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REHABILITATING RUSSELL

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In 1923 the *Australasian Journal of Philosophy and Psychology* was born. A notable contributor to its first issue was Bertrand Russell. Therein he presented for publication a lecture he had earlier given to the Jowett Society in Oxford entitled 'Vagueness'.(1) Whilst his thoughts on the matter may have been clear, exactly what his account of vagueness amounted to, in detail, was far from clear; it seemed to attract only criticism or silence. The fact that it has been little discussed is, I think, unfortunate since, as I will aim to show, it provides a new and enlightening analysis of the orthodox philosophical approach to vagueness. This is of particular relevance now as there has been something of an explosion of work on vagueness appearing in the literature in recent times.

Three major theories have come to fill the theoretical landscape and are defined by the two major watersheds which bisect it. The first — an epistemological theory — accepts the apparent semantic indeterminacy that characterises vague terms as just that, merely apparent. The remaining two theories take the appearance to reflect the reality; vagueness, in at least some cases (if not all), is actual semantic indeterminacy, accounted a property of the term or phrase in question. The first of this pair — the representational theory — nonetheless agrees with the epistemic theorist that vagueness is in no way a reflection of any underlying ontological indeterminacy, it is merely semantic; whilst the other — the ontological theory — distinguishes itself from both by claiming that vagueness is both a semantic and ontological phenomenon.

It will be my contention that a proper understanding of Russell's notion of vagueness offers a radically new analysis which captures the very essence of the representational theory. But we get much more from the rehabilitation of Russell's view than dry definitions; Russell's characterisation offers a new paradigm for representational vagueness and in this new light we are better able to understand what is implicit in many current accounts of vagueness.

(1) Russell, B., 'Vagueness', *Australasian Journal of Philosophy and Psychology* 1 (1923): 84-92.

1. *Russell as Representationalist*

In the first instance the representational theory treats vagueness as a semantic phenomenon. The semantic view of vagueness has dominated twentieth century philosophy (so much so that the epistemic view is often precluded by definition(2)). Russell himself speaks for many when, by way of an initial explanation of vagueness, he asks us to

> consider the various ways in which common words are vague, and let us begin with such a word as "red." It is perfectly obvious, since colours form a continuum, that there are shades of a colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word "red," but because it is a word the extent of whose application is essentially doubtful. This, of course, is the answer to the old puzzle about the old man who went bald. It is supposed that at first he was not bald, that he lost his hairs one by one, and that in the end he was bald; therefore, it is argued, there must have been one hair the loss of which converted him into a bald man. This, of course, is absurd. Baldness is a vague conception; some men are certainly bald, some are certainly not bald, while between them there are men of whom it is not true to say they must either be bald or not bald.(3)

Russell's conception not only typifies the view of vagueness as semantic but exemplifies what Bertil Rolf has described as *the traditional concept of vagueness*: as well as describing vagueness as a semantic phenomenon of natural language, this concept endorses the paradigmatic view of vagueness as a phenomenon affecting predicates and characterised by the presence of border cases.(4)

(2) See, for example, W.P. Alston's entry 'Vagueness' in P. Edwards (ed.), *The Encyclopedia of Philosophy*, MacMillan (1967), p. 218.

(3) Russell, *op. cit.*, pp. 85-6.

(4) Rolf, B., *Topics on Vagueness*, PhD Thesis, Lund (1981), p. 74. This concern with predicate-vagueness, a common feature of discussions of vagueness, is rooted, I think, in the fact that the modern problem of vagueness in general really arose with the sorites paradox of Greek antiquity where the puzzle *is* one of predicate-vagueness and the presence of border cases.

One further, and final, feature of the traditional concept is its overly weak conception of

The traditional concept however, whilst admitting vagueness as semantic, does not describe vagueness as *merely* semantic; the vagueness of natural language might be grounded in the world. Though I support such a position, I find myself, along with a few others, dwarfed by a massive majority who would deny any such metaphysical claim. This majority advocate a representational theory of one form or another. The reasons for its popularity are, I think, obvious. It accords with the prevalent intuition that the feature of natural language that initiated the problem in the first place — vagueness — is semantic, whilst denying that this has any import at all for metaphysics and promises to be of no more than superficial import for logical theory. Of course, some account is required of the relation between vague natural language and the world but this will be constrained by the fact that the world is precise.

One is not forced to look very far to find claims which suggest this metaphysical position. In what is considered by many to be the *locus classicus* on vagueness, Russell claims:

> There is a certain tendency in those who have realized that words are vague to infer that things also are vague. ... This seems to me precisely a case of the fallacy of verbalism — the fallacy that consists in mistaking the properties of words for the properties of things. Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. *They have to do with the relation between a representation and that which it represents.* Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; *things are what they are, and there is*

predicate-vagueness, characterised by the presence of any border case as opposed to the more recent stricter characterisation in terms of *precisely individuated* border cases; there are predicates which are vague according to the traditional concept of vagueness though precise according to more sophisticated accounts recently offered. (See B. Rolf, 'A Theory of Vagueness', *Journal of Philosophical Logic* 9 (1980), esp. §1; the spirit of this amendment is adopted by Mark Sainsbury in his 'What is a Vague Object?', *Analysis* 49 (1989): 99-103 and Michael Tye in his 'Vague Objects', *Mind* 99 (1990): 535-57.) The traditional concept then amounts to a naïve paradigmatic conception in so far as simple border cases are the defining characteristic, in conjunction with the view that the relevant vagueness is to be seen as a semantic property of the term involved. Most discussions of vagueness in the literature approach the phenomenon via this concept of vagueness — hence its descriptive title as 'traditional'. Issues surrounding the naïvete of this concept will become important later. For the moment we can ignore this complication.

> *an end of it. Nothing is more or less what it is*, or to a certain extent possessed of the properties which it possesses.([5]) [my italics]

One might reasonably interject at this point and point out that, in so far as the representationalist is to be understood to be making a substantive metaphysical claim, Russell need not be considered a representationalists. He might be understood as claiming that it is *meaningless* to ascribe vagueness to the world rather than false; though the world being represented in language can be talked about, 'vague' and its opposite 'precise' are, as Russell has said above, relational attributes that cannot be applied to the world as opposed to the represented world. On this view, it is akin to a category mistake to ascribe vagueness or precision to that which language represents; they are features that can only be applied in the realm of representations. If we accept this terminological point then, though 'vague' is not applicable to the world, there may nonetheless be a sense in which we may want to meaningfully inquire as to whether or not that feature of representations under discussion in the context of vagueness has any analogous correlate in discussions of things or the world.

Is there an analogue of 'vague' that can be meaningfully applied to the world? Well yes; 'fuzziness' understood to mean indeterminacy-in-extension will function as a meaningful analogue of 'vagueness' in discussion about the world, where this is simply intended as an ontological analogue of semantic indeterminacy of extension. So there *is* a sense in which the world might be said to be vague, namely its being fuzzy (though many think that ultimately this is logically incoherent) and a sense in which the world might be said to be precise, its not being fuzzy (which many think is logically

([5]) Russell, B., *op. cit.*, pp. 84-5. One might well wonder who Russell had in mind when describing this fallacy. Over fifty years later Margalit similarly protested that "things are what they are. ... The ascription of vagueness to objects may yield the quantities-turn-qualities kind of 'logic' ('dialectical' or otherwise), which commit the fallacy of verbalism." Margalit's criticism seems directed at the dialectical approach to vagueness endorsed by (at least) some Marxists; the turn-of-the-century Marxist scholar Plekhanov used just such a "quantities-turn-qualities" analysis of sorites situations and had no qualms in admitting the dialectical nature of the world — cf. G. Plekhanov, *Fundamental Problems of Marxism*, (ed. by D. Ryazanov) Lawrence & Wishart (1941), esp. pp. 114ff. Perhaps this was Russell's target too. (The paraconsistent nature of the Marxist analysis would also have appalled Russell, as it would some current theorists.)

necessary), which we can express as its being *sharp*.(6)

What we shall see is that, in the course of charitably interpreting Russell's definition of vagueness, he must be understood as claiming that the world is sharp. This is because, in spite of opinions to the contrary, he has (wittingly or not) provided a blueprint for representational vagueness. A charitable interpretation then sees him as committed to a representational account, including a sharp world.

2. *Russell's Definition of Vagueness*

In order to understand his analysis of vagueness further, a great deal of unravelling and clarification of his "definition" is required. The definition which I shall eventually put forward may seem remote from Russell's; however, my claim is that whilst it charitably interprets his remarks, thus making them more coherent and plausible, it nonetheless remains faithful to the spirit of Russell's views. Thus I see the ensuing reading as constrained by the following two principles of interpretation: Charity:— choose that interpretation which maximises the interest of the text; and Textual Fidelity:— choose that interpretation which most naturally fits the text. Of course these principles may pull against one another and we should aim, as I have done, for reflective equilibrium; there is the presumption that authors will mean what they say yet, if we respect their intelligence, we should hesitate to attribute foolish or trivial doctrines to them.(7)

Russell presented the following "definition" of vagueness:

> [A] representation is *vague* when the relation of the representing system to the represented system is not one-one, but one-many. For example, a photograph which is so smudged that it *might* equally represent Brown or Jones or Robinson is vague. A small-scale map is usually vaguer than a large-scale map, because it does not show all the turns and twists of

(6) I have adopted the term 'sharp' in preference to Rolf's term 'hard'; see his *Topics on Vagueness*, p. 13.

