A NOTE ON PRIEST'S "HYPERCONTRADICTIONS"

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Graham Priest's Dialetheian account of the paradoxes is infamous, for it takes the conclusion of various paradoxical arguments at face value and accepts their contradictory conclusions as being true. Thus, for example, it takes the Liar sentence; "This sentence is false" to be both true and false. Priest claims that his approach is superior to all consistent accounts of the paradoxes, which require us to unnaturally restrict the expressive power of our language. However one might think that Priest's approach runs into difficulties with the following strengthened version of the Liar sentence; "This sentence is only-false", for it seems as if we will be able to run a version of the Liar argument to the conclusion that this sentence is both true and only-false. And this seems to commit us to identifying truth and falsity, a preposterous consequence which Priest himself rejects.

In his [1984] Graham Priest sought to avoid this difficulty by developing a semantics which permits a sentence to take "impossible" truth values such as only-true and only-false. And recently in his [1993] Priest invokes this semantics as one possible response to a version of the strengthened Liar argument offered by Timothy Smiley.

It is the purpose of this paper to point out that even granted his [1984] semantics Priest is still susceptible to a version of the strengthened Liar argument noted above, and to argue that given his [1984] semantics hold Priest will always be so susceptible unless he artificially restricts the expressive power of his language. I consequently take this to undermine Priest's response to Smiley.

I

Priest's [1984] semantics can be seen as a straightforward extension of his [1987] semantics. Let S_1 be the range of truth evaluations possible in the [1987] semantics - that is let S_1 be $\{\{True\}, \{False\}, \{True, False\}\}$. First we extend the range of possible truth evaluations from S_1 to the power set of S_1, minus the empty set. Call this new set S_2. We now end up with seven possible truth value evaluations in place of the original three:

$\{\{True\}\}$, $\{\{True\},\{True,False\}\}$, $\{\{True\},\{True,False\},\{False\}\}$, $\{\{True\},\{False\}\}$, $\{\{True,False\}\}$, $\{\{True,False\},\{False\}\}$, and

$\{\{False\}\}$.

We now take the power-set-minus-empty-set construction by means of which we moved from S_1 to S_2, and iterate it up to ω We let $S_{n+1} = \mathscr{P}S_n - \varnothing$. And we let S be the union of all $S_{k\in\omega}$. Finally we take the range of truth evaluations for our new semantics to be S.

Which of these truth values are we to regard as designated? Priest introduces a function η such that:

$\eta(True) = \{True\}$,
$\eta(False) = \{False\}$,

and if $x \in S_n$ for some $S_{n\in\omega}$ then:

$$\eta(x) = \cup\left\{\eta(z)/z \in x\right\}.^1$$

Priest suggests that we should take x to be designated just in case $True \in \eta(x)$. He then goes on to show that the logical consequence relation remains essentially unchanged if we adopt the suggested semantics, and to explore various other technical properties of the new semantics.

Thus it might appear that if we take the range of possible truth evaluations to be given by S then we will be unable to construct a problematic version of our strengthened Liar argument. This is because for every pair of distinct values $v \in S_m$, $w \in S_m$ there will be a value $\{v, w\} \in S_{m+1}$ which allows us to evaluate a sentence as both v and w without our having to identify these values. Thus our sentence "This sentence is only-false" is rendered innocuous because it can simultaneously receive both the value only-false (i.e. $\{False\}$) and the value $True$ without our having to equate truth and falsity. For it could simply be evaluated as $\{True\},\{False\}\}$.

II. *A stronger strengthened liar.*

However, in fact, Priest's [1984] *still* provides the apparatus to assert of a sentence that it is solely false. We can generate a version of the Liar sentence which is *genuinely* monovalent, which cannot be evaluated by Priest's [1984] semantics as being both true and false unless we identify those two values. For recall that his semantics allows us to distinguish a sentence which receives no designated value from a sentence which does. We can therefore generate the following sentence (the "Monovalent Liar"):

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[1] Put rather loosely, this function maps a set A onto the set of Ur-elements which are either members of A; or of A's members; or of A's member's members .... etc: thus it maps $\{\{True\}\}$ onto $\{True\}$; $\{\{\{True,False\}\},\{\{True\}\}\}$ onto $\{True,False\}$; and so on.

ML: $\eta v_\alpha(\text{"ML"}) = \{False\}$.[2]

Given Priest's definition of η this sentence says that the only valuation which it receives from Priest's [1984] semantics is *False*. A simple argument by dilemma now shows that *True = False*. For given his definition of η Priest's semantics either evaluate "ML" so that $True \in \eta v_\alpha(\text{"ML"})$, or they evaluate it so that $\eta v_\alpha(\text{"ML"}) = \{False\}$. Suppose that $True \in \eta v_\alpha(\text{"ML"})$. Then it follows that either;

(1a) $\eta v_\alpha(\text{"ML"}) = \{True\}$,

or,

(1b) $\eta v_\alpha(\text{"ML"}) = \{True, False\}$.

