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EXPANDING THE TRADITIONAL SYLLOGISM

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I have developed a version of categorical logic[1] that includes the traditional syllogism, while it also has the capacity to handle such problems as: What follows from the premises below concerning the L's that are H? [2]

1. At most 5 non$T$'s are non$P$,
2. At most all but 40 non$X$'s are non$L$,
3. At least all but 17 non$H$'s are non$G$,
4. At most 4 non$X$'s are T, and
5. At least all but 10 P's are G.

The gist of that expanded version is presented in what follows.

Point of Departure. The conventional preparation of these premises for categorical logic presents them as a series of particular forms, or the conjunction of particular forms, such as

1. Some non$T$'s are non$P$	and	Some non$T$'s are not non$P$ =
$T'I\ P'$	and	$T'O\ P'$,
2. Some non$X$'s are non$L$	and	Some non$X$'s are not nonL =
$X'I\ L'$	and	$X'O\ L'$,
3. Some non$H$'s are non$G$	and	Some non$H$'s are not non$G$ =
$H'I\ G'$	and	$H'O\ G'$,
4. Some non$X$'s are T	and	Some non$X$'s are not T =
$X'I\ T$	and	$X'O\ T$, and
5. Some P's are G	and	Some P's are not G =
$P\ I\ G$	and	$P\ O\ G$.

[1] See Wallace A. Murphree, *Numerically Exceptive Logic: A Reduction of the Classical Syllogism* (New York: Peter Lang Publishers, 1991)

[2] I posed this problem to a class of ten students. One student proposed an answer after working for 35 minutes, but it was incorrect; and all the others eventually gave up "without a clue." However, when we came to such problems toward the end of the semester, all the students solved them routinely in 5-10 minutes.

But from the renditions given in 1'-5' no conclusion follows, because elements essential to the argument are pruned away to achieve the fit with standard form. My alternative approach involves expanding the traditional standard form to that of the format exhibited by the premises as they stand in 1-5 above. The result is a single logic which handily accommodates both the classical syllogism, and also such extended arguments as this one.

The expanded rendition of the traditional *A, E, I*, and *O* propositions are as follows:

S A P = At least all but *0 S* 's are *P*
S E P = At most *0 S* 's are *P*
S I P = At least 1 *S* 's are *P*
S O P = At most all but *1 S* 's are *P*

Then, given this general format, it becomes apparent that quantities other than zero and one might be substituted into such expressions. This, accordingly, can be indicated by the convention,

(x)S A P = At least all but x *S* 's are *P*,
(x)S E P = At most x *S* 's are *P*,
(x)S I P = At least x *S* 's are *P*, and
(x)S O P = At most all but x *S* 's are *P*.

Natural Language Expressions. Some typical instances of these types of expressions are as follows:

(9)*S A P* = At least all but 9 students will pass,
At minimum, all but 9 students will pass,
Either exactly all but 9 students will pass, or more will pass,
At most 9 students will not pass,
Etc.

(8)*S E P* = At most 8 students will pass,
At maximum, 8 students will pass,
Exactly 8 students will pass, or fewer will pass,
Fewer than 9 students will pass,
Etc.

(7)*S I P* = At least 7 students will pass,
At minimum, 7 students will pass,
Exactly 7 students will pass, or more will pass,
More than 6 students will pass,
Etc.

(6)*S O P* = At most all but 6 students will pass,
At maximum, all but 6 students will pass,
Either exactly all but 6 students will pass, or fewer will pass,
At least 6 students will not pass,
Etc.

Such expressions may seem to instantiate these general forms more readily than do the traditional propositions. But, if so, I submit the difference is only one of familiarity of expressions, and that conceptually the traditional propositions also constitute perfect instances of them. Each traditional form is considered in turn below.

The expanded version of the traditional *A*-form,

S A P = All students will pass,

is rendered as

(0)*S A P* = At least all but 0 students will pass.

Here the expansion involves the introduction of "...but 0...," and the qualification of the expression by "At least....". But these modifications do not alter the concept since, first, appending "...but 0..." to "all" simply serves to make part of the original meaning of "all" explicit, since "all" means "all, with no exception," or "all but 0."

