Logique & Analyse **135-136** (1991), 293-296

SORITES AND IDENTITY

Graham PRIEST

1. *Sorites: standard and non-standard*

Sorites paradoxes are of a piece with the Liar paradox, both in their origin (in the work of Eubulides) and in the difficulty of solving them (as judged by the number of different attempts made to solve each). Discussions of the Sorites tend to focus on examples of a certain standard form. The purpose of this brief note is to point out that there is also a non-standard form, and tentatively draw a few conclusions from this. I do not claim that any of the observations is novel; but it does seem to me that there is a certain value in bringing them starkly together.

Let us start with a couple of paradigms of a standard Sorites argument. Today is John's fifth birthday; he is a child. If someone is a child on one day, they are a child on the next day. Hence, John will be a child in 27,375 days' time, when he is 80 (give or take a few leap-days). Or: consider a colour continuum between red and blue. Divide this up into a number of adjacent segments in such a way that the colour of each is phenomenologically indistinguishable from those of its immediate neighbours. The first segment is red. If any segment is red its neighbour is (since it is indistinguishable from it). Hence, the last segment (which is blue) is red.

Standard Sorites can always be knocked into the form:

$$Fa(0),\ \forall x(Fa(x) \rightarrow Fa(x+1)) \vdash Fa(m)$$

(where F is a monadic predicate, a is a monadic function symbol, quantifiers range over numbers, and m is some large number). For example, in the first paradigm, a(n) is John on the nth day after today and Fx is 'x is a child'; in the second, we suppose that the segments are numbered from 0 to m, a(n) is segment number n and Fx is 'x is red'. In fact, the paradoxes can be stripped down further by taking as premises each of the conditionals obtained by instantiation: these seem of equal, if not greater, plausibility. We can also then dispense with functional notation in favour of constants, and obtain:

$Fa_0, Fa_n \rightarrow Fa_{n+1}\ (0 \leq n < m) \vdash Fa_m$

A couple of examples of the non-standard Sorites argument are as follows. Consider, again, the segmented colour spectrum. The colour of the first segment is red. Any two adjacent segments have the same colour. Hence, the colour of the last segment is red (which is false). Or: the ship of Theseus was rebuilt by replacing a plank each day. As the planks were removed, they were reconstructed until, by day m, the original ship had been reconstituted. Each day the ship being rebuilt was given a different name (say, by the different builders who worked on it): $ship_0$, $ship_1$,... Each of the sentences '$ship_n = ship_{n+1}$' is true. Hence $ship_0 = ship_m$; but this is impossible since on day m $ship_0$ and $ship_m$ are quite distinct objects.

Sorites paradoxes of this kind have the form:

$Fa_0, a_n = a_{n+1}\ (0 \leq n < m) \vdash Fa_m$

Thus, in the first example, 'a_n' is a name referring to the colour of segment n, and Fx is 'x is (a hue of) red'. In the second, 'a_n' is just '$ship_n$' and Fx is '$x = ship_0$'.

Although we have noted two kinds of paradox here, they are, in fact, intimately related (as was noted by Jean Paul van Bendegem in correspondence). Modulo certain reasonable assumptions, a paradox of each kind generates one of the other. It is easy to see that a paradox of the non-standard kind generates a paradox of the standard kind. For if $a_n = a_{n+1}$ then, intuitively, in extensional contexts of the kind that are in question, $Fa_n \rightarrow Fa_{n+1}$. Conversely, suppose that F is a determinate of some determinable, D, as *being red* is a determinate of the determinable *colour*. It would seem that this can, in fact, always be achieved, though it may require a little artifice. Thus, *being a child* is a determinate of *maturity*, *being bald* is a determinate of *hairyness of head*, etc. Next, consider a premise of a standard kind of Sorites, $Fa_n \rightarrow Fa_{n+1}$. Observe that, in fact, the corresponding biconditional, $Fa_n \leftrightarrow Fa_{n+1}$, is always true. (If John is a child on day n+1, he is a child on day n, etc.) Moreover, the same is clearly true for any other determinate in the family. In virtue of this, the premise can be rephrased as: the D of a_n is the same as the D of a_{n+1}. Write b_n for 'the D of a_n', rework F appropriately, and we have a Sorites of the second kind.

2. *Solutions*

One of the beauties (and frustrations) of Sorites paradoxes is that they leave so little scope for solution. If the argument fails, as it must, either an invalid inference is employed, or a premise is false. In standard Sorites arguments (or at least, their stripped-down form), the only inference used is *modus ponens*. Hence, rejecting this is one possibility. Call this a solution of *kind 1*. The alternative is to reject the truth of a premise. The first premise of the argument hardly seems questionable. Hence at least one conditional must fail. Some have suggested that all the conditionals fail. Call this a solution of *kind 2*. Others have suggested that only *some* of the conditionals fails (namely those that straddle some crucial dichotomy). Call this a solution of *kind 3*.

In the non-standard kind of Sorites, we have the same basic options. A solution of kind 1 rejects the validity of the form of inference involved. In this case, it is not *modus ponens*, but the substitutivity of identicals in the form:

$$a=b,\ Fa \vdash Fb$$

A solution of kind 2 rejects all of the identity statements. A solution of kind 3 denies just some of the identities (those which straddle a crucial change).

It is, of course, possible that the two kinds of Sorites have different kinds of solutions. However, the close structural similarity between the two kinds, makes this feel very implausible. Indeed, the fact that paradoxes of the two sorts are interderivable makes it very reasonable to suppose that the same sort of solution will apply to both kinds. This being the case, we can draw some general conclusions from the non-standard Sorites. Though anything one might say is liable to be challenged, the most plausible conclusions seem to me to be as follows.

Solutions of kind 3 (rejecting only some identity statements) look implausible, just because of symmetry considerations. The intrinsic relationship of a_n to a_{n+1} is exactly the same for all n, and since no other kinds of considerations are relevant to identity statements (presumably), this seems to rule out this option. Solutions of kind 2 (rejecting all identity statements) also looks implausible. In the colour case, phenomenological identity would seem the only natural candidate for the identity criterion of (phenomenological) colours. In the ship case, if we took the christenings to be giving names for the temporal slices of the boat on each day, then the identities would, of course, be false. But this is not how it was set up. On each day *the boat itself* is christened. To suppose that we cannot do this (even when minor changes occur) is, in effect, to suppose that there are no objects that endure through space/time. We could not say that Richard Routley is the same person as Richard Sylvan, or even that the Morning Star is the same as the Evening Star.

This leaves only solutions of kind 1 (rejecting the validity of the inference used). This certainly has the right kind of *feel* about it. For both the standard and the non-standard cases, it looks as though a few applications of the principle of inference in question may be acceptable, but an indefinite number of applications (and so the absolute validity of the form) of the inference, is not. Several non-classical logics of vagueness, which reject *modus ponens*, have been proposed. None (that I know of) reject the substitutivity of identicals in the form I have identified. It may well, of course, be possible to modify their semantics to do this. However, as a matter of fact, this has not been thought necessary. Doing so, would have all kinds of implications for the debate about the existence of fuzzy objects — which it is not my intention to go into here. Suffice it for the present to conclude (what we knew already for other reasons) that no extant solution to the Sorites paradox works.

University of Queensland