Logique & Analyse **139-140** (1992), 349-374

GRIM TALES RETOLD: HOW TO MAINTAIN ORDINARY DISCOURSE ABOUT —AND DESPITE— LOGICALLY EMBARRASSING NOTIONS AND TOTALITIES

Richard SYLVAN

Mainstream logical theory still conceals some very nasty shocks, right in the vicinity of the crisis that enveloped the subject at the beginning of the century, with deep problems in logical and set-theoretical foundations. While the mainstream theory and practice proceeds as if the crisis were resolved, somehow, from outside it looks different. It looks as if crisis continues, though under censorship coupled with an elaborate cover-up, where stories of crisis are downplayed or suppressed; it appears rather that practitioners have become accustomed to skirting around logical volcanoes while also exploiting, dangerous regions, pretty geysers, nice mud pools, which *persist*. Among the continuing shocks are not just that many objects of ordinary theoretical discourse are rendered problematic or worse, but further that some of those items are rendered out of bounds or beyond expression altogether. Such objects include not only many familiar totalities, comprehensive sets such as those of all truths, all truths of various general kinds, all facts, all propositions, ..., but also truth, totality, knowledge, omniscience, possible world, actual world, the universe, reality itself. The utter mainstream problematicness of all these familiar totalities and notions has recently been emphasized, in a shocking assemblage of material and results, by Grim in *The Incomplete Universe* (an object he himself appears unable to discuss coherently)([1]). What is assembled is not really *news*, because most of the arguments have been previously, if separately, noticed; it is the assemblage of so much, so forcefully, with mainstream escapes shown unviable or carefully sealed off, that produces the *grand effect*. It really looks as if 'we are forced to abandon a range of basic philosophical precon-

([1]) See esp. p.123, discussed below. Naturally Grim is not alone in his predicament, experienced earlier by Russell, Wittgenstein and other luminaries. As Grim helps to show, anyone committed to classical logic faces insuperable problems concerning grander totalities and comprehensive claims. The sorts of bizarre outcomes that genuine philosophical adoption of mainstream logic induces can be witnessed in a striking succession of recent practitioners.

ceptions' (p.4); it looks as if ordinary semantics is gutted.

Condensed to basics, what we see is outright confrontation between *mainstream logical* theory, none of the repairs or epicyclings of which leave the familiar totalities and notions intact, and *ordinary* practice, which retains the familiar items. Formal inclination and sustained experience combine to persuade some of us to align with a *refined* common-sense, to maintain ordinary classically untutored and classically uncorrupted discourse, by way of refined logical theory. The present exercise aims to retell Grim's tales that way. After all, there are viable options to mainstream logical ways, which are but dubiously correct ways.

A prime matter raised no doubt concerns what takes priority; in particular—what are not exhaustive alternatives—retaining classical logical theory, or retaining refined ordinary discourse. For all the in-house hullabaloo, classical logical theory does not have a really powerful claim to priority or precedence. It is a comparatively recent development, only having arisen late last century (though with certain much older roots(2)), and only having achieved logical dominance since mid-century. It is furthermore a development that has brought in its train, or exacerbated, many problems, paradoxes of one sort or another, and many anomalies. It represents, in short, a highly problematic recent development. It is certainly not sancrosanct; for it should answer at least to correct practice, which is naturally intertwined with rectified ordinary discourse.

The utter dependence of the shocking results upon a particular kind of logic, while not unnoticed peripherally, is strangely diverted. At the beginning and end of his exercise, Grim introduces a curious proviso, which goes far beyond regular scholarly caution:

1. 'the work ... clearly employs and relies on contemporary logic as we know it. Surely there is no alternative' (p.3, repeated p.127)

That suggests that logic is considerably more monolithic than it is, even that there is a single logic (at least in contemporary times!) to which there is no alternative. But of course there are alternatives, some of them moderately well developed; intuitionism is one, relevantism is another. (Intuitionism, though a major twentieth century philosophy of mathematics, receives no

(2) As for instance in a minority Philonian position within Stoic logic. Certainly the demand for consistency, and general hostility to contradictions, is even older, and stronger. But that demand can be met by a variety of logical developments, including such now superseded theories as that of connexive logic.

notice in Grim; it is not even indexed. It would hardly underwrite his main contentions, given his heavy reliance on informal reductive arguments and its heavy criticism of these.)

2. 'Nonetheless, conclusions throughout should perhaps be phrased so as to make that reliance explicit: within any logic we have, we should perhaps say, a totality of truths or a notion of total knowledge seems to prove incoherent' (p.3 continued, also repeated p.127).

By expanding claim 2 beyond "the only logic we know" of claim 1, claim 2 lapses (heavy modal qualification notwithstanding). A much more careful qualification is required. Grim's shocking conclusions hold, so far as they do, within a certain band, so far unspecified, of classical logical theories (that is what "any logic we have" comes to). They are an indictment not of 'basic philosophical preconceptions' and of informal semantics, but ultimately of those logical theories and their formalisations.

Rather inevitably, there is also an embarrassing Tractarian fall-out from thorough-going classical resolution: those who ascend by classical means have to throw away the expression ladder by which they ascended. How do they get down again? And down and up again? Like Wittgenstein, Grim is in serious expressive trouble. The notions he appeared to have been expressing claims concerning — all truths, all propositions, all possible worlds, and so on — he is not really alluding to at all. All these notions should be alluded to only indirectly, using quotation and other formal mode techniques. All these notions that he seems to have been reasoning about are 'incoherent' (pp. 122-4), *nonsense* in older terms.([3])

It is now widely recognised, not least by Wittgenstein himself, that such expressive costs are far too heavy. If there are logical alternatives, they should be taken seriously and investigated. Of course Wittgenstein imagined there were no such alternatives (whence one influential source of the demotion of logic within philosophy and its curricula). Like too many others, who should have taken a much wider look around and become informed about logical alternatives, he presumed that classical logic and limited variations upon it, variations not interfering with the (defective) inferential base, were *logic*. Wrong; dead wrong. (What is killed are significant parts of common-sense practice.)

([3]) Wittgenstein's Tractarian notion of nonsense closely resembles Grim's notion of incoherence. Simply compare what is said respectively. Issues of "incoherence" are pursued further below, in section 4.

There are occasional intellectuals, not such a common variety, who really enjoy surprises, who obtain a perhaps guilty pleasure or even anguish from shocking others with seemingly outrageous propositions. Nihilists, sceptics, classical miscreants, are among them. Most academics nowadays seem to prefer comfortable, rather staid (even sterile) and quiet lifestyles, supporting a *status quo*. Logically that status quo remains classical logic. But classical logic does not permit a quiet comfortable life. Insisted upon, it forces surprises and discomfort, and death. 'The surprise is not merely that "all the truths fail to form a set" or that "all propositions fail to form a totality". The surprise is that at the point we are tempted to speak of "all truths" or "all propositions", we already face incoherence', Tractarian nonsense. These are certain *classical* surprises. With the tales retold in different settings, *these* surprises disappear. More comfortable coherent lives can resume, at least for those who can live easily with benign contradictions.