Likewise, the qualification of "At least..." also merely serves to make part of the original quantifier explicit. That is, the expression, "...all but x *S*'s are *P*," is ambiguous, since it could mean "At least all but x *S*'s are *P*"$[(x)S\ A\ P]$," At most all but x *S*'s are *P*"$[(x)S\ 0\ P]$, or "Exactly all but x *S*'s are *P*," which is tantamount to " At least all but x S's are *P*, and at most all but x *S*s are *P*": $[(x)S\ A\ P$ and $(x)S\ O\ P]$. So, for example, when $x = 9$, as in the case above, "...all but 9 *S*'s are *P*" must be completed with "At least...," "At most...," or "Exactly..." ; however, when $x = 0$ the qualification can be omitted since "At most all [but 0] *S*'s are *P*" is tautologously vacuous, since it holds for every possible *S* and *P*. That is, "At most all [but 0] *S*'s are *P*" is tantamount to "All, or fewer than all, *S*'s are *P*," and this covers every possibility. Accordingly, "All *S*'s are *P*" must be taken to mean either "At least all [but 0] *S*'s are *P*" or "Exactly all [but 0] *S*'s are *P*."

But since "Exactly all [but 0] *S*'s are *P*" is equivalent to the conjunction of "At least all [but 0] *S*'s are *P*, and at most all [but 0] *S*'s are *P*," it is clear, since the latter conjunct is vacuous, that the former conjunct is the significant one. So, since whatever goes without saying is not said in the economy of natural language, the "At least..." is always omitted and the conjunct is simply expressed by "All *S*'s are *P*."

The expanded version of the traditional *E*-form,

S E P = No students will pass,

is rendered as

(0)*S E P* = At most 0 students will pass.

Here the expansion involves substituting "0" for "no," and qualifying the expression by "At most...." The substitution seems acceptable since "No *S*'s are *P*" is clearly equivalent to "Zero *S*'s are *P*." And the justification for the qualification of "At most..." is, *mutatis mutandis,* the same as the justification for the qualification of the *A*-form by "At least...," given above. That is, "...*x S*'s are *P*" must be disambiguated with the prefix of "At least...," "At most...," or "Exactly..." (which is "At least... and at most..."); but, when $x = 0$ it is clear that the significant alternative is "At most...," since "At least 0 *S*'s are *P*" is a tautology. Accordingly, the qualification of "At most..." simply makes explicit "what goes without saying" in the natural language.

The expanded version of the traditional *I*-form,

S I P = At least 1 student will pass,

is rendered identically as

(1)*S I P* = At least 1 student will pass;

but the expanded version of the traditional O-form

S O P = At least 1 student will not pass,

is rendered quite differently as

(1)*S O P* = At most all but 1 student will pass.

Here the entire form of the expression is altered from "At least 1 ...is not..." to the equivalent "At most all but 1 ...is...." The latter mode is adopted simply to achieve a consistency in the quantifiers, so that the negative expressions,

O: At most all but *x*...
E: At most *x*...
have a parity with the affirmative ones:
A: At least all but *x*...
I: At least *x*.... [3]

The result is that the affirmative and negative qualifiers ("at least" and "at most") now consistently constitute part of the quantifiers. But this consistency is somewhat arbitrary, and the familiar, but odd, *O*-form could be maintained without harm to the expanded rendition. Or, an alternative, consistent set that patterns the *E*-form after the familiar *O*-form could be used, such as:

O: At least *x*...are not...
E: At least all but *x*...are not...
and
A: At least all but *x*...are...
I: At least *x*....are.... [4]

So, again, I submit that the traditional propositions —now (0)*S A P*, (0)*S E P*, (1)*S I P*, and (1)*S O P*— constitute only one of infinitely many possible instantiations of the more general forms, (*x*)*S A P*, (*x*)*S E P*, (*x*)*S I P*, and (*x*)*S O P*, respectively.

[3] In fact, to achieve even greater parity I use "none but" in the quantifiers of the *E* and *I* propositions in *Numerically Exceptive Logic*, so that the forms appear as:
A: At least all but...
E: At most none but...
I: At least none but...
O: At most all but...

[4] George Englebretsen proposes a similar pattern in, for example, "The Myth of Modern Logic," *Cogito*, Autumn 1990, where he suggests
O: Some...is not...
E: Every...is not...
and
A: Every...is...
I: Some...is....

