Logique & Analyse **139-140** (1992), 341-348

FORM AND REPAIRING INVALIDITY

P.J. STAINES

Alethic approaches to the evaluation of argument identify two ways of criticising an argument: (i) it is invalid and (ii) at least one of its premisses is false.(1) If an argument fails in either of these ways it is unsound and does not prove its conclusion. Those engaged in criticising an argument will sometimes point out that it is invalid as it stands. Sometimes they will go further and identify a premiss "needed" to repair the invalidity, perhaps arguing that this "required" premiss is false so that although the repaired argument is valid it remains unsound.(2) The purpose of this paper is to examine some logical features of a mistaken view that can lie behind this practice — that there is a unique repair for invalidity.

If an argument is invalid, adding an extra premiss or modifying a premiss, or replacing it will suffice to make it valid. Given the specific premisses of an invalid argument it can be tempting to think that there is only one way this can be done. That there is no unique way to repair the invalidity of an argument is so evident and readily seen in the abstract that is surprising that the mistake is made.(3) One extreme repair which will always work is to adjust the premisses so that the conclusion is among them. Another is to add a premiss whose antecedent is the conjunction of the existing premisses and whose consequent is the conclusion of the argument. That these repairs are always distinct establishes that there is never a unique repair.(4) That in both of these repairs and in others arbitrary conjuncts could be added to the new premiss shows that there is no finite bound to the number of ways an

(1) We are using the classical modal account of validity.

(2) Teachers of philosophy will sometimes do this with famous arguments from the history of philosophy.

(3) It is made in part because there are other demands than validity on good argument. The point here is logical. The interpretive partly psychological task of identifying what assumptions arguers may have made but not expressed in arguing the way they did may be much more determinate. (See note 2)

(4) That they are syntactically distinct does not imply that they are logically inequivalent. Consider an example due to Huw Price: an argument of the form 'p-so-$\sim$p' supplemented with a premiss of the form 'p $\rightarrow$ $\sim$p'.

invalid argument can be made valid. We shall now illustrate the mistake with three examples taken from (meta-)philosophical writings.

Our first example is straightforward. David Stove writes (Stove, 1986, p. 121) of two arguments of the same form, the second of which has true premisses and false conclusion:

> The second argument's having true premisses and false conclusion proves, indeed, that *it* is invalid. But the invalidity of the given argument follows from the invalidity of the second one, only if that is conjoined with the assumption that any argument which shares its logical form with an invalid argument is invalid itself.

We can note that there are many other assumptions which would suffice when so conjoined. One such, which suffices, is the assumption that if the second argument is invalid then the first is.

Our second example is more complicated. The mistake occurs in a plausible reconstruction from an enthymeme. Brian Skyrms, in *Choice and Chance* (1966) makes the mistake when he is arguing the invalidity of an argument. He writes (p. 8):

> We can now see why the following argument is not deductively valid:
>
> George is a man
> George is 100 years old
> ⋮
> George has arthritis
> ---
> George will not run a four-minute mile tomorrow.
>
> The factual claim made by the conclusion is *not* (his emphasis) implicit in the premises, for there is no premise stating that no 100-year old man with arthritis can run a four-minute mile. Of course we all believe this to be a fact; if we *added* a premise to this effect, *then* we would have a deductively valid argument.

The invalid argument is presented in the sentence 'The factual ... for ... mile'. It is tempting to say in turn that this argument — the argument he uses, as distinct from the argument he discusses — is not valid because there is no premise stating that 'No 100-year old man with arthritis can run a four-minute mile' is the only premiss which would make the conclusion of

Skyrms' argument implicit in the premisses. But this would only continue Skyrms' error. If there were such a premiss then the argument would be valid but its absence is not necessarily the trouble. In Skyrms' case the different premiss

> No 100-year old man will run a four-minute mile tomorrow.

will also do, as will an infinite number of others.

A more technical example comes from Ingmar Porn (1977, p. 13).

> In virtue of the fact that R1 does not hold for Ea it is not the case that Eap $\models$ Ea(p $\vee$ q), though of course, p $\models$ (p $\vee$ q).