1. *Liar-like tales.*

A form of the Liar paradox 'seems to offer ... an argument against *any* totality of truths' (p.6). That argument takes the form of a classical *reductio* (CR). 'The assumption that there is a set of all truths leads to a contradiction' (p.6). Thus exposed is a crucial classical inference pattern:

CR. A leads to a contradiction $/\sim A$

There are many logical theories which do not endorse CR, notably where the "leading to" is enthymematic and perhaps only classical.

Dialethic theory goes a quantum step further. It concedes, what independent arguments show, that there is a totality Q of truths, and therefore, given that it does lead to a contradiction, that a certain contradiction stands. More, the totality Θ itself demonstrably has inconsistent features. It is worth looking at that familiar demonstration in due detail.

Let Θ be the set of all truths. That is, in standard symbolic notation, $\Theta = \{A : Tr(A)\}$, where it is presumed that the truth functor *Tr* conforms to the T-scheme biconditional: $Tr(A) \leftrightarrow A$, for arbitrary assertion A. Then, eliminating the here redundant functor *Tr*,

$A \in \Theta \leftrightarrow A$. (So, in particular, $L \in \Theta \leftrightarrow L$, for L as next characterised).

Now consider the following set-theoretic version of the Liar, the assertion L that says that L itself is not a member of Θ. That is, in stock formulation:

L. L is not a member of Θ.

Therefore, $L \leftrightarrow L \notin \Theta$. This biconditional follows, for instance, by substitution using L as just characterised in the identity, $L \leftrightarrow L$ (specificly on the right hand side).

That significant set of presumptions leads, directly and impeccably, to a contradiction, as follows:
As both $L \in \Theta \leftrightarrow L$ and $L \leftrightarrow L \notin Q$, so by transitivity,
$L \in \Theta \rightarrow L \notin \Theta$ and $L \notin \Theta \rightarrow L \in Q$. Now, to obtain an explicit conjunctive contradiction, apply the following relevant reductio rules:

RR. $A \rightarrow \sim A / \sim A \qquad \sim A \rightarrow A / A$

In relevant logic these are tantamount to the LEM, $A \vee \sim A$. By RR, $L \in \Theta$ and $L \notin Q$, whence adjoining, $L \in \Theta \,\&\, L \notin Q$.

Thus, given the acceptability of assertions like L, Θ *does* have inconsistent features: $L(\Theta) \,\&\, \sim L(\Theta)$, where $L(\Theta) \leftrightarrow L \in \Theta$. *But that is as far as such matters proceed.* Because CR is not admissible, there is no legitimate inference, via CR, to the sort of conclusion Grim seeks, that there is no such set as Θ. No classical inference pattern, CR, no Grim Liar tale. Nor should an analogue of CR be installed (for contraposition does not operate on inferential linkages, and its enthymomatic expansion, antilogism, is relevantly invalid). So no discomfort need ensue from this set-theoretic Liar.

That is by no means as far, however, as investigations can be pushed, or should. There is no reason, in particular, why the curious self-referential character of L should not be further investigated, the ungrounded and non-propositional features of L logically exposed, indeed the machinery of *self-referential puzzles analysed in detail within a dialogical setting*. Thus, for example, the "superficial" dialethic approach can be fruitfully combined with the "deep" propositional approach to paradoxes for which Grim finds much space.(4) Inconsistent and incomplete assertions do not yield well-behaved sharply two-valued propositions.

Part of the main strategy, the dialethic strategy, is simply accepting certain contradictions, letting them stand as (derived) assertions of a purportedly correct theory. Naturally that need not be and should not be, the whole of

(4) Grim hardly works to disguise his logical predelictions and prejudices. As to space, while the 'initially attractive' propositional response gains more than 7 pages, and the fashionable hierarchical approach more than 9, 'accepting inconsistency' ranks a threadbare 2.

the strategy, by any means.(5) There not merely may be, but are, many other parts to the business, such as a complementary theory explaining how those special contradictions, paradoxes, arise (e.g. through looping and cross-over effects), and such as a supplementary theory within the higher-valued logic of two-valued propositions. Thus approaches Grim takes as exclusive may be combined, and all the sophistication he imagines in a propositional theory can be enjoyed *within* a dialethic strategy. In fact there are further advantages to such an embedding. For, if we can characterize these *in* the logic then we escape such objections as those from failure of effectiveness that tell against direct attempts in these directions.

2. *An improved model for the place and role of contradictions.*

Rendering a dialethic strategy more palatable after thousands of years of outright hostility to contradictions, will not be easy nor accomplished overnight; for it calls for a drastic paradigm shift in logical theory.(6) As one part of trying to change attitudes, new comparisons and analogies are needed. The prevailing notions of contradictions as uniformly disastrous have to be countered: notions of contradictions as one and all unmitigated evils; as logical evils; or of contradictions as serious sickness, or disease of a

(5) A fuller *dialethic* plus strategy has been explained before, in JB appendix (which this exercise continues). What has not yet been published is its fuller implementations. But for now see JB pp.908-9 on context self-dependence, and pp.925-7 on inconsistent sets. Note that it is not, and never was, claimed that dialethism (by contrast with basic noneism, object-theory of JB, within a consistent setting) is mere common-sense.

Worth recording are further considerations bearing on *combination* resolutions of logical paradoxes, such as augmented dialethism. Already assembled in the literature are most of the main ingredients of self-referential paradoxes and their "vacuous" images ("the truth-teller" and so on), documenting *special* features of those contradictions that yield paradoxes. What may be lacking are certain relevant distinctions (e.g. as between types of self-reference and reflexiveness), which would enable a sharper story (and easy disposal of Goldstein's recreational puzzles, p.110; e.g. 'this sentence contains spelling errors' is simply true, while 'this sentence is not in English' is simply false, not both or neither). Paradoxical outcomes derive from a combination of factors, and exhibit holistic features. In a satisfactory story, several ingredients should be playing an appropriate summative role: not just inconsistency but content self-reference, and so on.

(6) All these large claims are much elaborated, and defended, in PL. For explanation too of nonstandard terminology and notions already infiltrated, such as those of dialethism, see further PL.

system. Instead we want to be able to show that certain contradictions may be part of a healthy functioning system. (Perhaps some may even be advantageous.)