Terminology.
(i) The value of *x* will be called the "deviation" of the quantifier (since the quantity of "...all but *x*" is a quantity that *deviates* from "all" by the amount of *x*, while the quantity of "... *x*" is a quantity that *deviates* from "none" by the amount of *x*.)
(ii) Statements containing a variable, *x*, will be said to express general "proposition types," rather than propositions.
(iii) The quantity of propositions and proposition types will be said to be "global" and "local," rather than universal and particular, since *A* and *E* propositions having a deviation greater than zero, such as "At least all but 2 *S*'s are *P*" [(2)S *A P*], or "At most 2 S's are *P*" [(2)S *E P*], are not fully universal.

Immediate Inference. The equivalent statements of the general proposition types (as well as of each new instantiation) is the same, *mutatis mutandis,* as it is for the traditional instantiations. Accordingly, the four equivalent statements of the global proposition types are:

1. (*x*)*S A P*	=	At least all but *x S* 's are *P*	(original)
2. (*x*)*S E P'*	=	At most *x S* 's are non*P*	(obverse of 1.)
3. (*x*)*P'E S*	=	At most *x* non*P*'s are *S*	(converse of 2.)
4. (*x*)*P'A S'*	=	At least all but *x* non*P*'s are non*S*	(obverse of 3, or contrapositive of 1.)

1. (*x*)*S E P*	=	At most *x S* 's are *P*	(original)
2. (*x*)*P E S*	=	At most *x P*'s are *S*	(converse of 1.)
3. (*x*)*S A P'*	=	At least all but *x S* 's are non*P*	(obverse of 1.)
4. (*x*)*P A S'*	=	At least all but *x P*'s are non*S*	(obverse of 2, or contrapositive of 3.)

That these equivalences actually hold should be apparent upon inspection. For the *A*-type the obverse must hold since if "At least all but *x S*'s are *P*" then the greatest number of *S*'s that can possibly be non*P* is *x*; hence, "At most *x S*'s are non*P*." And in return, if "At most *x S*'s are non*P*" then at least all the rest of them must be *P*, or "At least all but *x S*'s are *P*." Then the *E*-type, formed by the obversion of the original, has to be convertible since both "At most *x S*'s are non*P*" and "At most *x* non*P*'s are *S*" assert that classes *S* and non*P* have *x* members in common at maximum. And the same considerations hold, *mutatis mutandis,* when the *E*-type is the original statement.

The four equivalent statements for the local proposition types are:

1. $(x)S\ I\ P$	= At least x S's are P	(original)
2. $(x)P\ I\ S$	= At least x P's are S	(converse of 1.)
3. $(x)S\ O\ P'$	= At most all but x S's are non$P$	(obverse of 1.)
4. $(x)P\ O\ S'$	= At most all but x P's are non$S$	(obverse of 2, or contrapositive of 3.)

1. $(x)S\ O\ P$	= At most all but x S's are P	(original)
2. $(x)S\ I\ P'$	= At least x S's are non$P$	(obverse of 1.)
3. $(x)P'I\ S$	= At least x non$P$'s are S	(converse of 2.)
4. $(x)P'O\ S'$	= At most all but x non$P$'s are non$S$	(obverse of 3, or contrapositive of 1.)

Here it is obvious that the converse of the *I*-type holds since "At least *x* *S*'s are *P*" and "At least *x* *P*'s are *S*" both assert that classes *S* and *P* have *x* members in common at minimum. And the obverse holds since, if "At least *x* *S*'s are *P*" then at most all the rest of them are non*P*; hence, "At most all but *x* *S*s are non*P*." And in return, if "At most all but *x* *S*'s are non*P*" then *x* *S*'s must be *P;* hence, "At least *x* *S*'s are P." And the same considerations hold, *mutatis mutandis,* when the *O*-type is the original statement.

It now may be clearer why "At most all but 1 *S* is *P*" is a proper rendition of the traditional *O* proposition, "At least 1 *S* is not *P*," since they both obvert into "At least 1 *S* is non*P*."

