Logique & Analyse **131-132** (1990), 311-329

STALNAKER-LEWIS CONDITIONALS: THREE GRADES OF HOLISTIC INVOLVMENT

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1. *Introduction.*

The aim of the present paper is to give an analysis of three logics of conditionals — Lewis' system C1, Stalnaker's system C2 and a probabilistic extension of the latter which will be named C2Pr — in order to show that each one of them can be interpreted as a formal representation of conditionals from an *holistic* standpoint, the differences depending on weaker or stronger senses of the term "holistic".

In Section 2, 3, 4 the notion of *deviance* of the conditional operator *toward* the material conditional and the strict conditional is introduced and discussed with reference to each of the above mentioned systems. The notion of deviance is related to the one of collapse of explicit conditionals, and it will be proved that the deviance increases moving from C2 to C2Pr. In Section 5 the increasing approximation to the collapse of modalities which may be found in the three different systems is shortly discussed from the viewpoint of its holistic implications.

2. *Explicit and non-explicit conditionals.*

System C1 has been axiomatized in a variety of ways. The following axiomatization — where the capital letters stand for arbitrary wffs — has been introduced by D.K. Lewis in [6] (p. 132):

PC. Every truth-functional tautology
A1. $A \mathrel{\Box\!\!\rightarrow} A$
A2. $(\neg A \mathrel{\Box\!\!\rightarrow} A) \supset (B \mathrel{\Box\!\!\rightarrow} A)$
A3. $(A \mathrel{\Box\!\!\rightarrow} \neg B) \vee (((A \wedge B) \mathrel{\Box\!\!\rightarrow} C) \equiv (A \mathrel{\Box\!\!\rightarrow} (B \supset C)))$
A4. $(A \mathrel{\Box\!\!\rightarrow} B) \supset (A \supset B)$
A5. $(A \wedge B) \supset (A \mathrel{\Box\!\!\rightarrow} B)$

Rules:
Modus Ponens (MP) : If $\vdash A$ and $\vdash A \supset B$ then B.
Conditional Necessitation (CN): If $\vdash (B_1 \wedge B_2 \wedge \ldots \wedge B_n) \supset C$ then $\vdash ((A \Box\!\!\rightarrow B_1) \wedge (A \Box\!\!\rightarrow B_2) \wedge \ldots \wedge (A \Box\!\!\rightarrow B_n)) \supset (A \Box\!\!\rightarrow C)$.

Definitions:
$\Box A =_{Def} \neg A \Box\!\!\rightarrow A$;
$\Diamond A =_{Def} \neg \Box \neg A$;
$A \Diamond\!\!\rightarrow B =_{Def} \neg(A \Box\!\!\rightarrow \neg B)$;
$A \Box\!\!\Rightarrow B =_{Def} \Diamond A \wedge (A \Box\!\!\rightarrow B)$;
$A \Diamond\!\!\Rightarrow B =_{Def} \neg(A \Box\!\!\rightarrow \neg B)$;

For sake of simplicity, in what follows we will speak of axioms rather than of axiom schemata and of theorems rather than of theorem schemata.

The last two definitions are introduced by Lewis in [6] as definitions of "non-vacuous" conditional operators. By virtue of them $A \Diamond\!\!\Rightarrow B$ is proved to be equivalent to $\Diamond A \supset (A \Diamond\!\!\rightarrow B)$. It is a peculiarity of non-vacuous conditionals that $A \Box\!\!\Rightarrow A$ is not a thesis in any extension of C1, while $A \Diamond\!\!\Rightarrow A$ is such. It is also simple to prove that in any extension of C1 the following wffs are theses :