As alternative to the prevailing, unsatisfactory, fatal disease picture, we propose a different working analogy, also drawn from medicine: namely, a *tumour model* of contradictions within logical bodies.([7])

A tumour is simply a swelling of some part of a body or (biological) system. It may or may not be due to morbid growth. That is to say, not all tumours found in a body are *malignant*; they can be *benign*. There was a time when medical practice tended to regard all tumours found in a body as malignant, comprising abnormal or cancerous cells. There is certainly ground for concern regarding cancerous cells. For these cells are capable of multiplying rapidly, invading tissues and organs, destroying healthy cells and tissues, and even spreading all over the body—a process called metastatis([8]), a medical equivalent of trivialization. In treating such malignancies, logic appears poor by comparison with medicine where there is a range of (generally fairly drastic) methods of intervention: surgery, chemotherapy, radiation theory, inhibitor drugs, ... Received medical practice, however, unlike dominant logical practice, has advanced, in two different ways: in beginning to acknowledge alternative medical methods, but more important for present purposes, in recognising that many tumours are not cancerous, but benign. The benign swellings may require no treatment at all, no intervention. Obviously, now, *much more damage may result by trying to excise benign tumours than by letting them be*. This, we claim, is precisely the situation with logical and semantical paradoxes occurring in logical bodies.

There is a further stage of advance, to recognition that some tumours may be not merely benign, but natural, part of normal functioning of a body.

([7]) A disease picture of contradictions is stood up, then destructively questioned, in Wittgenstein's later work on philosophy of mathematics. Wittgenstein aimed to alter entrenched attitudes towards contradictions, in particular the received idea that a contradiction in a system constitutes a systemic sickness, totally spoiling it (see PL, esp. p.41). As to difficulties in changing such logical attitudes, witness hostility to Wittgenstein's suggestions on contradiction, even from apparent liberals. Thus, for example, Putnum's standard North American put-down: 'Wittgenstein had better have something *better* than this to say ... or he is done for (as a philosopher of logic and mathematics)' (Putnum's put-down is presented and briefly discussed in a note in DP p.281). Still more remarkably, similar sentiments were published by Anderson, founder of the American tradition in relevance logic.

([8]) More strictly, *metastatis* (*path.*): a change in the sort of a disease from one organ to another; in *location* from one part of a body to another.

Such swellings in a body as certain glands furnish evident examples (even though not generally accounted tumours they fit the general characterisation). It is perhaps in this way that we should come to regard certain paradoxes, such as Russell's and the Liar. They are natural benign logical tumours, self-referential contradictions, within the comprehensive bodies, theories, of sets and truth. That at least is part of the dialethic strategy being pursued here.

There may be room for an even more positive attitude to certain tumours: that they are not merely benign, but *beneficial* (perhaps in the fashion of recessive sickle cell carrying in West Africa, perhaps still more favourably). Speculative examples include outer swellings on early aquatic creatures that duly evolved into fins or limbs. (Fanciful logical analogues are readily glimpsed, as of the programmed contradiction that destroys the dreaded machine or disables some logical monster.) But for the present we can rest content with a two-fold distinction rather than a three-fold classification, content with belated recognition of occasional benign tumours.

In order to apply the tumour model, translation of the motions of benignness and malignancy into logico-mathematical terms is first required. A malignant tumour is one that leads to ill-health, morbidity or mortality, of the system; a benign tumour is one that does not, the health of the system concerned is not upset. *Leading to* is readily translated logically: in terms of derivation, symbolized /. Then, using obvious notation, these rules emerge:

$$\frac{A/B \qquad Ben\ B}{Stet!} \qquad \frac{A/B \qquad Mal\ B}{\text{?}A}$$

It is easy, furthermore, to approximate logically the subediting symbolism of the conclusions. Letting things stand amounts to doing nothing. What results is a no-action principle, displacing classivcal action. However erasing A amounts, at least on connexive accounts of the matter, to setting down the negation of A, i.e. to $\sim A$(9) Thus the rules become:

(9) On connexive and other accounts of negation, see R. and V. Routley, and also RLR. Erasure has been more extensively used, as an account of negation generally; thus e.g. Strawson. Note that "erasure" covers treatment generally of the malignant tumour, the object of which is removal by one treatment method or another.

$$\frac{A/B \quad \text{Ben } B}{A} \qquad \frac{A/B \quad \text{Mal } B}{\sim A}$$

The first rule gives nothing new, the inferential pattern leads nowhere. The interesting issue for the critical second scheme lies in explication of Mal.

There are helpful precedents for paradigmatic cases of logical malignancy deriving from pragmatism and positivism. Consider, just as in pragmatism, the bearing of outcome B on practical matters, its practical effects. To be rectified or ameliorated, no doubt, are outcomes that have a morbid impact on daily practice; for instance, those that result in bridges or buildings collapsing (to take examples from the famous Turing-Wittgenstein discussions on contradictions). One contradiction which does have just that sort of practical effort is Hilbert's paradigmatic malignancy, $0 = 1$. Plainly substitution of 0 for 1 in specifications for bridges, budgets, and so on, can have severe practical bearings. The procedure applies the identity scheme:

IE. $0 = 1$, $A(1) / A(0)$ granted for suitable $A()$

Consider such examples (of $A(1)$) as : with further taxes of 1 billion dollars the state's budget will balance; with this one critically located strut the bridge will stand. When construction proceeds without the strut, the bridge duly collapses.

Observe that it not $0=1$ on its own that is problematic, but $0=1$ as an arithmetic thesis where intersubstitution is fairly freely presumed (as in IE, or through some variant thereof such as extensionality or functionality). As with tumors, location or setting can be critical. In some settings, such as those of wall-paper patterns and (Wittgensteinian) games where arithmetic substitutivity is not ensured, $1=0$ need exhibit no malignancy. Again in paraconsistent *real* number aritmetic, where $0=1$ only means that the inconsistent subspace has a circumference of 1, the equation does not spread damagingly or seriously interfere with functionality.([10])

With information already gleaned, we are placed to *initiate* a tabulation of contradictions:

([10]) For details see Mortensen, chapter 6.

Benign	*Malignant*
In suitable nonclassical settings:	In suitable arithmetical contexts:
$L(\Theta)\&\sim L(\Theta)$, i.e. previous Liar paradox statement	$0 = 1$ $m = n$, for m and n distinct (smaller) numbers([11])
$R \in R \,\&\, R \notin R$, where $R = \{x: x \notin x\}$ i.e. Russell paradox statement	
One Classificatory principle: having merely theoretical impacts	*Notional Principle*: having negative practical bearings.

While such a rough initial classification leaves a great deal to be sorted out, it enables a worthwhile start to be made on present issues, and indeed upon much of established theory. It does not matter for such applications that the benign-malignant distinction remains so far excessively indeterminate, as indeterminate as the bases of pragmatism. It would matter if there were serious leakage from all (merely) theoretical concerns to practical ones, as happens as regards inconsistency with mainstream logics. Here we rely upon the limited spread features of relevant logics (features admittedly, like conjectured cases of benignness, not fully established). Admittedly too, the notion of negative practical bearings will require honing. As in medicine (by contrast with economics), let us not count the taking, or wasting, of time as exhibiting malignancy. Spending a lot of time trying to square the circle by rule and compass methods, because not appreciating an impossibility result, as Hobbes apparently enjoyably did, is not a negative practical outcome in the intended sense. Nor is wasting time trying to solve the Halting problem classically.