Truth. An *affirmative* categorical proposition is false if it asserts that *more* *S*'s are *P* than there are actual members of the *S* class that are also members of the *P* class; otherwise it is true. For example, suppose a class of 20 Students, of which exactly 9 are Protestants. Then, for the *I*-form, it is false to assert that "At least 10 (11, 12, ...n) *S*'s are *P*," while it is true to assert that "At least 9 (8, 7,...0) *S*'s are *P*." (Again, "At least 0 *S*'s are *P*" is necessarily true.) Likewise, for the *A*-form, it is false to assert that "At least all but 10 (9, 8,...0) *S*'s are *P*" (since all but 10 [9, 8,...0] of the 20 students amounts to 10 [11, 12,...20] of them), while only 9 of them are *P* in fact; and it is true to assert that "At least all but 11 (12, 13,...20) *S*'s are *P*" (since all but 11 [12, 13,...20] of the 20 students amounts to 9 [8, 7,...0] of them.

A *negative* categorical proposition is false if it asserts that *fewer* *S*'s are *P* than there are actual members of the *S* class that are also members of the *P* class; otherwise it is true. Hence, for the *E*-form, it is false to assert that "At most 8 (7, 6,...0) *S*'s are *P*," since exactly 9 are; and it is true to assert that "At most 9 (10, 11,...n) *S*'s are *P*." (Note that if "At most 9 *S*'s are *P*" is true, then "At most 10 (11, 12,...n) *S*'s are *P*" must also be true. That is, "at most 9..." is "9 or fewer"; so, if "9 or fewer *S*'s are *P*" is true, then "10 (11, 12, ...n) or fewer *S*'s are *P*" is true *a fortiori.*) Likewise, for the *O*-form, it is false to assert that "At most all but 12 (13, 14,...20) *S*'s are *P*," (since all but

12 [13, 14,...20] of the students amounts to only 8 [7, 6,...0] of them) while, again, 9 of them are *P*. And it is true to assert that "At most all but 11 (10, 9,...0) *S*'s are *P*" (since all but 11 [10, 9,...0] of the students amounts to 9 [10, 11,...20] of them), and so the claim is that 9 (10, 11,...20) or fewer of the *S*'s are *P*.

As the equivalences of immediate inference illustrate above, these two truth conditions (for the affirmative and negative statements) are complementary, since an affirmative statement is false (or true) when, and only when, its obverse, negative statement is the same, and *vice versa*. Hence, for the I-type, the "at least *x*" makes a claim about too many when and only when the complementary "at most all but *x*" makes the claim about too few in its O-type obverse; and for the A-type, the "at least all but *x*" makes a claim about too many when and only when the complementary "at most *x*" makes the claim about too few in its *E*-type obverse.

Existential Commitment. In keeping with the modern interpretation of categorical logic, the rendition I propose requires existential commitment for the local (particular) propositions, but it does not require such commitment for the global (universal) ones, although existence may be assumed when global claims are made. The principle dictating this perspective is: *To assert a statement is tacitly to affirm the conditions under which it is possible for it to be true.* Hence, to assert "At least *x S*s are *P*" is tacitly to affirm that there exist at least *x S*'s, since otherwise it would be impossible for the assertion to be true. That is, otherwise the statement would claim that more *S*'s are *P* than there are actual *S*'s. And the same is the case for "At most all but *x S*'s are *P*" (or "At least *x S*s are non*P*). Accordingly, the traditional "At least 1 *S* is *P*(non*P*)" is taken to affirm the existence of at least one *S*.

However, it is possible for global propositions of any deviation, *x*, to be true whether membership is supposed for the class indicated by the subject term or not. An example sometimes offered to show that traditional universals need not require an existential commitment is "All trespassers will be prosecuted," since clearly it can be true even if there are no trespassers. But, by the same token, the property owner (who intends to extend grace to the first violator) can also truly assert "At least all but 1 trespasser will be prosecuted," even if there are no trespassers. Or, the executive who has 10 positions to fill can truly assert "At least all but 10 applications will be rejected" (or in the obverse, "At most 10 applications will be accepted"), whether there are to be many applications or none. So, since the truth of these propositions does not depend on the existence of trespassers or applications, to assert them is not tacitly to affirm that some *S*'s (trespassers or applications) exist. And such is the case generally for the global types.

Of course one may assume that some *S*'s exist, as in the example of the students who are protestants. And when such assumptions are appropriately made the *A*-types can be converted, and the *E*-types contraposed, "by limitation." However, for the sake of simplicity the following development will proceed without such existential assumptions for the global types. [5] (An alternative way of handling syllogisms that are valid by virtue of existential presupposition will be advanced later.)