1) $(A \Box\!\!\Rightarrow B) \supset (A \Box\!\!\rightarrow B)$
2) $(A \Box\!\!\rightarrow B) \supset (A \Diamond\!\!\Rightarrow B)$
3) $(A \Diamond\!\!\rightarrow B) \supset (A \Diamond\!\!\Rightarrow B)$
4) $(A \Box\!\!\Rightarrow B) \supset (A \Diamond\!\!\rightarrow B)$

while no one of the converse implications holds. Now, since we have by A5

5) $(A \wedge \neg B) \supset (A \Box\!\!\rightarrow \neg B)$

by contraposition we obtain

6) $(A \Diamond\!\!\rightarrow B) \supset (A \supset B)$

and by virtue of the theorem

7) $\neg \Diamond A \supset (A \supset B)$

we further obtain

8) $(\neg \Diamond A \lor (A \Diamond\!\!\rightarrow B)) \supset (A \supset B)$

which by definition of $\Diamond\!\!\Rightarrow$ equals

9) $(A \Diamond\!\!\Rightarrow B) \supset (A \supset B)$

Since A $\dashv$ B implies A $\Box\!\!\rightarrow$ B, and the converse of 9) is a non-theorem, the above result shows that, $\dashv$, $\Box\!\!\rightarrow$, $\Diamond\!\!\Rightarrow$, $\supset$, stand for relations of decreasing strength.

Axiom A5 may be called "law of conditional factuality" and symbolized by CF. It is commonly held that an intuitive justification for it is provided by the possible worlds semantics which is commonly associated to C1. In particular, Lewis proposes to read "A $\Box\!\!\rightarrow$ B is true at a world w_i" as "all the worlds most similar to w_i at which A is true are worlds at which B is true". Since Lewis assumes that each world is the most similar to itself, we may reason as follows: if A $\land$ B is true at w_i, B is true at w_i, hence it is true at the world which is the most similar to w_i at which A is true : so A $\Box\!\!\rightarrow$ B is true at w_i.

The possible worlds interpretation of conditionals does not expressly suggest a reading of the conditional operator in terms of a necessary relation between the clauses. But such an interpretation follows from the convention proposed by Lewis of reading A $\Diamond\!\!\rightarrow$ B as "if it were the case that A, it *might* be the case that B". Since A $\Box\!\!\rightarrow$ B equals $\neg$(A $\Box\!\!\rightarrow$ $\neg$B), the latter formula is to be read (i) "if it were the case that A, it is false that it might be the case that $\neg$B". But intuitively this is only a complicated way of asserting (ii) "if it were the case that A *necessarily* it would be the case that B".

In [15] Stalnaker observes that *might* must be intended as an operator having as its scope the whole statement and insists on the epistemic meaning of it (p. 143). This however does not affect the preceding analysis since "it is false that it might" is an operator which is also to be intended *sensu composito* in (i), just as the adverb "necessarily" in (ii).

Thus *would-conditionals* instantiate a particular case of necessary implication (they are "variably strict" conditionals, as it sometimes said) and *might*-conditionals instantiate the so-called relation of cotenability. Reading conditionals in this key leads us to see that the law of conditional factuality asserts that any two true propositions are necessarily interdependent in the above mentioned sense.

Stalnaker's system C2 is obtaind by adding to C1 the axiom

$$\text{A6 } (A \mathrel{\Box\!\!\rightarrow} B) \vee (A \mathrel{\Box\!\!\rightarrow} \neg B)$$

which is also known as law of "conditional excluded middle" (CEM) and by a well-known result (see Lewis [6]) it can be proved to be independent from C2.

In C2 A4, CF and CEM are related in the following way. Thanks to $(A \mathrel{\Box\!\!\rightarrow} B) \supset (A \mathrel{\Diamond\!\!\Rightarrow} B)$ and CF, we have $(A \wedge B) \supset (A \mathrel{\Diamond\!\!\Rightarrow} B)$, hence $\neg(A \mathrel{\Diamond\!\!\Rightarrow} B) \supset \neg(A \wedge B)$, hence $\Diamond A \supset ((A \mathrel{\Box\!\!\rightarrow} \neg B) \supset (A \supset \neg B))$. Since we have $\neg \Diamond A \supset ((A \mathrel{\Box\!\!\rightarrow} B) \supset (A \supset B))$ it follows that $\vdash (A \mathrel{\Box\!\!\rightarrow} B) \supset (A \supset B)$. It is so proved that A4 follows from CF. But it can also be shown that, given CEM, A4 implies CF. CEM in fact is equivalent to $\neg(A \mathrel{\Box\!\!\rightarrow} \neg B) \supset (A \mathrel{\Box\!\!\rightarrow} B)$, which conjoined with A4 yields $\neg(A \mathrel{\Box\!\!\rightarrow} \neg B) \supset (A \supset B)$, hence by contraposition $(A \wedge \neg B) \supset (A \mathrel{\Box\!\!\rightarrow} \neg B)$, which is an instance of CF.