More will be said in developing this idea of "indeterminacy-of-extension" later, when required.

(7) Paraphrasing Peter Carruthers who cites these principles as guiding his interpretation of Wittgenstein's *Tractatus*. Cf. Carruthers, P., *The Metaphysics of the Tractatus*, CUP (1990), p. xii.

> the roads, rivers etc., so that various slightly different courses are compatible with the representation that it gives ... Passing from representation in general to the kinds of representation that are specially interesting to the logician, the representing system will consist of words, perceptions, thoughts, or something of the kind, and the would-be one-one relation between the representing system and the represented system will be *meaning*. In an accurate [precise] language, meaning [denotation] would be a one-one relation; no word would have two meanings ... In actual languages, as we have seen, meaning is one-many. [my italics][8]

The most thoroughgoing treatment of Russell's account is, to my knowledge, that presented by Rolf in his 'Russell's Theses on Vagueness', *History and Philosophy of Logic 3* (1982): 69-83. Therein Rolf presents two interpretations of Russell's definition;

D1 The representation r is vague if and only if there are two different entities, x and y, such that it is logically possible that r represents x and it is logically possible that r represents y.

or

D2 The representation r is vague if and only if it is logically possible that there are two different entities, x and y, such that it is logically possible that r represents x and it is logically possible that r represents y.

(The distinction between D1 and D2 is just that between extensional vagueness, actual indeterminacy, and intensional vagueness, the possibility of actual indeterminacy, now standard in the literature. In the context of the current discussion this distinction is of no import and I shall speak of *an* interpretation.)

This interpretation may be thought inadequate as regards its use of mere 'logical possibility'. Representations that are intuitively precise are such that it is *logically* possible that they might represent a whole range of things; mere logical possibility is too extensive. In fact (and this seems to be what

[8] Russell, *op. cit.*, pp. 89-90.

Rolf has in mind anyway) the logical possibilities are constrained. As Rolf explains it, the right-hand sides of D1 and D2 are true just when a representation r has certain "essential properties" but that these do not determine whether or not r represents x or y — for two, perhaps non-actual but logically possible, entities x and y.(9) Both alternatives are possible in as much as x and y only differ on some property that is not determined either way by the properties essential to the representation; that is, *both alternatives are logically possible to the extent that they are consistent with the essential properties of the representation*. What then is this notion of an "essential property"?

Let us say that an *essential property* of a representation r is any property constitutive of r itself — qua representation; that is, one that could, in principle, be directly given to or known by an agent to be a property of r on the basis of the representation r itself (which might be a painting or photo, knowledge, beliefs, propositions, or other linguistic items depending on whether the representation is mechanical, cognitive or linguistic). For example, a colour photo, p, showing a blue ball has as one of its essential properties that what is represented is represented as being blue and, though it may, as it happens, in fact be a photo of a green ball this is simply an accidental property of p.

As another example, consider a predicate. Properties essential to a predicate (qua representation) are just the semantic properties of the predicate; for example, that the predicate 'P' denotes the property α, or applies to some object a, say.

In focussing on the essential properties of a representation we are interested merely in how that which is actually represented is represented as being — something internal to the representation itself. In the above example of the photograph, the ball that is actually photographed is green yet represented as being blue. In this case the represented — the ball — is represented as being a colour which it in fact is not and to this extent the photograph is a *misrepresentation*, however such a feature has no bearing on whether or not the representation is vague — external considerations figure in determining whether or not a representation misrepresents. A representation of x counts as a misrepresentation just in case it represents x as being something it is not — it is a property of the representation which obtains on the basis of the relation between x and the essential properties of the

(9) Rolf, 'Russell's Theses on Vagueness', *History and Philosophy of Logic* 3 (1982), p. 70.

representation of x; a representation is vague, on the other hand, by virtue of features internal to the representation itself — the representation's essential properties.

Consider Russell's smudged photo. Its essential properties do not have the distinctness which would entail either that the photo does not represent Brown or that it does not represent Robinson. It is therefore possible relative to the essential properties of the photo that it is a photo of Brown (the essential properties do not rule this out) or of Robinson (the essential properties do not rule this out either). The photo underdetermines its referent and it is this, according to Russell, which constitutes its being vague.(Henceforth, unless stated otherwise, I shall restrict my attention to veridical representations of the world; this enables us to say that if a representation has, as an L-determined property, that the represented is so-and-so then we can say that the world is so-and-so. This simplifying assumption enables us, where necessary, to conflate properties of a representation r, e.g. 'representing the represented as being ...', with facts about the world, e.g. 'being ...'.)

So, in the interpretation of Russell's definition above, x and y are not just logically possible referents of the representation r; we can be more specific. They are logical possibilities constrained by the fact that they must be consistent with what is already represented as being the case or *logical possibilities relative to the essential properties of r*. Letting Σ_r stand for the set of essential properties of the representation r, this type of possibility can be written as '$\Diamond_{\Sigma r}$', by means of which we can define $\Box_{\Sigma r}B =_{df} \sim \Diamond_{\Sigma r} \sim B$ and $\nabla_{\Sigma r}B =df \Diamond_{\Sigma r}B \ \& \ \sim \Box_{\Sigma r}B$.([10])

Returning (finally) then to the Russellian definition of vagueness, x and y are logically possible referents of r subject to the restriction that they be consistent with what is already represented as being the case.

Thus we may now interpret Russell's definition more adequately as follows:

([10]) Naturally 'consistency' is logic-relative, however, since nothing turns upon the use of different logics here, I shall speak of 'consistency' *simpliciter*. (Russell would, of course, use 'consistency' in the sense of 'classical-consistency'.) Furthermore, Russell would have objected to treating '$\Diamond_{\Sigma r}$' as a properly logical notion so we must be careful not to attribute such a view to him; we can make use of the notion — Russell would just have added that, in so doing, we are not engaging in any new logic.

D3 The representation r is *(extensionally) vague* if and only if there are two different entities, x and y, such that it is logically possible relative to Σ_r that r represents x and it is logically possible relative to Σ_r that r represents y.

or

D4 The representation r is *(intensionally) vague* if and only if it is logically possible that there are two different entities, x and y, such that it is logically possible relative to Σ_r that r represents x and it is logically possible relative to Σ_r that r represents y.

Less formally we can say that a representation is vague just if there are, or could be, various different referents compatible with the representation given. On the basis of the representation itself one cannot determine its referent — various possibilities remain open, the representation lacking the distinctness which would entail any particular referent as being represented.

Comparing Russell's all too brief "definition" cited earlier with that given above, I claim that the definition of vagueness given above is faithful to Russell, thereby satisfying the interpretive principle of Textual Fidelity. It spells out in more detail the sense in which Russell took representations to be vague — when the relation of the representing system to the represented system (which, in the case where the representing system is linguistic, will be the relation of *meaning* or *denotation*) is *one-many*. Moreover, by the time we reach the end of §3 it will become clear that we have, in the above definition, the essence of an account of representational vagueness and so, in accord with the interpretive principle of Charity, Russell can be seen as the paradigmatic representationalist endorsing these definitions.

2.1 *The Adequacy of Russell's Definition*

Russell's definition remains problematic for the moment however, even given the above refinement. The problem concerns the logical type of the 'entities' referred to in D3 and D4. The narrow construal of them as items of the lowest logical type, i.e. individuals — items of type 0, causes problems for the definition(s). It is to this problem that I now turn.

Rolf discusses Russell's definition of vagueness as he understands it and concludes that "Russell's definition of vagueness as denotation being one-

many is not an attempt to account for ordinary usage of the word 'vague'".([11]) Russell has in effect, says Rolf, confused vagueness with lack-of-specificity from the very outset and therein lies the root of the problem whereby he cannot make the requisite distinctions one requires of a definition of vagueness, e.g. that between vagueness and generality. Similarly, Max Black and Marvin Kohl seem to take this line in their discussions of Russell's theory, as will become apparent.

Whilst it is true that Russell's discussion as to the source of vagueness conflates vagueness with lack-of-specificity or generality (cf. esp pp. 87, 91), this is a common conflation inessential to his subsequent analysis and the relevant remarks can and should be simply abandoned.([12]) Rolf has only considered one of (at least) two possible interpretations of Russell's claims (namely — he has assumed the 'entities' to be of type 0) and we shall see that, with appropriate repair, Russell's definition can make good those distinctions that seem necessary to an adequate account of vagueness. Thus it is worthwhile considering Rolf's objections and to see how Russell's definition needs filling out in order that it characterise 'vagueness' — at least as traditionally understood.

So what problems does Rolf see for Russell's definition?

