 1/4 of the S are M
- (21) More than 3/4 of the S are non-M
- (22) Many more than 3/4 of the S are non-M
- (23) Not *MANY* more than 3/4 of the S are non-M

In order to prove that (17) is inequivalent to (15), I have had to argue below that its denial is part of a conjunction where the other conjunct has exactly the same form (*vis à vis* "not *many*" vs. "many...") and where the conjunction entails a contradiction. Since what is at issue in each conjunct is the same, I conclude that *both* conjuncts are false (even though the most that is formally proved is that one of them is).

Proof:

1. $((17) \equiv \sim(15)) \& ((23) \equiv \sim(22))$ RAA hypothesis
2. $(17) \equiv \sim(15)$ From 1
3. $(17) \supset (18)$ From English semantic data
4. $\sim(15) \supset (18)$ From lines 2 & 3
5. $(19) \supset \sim(15)$ Column entailments; cf. (5) below

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|-----|---|--|
| 6. | $(19) \equiv \sim(18)$ | From Squares of Opposition (5) below |
| 7. | $(19) \supset (18)$ | From line 4 & 5 |
| 8. | $\sim(18) \supset (18)$ | Sub. in 7 via 6 |
| 9. | (18) | From line 8. |
| 10. | $(18) \supset (20)$ | Column entailments; cf. (5) below |
| 11. | (20) | From lines 9 & 10 |
| 12. | $(20) \equiv \sim(21)$ | From Squares of Opp. (5) below |
| 13. | $\sim(21)$ | From lines 11 & 12 |
| 14. | $(22) \supset (21)$ | Column entailments; cf. (5) |
| 15. | $\sim(22)$ | From lines 13 & 14 |
| 16. | $\sim((22) \equiv (23))$ | From line 1 |
| 17. | (23) | From lines 15 & 16 |
| 18. | $(23) \supset (21)$ | English sem. data (parallel to line 3) |
| 19. | (21) | From lines 17 & 18 |
| 20. | $(21) \supset (19)$ | Column entailments; cf. (5) |
| 21. | (19) | From lines 19 & 20 |
| 22. | $\sim(18)$ | From lines 6 & 21 |
| 23. | $(18) \& \sim(18)$ | From lines 9 & 22 |
| 24. | $((17) \equiv \sim(15)) \& ((23) \equiv \sim(22)))$ | <i>Reductio ad absurdum</i> , lines 1-23 |

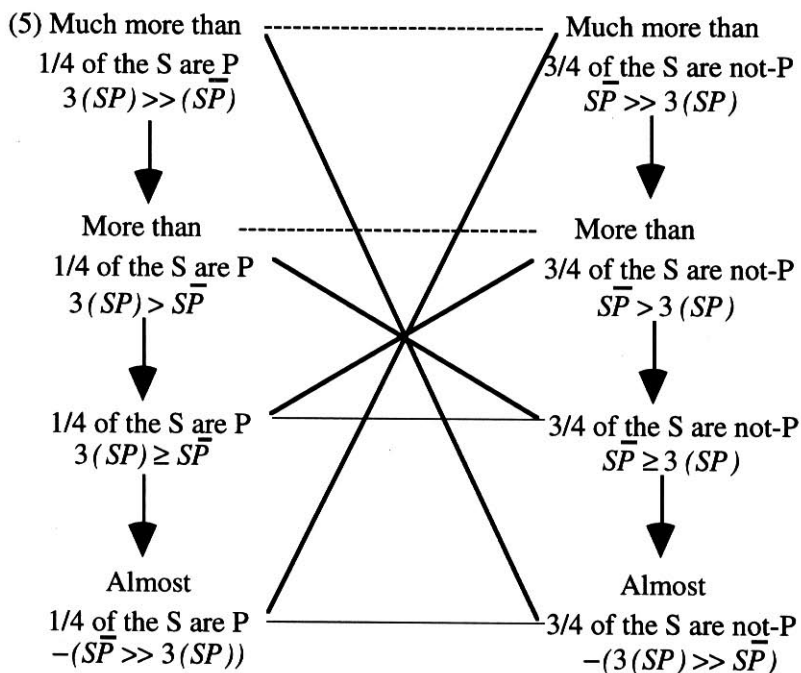
This is less than perfect, but not too much [*sic*] less. I have proved the denial of a conjunction. So, one conjunct or the other must be false. But *both* might be false too. I believe they are —i.e., what's wrong with one is the same thing that is wrong with the other (which is assuming "not *many* more..." is equivalent to "not (many more...)").

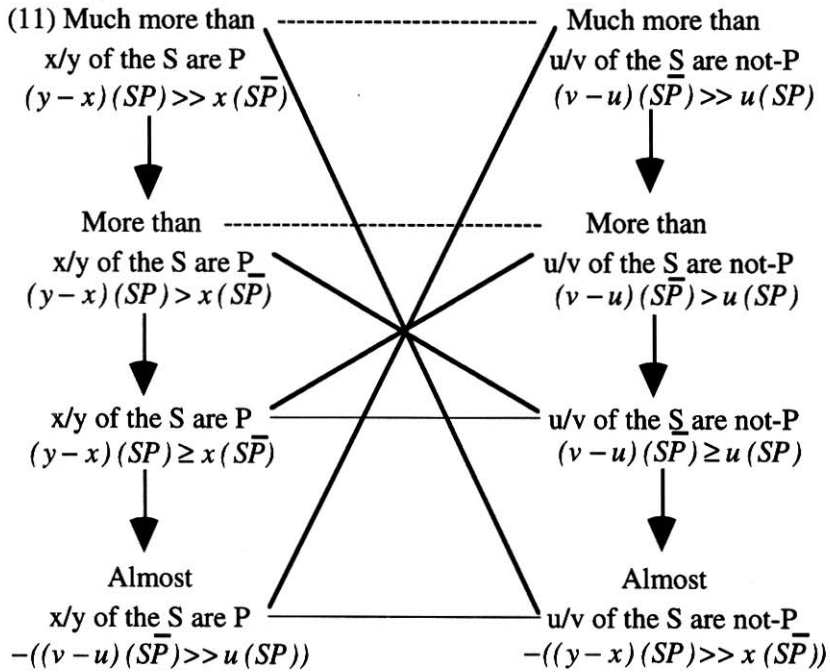
Finally, I do not believe that "Not any S is P" is a counter-example (i.e., that prefixing "not" produces a contrary, rather than, a contradictory here). The reason is that "Not any S is P" is formed by prefixing "Not" to "Some S is P"; i.e., "some" is *pronounced* "any" when preceded by "not". In any case, "any" is a peculiar word. For my *argument* that "Not any S is P" (and even "No S is P") is the product of prefixing "not" to a particular affirmative, see my "Syllogistic Logic and the Grammar of Some English Quantifiers", Indiana U. Linguistics Club Publications, 1988.

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Appendix

Here is the above referred to (5) from "Complexly Fractionated Syllogistic Quantifiers" (*op. cit.*) together with the generalization of it, (11), from the same paper:





where if $x/y = m/n$, then $u/v = (n-m)/n$, and if $u/v = m/n$, then $x/y = (n-m)/n$, AND m and n are integers $m/n \leq 1/2$.

(11) only holds for $n > m > 0$; i.e., n/n is not taken to be "all" and "0/n" is not taken to be "none".