Distribution. The distribution pattern of the terms in the general proposition types (as well as in each new instantiation) is the same as it is in the traditional instantiation. That is, the subject terms of the global statements are distributed as are the predicate terms of negative statements, while all other terms are undistributed.

This claim may at first appear questionable since it would make *S* distributed in, for example, "At least all but 11 students are protestants," even though this proposition does not predicate *P* of every *S* as it presumably should if it were actually distributed. However, the proposed pattern of term distribution clearly holds when distribution is conceived as a matter of term-replacability, as is outlined below. [6]

First, let terms of progressively decreasing intension be called progressively "wider terms," as in "cat-mammal-animal," and terms of progressively increasing intension be called progressively "narrower terms," as in "animal-mammal-cat." Then a term is distributed in a proposition if, and only if, it can validly be replaced by any narrower term (in the sense that the proposition formed by the replacement is implied by the original proposition by virtue of the replacement [7]), and it is undistributed if, and only if,

5 See *Numerically Exceptive Logic*, pp. 16-20 for a treatment of existential presupposition for global propositions, and Chapter 7 for a treatment of inferences that are valid by virtue of such presupposition.

6 This is essentially the concept of distribution advanced by Stephen F. Barker in *The Elements of Logic* (New York: McGraw-Hill, 1965), pp. 43-44.

7 The implication must hold *by virtue of* the scope of the replacing term because, for example, otherwise, "cats" would be ruled undistributed in "All cats are animals" since it implies "All animals are animals." But here the implication does not hold *by virtue of* the fact that "cats" is replaced by a wider term but, instead, by virtue of the fact that "All animals are animals," being necessarily true, is implied by every sentence, whatever. Analogously, "animals" may not be ruled distributed in "Some square circles are animals" simply because this implies "Some square circles are cats." That is, here the implication does not hold *by virtue of* the fact that "animals" is replaced by the narrower term but, instead, by virtue of the fact that "Some square circles are animals," being necessarily false, implies every sentence, whatever.

it can validly be replaced by any wider term (in that sense).

Now, "people (in the class)" —which would include the teacher and any visitors— is a wider term than "students (in the class)," while "juniors (in the class)" is a narrower term; and "Christians" is a wider term than "protestants," while "Methodists" is a narrower term. Accordingly, for the expanded rendition of the *A* proposition, as well as for the traditional version, the subject term, student, can validly be replaced with the narrower term, juniors, while it cannot validly be replaced with the wider term, people. That is, from the original claims,

> At least all but *x* students are protestants, and
> All students are protestants,

it follows that

> At least all but x juniors are protestants, and
> All juniors are protestant;

but it does not follow that

> At least all but x people are protestants, or
> All people are protestant.

This illustrates that the subject is distributed in both occurrences. On the other hand, the predicate, protestants, can validly be replaced with the wider term, Christians, while it cannot validly be replaced with the narrow term, Methodists. That is, it follows from the original claims that

> At least all but *x* students are Christians, and
> All students are Christians,

but it does not follow that

> At least all but x students are Methodists, or
> All students are Methodists;

and this illustrates that both predicate terms are undistributed. And the application of this test to the *E*, *I*, and *O*, confirms that the pattern of term distribution for each of the expanded types is the same as it is for the traditional forms, given this concept of distribution.

Rules of Syllogistic Validity.[8] The rules of distribution and quantity below are essentially those proposed by Fred Sommers.[9] These four rules are sufficient to identify the valid traditional syllogistic forms of the hypothetical perspective. And the final two rules are sufficient to identify any other valid deviation for these moods and figures occurring in the expanded version.

Rules of Distribution

1. The middle terms are to have opposite distribution values.
2. The extreme terms are to have the same distribution values in the conclusion that they have in the premises.

Rules of Quantity

3. At least one premise is to be global.
4. The conclusion is to be local if and only if a premise is local.

Rules of Deviation [10]

5. The deviation for a global conclusion is to be no less than the sum of the deviation of the premises.
6. The deviation for a local conclusion is to be no greater than the deviation of the local premise minus the deviation of the global premise.