The first thing that Rolf notes about Russell's definition of vagueness (as it stands) is that *it apparently fails to distinguish vagueness from generality*.([13]) This charge is echoed by Black when he asserts that "Russell's definition of vagueness as constituted by a one-many relation between symbolising and symbolised systems is held to confuse vagueness with generality" whereas vagueness is a "feature of the boundary of a term's extension and

([11]) Rolf, *op. cit.*, p. 81.

([12]) It also occurs most notably in A. Rosenberg, 'The Virtues of Vagueness in the Languages of Science', *Dialogue* 14 (1975): 281-311, resulting in the potentially misleading claim that "unfalsifiability ... follows in the train of vagueness" (Rosenberg, pp. 299-300). Thus vagueness is portrayed as an enemy of Popperian scientific theorising - a view echoed by Russell on page 91. The scientific "second-gradeness" of vague claims that makes them a target for elimination in the present philosophical climate follows largely from this conflation. Roy Sorensen has recently campaigned to disambiguate these two colloquial senses of the word 'vague' thereby making "the streets of speculation just a little bit safer for the philosophers of tomorrow" - see R. Sorensen, 'The Ambiguity of Vagueness and Precision', *Pacific Philosophical Quarterly* 70 (1989): 174-83.

([13]) Rolf, *op. cit.*, p. 71.

is not constituted by the extension itself".([14]) And Kohl objects to Russell's view of vague language in part "because it confuses vagueness with generality".([15]) It is then, it would seem, a fairly common (and, I think, misguided) critique of Russell's definition.

What then is generality? Taking that type of representation that is of primary interest in this debate, language, we may say that: a term is *general* (not to be confused with 'ambiguous'([16])) when it is understood to be applicable to a number of different objects in virtue of some common property, or, alternatively, when it designates or denotes a class that is its extension. It is important to note that, in this sense, a term denotes its extension and not the members of its extension so, though a general term is said to *apply* to a number of different objects, it does not *denote* a number of different objects (the members of its extension, items of type 0) but denotes the class having those objects as members. For example, 'man' is a general term since it denotes the class, mankind, whose members (to which the term is applicable but which the term does not denote) all possess the property of 'masculinity'. It is this crucial and rather elementary distinction, whereby a general term applies to a number of different objects but denotes the class comprising those objects, that paves the way for a suitable interpretation of Russell's definition.

Rolf argues that if r is a general word like 'man', it can represent — denote — many different men and would therefore count as vague on Russell's definition. But a general word can, at least *prima facie*, be precise. So "Russell's definition of vagueness cannot by itself distinguish generality from vagueness".([17]) However, we have just seen how the denotation relation between a general term and its denotatum is one-one. Rolf is confused about the denotative properties of general terms and consequently argues

([14]) Black, M., 'Vagueness: An Exercise in Logical Analysis', *Philosophy of Science* 4 (1937); reprinted (including a reply to Hempel) in M. Black, *Language and Philosophy*, Cornell University Press (1949). This reference is from the latter, p. 29.

([15]) Kohl, M., 'Bertrand Russell on Vagueness', *Australasian Journal of Philosophy* 47 (1969), pp. 37 ff.

([16]) Cf. Quine, W.V.O., *Methods of Logic*, 2nd edn RKP (1962), p. 203. Ambiguous terms can be considered to be (at least) two quite distinct terms individuated by two distinct sets of semantic properties, having distinct (though perhaps indeterminate) extensions. E.g. 'bank' has at least two distinct senses, 'river bank' and 'financial institution', each having its own seperate extension.

([17]) Rolf, *op. cit.*, p. 71.

fallaciously.

In Rolf's defence, one might be excused for thinking 'man' is "vague" to Russell's way of thinking since Russell himself sometimes seemed confused as to whether general terms were vague. In *The Analysis of Mind* for instance he does say "'I met a man' is vague, since any man would verify it"([18]) but then goes on to contradict himself by claiming that "[a] vague word is not to be identified with a general word, though in practice the distinction may often be blurred."([19]) Nevertheless, in text preceding his definition of vagueness he makes the following declaration: when we speak of a proposition having a certain degree of vagueness,

> there is not one definite fact necessary and sufficient for its truth, but a certain region of possible facts, any one of which would make it true. And this region is itself ill-defined: *we cannot assign to it a definite boundary*. This is the difference between vagueness and generality. A proposition involving a general concept — e.g. "This is a man" — will be verified by a number of facts, such as "This" being Brown or Jones or Robinson. But if "man" were a precise idea, the set of possible facts that would verify "this is a man" would be quite definite. Since, however, the conception "man" is more or less vague, *it is possible to discover prehistoric specimens concerning which there is not, even in theory, a definite answer to the question, "Is this a man?"* As applied to a such specimens, the proposition "this is a man" is neither definitely true nor definitely false. [my italics]([20])

So Russell *does* informally distinguish vagueness from generality; lack of determinate or definite boundaries and border cases are invoked to do this, as the above italicised claims show. Now, whether the definition is successful or not depends on its ability to formally incorporate these insights regarding the distinction.

This brings us to a second criticism levelled at Russell's definition by Rolf: *the definition makes no use of the notion of a border case.*([21]) As we have just seen though, Russell invokes the notion informally in order

([18]) Russell, B., *The Analysis of Mind*, Allen and Unwin (1921), p. 182.

([19]) *Ibid.*, p. 184.

([20]) Russell, 'Vagueness', *op. cit.*, p. 88.

([21]) Rolf, *op. cit.*, pp. 71-2.

that a requisite distinction concerning vagueness and generality can be made; furthermore he explicitly says, in his initial consideration of just what one means when one speaks of vagueness, that common words such as 'red' are vague since "there are shades of colour concerning which we shall we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word 'red', but because it is a word the extent of whose application is essentially doubtful [indeterminate]."([22]) So Russell is certainly working with the notion of a border case. My claim will be that it also does in fact figure implicitly in his definition and to this extent is "used". This traditional signature of vagueness — the presence of border cases — is absorbed into Russell's definition and, though transformed in the process, it is nonetheless operative thus enabling Russell's definition to distinguish vagueness from generality.

This is the dialectical position then. Initially Rolf and others argued that Russell's definition of vagueness as one-manyness couldn't make a necessary distinction between vagueness and generality. In the process of rebutting Rolf's second criticism — in showing *that* the definition makes use of border cases — his first criticism will be seen to be not only unwarranted (since based upon a confusion, as described above) but false, since once we understand *how* the existence of border cases relates to the one-manyness of denotation we will see that the definition is able to distinguish vagueness from generality. So how do border cases figure in the definition?

We have already seen that the one-manyness of the representation relation in Russell's definition arises, not as a result of the one vague representation r *actually* representing many referents, but as a result of each of a plurality of distinct referents being *possible relative to the essential (semantic) properties of* r — Σ_r. What I propose is that this modal relation — 'what r might represent given Σ_r' — is one-many just when reference varies depending whether or not the representation's border case(s) is (are) to be included in or excluded from the range of the representation function. In other words, that characterising feature of vague representations on Russell's account has its source in exactly that which characterises representations as vague on the traditional account — namely, border cases. The only way in which his definition differs from the more common account in terms of border cases is that, as I said earlier, his definition incorporates the concept of border cases into his specific (representational) theory of vagueness; it

([22]) Russell, 'Vagueness', p. 85.

thereby becomes subsumed into the more specific account and no longer features explicitly.

How then are border cases responsible for the one-manyness of the representation relation and *vice versa*? This question is best answered, I think, by contrasting the traditional conception of vagueness, border case (semantic) vagueness, with another notion employed by Russell — "accuracy". In the preamble to his definition of vagueness he says the following:

> One system of terms related in various ways is an *accurate* representation of another system of terms related in various other ways if there is a one-one relation of the terms of the one to the terms of the other, and likewise a one-one relation of the relations of the one to the relations of the other, such that, when two or more terms in the one system have a relation belonging to that system, the corresponding terms of the other system have the corresponding relation belonging to the other system. Maps, charts, photographs, catalogues, etc., all come within this definition in so far as they are accurate. [my italics](23)

In other words, one structured system is an accurate representation of another if there is a one-one relation correlating elements of one to elements of the other by means of which the former can be interpreted as a model of the latter. When such a structure-preserving correlation obtains between a system A and another system B, A is said to be isomorphic to B.

Consider, for example, a map which consists of various marks on paper related in various ways, a legend saying what each type of mark represents, a scale, contours, and so on. If this map is accurate or isomorphic to that which it purports to represent then each mark correlates one-one with (denotes, represents) something in the region mapped, and the relations that obtain on the map, when interpreted and scaled, correspond to relations that obtain in the region mapped. If for instance a dot designated 'Black Mtn' is 1.5 centimetres from another dot designated 'ANU' in a direction designated as 'west' on a 1:100,000 map of Canberra then, using the intended or canonical one-one correlation, the map represents the elevation Black Mountain as 1.5 kilometres west of the Australian National University in the city of Canberra. What's more, the map is then accurate in so far as this is the case.

(23) *Ibid.*, p. 89.