([11]) Among "smaller" numbers are included all those having practical application not merely in day-to-day and business activities but in science and technology. At the University of Cape Town where this sort of modelling was first exposed, I was informed of Skewes number, a sizeable but decidedly finite number claimed to be the largest number to have practical use. Note well that it is not intended to exclude dialethic models for arithmetic where, for uncommonly large m and n, larger even than the power set of all individual entities, $n = m$. On such models of arithmetic, see esp. Priest.

When improvements in medical diagnoses occur, often accompanying improvements in medical practice and perhaps in technology are required. For example, old practices which spread diseases, viruses or whatever, about, need to be replaced by newer procedures where instruments are duly replaced or adequately cleaned between uses, and so forth. So it is with logical practices. Old practices may no longer be satisfactory, and may have to be superseded. One important example concerns use of definitions, where old practices can facilitate, or guarantee, unjustified spread of data, perhaps degrading the material. Take, for instance, use of characteristic functions, a widely deployed classical technique, which, while admissible in classical settings where types of logical tumours are not distinguished, prove no longer satisfactory, but assume creative roles. Let T be some paradoxical set, such as the Russell set R. and define a characteristic function t on elements of T as follows: $t(a)=1$ iff $a \in T$; $t(a)=0$ iff $a \notin T$. Since $T \in T$ and $T \notin T$, $1=t(T)=0$.([12]) So, in familiar contents, t will effectively convert *any* benign contradiction to malignancy, and therewith collapse a central distinction. As it happens, such definitions are already outlawed, on entirely familiar grounds, that they are creative. They enable derivation of what is not otherwise derivable in the original theory. For example, in dialethic set theory they would wrongly allow derivation of arithmetic inconsistencies from isolated results concerning the Russell set.

It will be evident that both classical and improved classical treatment of the second scheme have been rejected, in passing. Under classical treatment, Mal is explicated simply through falsehood, cashed out as negation, giving the derivational reduction scheme:

$$\frac{A/B \qquad \sim B}{\sim A}$$

But even classically —though normally this is not adequately flagged— this may not hold valid under applications with inferential rules exceeding conditions for the deduction theorem, which supports such a rule. (For consider such rules as generalization and necessitation.) And even relevantly the scheme holds where the premissed inference supports a good implication, as the scheme is then but a rule form of contraposition:

([12]) It would be better to arrive at something like [1] = [0] for distinguished "numbers" [1] and [0] which afford only limited substitution opportunities, i.e. where full functionality is bracked out.

$$A \rightarrow B,\ \sim B\ /\ \sim A.$$

Under improved classical or modal treatment, Mal is explicated through logical falsehood, cashed out as impossibility, giving the scheme:

$$\frac{A/B \quad \sim \Diamond B}{\sim A}$$

Strictly this too requires an improved inferential linkage in the premiss; that is, for the rule to be upgraded to a principle of basic modal system S1. Theoretically an adequate modal result could be achieved by checking and delineating inferential rules applied in stock *reductio* arguments; but in practice of course this is almost never attempted. In a normal modal setting where all impossibilities are equivalent to any chosen contradiction, say a malignant one, the rule returns us to the already rejected theorem that all contradictions are malignant.

An important corollary is that *informal proofs may be unreliable*, for all that they are regularly deployed in standard mathematics and in prevailing logic. That claim has been confirmed at *each period of crisis* in mathematics and logic. It has also been highlighted firstly by revelations of *assumptions* that have been implicitly made by practicising mathematicians (one stock example concerns use of the axiom of choice and of its equivalents, another involves use of abstraction principles), and secondly by alternative more rigorous practices, which can expose such assumptions and others, practices like Hilbert's exact axiomatic methods and like those recommended under intuitionism. Good relevant practice does the same, sometimes with superior justification.

One region where received practice has already been regularly questioned, by intuitionists and others, is that concerning *indirect* proofs, *reductio* methods in particular. These methods also come under question in dialethic theory, for more than one reason. The broad form of *reductio*, of present relevance, takes the form:

R. Where a statement A leads, by acceptable procedures, to a contradiction (of form $p \ \& \ \sim p$), infer its negation, $\sim A$.

A restriction to *acceptable* procedures is of course essential, to avoid sloppyness, skulduggery, and so on. An immediate problem is that standard classical practice itself is not acceptable. A comparison with relevant practice shows why it is not. Relevant implication is an acceptable procedure.

Then $A \rightarrow p \,\&\, \sim p$, whence by contraposition $\sim(p \,\&\, \sim p) \rightarrow \sim A$. So as $\sim(p \,\&\, \sim p)$, by the modus ponens rule, $\sim A$. Standard practice does not however establish implication, but only some sort of derivational linkage from A to $p \,\&\, \sim p$, commonly symbolised $A \,/\, p \,\&\, \sim p$. To get from there to $A \rightarrow p \,\&\, \sim p$ requires a deduction theorem, which is not relevantly permissible. It is unacceptable because its proof is characteristically intricated in implicational paradoxes. It has to allow, for instance, for the classical practice of ad-libing-in theorems in a proof sequence, and so for the principle $D \rightarrow t$, or contrapositively $\sim t \rightarrow E$. Such principles are entirely dialethically unacceptable, as for instance they lead to triviality from the negation of theorems, which however may also be theorems.

What is being proposed, then, is a double rectification of rule R: first, a narrowing of "acceptable" procedures to meet good dialethic standards, and second a confinement of contradictions to malignant forms. That is, the rectified rule is as follows

DR. Where A leads, by dialogically acceptable procedures, to a malignant contradiction, infer its negation, $\sim A$.

But there is *no licence* granted, where the contradiction reached is benign, to infer $\sim A$. Benign contradictions can stand, without inflicting systemic damage, for instance through rule like R. It is this proposal that will now be put to work, in further tale retelling, and eventually elsewhere. For example, the differences in inference licences yield a distinction between two types of diagonal arguments.([13])

As well as anticipating later entertaining developments, it will serve present motivational purposes to release a further fat cat from this dialethic bag. The distinction between malignant and benign forms can be operated (manipulated, some may say) to admit Cantor's paradigmatic diagonal argument that there are more real numbers than rational (as the diagonal d differs *numerically* from all enumerated rationals, as represented for instance by infinite decimals), while immobilizing theoretical Cantorian analogues, notably paradoxes of set theory such as that concerning Cantor's set of all cardinals. Part of the interim argument will be that real number questions appear, along with natural numbers, to have practical bearing, as numerous

([13]) As is argued in my 'On relativity of computability', the main diagonalizations of computing science establishing unsolvability and uncomputability depend upon *reductio* procedures delivering benign contradictions; accordingly they do not automatically succeed (dialethically that is).

applications of analysis appear to attest.