Applications. So, for example, the following renditions of *AAA*-1 and *AII*-1 are valid (since they conform to each of the six rules),

[8] In *Numerically Exceptive Logic* a system of schematics is employed to illustrate the categorical entailment, and three criteria of validity that are closely tied to the schematic representations are used, instead of the rules given here.

[9] Fred Sommers, "Distribution Matters," *Mind* LXXXIV (1975), p. 34.

[10] The deviation of the strongest global conclusion entailed is equal to the sum of the deviation of the premises, but weaker conclusions, having greater deviations, are also entailed; and the deviation of the strongest local conclusion entailed is equal to the deviation of the local premise, minus the deviation of the global premise, but weaker conclusions, having smaller deviations are also entailed.

Traditional Instantiation	One Expanded Instantiation	Another Expanded Instantiation
(0)*M A P*	(4)*M A P*	(4536278)*M A P*
(0)*S A M*	(5)*S A M*	(7389502)*S A M*
———	———	———
(0)*S A P*	(9)*S A P*	(11925780)*S A P*
(0)*M A P*	(3)*M A P*	(9564632)*M A P*
(1)*S I M*	(7)*S I M*	(13586957)*S I M*
———	———	———
(1)*S I P*	(4)*S I P*	(4022325)*S I P*

as are any other instantiations of the general form types:

(*x*)*M A P*	(*x*)*M A P*
(*y*)*S A M*	(*y*)*S I M*
———	———
(*y*+*x*)*S A P*	(*y*-*x*)*S I P*

And the same is the case, *mutatis mutandis*, for each of the thirteen other valid traditional forms, and their expanded renditions. [11].

Moreover, each invalid form stands in violation of at least one of the six rules. Sets of premises that yield no conclusion whatever stand in violation of rule 1 or 3 (or both), and arguments that are invalid because their conclusions are of the wrong quality or quantity (or both) stand in violation of rule 2 or 4 (or both). And arguments that are invalid only because the conclusions are of the wrong deviation stand in violation of either rule 5 or 6. For example, if the conclusions of the expanded global forms above had been (8)*S A P* and (11925779)*S A P* —or any smaller deviation— they would be in violation of rule 5, while if the local conclusions had been (5)*S I P* and (4022326)*S I P* —or any greater deviation— they would be in violation of rule 6.

These, again, are rules for the hypothetical perspective and, accordingly, prohibit such syllogisms as *AAI*-1 and *EAO*-3, although these are valid from the existential perspective. Sommers[12] casts these cases as enthymematic sorites in which the existential premises are suppressed. So he would make the suppressed premises explicit, and then work the resulting

[11] See *Numerically Exceptive Logic*, Chapters 4 and 5, for an exhaustive treatment of these valid form types.

[12] Sommers, p. 35.

sorites for the particular conclusions. That is, while

M A P		*M E P*
S A M	and	*M A S*
———		———
S I P		*S O P*

are ruled invalid as they stand (since they are in violation of rules 2 and 1, respectively), they turn out to be valid sorites when "At least 1 *S* is an *S*" (*S I S*) is added to the first, and "At least 1 *M* is an *M*" (*M I M*) is added to the second. (Note that unlike "All *S*'s are *S*," which is a tautology, "At least 1 *S* is an *S*" is a contingent form.) Now, with the existential premises made explicit, the ingredient syllogisms of the sorites can appear as

S A M			*M I M*		
S I S			*M A S*		
		M A P			*M E P*
———			———		
S I M	----->	*S I M*	*S I M*	----->	*S I M*
		———			———
		S I P			*S O P*

and these ingredient syllogisms conform to the rules. And the validity of the other syllogisms valid only from the existential perspectives can be accounted for in like fashion.