If our geographical world is however not as described then the correlation, though one-one, is not structure-preserving and we may suppose the map to be inaccurate in this regard. The map is a factually inaccurate representation of our world because it *misrepresents* our world, nonetheless there is some world it can be taken to accurately represent — namely a world which, were it to exist, would be as the map describes. Though factually inaccurate, the map is isomorphic to some possible world.

Thus, factually inaccurate representations are contrasted with accurate ones. But there is another contrast we can draw. When we come to consider vague natural language descriptions we are, on the representationalist view, forced to concede that such descriptions could never be isomorphic to the world. There is a different type of inaccuracy inherent in vague language — *logical inaccuracy*.

Were vague language ever isomorphic to the world, say, then the world would have to exhibit that feature analogous to vagueness yet this, according to representationalists (and, I shall argue, Russell in particular) is impossible. As a matter of necessity, that represented by language is never vague or fuzzy so any possible candidate referent for a vague term will fail to share that logical property analogous to vagueness — fuzziness; any candidate for reference, anything in the range of the denotation function, is sharp. So vague descriptions can never accurately represent precise or sharp referents and hence don't stand in a one-one relation to that represented.

Russell's smudged-photo example is an apt illustration of this inaccuracy and the paradigm for vagueness, in particular for vagueness in language. "[A]ll vagueness in language and thought is essentially analogous to this vagueness."([24]) Being smudged is, like vagueness on Russell's view, a property a photograph possesses by virtue of the nature of its relation to that photographed. The indistinctness in the photograph amounts to the necessary inaccuracy of the representation rather than the (impossible) indeterminacy of the represented. Just as the represented is never smudged in itself, so too the represented can never be vague, nor anything analogous, in itself. Smudged or vague representations are inaccurate in a way that mere factually inaccurate representations are not.

Using that example of vagueness where our intuitions seem strongest — the case of a vague predicate 'P' — we can show, more specifically and firstly, that representational vagueness results in one-manyness. One-many-

([24]) *Ibid.*, p. 91.

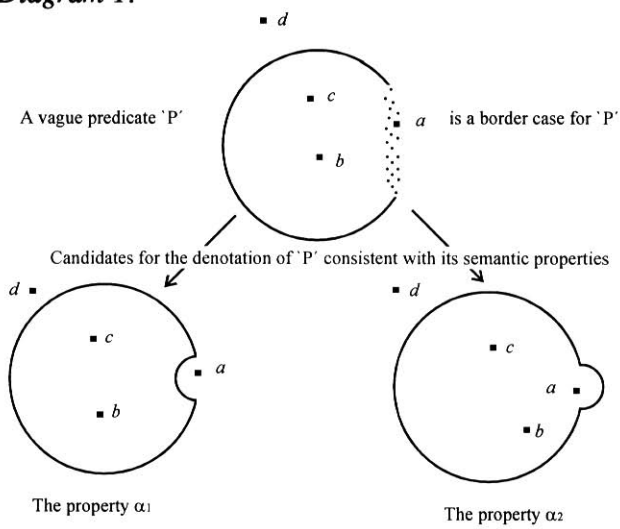
ness is a necessary condition for representational vagueness.

To say that the predicate is vague as traditionally understood is to say that there is some border case for 'P', *a* say. Suppose further that this is the predicate's sole border case, with *d* being such that 'P' determinately does not apply, and objects *b* and *c* being such that the predicate determinately does apply. So what property might 'P' denote? Since it's denotation must be sharp there are *two* candidates consistent with it's semantic properties, that is, consistent with the fact that *b* and *c* must determinately possess the property denoted by 'P' whilst *d* must determinately not possess the property denoted by 'P'. Either the border case *a* determinately possesses the property denoted by 'P' or it determinately does not. The predicate's border case must be resolved at the ontological level, one way or the other.([25])

([25]) Thus, for the sake of the argument, we are simply assuming that property-sharpness is the ontological analogue of predicate-precision — there are no border cases for the property.

Again, I am setting aside problems surrounding the simple border-case characterisation of predicate-vagueness for the moment (cf. n. 4). So, having already admitted earlier that our current concerns with vagueness are really concerns with the traditional, naïve conception of vagueness, we now note that talk of fuzziness, in so far as it is simply an analogue of vagueness, is really talk of a naïve conception of fuzziness. More sophistication can be added, but to do so now would simply confuse things.

Diagram 1.



The vague predicate's denotation might consistently be said to be the property α_1 or the property α_2. Moreover, this one-manyness regarding what the predicate might represent is one-manyness about whether or not to include the border case in the property denoted. Since the vagueness in the representation has no analogue in what is represented the vagueness can (must) be resolved one of the two ways described, each being consistent with the essential properties of the representation (in this case, the semantic properties of 'P') though neither being necessary.

In the case of predicate vagueness, then, we can see that not only does the presence of *some* border case give rise to some one-manyness, thereby contrasting vagueness with accuracy; the very border case *itself* is the source of the referential indeterminacy.

Secondly, with predicates at least, we can do more than merely contrast vagueness with accuracy by showing that border cases give rise to one-manyness — the contrasting notion instantiated, one-manyness, is instantiated *solely* by terms that are border-case vague. That is to say, we can do more than merely show that vagueness is contrasted with accuracy by showing that one-manyness is a necessary condition for vagueness — we can also show that one-manyness is sufficient for vagueness.

The simplest way to show this is to establish that precise predicates can stand in a one-one correlation with a sharp property, so precision is sufficient for one-oneness. A one-one correlation (that will also be factually

accurate) between a predicate 'P' and a sharp property can be obtained by simply taking the property, α, to be that which all objects describable as 'P' have in common; hence the set of objects having the property α (this set often being identified with the property α itself) is the set of objects satisfying 'P'. So by definition the property α is correlated one-one with the predicate and α is sharp since the fact that everything either determinately satisfies the predicate or determinately doesn't entails that all objects instantiate α determinately or determinately not — there are no border cases for α. (See Diagram 2a.)

So, conjoining the two claims arrived at so far, one-manyness of denotation is both necessary *and* sufficient for traditional predicate-vagueness on the assumption that the represented is sharp; hence, Russell's definition is a definition of traditional predicate-vagueness *if* the represented is sharp.

Diagram 2a.

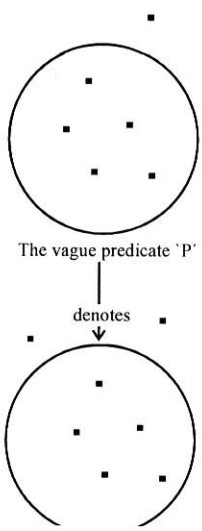
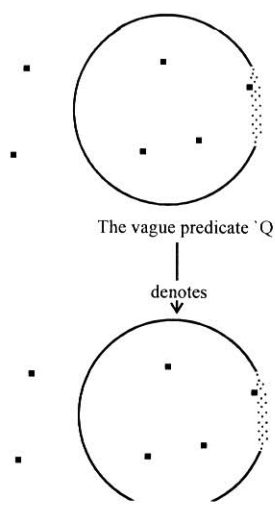


Diagram 2b.



On Russell's account the above mentioned one-one correlation is not possible with vague predicates because they could only ever be so correlated with a fuzzy property and there are none according to his view. However, if there could be fuzzy properties then the same one-one correlation could then be employed. (See Diagram 2b.) In other words, if the represented could be fuzzy then not all vagueness need result in one-manyness; a vague predicate 'Q' could accurately represent or denote some fuzzy property β,

say. So we can further add that Russell's definition is a definition of traditional predicate-vagueness *only if* the represented is sharp.(26)

The following claims seem plausible then: (i) If the represented is sharp then all traditional predicate-vagueness results in one-manyness and *vice versa*; and (ii) if all traditional predicate-vagueness results in one-manyness and *vice versa* then the represented is sharp. So, conjoining these two claims, it would follow that:

Conjecture: Russell's definition of predicate-vagueness and the traditional definition of predicate-vagueness are equivalent if and only if properties are sharp.

If this conjecture is correct (a proof follows shortly) then Russell has successfully defined traditional vagueness for all and only those predicates where vagueness is semantic but not ontological; that is, he has given a definition of traditional *representational* predicate-vagueness.

If, furthermore, the definition can be amended to avoid that traditional aspect of vagueness whereby some intuitively precise predicates are counted as vague by virtue of their possessing border cases(27) and, if it can simultaneously be shown to extend to denoting phrases in general (predicates, singular terms and sentences), then Russell's definition provides the foundation for any adequate representational account of vagueness.

I think all these conditions can be met. This is a far cry from charges of confusion often levelled at the Russell account; with a few minor adjustments to avoid some counter-intuitive consequences of the traditional concept of vagueness (which, it should be said, most theories stand in equal need of) Russell has gotten the semantic analysis of representational vagueness exactly right.

Actually proving such a result, though satisfying in its generality, would, I think, be unnecessarily complex. I shall content myself with proving the above conjecture, thus establishing Russell's definition as an adequate definition of traditional representational vagueness for predicates. A subsequent sketch of how to extend the result to vague denoting phrases in gene-

(26) His definition can be seen to distinguish vagueness from ambiguity since an ambiguous predicate is one-many only in the sense that its semantic properties determine a plurality of distinct referents, whereas Russellian one-manyness amounts to a predicate's semantic properties failing to determine any referent.