Observe that this diagonal argument, distinguishing the reals, differs in *two* major respects from arguments that will feature in paradoxes: Firstly, it is *hypothetically* advanced, and secondly it *would* (in the setting speculated) produce malignancy. However everything works out nicely: malignancy operates to negate the hypothesis of enumerability. Similarly but oppositely, so it is conjectured, things work out appositely with paradoxical diagonal arguments. While the arguments are not hypothetically advanced, they do not lead to malignancy.([14])

3. *Dissolving defective reduction and diagonal arguments.*

Among targetted items are those of *complete universe, universal knowledge*, and a *totality of truths*. Paradox analogues and diagonal arguments have been applied to eliminate all these and like problematic items (e.g. totalities of possible worlds, totalities of falsehoods, etc.). Let us concentrate upon those concerning a totality of truths, as other arguments are very similar. Although there are *many* sorts of totalities, it is standarly assumed that totalities can be traded in as sets.

- *An old-fashional set of all truths?* Intuitively there is such a set, formed for instance by amassing all truths. Straightforward set logic backs up intuition (though not Zermelo-Fraenkel set theory). For instance, all subsets of truths can be combined into one big set. More satisfactorily, the set abstraction axiom can be applied, using a T-predicate such as 'that ... is true', *Tr* say. Let *T* be the resulting set of all truths. So, by abstraction:

$$t \in T \leftrightarrow .\ Tr\ (t)$$

Now let us investigate further the character of this set *T*. Our investigation can usefully begin by following through 'the proof that there can be no set of all truths'([15]), a partially formalised, essentially classical proof of

([14]) A grander proof that all is hunky-dory is yet to be delivered; indeed it can hardly be reached in advance of conceptual preliminaries, such as the present exercise starts to address. More on these matters below.

([15]) Grim p.91, where it is maintained that 'what follows may be the cleanest and most concise form in which we have yet seen it' namely the 'problem that is at issue', essentially 'the *same* problem' for all problematic items considered, 'a single deep problem'.

course. This classical proof (*suspending* abstraction) starts from the *supposition* that there is a set of all truths T (or that T is acceptable); whereas we already have T as a datum. It is not difficult to guess what is going to happen. When the classical proof arrives inevitably at a contradiction, reductio is invoked to reject T; whereas our parallel tracking of the "proof" yields not the rejection of T (as Rule R is not available), but result that T exhibits inconsistent features. It is an inconsistent object like the Russell set; that is part of its consequential character.

Now consider this Cantorian argument against T. For graphic purposes (though no enumeration is presumed), let T be represented $T = \{t_1, t_2, t_3 \ldots\}$. First, consider all subsets of T, i.e. elements of the power set PT, graphically: $\{\}$, $\{t_1\}$, $\{t_2\}$, $\{t_3\}$ ... $\{t_1, t_2\}$ ... $\{t_1, t_2, t_3\}$ .... To each element of this power set there will correspond at least one truth, for example that the element t_1, does, or does not, belong to a subset. Under the graphic depiction there are the following truths: $t_1 \in \{t_1\}$, $t_1 \notin \{t_2\}$, $t_1 \notin \{t_3\}$.... $t_1 \in \{t_1, t_2\}$ ... $t_1 \in \{t_1, t_2, t_3\}$ .... Whence a first subconclusion

• There are at least as many truths as there are elements of the power-set PT. Now, *given* Cantor's theorem, for any set α, $P\alpha$ is larger than α. Thus PT exceeds T in cardinality. But, by the preceding, there are as many truths as there are elements of PT. Hence

•• PT contains more truths than T.

Therefore, there are more truths than there are members of T, that is more truths than all truths. Thus T both comprehends all truths (by initial characterization) and does not (because there are more truths). In classical terms at least, very rum.

But the outcome, though reached by a more elaborate route and yielding more information, does not differ in dialethic substance from that of the set-theoretic Liar. T, like its identical counterpart Θ, has inconsistent features, though not of malignant character. So that is again as far as matters appear, or need, to proceed. No inference, such as Grim urges, to rejection of T is supplied, or so far warranted. T is admissible: just inconsistent, in both containing all truths and also not containing certain self-referentially or diagonally-defined truths. It is, as ordinarly conceived (in advance of consistencizing stratagems, which change it), an inconsistent totality.

There are however gaps in this pretty but informal reasoning. One issue concerns whether the argument, with its set and cardinality theory, can be satisfactorily formalised within a dialethic setting, exposing assumptions as need be. Tied up with that, but more accessible, is the matter of the adequacy of Cantor's theorem in this setting. Looked at one way, the "theo-

rem" proceeds to locate a contradiction concerning a set T whose benign inconsistency is already appreciated, but to regard it as malignant. If instead, we take the proof as (further) evidence that T is inconsistent, rum results, such as the PT is larger than T and contains more truths, are averted.

Consider pertinent parts of the argument that PT is larger than T. PT is at least as large as T, since it includes T. The argument that it is larger proceeds by assuming (what we already saw was the case) that PT is the same size as T.([16]) By virtue of equicardinality there is a 1-1 function f from T onto PT, taking each t in T to $f(t)$ in PT, so that for each element of PT some element of T is assigned (call this 1-1 feature $\oslash$).

Now define a "diagonal" subset T^{d} of T as follows:

$T^{d} = \{t : t \notin f(t)\}$, that is T^{d} consists of the set of all and only those elements of T that are not members of the set assigned to them under f in PT (effectively a Russell set at a functional remove, or where the function is identity). Now by feature $\oslash$, there is some t_{d} in PT such that

$$f(t_{d}) = T^{d}.$$

Thus combining equations,

$$t \in f(t_{d}) \text{ iff } t \notin f(t),$$

Whence in particular,

μ

$$t_{d} \in f(t_{d}) \text{ iff } t_{d} \notin f(t_{d}).$$

So $t_{d} \in f(t_{d})$ and $t_{d} \notin f(t_{d})$: contradiction (given of course that the logical apparatus will support inferences made).

The usual classical conclusion is that there is no such 1-1 function as f (which presupposes in turn that there is not such singular set as T). But here an *alternative* explanation is open; that diagonal function f serves to expose an inconsistency in T, namely T contains members like t_{d} with contradictorial features such as Q, where $Q(t)$ iff $t \in f(t)$.

It would be a serious mistake to view the outcome as an emasculation of

([16]) There may be gaps here in the argument that relevant cardinality theory has to fill, depending upon how it is done.

set theory, by removal of power set principles, in particular of Cantor's theorem. Firstly, nothing impedes formation of power sets, quantification over them all as items, and so on. Secondly, Cantor's theorem does not emerge intact, unscathed, elsewhere; but matters have been logically engineered so that it tends to break down for very large sets ("classes" or whatever). Here it fails, in a different way, for inconsistent sets. Not only has it been conceded that Cantor's argument may be retained for that most important class, the reals; but it can be admitted for consistent sets generally. That is, *provided* α is a consistent set, the cardinality of $P\alpha$ exceeds that of α—which is what the argument in any case shows. Such a *consistency-circumscribed theorem* is one of the main lessons from tale retelling.