Where R1 is the following:

> If p1, p2, ..., pn $\models$ q then Op1, Op2, ..., Opn $\models$ Oq.

If R1 held for Ea then of course Eap $\models$ Ea(p $\vee$ q). But R1's not holding does not imply that it is not the case that Eap $\models$ Ea(p $\vee$ q) unless the system has special properties.(5) Notice that we could give the analogous argument:

> In virtue of the fact that R1 does not hold for Ea it is not the case that Eap $\models$ Eap, though of course, p $\models$ p

which has true premisses (the standard model of argument would take p $\models$ p as one of the premisses) and false conclusion. But this, while it may help someone see that the other argument is invalid, does not establish that it is.

We have observed that it is not uncommon for someone to combine the claim that a particular premiss is needed to make an argument valid with the claim that that premiss is false and to conclude thereby that the argument cannot be made sound. It cannot be both valid and have true premisses. Stove does exactly this. The passage quoted above continues 'But this is precisely the assumption that all instances of a form which has invalid instances are invalid: the assumption which (b) and (c) suffice to show is false'. We have seen that it is a mistake to suppose that there is a unique

(5) Ingmar Porn accepted this criticism.

premiss needed, but if there is a weakest premiss which makes the argument valid and it is false then this is a reasonable strategy for if it is false all the others must be too and so the argument cannot be made sound. The premiss Stove identifies is not the weakest possible as can be seen from our example above.

It is generally agreed that conclusions can be argued for from many different sets of premisses. It seems a remarkable fact that when specific premisses and conclusion are given which need augmentation or replacement to make the argument valid, the awareness of multiplicity vanishes. Perhaps the explanation has to do with familiar patterns or forms of reasoning. When someone argues that an argument would be valid only if it had a particular premiss, or as it is sometimes put, that a certain premiss is needed to make an argument valid, the premisses offered as augmentations or replacements are typically ones which fit well-known valid patterns. For example, Stove chose the pattern of the singular syllogism. We have noticed, in effect, that he could have chosen Modus Ponens and his supplementing premiss would have been weaker.([6])

It is a surprising fact, when one looks at the familiar patterns of inference from natural deduction systems that the supplementing premisses they suggest are almost all unnecessarily strong.([7]) We can illustrate this with some examples.([8])

Consider four argument forms

$P \rightarrow Q$	$P \vee Q$	$P \rightarrow Q$	All A are B
So $P \rightarrow R$	So Q	So Q	So no A are C

In the first example given '$P \rightarrow Q$' as premiss, with conclusion '$P \rightarrow R$' the natural supplementation to occur to anyone would be

$$Q \rightarrow R,$$

([6]) on an existential reading of his premiss.

([7]) Perhaps not so surprising when one looks at features of some of the alternatives and thinks of what is likely to be used in turn to argue for them.

([8]) Here we are talking throughout about forms, for if we were talking about concrete arguments of these forms, when we say 'weaker' we would have to say 'no stronger' — an alternative is make the arguments concrete but require P and Q etc. to be logically independent.

fitting the pattern of Hypothetical Syllogism. However, a different, but weaker supplement that suffices to make the argument valid is

$(P \& Q) \rightarrow R$.

Similarly, given '$P \vee Q$', the obvious suggestion for validly deducing 'Q' is '$\sim P$', fitting the pattern of Disjunctive Syllogism, but a weaker alternative premiss is '$\sim P \vee Q$'. Similarly (and symmetrically), given '$P \rightarrow Q$' as premiss, with 'Q' as conclusion, the natural supplement would be 'P', but a weaker sufficient one would be '$P \vee Q$'.

In syllogistic with its constraints limiting the form of the component statements and the permissible patterns of argument it may be that there is only one form of statement which will fit the constraints. Thus, if the premiss is of the form 'All A are B' and the conclusion is 'No A are C' then the premiss suggested by syllogistic is 'No B are C', or the equivalent 'No C are B'. But throw off these constraints and the weaker form 'No B that are A are C' suffices. Venn diagrams can be a helpful source for weaker but sufficient augmenting premisses.