The conclusion we reach is then that, given CEM, A4 is logically equivalent to CF, so that in C2 we cannot give up the latter without giving up also the former. Anyone who feels puzzled by CF is then forced to admit that in C2 CF cannot be removed from the axiomatic basis without removing also the minimal property of having $A \supset B$ logically implied by $A \mathrel{\Box\!\!\rightarrow} B$(1).

The language of C1 and C2 allows us to draw a commonly neglected distinction, the one between *non-explicit* conditionals and *explicit* conditionals. A conditional is explicit when it is stated in conjunction with the truth or falsity of one of its clauses, and non-explicit in the contrary case (which is the case of what usually logicians call conditionals).

It is not easy to draw the distinction between explicit and non-explicit conditionals in ordinary speech, since either their meaning or the particular verbal moods and tenses used in them often suggest that the speaker believes that the clauses are true or false: so that it would be a redundancy, from the speaker's viewpoint, to explicit the truth or falsity of the clauses. From a formal viewpoint however the distinction can be rendered fully clear. We define an *explicit conditional* to be a conjunction $\delta \wedge (A \mathrel{\Box\!\!\rightarrow} B)$, where δ is to stand for a wff in the set $\{A, \neg A, B, \neg B\}$. In a more extended sense

(1) In Chellas' [2] A4 is seen as a "principle of Modus Ponens for $\Box\!\!\rightarrow$" and viewed as one of the basic laws of conditional Logic.

which for sake of simplicity will not be treated here, an explicit conditional has the form $\delta' \wedge \delta'' \wedge (A \mathrel{\Box\!\!\rightarrow} B)$, where $\delta' \wedge \delta''$ is a (possibly degenerate) conjunction whose first member belongs to the set $\{A, \neg A\}$ and whose second member belongs to the set $\{B, \neg B\}$.

The following table gives a list of all explicit conditionals having A and B as clauses:

1. $A \wedge (A \mathrel{\Box\!\!\rightarrow} B)$
2. $B \wedge (A \mathrel{\Box\!\!\rightarrow} B)$
3. $\neg A \wedge (A \mathrel{\Box\!\!\rightarrow} B)$
4. $\neg B \wedge (A \mathrel{\Box\!\!\rightarrow} B)$

The problem of giving a correct reading of 1.- 4. will not be discussed in these pages. In C1 1. is equivalent to $A \wedge B \wedge (A \mathrel{\Box\!\!\rightarrow} B)$ and Goodman calls both *factual* conditionals in [3]. Pollock in [11] sees 2. as a correct rendering of an "even if" conditional (the present author has however suggested the possibility of reading it as an "as if" conditional (see [9]). 3. is a full expression of what is normally called a counterfactual conditional. The importance of explicit counterfactuals can be seen in discussing the question of the collapse of the conditional operator on the remainig three operators we have taken into consideration, namely $\dashrightarrow$, $\Diamond\!\!\Rightarrow$, $\supset$. It is straightforward to observe that in C1 $\mathrel{\Box\!\!\rightarrow}$ does not collapse on any of the other three operators. Let us call the following three metatheoretical statements *downward collapse, upward collapse* and *intermediate collapse*:

(DC) $\vdash (A \supset B) \equiv (A \mathrel{\Box\!\!\rightarrow} B)$
(UC) $\vdash (A \mathrel{\Box\!\!\rightarrow} B) \equiv (A \dashrightarrow B)$
(IC) $\vdash (A \mathrel{\Diamond\!\!\Rightarrow} B) \equiv (A \mathrel{\Box\!\!\rightarrow} B)$