But in either case as $True \in \eta v_\alpha(\text{"ML"})$ then $T(\text{"ML"})$. So by the truth-schema, ML must be the case. And from our definition of "ML" this entails that;

(2) $\eta v_\alpha(\text{"ML"}) = \{False\}$.

From this follows the unfortunate consequence that either $\{False\} = \{True\}$ or $\{False\} = \{False, True\}$. And of course the standard interpretation of the set-membership relation entails that these equalities are only possible if *True = False*.

Suppose then that $\eta v_\alpha(\text{"ML"}) = \{False\}$. Then by our definition of η, ML will hold. It then follows from the truth-schema that;

(3) $T(\text{"ML"})$.

And once more the argument can proceed as above.

We appear to have assumed only the most basic truths of set theory, modus ponens, the substitutability of logically equivalent sentences within the scope of the truth-schema, and the semantic apparatus of Priest [1984]. On the assumption that "ML" is true we then proved that *True = False*. And on the assumption that "ML" is false we proved the same thing. Granted bivalence it therefore follows by dilemma that truth is falsity. And Priest's language is rendered trivial.

III.

How might Priest respond to this argument? Priest could certainly block our argument by denying that η is really a function, so allowing $\eta(x)$ to take two or more distinct values. But to do this would be to destroy the apparatus he requires for distinguishing designated evaluations from non-designated ones. And I think that Priest's language needs to be able to express this distinction for three reasons. In the first place we seem able to

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[2] Here the "α" indicates that we are evaluating "ML" with respect to the actual world-- see Priest [1987].

express the distinction in English, and so it must be expressible in Priest's language if he is not to unnaturally restrict its expressive power and so fall victim to exactly the same criticisms he levels against consistent accounts of the paradoxes.

In the second place Priest surely needs to be able to express the claim that certain absurd sentences are just plain false - to express the fact that his intended semantics evaluates such sentences as purely false and fails to evaluate them as true. Otherwise there would be no way in which we could rule out the possibility of the states of affairs which they describe obtaining. We need to be able to express the distinction between such sentences as "0=1" which are unreservedly false, and those Dialetheia and Hypercontradictions which are both true and false or which take impossible values.[3],[4]

Finally and most importantly note that Priest himself needs to make use of these notions if he is to avoid triviality. For there are a number of principles which threaten to render Priest's language trivial, including the Disjunctive syllogism and the Absorption principle.[5] Now prime facie these are plausible principles and if Priest is to reject them then he must provide some independent justification for these rejections, otherwise he will be subject to the very charges of ad hockery and unnaturally restricting the expressive power of his language that he levels against consistent accounts of the paradoxes. Priest attempts to provide such justification by producing counter-models to the principles. However it is not sufficient for Priest that these principles be false, for if they were both true and false or if they were Hypercontradictions they would still serve to render his language trivial.[6] For Priest, merely demonstrating that a principle is invalid will tell us *nothing* concerning whether that principle may be validly employed in inference. To do this Priest will need to further establish that the problematic principles also fail to be valid. To do this Priest must construct counter-models in which they are not Dialetheia or Hypercontradictions. And of

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[3] As traditionally conceived the act of denying *P* is identical to that of asserting *P*'s negation. However if there are true contradictions these two acts come apart. For if *P* is a Dialetheia then we will want to assert both *P* and its negation without denying either. See Parsons [1984] for a further argument that (even without Dialetheia) we should acknowledge that asserting *P* and denying its negation are separate acts.

[4] Note in fact that if Priest bars the apparatus necessary to construct the Monovalent Liar from his language then he will then be left in a *worse* position that those orthodox accounts of the Liar paradox he criticises. For he will not merely be subject to the same criticisms of ad hockery and expressive incompleteness, but he will also be saddled with nasty and counter-intuitive Dialetheia as well.

[5] See for example Priest's [1987] and [1990].

[6] For some further discussion of this point see my [1994].

course to do this Priest's semantics must be capable of expressing the notion of these principles not being Dialetheia or Hypercontradictions.

However if Priest's language can do this then we can employ the relevant apparatus to construct a version of the Monovalent Liar sentence and to generate a version of the argument outlined above.

I conclude that Priest cannot avoid a version of the Monovalent Liar argument without doing serious damage to his own position.

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