Also in like fashion, the extended syllogism can handle global premises with the existence of any number of members presupposed. According, for example, the following forms,

(8)*M A P*		(20)*M E P*
(7)*S A M*	and	(30)*M A S*
———		———
(9)*S I P*		(10)*S O P*,

are invalid as they stand; but they turn out to be valid sorites, when the existence of 24 *S*'s is presupposed for the first, and 60 *M*'s for the second, as follows:

$$
\begin{array}{c} (7)S\ A\ M \\ (24)S\ I\ S \\ \hline (17)S\ I\ M \end{array}
\ \text{----->}\
\begin{array}{c} (8)M\ A\ P \\ (17)S\ I\ M \\ \hline (9)S\ I\ P \end{array}
\qquad
\begin{array}{c} (60)M\ I\ M \\ (30)M\ A\ S \\ \hline (30)S\ I\ M \end{array}
\ \text{---->}\
\begin{array}{c} (20)M\ E\ P \\ (30)S\ I\ M \\ \hline (10)S\ O\ P \end{array}
$$

And, in general,

$$
\begin{array}{c} (x)M\ A\ P \\ (y)S\ A\ M \\ \hline (z)S\ I\ P \end{array}
\quad \text{and} \quad
\begin{array}{c} (x)M\ E\ P \\ (y)M\ A\ S \\ \hline (z)S\ O\ P \end{array}
$$

are valid when the existence of $(x+y+z)$ members is presupposed for the relevant term:

$$
\begin{array}{c} (y)S\ A\ M \\ (x+y+z)S\ I\ S \\ \hline (x+z)S\ I\ M \end{array}
\ \text{----->}\
\begin{array}{c} (x)M\ A\ P \\ (x+z)S\ I\ M \\ \hline (z)S\ I\ P \end{array}
\qquad
\begin{array}{c} (x+y+z)M\ I\ M \\ (y)M\ A\ S \\ \hline (x+z)S\ I\ M \end{array}
\ \text{----->}\
\begin{array}{c} (x)M\ E\ P \\ (x+z)S\ I\ M \\ \hline (z)S\ O\ P \end{array}
$$

And the same holds, *mutatis mutandis,* for the other sets of premises that yield conclusions by way of existential presupposition.

Having introduced sorites, the problem posed initially can now be solved by setting up ingredient arguments that conform to the rules. The premises (1-5) are symbolized below in column"a," and an equivalent statement of each is given in column "b." Then as the premises are introduced in the ingredient syllogisms they are identified by number and column (as, e.g.,"4b").

	a	b
1. At most 5 non*T*'s are non*P*	$(5)T'E\ P'$	= $(5)T'A\ P$ (obverse)
2. At most all but 40 non*X*'s are non*L*	$(40)X'O\ L'$	= $(40)X'I\ L$ (obverse)
3. At least all but 17 non*H*'s are non*G* ..	$(17)H'A\ G'$	= $(17)G\ A\ H$ (contrap.)
4. At most 4 non*X*'s are *T*	$(4)X'E\ T$	= $(4)X'A\ T'$ (obverse)
5. At least all but 10 *P*'s are *G*	$(10)P\ A\ G$	= $(10)G'A\ P'$ (contrap.)

Perhaps the most straightforward solution is as follows:

AII-3

4b. (4)*X'A T'* *AII*-1
2b. (40)*X'I L*
——— 1b. (5)*T'A P* *AII*-1
(36)*L I T'* -----> (36)*L I T'*
——— 5a. (10)P *A G* *AII*-1
(31)*L I P* ------> (31)*L I P*
——— 3b. (17)*G A H*
(21)*L I G* -------> (21)*L I G*
———
(4)*L I H*

The following solution, however, illustrates additional moods and figures.

AAA-1

5b. (10)*G'A P'*
3a. (17)*H'A G'* *AEE*-2
———
(27)*H'A P'* --> (27)*H'A P'*
1a. (5)*T'E P'* *EAE-1*
———
(32)*T'E H'* ---> (32)*T'EH'*
4b. (4)*X'A T'* *AII*-3
———
(36) *X'E H'*--(ob.) -> (36)*X'A H*
2b. (40)*X'I L*
———
(4)*L I H*

As both solutions show, the answer is that at least 4 *L*'s are *H*.

Significance of the Rendition. Perhaps the question of the importance of the expanded version of the syllogism can be appropriately raised from the basis of this introductory sketch of it. I suggest that it is important in two general ways. The first, as is illustrated in the examples above, is that it provides an immensely more powerful logic than the traditional syllogism, since it utilizes the "entire numerical iceberg," whereas the traditional logic accesses only the "tip." But also of importance, I submit, is that this very fact puts the traditional syllogism in a completely new light and thereby reveals one of its long-hidden secrets, viz., that its propositions and inferences are inherently numerical.

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