One half of these equivalences (the one from right to left) is a thesis of C1, but the one from left to right is not. Taking for granted that the collapse formula $A \supset \Box A$ cannot be proved in C1 nor in any of its extensions, C2 and C2Pr, the fact that in C1 $\mathrel{\Box\!\!\rightarrow}$ does not collapse on $\dashrightarrow$, $\Diamond\!\!\Rightarrow$, $\supset$ can be proved in the following way:

1) From $\vdash (A \supset B) \supset (A \mathrel{\Box\!\!\rightarrow} B)$ one obtains $\vdash (\neg A \supset A) \supset (\neg A \mathrel{\Box\!\!\rightarrow} A)$, from which we have $\vdash A \supset \Box A$.
2) From $\vdash (A \mathrel{\Box\!\!\rightarrow} B) \supset (A \dashrightarrow B)$ it follows $\vdash ((A \vee \neg A) \mathrel{\Box\!\!\rightarrow} A) \supset ((A \vee \neg A) \dashrightarrow A)$. Thus, thanks to the equivalence between $(A \vee \neg A) \mathrel{\Box\!\!\rightarrow}$

A and A and the one between $(A \vee \neg A) \prec A$ and $\Box A$ we would have $\vdash A \supset \Box A$.
3) If (IC) were a C1-thesis, also $(\neg \Diamond A \vee (A \Diamond\!\!\rightarrow B)) \supset (A \Box\!\!\rightarrow B)$ would be such. In this case, by PC laws, also $(A \Diamond\!\!\rightarrow B) \supset (A \Box\!\!\rightarrow B)$ would be a C1- thesis, but this wff is equivalent to CEM, which we already know to be independent of C1. End of the proof.

It is also straightforward to prove that any extension of C1 containing the collapse of modalities contains (UC) and (DC). So having (UC) and (DC) as theorems in any extension of C1 is the same as having in it the collapse of modalities. Therefore, the preceding result amounts to a proof that (UC) and (DC) are not theorems in any extension of C1.

Remark. Notice that $(A \Diamond\!\!\Rightarrow B) \supset (A \prec B)$ and $(A \supset B) \supset (A \Diamond\!\!\Rightarrow B)$ are also non-theorems of Cl. From the first wff we would have by transitivity $(A \Box\!\!\rightarrow B) \supset (A \prec B)$, hence (UC). The second one equals $\neg(A \Diamond\!\!\Rightarrow B) \supset (A \wedge \neg B)$ and $(A \Box\!\!\Rightarrow \neg B) \supset (A \wedge \neg B)$, i. e. $(\Diamond A \wedge (A \Box\!\!\rightarrow \neg B) \supset (A \wedge \neg B)$. If B is instantiated by $\neg A$, we would have $\vdash \Diamond A \supset A$, hence the collapse of modalities.

3. *Weak and strong deviance of conditional operators.*

It seems now to be a natural step to extend the notions of downward, upward and intermediate collapse also to explicit conditionals. We will apply these three labels in an obvious way to instances of the metatheorem $\vdash (\delta \wedge (A \Box\!\!\rightarrow B)) \equiv (\delta \wedge (A \circ\!\!\rightarrow B)$, where "$\circ\!\!\rightarrow$" stands for "$\supset$", "$\prec$" or "$\Diamond\!\!\Rightarrow$". Looking at explicit conditionals we may conjecture that in any conditional logic the following rules should hold for every value of δ:

(a*) $\vdash (A \supset B) \equiv (A \Box\!\!\rightarrow B)$(DC) follows from $\vdash (\delta \wedge (A \supset B)) \equiv (\delta \wedge (A \Box\!\!\rightarrow B))$

(b*) $\vdash (A \Box\!\!\rightarrow B) \equiv (A \prec B)$(UC) follows from $\vdash (\delta \wedge (A \Box\!\!\rightarrow B)) \equiv (\delta \wedge (A \prec B))$