(27) Cf. n. 4.

ral whilst simultaneously avoiding problems that beset the overly weak traditional conception ought to be convincing enough. Then, with the addition of a recursive extension of the concept of vagueness to include any grammatical category, Russell's definition can be used to generalise to a definition of representational vagueness *simpliciter*.

So, let's begin with a proof for the special case of traditional predicate-vagueness; that is, let's firstly prove the Conjecture.

A Digression: Formulating The Notion Of 'Sharpness'

In order to appreciate the proof in support of this restricted claim, some formulation is required of what it is for properties to be 'sharp'. Of course this is not easily done without opening up a veritable Pandora's Box. Any account of 'sharpness' will, by implication, commit one, at least partially, to a theory of ontological vagueness (as coherent thus-and-so, or as defective thus-and-so, or ...) and as such is bound to be controversial — there being few, if any, agreed principles in this area. However, some understanding is currently required of what it is for properties to be sharp. It is this task to which we now turn.

At present all that is meant by the ascription of sharpness is the *ontological* analogue of the ascription of precision. So, taking the case of properties, to say that a property is sharp is just to ascribe to the property that ontological analogue of ascriptions of precision to predicates; that is (ignoring problems discussed in n. 4), a property is sharp if and only if it has no border cases.

Let us characterise the sharpness of properties then as follows: for any property ψ and any object x, either x determinately instantiates ψ or determinately does not.([28]) Call this principle P(roperty) S(harpness).

(PS) $\forall\psi\forall x(D\psi x \vee D\sim\psi x)$

Of course, for any property ψ and object x, if it is determinately the case that ψx, then ψx. Call this principle D.

(D) $\forall\psi\forall x(D\psi x \rightarrow \psi x)$

([28]) The variable 'ψ' is taken to range over properties.

Given D, PS is equivalent to the claim that every property is instantiated determinately if at all. I shall refer to this as PS*.([29])

(PS*) $\forall\psi\forall x(\psi x \rightarrow D\psi x)$

So, if properties are sharp then principles D and PS* jointly hold and they are jointly equivalent to its being the case that a property is instantiated *if and only if* it is instantiated determinately. Call this principle DPS*:

(DPS*) $\forall\psi\forall x(\psi x \leftrightarrow D\psi x)$.

Which is just to say that with regard to property instantiation we have a 'D'-Redundancy Thesis; though, of course, with regard to properties-as-represented we don't have such a thesis — that predicates are vague is one of the reasons why the 'D' operator was invoked in the first place.

The extension of the concept of 'sharpness' to other ontological categories will simply mirror the extension of 'vagueness' to other relevant grammatical categories.

Having digressed to formulate the sharpness of properties, we are ready to attempt a proof of the Conjecture above.

The proof will contain two sub-proofs: (I) If properties are sharp then a predicate is vague on the traditional conception if and only if it is vague on the Russellian conception; and (II) If every predicate that is vague on the traditional conception is vague on the Russellian conception and *vice versa* then properties are sharp.

(I) If properties are sharp, then a predicate is vague on the traditional conception if and only if it is vague on the Russellian conception.

Informally we can argue as follows. Assume a predicate, $\underline{P}$ say, is vague as traditionally understood; there is something for which it is indeterminate whether it satisfies $\underline{P}$ [which I shall represent as: $\exists x I\underline{P}x$].([30]) That is, there

([29]) The equivalence between PS and PS* is established in Appendix A.

([30]) I shall proceed using a one-place predicate. The result for any n-place predicate is an obvious generalisation. For formatting reasons I shall adopt the convention whereby predicates are written underlined when named (e.g. $\underline{P}$) and not when used (e.g. P); when particular properties are named I shall continue to use Greek letters (α, β, ...). 'I' and 'D'

is some (possible) border case, *a* say, such that it is neither determinately the case that *a* satisfies $\underline{P}$ [$\sim D\underline{P}a$] nor determinately not the case that *a* satisfies $\underline{P}$ [$\sim D \sim \underline{P}a$]. This indeterminacy is an intrinsic semantic feature of the predicate $\underline{P}$ so it follows that the semantic properties of the predicate do not entail that *a* determinately instantiates the property denoted by $\underline{P}$ [$\sim \Box_{\Sigma\underline{P}} D(a \text{ i } \iota\psi(\psi = \text{den}\underline{P}))$, which I shall shorten to: $\sim \Box_{\Sigma\underline{P}} D\underline{P}a$] nor whether it determinately does not instantiate the property denoted by $\underline{P}$ [$\sim \Box_{\Sigma\underline{P}} D \sim (a \text{ i } \iota\psi(\psi = \text{den}\underline{P}))$, which I shall shorten to: $\sim \Box_{\Sigma\underline{P}} D \sim Pa$]. Yet, though 'satisfaction' may be a vague relation, 'instantiation' is not; by sharpness (DPS*), any property which $\underline{P}$ actually denotes is determinately instantiated by *a* if and only if it is instantiated by *a simpliciter* [$D(a \text{ i } \iota\psi(\psi = \text{den}\underline{P})) \equiv a \text{ i } \iota\psi(\psi = \text{den}\underline{P})$, i.e. $Pa \equiv DPa$]. So, it can't follow from the semantic properties of $\underline{P}$ that $\underline{P}$ denotes a property instantiated by *a* [$\sim \Box_{\Sigma\underline{P}} Pa$] and consequently $\underline{P}$ might be consistently said to denote a property not instantiated by *a* [$\Diamond_{\Sigma\underline{P}} \sim Pa$]. Similarly, it might denote a property that is instantiated by *a* [$\Diamond_{\Sigma\underline{P}} Pa$]. The semantic properties of the predicate leave it undetermined. That is to say, it is essentially contingent, relative to those properties of the predicate $\underline{P}$ determined by its semantic properties, whether the denotation of $\underline{P}$ is instantiated by *a* or not [$\nabla_{\Sigma\underline{P}} Pa$]. In this sense the denotation of $\underline{P}$ is one-many — that is, not unique — and thus satisfies Russell's definition.

Conversely, if the denotation of $\underline{P}$ is one-many in the above sense — for all that is determined by its semantic properties it might denote the property instantiated by *a* yet it might not [$\Diamond_{\Sigma\underline{P}} Pa \;\&\; \Diamond_{\Sigma\underline{P}} \sim Pa$] — then the semantic properties of $\underline{P}$ could never warrant our claiming *a* as a determinate satisfier of $\underline{P}$ nor as a determinate non-satisfier of $\underline{P}$ [$\sim \Box_{\Sigma\underline{P}} DPa \;\&\; \sim \Box_{\Sigma\underline{P}} D \sim Pa$]. And so, argues the representationalist, it is essentially semantically indeterminate whether a satisfies $\underline{P}$ and *a* is therefore a border case for the predicate $\underline{P}$. QED.

Given that DPS* holds, then the formal equivalence between the traditional conception of vagueness and Russell's definition can be established — that is, we can establish

$$(\text{DPS}^*) \vdash (\exists x I\underline{P}x \leftrightarrow \exists x \nabla_{\Sigma\underline{P}} Px)$$

— as follows:

are used here as operators on meta-linguistic sentences.

The predicate $\underline{P}$ is vague as traditionally understood iff $\exists x I\underline{P}x$.
Assume $\underline{P}$'s border case is a;

that is,	$I\underline{P}a$.	
iff	$\sim D\underline{P}a \;\&\; \sim D\sim \underline{P}a$	(by definition of 'I')
iff	$\sim \Box_{\Sigma\underline{P}} DPa \;\&\; \sim \Box_{\Sigma\underline{P}} D\sim Pa$	(by meaning of '$\Box_{\Sigma\underline{P}}$')
iff	$\Diamond_{\Sigma\underline{P}} \sim DPa \;\&\; \Diamond_{\Sigma\underline{P}} \sim D\sim Pa$	(by definition of '$\Diamond_{\Sigma\underline{P}}$')
iff	$\Diamond_{\Sigma\underline{P}} \sim Pa \;\&\; \Diamond_{\Sigma\underline{P}} Pa$	(by DPS*)
iff	$\nabla_{\Sigma\underline{P}} Pa$	(by definition of '$\nabla_{\Sigma\underline{P}}$').
So,	$\forall x(I\underline{P}x \text{ iff } \nabla_{\Sigma\underline{P}} Px)$	(by universal generalisation). ‖

I shall refer to this claim as '(RUSS)'.([31]) (RUSS) says effectively that a predicate $\underline{P}$ is vague as traditionally understood if and only if, for all that we could know, it might include its border case in its extension and it might not; that is, $\underline{P}$ might denote the property instantiated by a and it might denote some other property not instantiated by a. In this sense its denotation relation is "one-many".

(RUSS) entails $\exists x I\underline{P}x$ iff $\exists x \nabla_{\Sigma\underline{P}} Px$.