Similar "Cantorian" arguments can be rerun with much less logical technology presumed; for instance, rather like virtual set theory, in little more than quantificational logic. The strategy Grim pursues, after adapting the Cantorian argument to variant set theories, including interestingly a version of *ZF* without the power set axiom (but with *aboutness* imported), is to adopt a similar form of argument to settings with much less logical machinery. For the richness of set theory, concentrated upon membership, are substituted the so far poorer notions of application and aboutness. Diagonal arguments formulated in terms of functionality and membership are replaced by analogous arguments formulated in term of functionality and the substitute for membership.

There are several of these lower-tech arguments, using one replacement or another for sets. Main further apparatus can be driven down to near quantifiational levels (thus e.g. 1-1 functions into certain sorts of relations, p.118).

• A *predicate*(or *property*) *version*?([17]) This version considers any predicate τ satisfied by just those propositions that are true (thus τp iff $p \in \Theta$). Call the satisfiers ts; so τt_i for any such t_i. Consider those predicates, *C predicates* (or convenient predicates) comprising

1. any predicate that holds of nothing, and
2. all predicates that hold of one or more ts. I.e. for one or more of the things which satisfy τ.

First, there are at least as many true propositions as there are C predicates.

([17]) Grim advances this version in terms of properties pp.116-117, but use of predicates gives the appearance of reducing commitments still further (even if a semantical notion, satisfaction, appears to be imported).

For corresponding to each distinct predicate there will be a distinct true proposition, e.g. trivially 'predicate ... is a predicate'. But secondly, there are more C predicates than there are ts. Again then, there are more truths than comprehended as all true propositions.

The second claim is established, once again, by a diagonalization procedure. Suppose, to the contrary, there were not more C predicates. Then, given the first claim, the cardinalities must be the same. Accordingly there is a 1-1 mapping f of ts to C predicates. Now consider a predicate d characterized as follows:

t satisfies predicate d iff t does not satisfy f(t);

in symbols, sat(t,d) iff $\sim$sat(t,f(t)). As d is a C predicate, there is some t^* which f maps to d, i.e. f: $t^* \rightarrow$ d. Thus sat(t^*,d) iff $\sim$sat(t^*, f(t^*))
iff $\sim$sat (t^*, d).
So sat (t^*,d) and $\sim$sat(t^*,d), i.e. t^* satisfies d and t^* does not satisfy d. Or, in application terminology, (property) d applies to t^* and does not. An apparently benign contradiction is accordingly displayed. (Moreover in simpler settings, benignness can be established.)

As Grim tells it, still forcing inconsistency back through rejection of assumptions, there can be no such mapping as f doing what f is supposed to, and no such satisfaction predicate, no such property as applying to all truths. We retell it in terms of *inconsistency without rejections*; f and τ stand, simply applying to inconsistent items. What arguments of this well-known sort show, reworking variants of logical paradoxes using notions of satisfaction and application, is that such sums or collections as those of all true statements, are inconsistent. The version Grim supplies is but a more sophisticated variant, proceeding on a (pseudo-)hypothetical base through a functional diagonal linkage. But Grim too will be forced to revise his story, as he proceeds step to step to trap himself in his own scope.

• *Raw quantification forms?* 'The argument appears with full force phrased quantificationally throughout ... raw quantification proved insufficient to avoid Cantorian conclusions. ... global properties, sets, class or other collections are in the end simply inessential to the basic mechanism of the argument'.([18]) As it happens, several of the more minimal arguments are less pure than Grim makes out. Rather they operate in adjacent applied settings containing predicates with recognised self-referential properties.

([18]) Grim p.120, sentence order reversed. These matters were clear to Russell, almost a century ago: see e.g. his Introductions to editions of *Principia Mathematica*.

Thus the predicate 'α' signifying *aboutness*, which is slipped in both in the argument faulting set theory without the power set axiom (ZF-Power specifically) and these 'raw' quantificational arguments. Thus an alternative conclusion is that such predicates, bearing problems, cannot be systematically adjoined.

In any event, it can be argued persuasively enough that aboutness in combination with (restricted) quantification is classically fatal. Observe that some propositions are about themselves, and some are not. Consider then a proposition about all propositions not about themselves. It seems evident that there is such. Ask, in a Russellian way, whether it is about itself? It is if it isn't, and isn't if it is. *If* moreover the intuitive argument fails, it can be refashioned in Grim's interesting way. Aboutness exhibits semantical features sufficiently similar to membership and instantiation, application and satisfaction.([19])

Essentially similar diagonal arguments are advanced by Grim to show
1. There is no 'proposition that is genuinely *about* all propositions', such as universal propositional quantification $\forall p$ $(...p...)$ is presumed to be, a 'way of speaking about all propositions' (p.119).
2. There is no one-to-one and onto mapping from propositions to propositions.

Argument 2 is taken to lead to a striking inconsistency in the idea 'that we *can* speak of *all* propositions'. For, on the one hand, by 2 itself, there is no 1-1 correspondence of propositions onto themselves. On the other, there are such correspondences, e.g. trivially by a identity relation. What results is a tight technical elaboration of an aboutness paradox.

Suppose there were such a proposition P, as in 1, and consider all the propositions it is about, called P-propositions. 'Were P genuinely about *all* propositions [as it *is* if it represents universal quantification] ... there would be a one-to-one mapping *f* from P-propositions [on]to propositions *simpliciter*.... But there can be no such mapping' (p.119). As we retell it, the mapping-argument once again reveals a benign inconsistency, reflecting on the initial proposition. But more surprising, even spectacular, conclusions are reached by assuming malignancy. As any such mapping is then shown to leave some proposition out, 'there are *more* propositions than P-propositions. ... Thus *whatever* proposition any proposition is about there are more.

([19]) As Goodman, Putnam and others have emphasized, typically in demolition and disposal enterprises. All admit nice dialethic rehabilitation.

No proposition can genuinely be about them all'.

As for the mapping argument, it is another functional diagonalization, now of an aboutness puzzle. Suppose there were such a mapping f. Consider all P-propositions p such that their $f(p)$ is not about them, and form a proposition P_d about precisely these. (Classically such a proposition might take the form $\forall p(P_p \,\&\, \sim(f(p))\alpha\, p) \supset \ldots p \ldots$. The corresponding form and argument in case 2 is exactly the same, except that P_p can be suppressed, as it is guaranteed because all propositions are being considered). Let f^* be a P-proposition for which P_d is $f(p^*)$; the mapping f guarantees such.