(a*) and (b*) may be summarized as the thesis that the collapse of non-explicit conditionals follows from the collapse of explicit conditionals of the same kind. To argue in favour of this conjecture we may simply show that it works for an operator which is like $\Box\!\!\rightarrow$ and $\Diamond\!\!\Rightarrow$ in having an intermediate strength between $\prec$ and $\supset$. The reference is to Burks' operator for "causal" implication $\boxed{c}\!\!\rightarrow$ defined as $\boxed{c}(A \supset B)$, where for we $\boxed{c}$ have $\vdash \boxed{c}A \supset A$, $\vdash \Box A \supset \boxed{c}A$ and $\vdash \boxed{c}(A \supset B) \supset (\boxed{c}A \supset \boxed{c}B)$. Thanks to these axioms we obtain theorems, having $\boxed{c}\!\!\rightarrow$ as the only modal operator, which parallel the ones having $\strictif$ as the only modal operator (see [1]). In order to simplify the argument, let us list all the possible formulas expressing what we have called the downward and upward collapses of explicit conditionals:

DC1 $\vdash (A \wedge (A \supset B)) \equiv (A \wedge (A \Box\!\!\rightarrow B))$
UC1 $\vdash (A \wedge (A \Box\!\!\rightarrow B)) \equiv (A \wedge (A \strictif B))$

DC2 $\vdash (\neg A \wedge (A \supset B)) \equiv (\neg A \wedge (A \Box\!\!\rightarrow B))$
UC2 $\vdash (\neg A \wedge (A \Box\!\!\rightarrow B)) \equiv (\neg A \wedge (A \strictif B))$

DC3 $\vdash (B \wedge (A \supset B) \equiv (B \wedge (A \Box\!\!\rightarrow B))$
UC3 $\vdash (B \wedge (A \Box\!\!\rightarrow B)) \equiv (B \wedge (A \strictif B))$

DC4 $\vdash (\neg B \wedge (A \supset B)) \equiv (\neg B \wedge (A \Box\!\!\rightarrow B))$
UC4 $\vdash (\neg B \wedge (A \Box\!\!\rightarrow B)) \equiv (\neg B \wedge (A \strictif B))$

Let us hypothesize that the symbol for Burks' operator "$\boxed{c}\!\!\rightarrow$" replaces every occurrence of "$\Box\!\!\rightarrow$" in the table. Then we may reason as follows :

(i) Each of the wffs DC1-DC4 may be easily seen to be equivalent to some instance of $\vdash (\delta \wedge (A \supset B)) \supset (A \boxed{c}\!\!\rightarrow B)$. The four instances of $\delta \wedge (A \supset B)$ are

1) $A \wedge (A \supset B)$
2) $B \wedge (A \supset B)$
3) $\neg A \wedge (A \supset B)$
4) $\neg B \wedge (A \supset B)$

1) and 2) are implied by $A \wedge B$, 3) and 4) by $\neg A \wedge \neg B$. Thus, by using contraposition of $\boxed{c}\!\!\rightarrow$, from the latter two implications we derive $\vdash (\neg A \wedge \neg B) \supset (\neg B \boxed{c}\!\!\rightarrow \neg A)$, and from the former two we have, without using contraposition, $(B \wedge A) \supset (A \boxed{c}\!\!\rightarrow B)$, which equals an instance of the same theorem. Taking any truth functional tautology as an instance of A we have, from the latter theorem, $\vdash B \supset \boxed{c}B$, hence also $\vdash (A \supset B) \equiv (A \boxed{c}\!\!\rightarrow B)$.