And so, given the sharpness of properties, we can established the slightly weaker claim that any predicate vague on the traditional conception is vague according to Russell's definition. It is weaker because unlike (RUSS) it does not assert that the border case is *the* case giving rise to semantic contingency; merely that if $\underline{P}$ has some border case then it has some case giving rise to semantic contingency.

Having presented the proof, it is worth noting that the following sub-proof is valid:

(D) $\vdash \exists x \nabla_{\Sigma\underline{P}} Px \rightarrow \exists x I\underline{P}x$.

This holds independently of the truth of DPS*. A proof of the right-hand-side only requires the conditional '$\forall\psi\forall x(D\psi x \rightarrow \psi x)$' — something's determinately instantiating a property implies its instantiating the property *simpliciter*; that is, principle D. (DPS*) is an unnecessary premise.

([31]) The equivalence (RUSS) can also be expressed using the variant '$\forall x(D\underline{P}x \text{ iff } \Box_{\Sigma\underline{P}} Px)$'. See Appendix B.

In other words, whilst epistemic and representational theorists are committed to claiming *all* semantic vagueness as instancing one-manyness of denotation (vacuously so for strong epistemic theorists), every theorist is committed to instances of Russellian one-manyness being instances of traditional predicate-vagueness. Hence any ontological account of vagueness, whilst claiming that there can be cases of semantic vagueness without one-manyness of denotation, must admit instances of Russellian vagueness (if there are any) as cases of traditional vagueness proper and advocates of an epistemic account denying that there is any such thing as traditional (semantic) vagueness are compelled to deny that there are any instances of Russellian vagueness.

(II) If every predicate that is vague on the traditional conception is vague on the Russellian conception and *vice versa* then properties are sharp.

To show this I shall argue that if a property were fuzzy then any predicate uniquely or "accurately" denoting the property would be vague, in which case not all predicate vagueness need result in one-manyness of denotation.

Assume then that some property, α say, is fuzzy $[\exists x(\sim D\alpha x \ \& \ \sim D\sim\alpha x)]$ with border case a say. Then it is neither determinately the case that a instantiates the property α nor determinately not the case that a instantiates α $[\sim D\alpha a \ \& \ \sim D\sim\alpha a]$. Assume further that $\underline{P}$'s semantic properties are such as to make it the case that it accurately denotes α $[\Box_{\Sigma\underline{P}}(\iota\psi(\psi = \text{den}\underline{P}) = \alpha)$, which I shall shorten to: $\Box_{\Sigma\underline{P}}(P = \alpha)]$. Now if $\underline{P}$'s semantic properties are such that it accurately denotes α then (by # — see below) it follows from $\underline{P}$'s semantic properties that it denote something determinately (not) instantiated by a if and only if α is determinately (not) instantiated by a $[\Box_{\Sigma\underline{P}}D(a \text{ i } \iota\psi(\psi = \text{den}\underline{P})) \equiv D\alpha a$, and $\Box_{\Sigma\underline{P}}D\sim(a \text{ i } \iota\psi(\psi = \text{den}\underline{P})) \equiv D\sim\alpha a$, which I shall shorten to: $\Box_{\Sigma\underline{P}}DPa \equiv D\alpha a$, and $\Box_{\Sigma\underline{P}}D\sim Pa \equiv D\sim\alpha a]$. Since α is neither determinately nor determinately not instantiated by a then it neither follows from $\underline{P}$'s semantic properties that it denote something determinately instantiated by a $[\sim\Box_{\Sigma\underline{P}}DPa]$ nor that it denote something determinately not instantiated by a $[\sim\Box_{\Sigma\underline{P}}D\sim Pa]$. But this is just to say that $\underline{P}$ neither determinately applies to a nor determinately doesn't $[\sim D\underline{P}a$ and $\sim D\sim\underline{P}a]$. So a is a border case for $\underline{P}$ and $\underline{P}$ is vague (as traditionally understood). So, if Russellian predicate-vagueness is equivalent to traditional predicate-vagueness then properties must be sharp. QED.

(#) is a special case of a substitution schema which says that if the seman-

tic properties of a representation r determine that what is represented is A — if r is an accurate representation of A — then, for a range of contexts φ, the semantic properties of the representation r determine that what is represented φ's if and only if A φ's.

$\Box_{\Sigma r}(\iota\psi(\psi = \text{den}(r)) = A) \vdash \Box\Sigma_r\varphi(\iota\psi(\psi = \text{den}(r))) \equiv \varphi(A)$, for a range of contexts φ.

As an example, consider a map of Canberra. If it follows from the map that Canberra has an area of n^2 kilometres then it follows from the map that Canberra has a large area if and only if n^2 kilometres is a large area. (Of course, if the map is also accurate then Canberra is as described by the map.)

In cases where 'r' is a predicate $\underline{P}$ and 'A' is the property α, the substitution schema can be instantiated and, using the usual abbreviations, shortened to:

$\Box_{\Sigma\underline{P}}(P = \alpha) \vdash \varphi(\alpha) \equiv \Box_{\Sigma\underline{P}}\varphi(P)$, for a range of contexts φ.

In the special case where φ is of the form '.. is determinately instantiated by *a*' or '.. is determinately not instantiated by *a*' we can say:

(#) $\Box_{\Sigma\underline{P}}(P = \alpha) \vdash D\alpha a \equiv \Box_{\Sigma\underline{P}}DPa$ and
$\Box_{\Sigma\underline{P}}(P = \alpha) \vdash D\sim\alpha a \equiv \Box_{\Sigma\underline{P}}D\sim Pa$.

Having said all this we can now argue more formally as follows:

Assume	(1)	$\sim D\alpha a \;\&\; \sim D\sim\alpha a$	(by definition of fuzziness)
and	(2)	$\Box_{\Sigma\underline{P}}(P = \alpha)$	(by definition of one-oneness).
Then	(3)	$\Box_{\Sigma\underline{P}}DPa \equiv D\alpha a$	(by 2 and #).
and	(4)	$\Box_{\Sigma\underline{P}}D\sim Pa \equiv D\sim\alpha a$	(by 2 and #).
Yet	(5)	$\sim D\alpha a$	(by 1 and &-elimination).
Hence	(6)	$\sim\Box_{\Sigma\underline{P}}DPa$	(by 3 and 5)
and so	(7)	$\sim D\underline{P}a$	(by 6 and meaning of '$\Box_{\Sigma\underline{P}}$').
Also	(8)	$\sim D\sim\alpha a$	(by 1 and &-elimination).
Hence	(9)	$\sim\Box_{\Sigma\underline{P}}D\sim Pa$	(by 4 and 7)
and so	(10)	$\sim D\sim\underline{P}a$	(by 9 and meaning of '$\Box_{\Sigma\underline{P}}$').
Therefore	(11)	$\exists x I\underline{P}x$	(by 7, 10, definition of 'I' and $\exists$-intro).

So If $(\exists\psi\nabla_{\Sigma\underline{P}}(P = \psi)$ iff $\exists x I\underline{P}x)$ then properties are sharp.

That is, If $(\exists x \nabla_{\Sigma \underline{P}} Px$ iff $\exists x I\underline{P}x)$ then properties are sharp.([32]) ‖

Consequently, if Russell is right in characterising predicate-vagueness as he does then properties are sharp.

What I claim then, via (I) and (II), is that the Conjecture cited earlier is established — the Russellian conception of predicate-vagueness and the traditional conception of predicate-vagueness are equivalent if and only if properties are sharp.

Whether Russell intended to or not, what he has characterised is representational predicate-vagueness as standardly conceived, traditional predicate-vagueness. Given this fact then, we could say that he was providing an alternative characterisation of predicate-vagueness were we to suppose properties to be sharp — that is, on the assumption that all predicate-vagueness was representational. Thus, charitably interpreted, we should take Russell's remarks concerning the impossibility of vagueness in the world to mean that properties (at least) are not vague or fuzzy; the agnosticism contemplated at the end of §1 is not a coherent option, and Russell is a representationalist about traditional predicate-vagueness at least.

3. *Caveats*

So, can we generalise on this result and in the process show both: (i) that a charitable interpretation of Russell has him committed to the absence of *any* vagueness or fuzziness in the world — thereby suggesting that he was not agnostic at all about ontological vagueness or fuzziness and truly intended a representationalist account of vagueness; and (ii) more strongly, that all and only those so committed should endorse Russell's definition as providing necessary and sufficient conditions for vagueness *simpliciter*? In

([32]) This last move is warranted by the following equivalence: there is some property ψ such that it is contingent relative to $\Sigma\underline{P}$ whether or not the denotation of the predicate $\underline{P}$ is ψ if and only if there is some object x (instantiating the property ψ) such that it is contingent relative to $\Sigma\underline{P}$ whether or not the denotation of the predicate $\underline{P}$ is a property instantiated by x.

It is simply the analogue of the equivalence between the existence of a property which counts as a border case for the denotation of the predicate and the existence of a border case for the predicate.

other words, can we show that Russell's definition can be extended to provide a definition of representational vagueness in general? I think we can but, as I said earlier, I shan't prove the extended result; rather I shall sketch how such a proof might proceed.