There is not a one-way answer to the question, whether P_d is about p^*? It both is and is not; $P_d\ \alpha\ p^*$ is a dialetheism. In brief, since generally $P_d\,\alpha\,p$ iff $\sim f(p)\,\alpha\,p$, $P_d\,\alpha\,p^*$ iff $\sim f(p^*)\,\alpha\,p^*$; whence $P_d\,\alpha\,p^*$ iff $\sim P_d\,\alpha\,p^*$, so $P_d\,\alpha\,p^*$ and $\sim P_d\ \alpha\ p^*$.[20] The result appears remote from the health of the body practical; it lacks theoretical and benign. But it does render theoretical life extremely uncomfortable for some, not least Grim.

4. *A grim dilemma and beyond.*

Images almost abound to depict the sort of predicament in which Grim has landed himself: caught in his own net, hoist on his own petard, the ladder on which he is perched kicked away, succumbed to the Ismael effect, ... more literally suffered self-refutation. Grim's dilemma is familiar from the type theory and hierarchical exercises. On his presentation the dilemma appears as follows: that in contending that there is, and can be, no genuine propositional quantification, he has regularly deployed propositional quantification, for example he formulates main claims, such as that there is no totality of all propositions, in terms of it. Striking to behold are the squirming double-talking moves made in efforts to escape from such dilemmas. None of these moves is very satisfactory; Grim's attempt to extract himself is, if anything, worse than usual, partly because he does not treat the dilemma as serious, 'the tension is merely apparent' (p.122).

Grim tries to make his escape through an unexplicated notion of *incoherence*. His qualified denials, such as 'There can be no set of all truths', cannot be taken at face grammatical value, so he says, but 'should alternately be understood as [a] denial of the coherence of the basic notion of

[20] For a quasi-English spelling out of both cases 1 and 2, see Grim p.119 and p.121.

... a "set of all truths"' (p.123). Similarly for other claims such as that supposition that 'there is an omniscent being' or a totality of possible worlds leads to inconsistences through reduction arguments: all these are to be reconstrued in terms of the incoherency of notion thereby mentioned (no longer used). Conclusions should be rendered quotational. A use-to-mention shift has been infiltrated; and incoherence is not to be spelt out in terms of involving inconsistency, but rather functions as not making sense, that is as a nonsense or absurdity notion. There is no escape this way. Either Grim is forced back to one of the ways he has earlier rejected, in particular elements of a levels-of-language approach, which leads to fantastic hierarchical tales, or else he is forced on towards something isomorphic to a significance logic—an approach he has not considered—which however leads beyond classical logical resources.

Either way he cannot remain satisfactorily where he has arrived. His position itself is unstable, and literally incoherent. Classical logic, the "only logic" we have, know and cherish, has either to be wound in, for example through strait-jacketing hierarchialization, or else replaced, for instance through a significant logic analogue not investigated in Grim.([21])

5. *Despatching further popular criticisms of dialethism*

The theory as so far elaborated already takes care, obliquely, of several commonly presented objections to dialethism. More theory would do more; that no doubt is where main research should be directed. But there remains strategic point to direct responses to certain criticisms.

Most critics do not yet take dialethism seriously. They imagine they can rely on prevailing classical prejudice. So far as formal logical practice is concerned they are no doubt right; in other respects, including right reason, it is very different. So it is with the select recent critics deconsidered here and elsewhere.([22]) Grim, for one very typical critic, does not take dialethism seriously and does not get to appreciate its expressive possibilities. He is set against 'accepting inconsistency' (his heading) from the outset, dismissing it 'as a counsel of despair' in a couple of scrappy pages. Yet evidently, from the perspective of retaining everyday discourse and

([21]) For how different a suitable significance logic will be, consider in Ryle's *reductio* argument, discussed in R. and V. Routley 73.

([22]) Critics include Goldstein, Havas, Smiley as well as Grim.

common-sense principles, it is counsel of hope. For with dialethism we *can* satisfactorily say and do what we thought we could, before classical amputation practices became established. There is renewed prospect of full class abstraction, of total expressibility, of ordinary nonhierarchical semantics, and so on. So the grim classical tales of logical austerity and expressive calamity can recede into twentieth century history.

"Accepting inconsistency", itself exhibits a familiar ambiguity, between accepting a very few and perhaps very distinctive samples, through to accepting any and all. It is the latter unfavourable, and jejune, construal onto which certain critics have latched; it is the former end of the range, however, to which exponents have generally confined themselves and have tended to canvass. That Grim seeks to push dialethicians towards the unfavourable end of the range is revealed through his portrayal of contradictions quite generally as evil, by his dismissal of Priest's work as resulting in 'a bewildering flurry of paradoxical implications', and through his comparison of accepting contradictions with accepting triviality (p.27 also). When accepting certain contradictions is a highly discriminating business, it is hardly like accepting everything that is on formal offer. Grim has in effect confused gormets with gormands; it is that gross.

Observe next that Grim's argument to his shocking conclusions appears to be by way of elimination of alternatives; there is no real alternative, such as a rich menu seem to offer, to his digestion shockers, As it happens, then, his argument is vitiated, among other things, by a small circularity. Specifically he argues against letting contradictions stand, that it is not a last resort to which there is no alternative. But he argues this (p.27) on the ground that 'there clearly is an alternative', namely *that* he is committed to arguing for: 'we can abandon notions of a set of all truths, of set-theoretical possible worlds, and omniscience' and so on! Evidently, he is not really entitled this argument (or "easy option"), as he has to defeat dialethism on *independent* grounds for his elimination of alternatives to succeed.

As regards dialethic approaches, moreover (among *others*), Grim is very far from exhausting alternatives, considering only two problematic cases from the riches now on offer (namely he considers only Rescher and Brandon's 'half-hearted' consistency-based approach, which is immobilized immediately by strengthened paradoxes, and Priest's 'half-baked' 3-valued approach, where the unnatural restriction to three values, along with a truth-negation commutation principle, forces unsatisfactory outcomes). Out of those two unrepresentative cases Grim proceeds to draw various general conclusions, some of which even the cases themselves do not sustain, still

less a wider sweep of dialethic approaches. Take the expected universal evil admission: 'it must be admitted that contradiction *is* an evil to be avoided'. A similar expected concession is demanded two paragraphs later: 'surely ... contradiction is to be avoided whenever possible'. It is not admitted or conceded: as observed, contradiction may be benign, even beneficial. After the expected concession, Grim continues: 'As advocates of inconsistency-tolerant systems have standardly [?] argued, we *may* be forced to accept contradictions when there is no alternative' (p.27). That is not how advocates have tended to argued, it is not the way we are arguing (it may be a way Priest, Grim's main target here once argued). In a way, there are always alternatives (for example, restricting *modus ponens*, pursued by Fitch and others, is one Grim nowhere considers); and here there are nicer alternatives (though admittedly difficult ones) than Grim is trying to force us towards. Rather, dialethicians have contended that there are valid, indeed sound arguments, to contradictions.