(ii) Every instance of UC1-UC4 may be seen to be equivalent to some instance of $\vdash(\delta \wedge (A \; \Box_c\!\!\rightarrow B)) \supset (A \rightarrow B)$. Two of them are $\vdash(B \wedge (A \; \Box_c\!\!\rightarrow B)) \supset (A \rightarrow B)$ and $\vdash(A \wedge (A \; \Box_c\!\!\rightarrow B)) \supset (A \rightarrow B)$. From them, taking then A as a truth-functional tautology, we obtain $\vdash(B \wedge \Box_c B) \supset \Box B$ and $\vdash \Box_c B \supset \Box B$ respectively, which are both equivalent to the thesis $\vdash \Box_c B \supset \Box B$, which entails $\vdash(A \; \Box_c\!\!\rightarrow B) \equiv (A \rightarrow B)$. The other two instances, via contraposition, yield $(\neg B \wedge (\neg B \; \Box_c\!\!\rightarrow \neg A)) \supset (\neg B \rightarrow \neg A)$ and $(\neg A \wedge (\neg B \; \Box_c\!\!\rightarrow \neg A)) \supset (\neg B \rightarrow \neg A)$. As regards the former, a result which is analogous to the preceding one (namely $\vdash \Box_c \neg B \supset \Box \neg B$) may be proved by taking a truth-functional contradiction as an instance of B. As for the latter, take $\neg A$ for B. Since $\vdash \Box_c \neg A \equiv A \; \Box_c\!\!\rightarrow \neg A$ and $\Box \neg A \equiv A \rightarrow \neg A$, we reach the thesis $(\neg A \wedge \Box_c \neg A) \supset \Box \neg A$, so also $\Box_c A \supset \Box A$. We have hence again, in both cases, $(A \; \Box_c\!\!\rightarrow B) \equiv (A \rightarrow B)$, and this ends the proof.

The preceding result concerns an implicative relation which is simply defined as having a modal status which is intermediate between $\supset$ and $\rightarrow$ and makes it plausible to conjecture that a normality requirement for *any* implicative relation intermediate between $\supset$ and $\rightarrow$ should satisfy the property described in (a*) and (b*). As a further remark concerning $\Box\!\!\rightarrow$, let us look at (a*) and (b*) in the contrapositive variants : for every δ

(a**) $\vdash(A \supset B) \equiv (A \Box\!\!\rightarrow B)$ implies $\vdash(\delta \wedge (A \supset B)) \equiv (\delta \wedge (A \Box\!\!\rightarrow B))$
(b**) $\vdash(A \Box\!\!\rightarrow B) \equiv (A \rightarrow B)$ implies $\vdash(\delta \wedge (A \Box\!\!\rightarrow B)) \equiv (\delta \wedge (A \rightarrow B))$

Since the two premises are true, (a**) and (b**) amount to saying that the conclusion of the rules also hold. But this is very plausible at least in the paradigmatic case of first-degree conditionals, where δ stands for a truth-functional formula.

Suppose in fact that the conclusion does not hold. This means that we could derive as theorems $(\delta \wedge (A \supset B)) \supset (A \Box\!\!\rightarrow B)$ and $(\delta \wedge (A \Box\!\!\rightarrow B)) \supset (A \rightarrow B)$. But it is hard to see how the consistent conjunction of the truth-functional δ with $A \supset B$ or with $A \Box\!\!\rightarrow B$ may entail the modally stronger $A \rightarrow B$.

We stipulate to call *normal* any operator, intermediate between $\supset$ and, $\rightarrow$ which has the property described in (a*) and (b*), and *deviant* any such operator lacking the mentioned property. The deviance can be *weak* and *strong*. The deviance is strong when the premise of rule (a*) or rule (b*) is not only supposed to be a thesis of the system under discussion but is actually such, while the conclusion is not. The deviance is weak when it is not strong, namely when the premise is not a thesis of the system under discussion but is simply supposed to be such.

A more rigorous treatment of the notion of deviance may be given as follows in reference to the operator $\Box\!\!\rightarrow$.

We shall say that $\Box\!\!\rightarrow$ is *strongly deviant toward* $\supset$ *(*$\mathrel{-\!\!3}$*)* in a system X if
(i) some value of δ can be found which in X provides a counterexample to (a*)(b*), and
(ii) the premise of (a*)((b*)) belongs to X.

We shall say that $\Box\!\!\rightarrow$ is *weakly deviant toward* $\supset$ *(*$\mathrel{-\!\!3}$*)* in a system X if (i) some value of can be found which in X provides a counterexample to (a*)((b*)) and (ii) the premise of (a*)((b*)) does not belong to X but to some non trivial axiomatic extension of X.

We have to notice that it is possible in principle to assign a numerical measure to the degree of weak and strong deviance, which of course is given by the amount of values of δ which provide the counterexamples to the given rules, but this possibility will not be taken into consideration in the present paper.