It is interesting to note that in certain circumstances one can identify an augmenting premiss that any alternative must be at least as strong as. If we have a conditional construction in our language, call it '$\rightarrow$', for which the Conditionalizing rule holds viz.

If

P1, P2, ..., Pn-so-C

is valid so is

P1, ..., Pi-1, Pi+1, ..., Pn-so-(Pi $\rightarrow$ C) for any i from 1 to n with $n > 1$

then it is easy to see that if 'P1, P2, ..., Pn-so-C' is invalid then any augmenting premiss, 'X' which makes the argument valid is at least as strong as '$(P1 \rightarrow (P2 \rightarrow (P3 \rightarrow \ldots (Pn \rightarrow C)\ldots)$'. For if

P1, P2, ..., Pn, X-so-C

is valid then by this rule so is

P1, P2, ..., Pn-1, X-so-(Pn → C)

as is

P1, P2, ..., Pn-2, X-so-(Pn-1 → (Pn → C))

and so on until we have

X-so-(P1 → (P2 → (P3 → ... (Pn → C)...)

is valid and hence 'X' is at least as strong as

(P1 → (P2 → (P3 → ... (Pn → C)...).

This premiss is in turn seen to be sufficiently strong to make the argument valid if there is a form of Modus Ponens for '→' and we use a form of the Synthetic Rule viz.

> Any conclusion validly drawn from our original set of premisses may be added to our premisses to get new conclusions, which may in turn be added. All the conclusions so obtained count as conclusions from the first set of premisses.(9)

Where we have Exportation, relating '→' and conjunction

(P1 → (P2 → (P3 → ... (Pn → C)...)

will be equivalent to

(P1 & P2 & P3 & ... & Pn) → C

Since we have Conditionalizing, Modus Ponens and Exportation for the material conditional we can conclude that any augmenting premiss that makes an invalid argument valid must be at least as strong as the material conditional of the conjunction of its premisses and its conclusion. In the examples above the weaker premisses used in place of the "natural" ones

(9) Geach (1976, p. 65) calls rules like this 'themata'.

are equivalent to this conditional.

Some people think that Conditionalizing, Modus Ponens and Exportation hold for natural language conditionals. On their view it follows that augmenting premisses must be at least as strong as

If P1 and P2 and ... and Pn then C.

Given that this is also true for the material conditional it is readily seen how on these assumptions they can argue the equivalence of natural language and material conditionals.

The above is immediately adaptable to the case in which one makes the argument valid by replacing a premiss rather than adding a premiss. Where we have Conditionalizing and Modus Ponens and we are replacing 'Pn', then any replacing premiss must be at least as strong as

$$(P1 \rightarrow (P2 \rightarrow (P3 \rightarrow \ldots (Pn\text{-}1 \rightarrow C)\ldots)$$

In criticising an argument, when one knows a valid form which the argument would exemplify if a premiss were replaced, it can be very tempting to say, as we have seen, that what its author needs instead is the replacing form fitting premiss. We have been arguing that this is usually wrong because mostly there will be a weaker premiss than the one suggested by the form which will also suffice. But we can make a broader claim. There will frequently be circumstances in which it is better to augment the argument with an additional premiss, even though replacing a premiss would make it fit a familiar valid form. After all (under the same assumptions)

$$(P1 \rightarrow (P2 \rightarrow (P3 \rightarrow \ldots (Pn\text{-}1 \rightarrow C)\ldots)$$

is at least as strong as

$$(P1 \rightarrow (P2 \rightarrow (P3 \rightarrow \ldots (Pn\text{-}1 \rightarrow (Pn \rightarrow C)\ldots).$$

It will usually be the case that a stronger premiss is required when replacing a premiss rather than merely adding a premiss to those already there. One rather special circumstance in which this will not happen is when the other premisses imply the premiss in question.

University of New South Wales

Acknowledgements

Thanks to Jenny Lloyd and Huw Price for helpful comments on this material.

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