The only remaining substantial point Grim makes also concerns general avoidance, again of contradiction, now because of falsehood: 'contradiction is the short road to falsehood, and if falsehood is not to be avoided, it's not clear what is' (p.27). Remarkably, Grim himself suggests two answers to this rhetorical question: shortly before, his own alternative of gutting semantics (which is worse than *some* leakage of Falsehood), and soon after, the 'horror ... of a trivial system in which *anything* follows' (p.27). What is 'wrong with triviality' is 'not that it assures [as it does] that even falsehoods will appear or theroems', but evidently that it assures that *everything* will appear. So no exclusions are made, no requisite discrimination is feasible. Triviality, by contrast with isolated inconsistency, is a malignancy, often of immediately lethal form.

In addition, it has escaped sufficient notice that dialethism makes room for more than one line on falsehood. In particular, there is both a wide view (adopted by Priest) under which what is both true and not true is false, and a narrower view (implicit in Routley and others) which does not say this but reserves the term 'false' for something nearer to false only. Under narrower views contradiction is not 'a short road to falsehood', but not always a road at all. Malignant contradictions may be false, if that is how it is decided to classify them truth-value-wise; but *they* are not accepted, they are excluded and avoided.

As a parting shot, Grim makes an unreferenced appeal to Aristotle's authority.([23]) No doubt Aristotle has been a major force in establishing the consistency ideology, in logical theory and elsewhere. At this late stage in post-scholastic tradition, when so many of Aristotle's pronouncements have been overturned, many under impact of classical symbolic logic, it is too late to appeal to Aristotle. In any case, Aristotle's politically motivated case was elegantly dissolved by Lukasiewicz, who pulled the supporting arguments from under the authority.([24])

6. *Interim conclusion: towards a compleat universe.*

A much favoured approach, dominant nowadays, is brinkmanship, which fits perfectly with features of the dominant social paradigm, its risk-taking, opportunism and maximization: roughly maximize totalities, construed as sets or classes, within consistency constraints. The approach has blown out (of control), because there is not a unique way to so maximize, and every alternative taken seem to leave things out. Main alternatives pursued have certainly enabled proof of some spectacular results, especially as regards limitations. Brinkmanship represents but one approach. Another is safety-first, which intuitionism and constructivism are supposed to supply. Safety-first naturally leaves out more, mauls even more of discourse, than orthodox opportunism.

Accordingly a different approach is advocated here, an object-theoretic approach (a main early embodiment is JB). Under this approach objects are taken as data, and their types and character and features investigated. The well-known village barber serves to illustrate nicely certain differences. The barber, recall, shaves all and only those shaving villagers who do not shave themselves. Shavers in the village divide exclusively into self-shavers and non-self-shavers. An awkward problem appears to arise concerning the barber's location. A stock line is that, for all its similarities to self-referential paradoxes, this is a pseudo-problem. Supposing there were such a barber, inconsistency would result. So there is no such barber. This is the way Grim, pursuing brinkmanship, regularly proceeds concerning problematics totalities. An alternative is to ask what the barber—or any particular

([23]) The appeal is referenced by Goldstein, who also want to reinforce the authority, but does list some notable counter-authorities.

([24]) See PL, p.26 ff.

totality—is like. Characterising principles enable us to infer that the barber is self-shaving and also non-self-shaving, whence results, a consequential feature, that the barber is an inconsistent item. Accordingly, the barber does not exist, indeed cannot. The stock line is not contradicted, but confirmed and clarified. While there is such an object as the barber, that is a nonexistent object; there does not exist such a barber.

Similarly with grand totalities such as that of all truths, so it has been presumed throughout. There is such a totality T; but it is inconsistent, so it does not exist. A prime question again, is what is such an item as T like?([25]) There are two intertwined trains of investigation: phenomenological and formal. Much of the labour in the present exercise has been phenomenological, concerned for instance with features of T and other totalities that phenomenological investigations disclose, features that a complementary formal theory would confirm all going well. As it happens, much more work remains to be done, distinguishing and characterising different types of totalities, careful work often preceding formalisation, to which a premature rush should be avoided. No doubt formal theory should follow, to fill out, correct and validate more phenomenological pictures, to reveal residual difficulties, to show adequacy of rectifications, and so on.

While nice sorts of completeness have yet to be proved (indeed axioms pertinent to what modest sorts of completeness might be expected are still be unearthed), it is at least good to realise that there need be no barriers to completeness of the sorts addicts of classical logic have imagined or proclaimed. What incompleteness they have glimpsed, to the universe, to knowledge, to provability and so on, represents classical deficiencies, premissed on classical consistency and other classical features. Similarly, related limits are but classical limits.([26]) We can still reasonably reach for completeness.

More than one sort of incompleteness is dislodged:

- discourse-universe incompleteness, and correspondingly
- item incompleteness of worlds.

Both these collapse together, given classical existential assumptions and appropriate richness of discourse, into

- existential incompleteness of the (existance) domain of the actual world.

To the contrary, what is being maintained is this: that agents can discourse

([25]) Thereafter significant differences do enter, as T is not a ground-order object.

([26]) See Sylvan 93.

perfectly well about the totality of truths, all propositions, omniscient agents, and so on, all that Grim pronouces incoherent; furthermore, that many of the terms concerned signify items that are satisfactory and healthy enough, though exhibiting benign logical tumours. Pertinent universes are not rendered incomplete by their systematic absence. Suitably rich actual universes are, while displaying cases of benign inconsistency, coherent and complete.

Eco-Logical Organization, Bungendore

References

L. Goldstein, 'Smooth and rough logic', *Philosophical Investigations*, 15:2 (1992) 93-110.

P. Grim, *The Incomplete Universe*, Bradford, MIT Press, Cambridge, Mass., 1991; all page references without further citation are to this work.

G. Priest and others (eds.), *Paraconsistent Logic*, Philosophia Verlag, Munich, 1989; referred to as PL.

G. Priest, 'Is arithmetic consistent?', typescript, University of Queensland 1993.

C. Mortensen, *Inconsistent Mathematics*, Kluwer, Dordrecht, 1994.

R. Routley, *Exploring Meinong's Jungle and Beyond*, RSSS, Australian National University, 1979; referred to as JB.

R. Routley and others, *Relevant Logics and Their Rivals*, Ridgeview, Atascadero, California, 1982; referred to as RLR.

R. and V. Routley, 'Negation and contradiction', *Revisto Colombiana de Matemáticas* 19(1986) 201-231.

R. and V. Routley, 'Ryle's *reductio ad absurdum* argument', *Australasian Journal of Philosophy* 51(1973) 211-225.

P.F. Strawson, *Introduction to Logical Theory*, Methuen, London, 1959.

R. Sylvan, *Deep Plurallism*, typescript, Canberra, 1994; referred to as DP.

R. Sylvan, 'What limits to thought, inquiry and philosophy?', typescript, Canberra, 1993.

A.N. Whitehead and B. Russell, *Principia Mathematica* vol 1, Second edition, Cambridge University Press, Cambridge, 1927.