The first point to address, prior to extending beyond the category of predicates, is whether Russell's definition can be modified to provide necessary and sufficient conditions for predicate-vagueness *simpliciter* as opposed to traditional predicate-vagueness.([33]) The point of difference, remember, was that traditional vagueness is evidenced by the presence of border cases rather than the presence of *precisely individuated* border cases required for vagueness proper. So can the above proofs, (I) and (II), be reworked using such border cases as opposed to the simple border cases employed therein? Of course they can. What it amounts to is our claiming that, when we cite something as a border case in the proofs (*a* say, as we did), we must now suppose there to be no vagueness in identifying what counts as that thing. This caveat is then carried on through the proofs. Nothing in the proofs depended on the border case not being precise, so this caveat can be added without jeopardising any steps in the proofs.

In other words, Russell's account of what it is for a predicate to be vague, though it falls foul of the same objection that besets most accounts of predicate-vagueness (namely, requiring only border cases rather than the narrower requirement of precisely individuated border cases), can take on board the required amendment — the one-manyness of reference is one-manyness about possible candidates which are themselves precisely individuated.

Having seen that the required caveat can be added to his definition so that it captures predicate-vagueness proper, it remains to be seen whether it can serve as a definition of representational vagueness for any denoting phrase — predicate, singular term or sentence. The unifying aspect of all these categories of vagueness is vagueness in extension, which Russell's conception of vagueness is easily able to capture. Though I proceeded with an intuitive description, then an argument and proof for predicate-vagueness only, Russell's account dealt more generally with vagueness of denoting phrases. Given a border-case characterisation of vagueness for each of these categories, it is not hard to see that the equivalence between the presence of (precisely individuated) border cases and the underdetermination or one-manyness of reference is quite general. The discussion surrounding Dia-

([33]) Remember that when we speak of vagueness *simpliciter* we intend this to be understood as semantic vagueness, having put aside epistemic analyses.

grams 1 and 2 can simply be reworded to apply to any denoting phrase.

"A singular term naming a[n] ... object can be vague in point of the boundaries of that object", in other words, the term is vague if there are border cases for being part of the object named.([34]) By redescribing the predicate-extension of Diagram 1 as the extension of the relevant vague singular term we see that the fact that some part a is neither determinately part of the extension of 'N' nor determinately not part of the extension of 'N' means that there are two possible sharp candidates for the denotation of 'N' consistent with 'N''s determinate semantic properties (that is, consistent with *b*'s being a determinate part of the extension of 'N', *d*'s being a determinate non-part, and so on).

Moreover, redescribing Diagram 2, if the existence of border parts for the denotation of 'N' is tantamount to there being a plurality of possible referents then objects must be sharp, since were fuzzy objects logically possible then it would be logically possible that a vague term 'N' determinately denotes a fuzzy object having as an indeterminate part that which was neither determinately nor determinately not part of the extension of 'N'. Indeterminacy in extension makes for a plurality of possible sharp referents and a plurality of sharp referents entails there being some indeterminacy of extension.

A similar reworking can be given for sentences. Now we can redescribe the extension of 'P' as the extension of the sentence 'S' consisting of the set of possible worlds where 'S' obtains (in substituting talk of states-of-affairs with talk of a set of possible worlds I am following the common convention whereby a state-of-affairs is equated with the set of possible worlds where that state-of-affairs obtains). If 'S' is vague then there could be worlds where it is neither determinately true nor determinately false. That is, there is some possible world, *a*, such that it is indeterminate whether or not it is in the extension of 'S', the set of worlds where 'S' determinately obtains. The extension of 'S' has a border case, world *a*. Yet if states-of-affairs are sharp then, given the equivalence between a state-of-affairs and the set of worlds wherein it obtains, any set of possible worlds that is a candidate for the extension of 'S' is sharp; that is, every world, *a* included, is either a determinate member of the set or a determinate non-member. Hence there are two possible candidates for the extension of 'S' — the set determinately including world *a*, or the set determinately ex-

([34]) Quine, W.V.O., *Word and Object*, MIT (1960), p. 126.

cluding *a*.

Moreover, redescribing Diagram 2, if the presence of border cases for the denotation of 'S' is tantamount to there being a plurality of possible extensions (sets of worlds) then states-of-affairs must be sharp since were fuzzy states logically possible then it would be logically possible that a vague phrase 'S' determinately denotes a fuzzy state-of-affairs, having as its extension an indeterminate set of worlds that neither determinately included world a nor determinately excluded it.

In this way I think it can be shown that Russell's account of the (semantic) vagueness of denoting phrases gets it exactly right (given fine-tuning to take account of more recent restrictions) if and only if properties, objects, and states-of-affairs — that is, the world — are sharp. The definitions D3 and D4 adequately characterise extensional and intensional representational vagueness of denoting phrases.

Supposing the claimed success of Russell's characterisation of the representational vagueness of denoting phrases, it still remains to be seen how it can be further extended to characterise the representational vagueness of any grammatical category; that is, how it can be extended to characterise the representational vagueness of language in general.

Russell's definition only characterises the representational vagueness of *denoting* phrases — predicates, singular terms and sentences. He relied on an infection theory of vagueness to extend to talk of vagueness for non-denoting phrases like the logical constants. However it is now generally acknowledged that his infection theory is false and his generalised account (implicit in his article 'Vagueness'), according to which all language is vague, is misguided.([35])

The obvious way to proceed is firstly to find an *adequate* comprehensive pre-theoretical characterisation of vagueness (i.e. a characterisation neutral on further issues devolving from its analysis as epistemic, semantic or ontological) and then see if a Russellian-generalisation can be had by mimicking this. Such comprehensive treatments of vagueness are rare. Attention invariably focuses on predicate-vagueness since, typically, this is where the ancient sorites paradox arises and this paradox has generally been the main concern with the phenomenon of vagueness.

([35]) See Rolf, B., 'A Theory of Vagueness', *op. cit.*, esp. §§3-4, and 'Russell's Theses on Vagueness', *op. cit.*, esp. §2.

As it happens, Rolf has furnished us with just such a theory.([36]) The supplementation of Russell's account of the vagueness of denoting phrases with Rolf's recursive extension of this concept from the categories of denoting phrases to any phrase whatsoever will successfully complete a representational account of vagueness.([37]) (Of course, this latter extension result is not limited to Russell's account; any account of the vagueness of denoting phrases could make use of Rolf's recursive extension.)

Let us pause to consider what has been said in §§2-3. It has been suggested by a number of philosophers that Russell's definition of vagueness is defective for a variety of reasons. I believe they are mistaken to the extent that, though Russell himself may perhaps have had something else in mind, the reconstruction above is consistent with the spirit of his proposal for denoting phrases and preserves the essential ingredient of "one-manyness". Moreover, far from being an inadequate definition of vagueness, his definition (or, if you prefer, my detailed interpretation of his definition) is both sufficient and necessary to the task of characterising representational vagueness when it is conjoined with an appropriate theory extending the concept of vagueness from denoting phrases to any grammatical category in general.([38])

Thus Rolf's criticism (cited at the beginning of §2.1) that Russell's definition of vagueness in terms of the one-manyness of denotation cannot account for ordinary usage of the word 'vague' is seen to be false. Russell has accounted for the most popular use of 'vague', the representationalist's use, if not the most general and theory-free use.

([36]) Rolf, B., 'A Theory of Vagueness', *op. cit.*.

([37]) For details see my *Being Coherently Vague*, PhD Thesis, ANU (1993), §1.4 and p. 91.

([38]) We have only discussed Russell's theory of vagueness for linguistic items and language. But I see no reason to think that we could not extend his account as analysed above to include all forms of representation, linguistic or otherwise and thus finally arrive at a complete explanation of his theory of the vagueness of representations in general. This is not a task to be pursued here.

4. *Vagueness, Logic and the World*

It is not hard to see how Russell's philosophical analysis of vagueness, with one-manyness as the central metaphor, provides a canonical interpretation of the formal logical analysis of vagueness offered by supervaluation theorists. Consider his analysis as applied to the vague predicate 'is red'; it is representationally vague if and only if its denotation relation is one-many in the sense that there are, at least, two possible (sharp) properties it could denote consistent with its semantic properties. By associating each of the possible properties with linguistic items, namely precise predicates denoting these properties, we can shift to intra-linguistic talk of vagueness as a relation between vague and precise predicates. The vagueness of 'is red' will now be said to amount to there being, at least, two possible ways of making the predicate precise consistent with its semantic properties; that is, vagueness amounts to a plurality of "admissible precisifications", to use the terminology of the supervaluationist.

Russell, of course, was precluded from following this short path to what is now a common logical response to vagueness — a supervaluation account. His anti-intensional stance, reflected most notably in his rejection of modal logics and common earlier this century prior to innovations in formal semantics, ruled out his ever considering the operator '$\nabla_{\Sigma r}$' as a properly logical operator.

Instead he defended classical logic in much the same way Quine does; vagueness is admitted as an essential semantic feature of natural language, yet the supposedly sharp world is describable in a precise ideal language to which the scope of logic is limited. This ideal language plays the role that natural language plays for epistemic theorists. They claim natural language is precise and are sceptical about our ability to know the precise set of linguistic rules governing such language. Russell denies this: "It is perfectly obvious ... that there are shades of colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word 'red,' but because it is a word the extent of whose application is essentially doubtful."([39]) He is sceptical about our ability to know the precise set of ontological relations that obtain in the world, and so sceptical about our ability to know the precise set of semantic relations that obtain in the ideal language, but natural language is the expression of

([39]) Russell, *op. cit.*, p. 85.

what we know so we should not be linguistic sceptics with regard to natural language. Nonetheless he holds a view very close to theirs with regard to his ideal language which is precise though unknowably so and which is able to support classical logic. Were we able to occupy the epistemic high-ground (indeed heaven, the epistemically highest!) all would be clear ... and classical.

> All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial existence. Where, however, this celestial existence would differ from ours, so far as logic is concerned, would be not in the nature of what is known, but only in the accuracy [precision] of our knowledge.([40])

Arguing in this way for the retention of classical logic whilst satisfying the intuition that natural language is vague, Russell has distanced logic so much as to make it virtually useless. It is no longer appropriate for the assessment of the validity of many arguments in natural language — in particular, the sorites paradox (whose soritical expressions Russell considers beyond the scope of logic). Since the heady days of atomism and positivism, ideal language theories have come under a great deal of criticism and current philosophical fashions are such as to make one suspicious of theories that make essential reference to such ideals (though we may have no more faith these days in the ability of ordinary language to act as final arbiter either and have moved on from the antithesis to Russell offered to us in the nineteenfifties by Oxford).

What we should concern ourselves with is natural language as it is; after all, if logic is a theory of valid argumentation then surely a logic of those terms or concepts used in argument is what is of primary interest to us. The obvious step for the representationalist is simply to take the logic of '$\nabla_{\Sigma r}$' seriously, and we have just seen where that leads ... to supervaluationism.

The orthodox metaphysical position grounding a representational analysis of vagueness lacks adequate defence however. Claims encountered in the beginning of this paper to the effect that 'the world is what it is' do not constitute the required proof since one may simply ask: 'What is it that it

([40]) *Ibid.*, p. 89.

is, sharp or fuzzy?' In other words, such claims simply miss the mark.

Taking a closer look, one is left with the feeling that Russell has succumbed to a kind of deep-seated metaphysical prejudice (to borrow a phrase of Dummett's). To say that "things are what they are" is, presumably, tautologous; I may claim something to be what it is even though what it is is vague. Assertions of self-identity need not (perhaps even cannot) be indeterminate or false, and if this is what Margalit means when he says that "they are not what they are in degrees" one might agree and yet claim that he has missed the mark. Things are what they are, but what counts as that, beyond the tautological reply already proffered? To have any consequences for the advocate of a fuzzy world the above statements must be read not as denying the indeterminacy of being-self-identical, but as denying the indeterminacy of being-ψ, for some property ψ.([41])

Russell's claim that nothing is to a certain extent possessed of the properties it possesses surely only holds for those properties a thing possesses determinately, of which, self-identity may be thought to be one. Is it not possible that something may be to a certain extent possessed of a property if it possesses that property to a certain extent? Given that things are what they are, they are only precise if they are precise. (Hardly controversial metaphysics!) From this, the thesis that they are precise can only figure as the conclusion given the premise that they are precise.

Thus the metaphysical issue in question remains either question-beggingly assumed or simply asserted. More recent advocates of the representational account have attempted to provide arguments for this claim, most notably Gareth Evans, thus bolstering a representational view.([42]) But that is another story.(*)

([41]) A notable exception to this would be Peña's arguments to the effect that the fuzziness of something entails that thing's being distinct from itself to some extent. Peña, L., 'Identity, Fuzziness and Contradiction', *Nous* 18 (1984): 227-59.

Note that, according to Kathinka Evers (in her *Plurality of Thought*, Library of Theoria #18 (1991), p. 97), A. Hägerström also makes just this assimilation of determinacy to self-identity in his *Die Philosophie der Gegenwart in Selbstdarstellungen*, VII, Raymund Schmidt (1929).

([42]) Evans, G., 'Can There Be Vague Objects?', *Analysis* 38 (1978): 208.

(*) I would like to thank Bertil Rolf, Richard Sylvan and Graham Priest for their helpful comments.

Appendix A

The equivalence between PS and PS* can be established as follows.

Assume PS: every property is either determinately instantiated by an object or determinately not. Now assume some arbitrary property α is instantiated by a. Then it is not the case that a determinately does not instantiate α (given consistency and D); so, by PS, a must determinately instantiate α. And this is universally generalisable.

I.e.:	(1) αa	(for arbitrary α and a)
	(2) $\sim D \sim \alpha a$	(D, instantiated with '$\sim \alpha a$', 1 and modus tollens)
	(3) $D\alpha a$	(PS, instant. with 'αa', 2 and disj. syllogism)
	(4) $\alpha a \rightarrow D\alpha a$	('$\rightarrow$' intro., 1)
	(5) $\forall\psi\forall x(\psi x \rightarrow D\psi x)$	(4 and univ. generalisation)

Conversely, to show that every property is instantiated by every object either determinately or determinately not, let's assume PS*: that every property is instantiated determinately if at all. In particular, assume that for some property α that, if α is instantiated by anything then it is determinately instantiated by that thing; furthermore assume that α is not determinately instantiated by a. But then it is not instantiated by a at all; i.e., $\sim \alpha a$ is the case. But then, by PS* again, it ($\sim \alpha$) must be determinately instantiated by a. In other words, given PS*, if α is not determinately instantiated by a then it is determinately not instantiated by a, or, if you prefer, either α is determinately instantiated or it is determinately not. And this is universally generalisable since both α and a were arbitrary.

I.e.:	(1) $\sim D\alpha a$	(for arbitrary α and a)
	(2) $\sim \alpha a$	(PS*, instantiated with 'αa', 1 and modus tollens)
	(3) $D \sim \alpha a$	(PS* instantiated with '$\sim \alpha a$', 2 and modus ponens)
	(4) $\sim D\alpha a \rightarrow D \sim \alpha a$	('$\rightarrow$' intro., discharging 1)
	(5) $D\alpha a \vee D \sim \alpha a$	(4 and interdefinability of '$\vee$' and '$\rightarrow$')
	(6) $\forall\psi\forall x(D\psi x \vee D \sim \psi x)$	(5 and univ. generalisation) ‖

Appendix B

The equivalence (RUSS), $\forall x(I\underline{P}x$ iff $\nabla_{\Sigma\underline{P}}Px)$, can also be expressed as: $\forall x(D\underline{P}x$ iff $\Box_{\Sigma\underline{P}}Px)$.
That this latter formulation *is* a variant of the former is proven as follows:

(i)	Assume	(1) $D\underline{P}a$.	
	Hence	(2) $\sim I\underline{P}a$	(by definition and 1).
	Hence	(3) $\Box_{\Sigma\underline{P}}\sim Pa \vee \Box_{\Sigma\underline{P}}Pa$	(by (RUSS) and 2).
	Yet	(4) $\underline{P}a$	(by 1).
	And assuming	(5) $\Box_{\Sigma\underline{P}}\sim Pa$	
	then	(6) $\sim\underline{P}a$	(by 5).
	So	(7) $\sim\Box_{\Sigma\underline{P}}\sim Pa$	(by *reductio* on 4 and 5).
	Hence	(8) $\Box_{\Sigma\underline{P}}Pa$	(by disj. syll., 3 and 7).
	Hence	(9) $D\underline{P}a \rightarrow \Box_{\Sigma\underline{P}}Pa$	(by '→ Intro', 1 and 8).
	Hence	$\forall x(D\underline{P}x \rightarrow \Box_{\Sigma\underline{P}}Px)$	(by '∀ Intro' on 9).
(ii)	Assume	(1) $\Box_{\Sigma\underline{P}}Pa$	
	Hence	(2) $\sim\nabla_{\Sigma\underline{P}}Pa$	(by definition and 1).
	Hence	(3) $D\sim\underline{P}a \vee D\underline{P}a$	(by (RUSS) and 2).
	Yet	(4) $\underline{P}a$	(by 1).
	And assuming	(5) $D\sim\underline{P}a$	
	then	(6) $\sim\underline{P}a$	(by 5).
	So	(7) $\sim D\sim\underline{P}a$	(by *reductio* on 4 and 5).
	Hence	(8) $D\underline{P}a$	(by disj. syll., 3 and 7).
	Hence	(9) $\Box_{\Sigma\underline{P}}Pa \rightarrow D\underline{P}a$	(by '→ Intro', 1 and 8).
	Hence	$\forall x(\Box_{\Sigma\underline{P}}Px \rightarrow D\underline{P}x)$	(by '∀ Intro' on 9).

Combining (i) and (ii) then: $\forall x(D\underline{P}x \leftrightarrow \Box_{\Sigma\underline{P}}Px)$. ‖

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