3. *Deviance of* $\Box\!\!\rightarrow$ *in C1 and C2.*

The problem we have now to face is the following. Is $\Box\!\!\rightarrow$ a normal or deviant operator in C1 and in C2 ? The answer is that it is deviant in both systems, and it is interesting to see why and to which extent.

We may state the following theorems concerning C1, the third of which follows from an established result:

T1. *In C1* $\Box\!\!\rightarrow$ *is strongly deviant toward* $\supset$
T2. *In C1* $\Box\!\!\rightarrow$ *is weakly deviant toward* $\mathrel{-\!\!3}$
T3. *In C1* $\Box\!\!\rightarrow$ *does not collapse on* $\Diamond\!\!\Rightarrow$

The proof of the first two statements is the following:

T1. DC1 turns out to be a C1-thesis via CF (suffice it to observe that both $A \wedge (A \Box\!\!\rightarrow B)$ and $A \wedge (A \supset B)$ equal $A \wedge B$).

T2. UC2 and UC4 are not C1-theses but, if they were added as axioms to C1, this would not yield the collapse of modalities. The proof is semantic and is obtained by showing that UC2 and UC4 hold in every C1-model having two possible worlds, while it is well-known that the collapse-formula $A \supset \Box A$ is not valid in every model of this class.

A C1-model is a 3-ple ⟨W, *f*, V⟩ where W is a non-empty set of possible worlds, *f* is a selection function with the properties stated by Lewis in [6] (p. 58) for C1-models, V is a standard modal value-assignment with the additional property V(A □→ B, w_i) = 1 if and only if, for every w_j such that $w_j \in f(A, w_i)$, V(B, w_j) = 1. R is introduced by stipulating that w_i R w_j if, for some A, $w_j \in f(A, w_i)$ and it easily seen to be reflexive.

We may reason as follows:

1) Let (¬A ∧ (A □→ B)) ⊃ □(A ⊃ B)) — a wff which equals UC2 — be by *Reductio* false at a world w_i of some C1-model ⟨W, *f*, V⟩ such that W = {w_i, w_j} and $w_i \neq w_j$. This means V(¬A, w_i) = 1, V((A □→ B), w_i) = 1, V(□(A ⊃ B), w_i) = O. We have to consider two possible cases: either w_i R w_j or not w_i R w_j. In the latter case, since w_i R w_i, V((A ⊃ B), w_i)=0, but this is incompatible with V(¬A, w_i) = 1. If w_i R w_j, V(□(A ⊃ B), w_i) = O implies V(A ⊃ B, w_j) = O, hence V(A, w_j) = 1 and V(B, w_j) = O. Since V(A, w_i) = O, this means that f(A, w_i) = {w_j} and, since V(A □→ B, w_i) = 1, this implies V(B, w_j) = 1. Contradiction. Notice that in C1-models having more than two distinct possible worlds a contradiction does not necessarily follow (think for instance of a C1-model such that W = {w_i, w_j, w_l} in which V(A ⊃ B, w_j) = 1 and V(A ⊃ B, w_l) = O. Thus adding UC2 as an axiom to C1 implies that C1 + UC2 is sound with respect to the class of models having no more than two possible worlds. But a two-worlds model which is a countermodel to the collapse formula A ⊃ □A can easily be found by a Reductio argument.

2) Let us suppose that (¬B ∧ (A □→ B)) ⊃ □(A ⊃ B) — a wff which equals UC4 — is false at a world w_i in a C1-model in which W = {w_i, w_j} and $w_i \neq w_j$. This means V(B, w_i) = O and V(A □→ B), w_i) = 1, but this implies V(A, w_i) = O and V(A ⊃ B, w_i) = 1. Since V(□(A ⊃ B), w_i) = O, it follows that V(A ⊃ B, w_j) = O, and the argument runs as in the preceding case. End of the proof.

To sum up, 1) and 2) prove that UC2 and UC4 do not yield the collapse of modalities. Since C1-theorems are validated by models having an arbitrary number of worlds, it follows that UC2 and UC4 cannot be C1-theses since they are refuted in C1-models having more than two possible worlds. So we conclude that in C1 □→ is not strongly but weakly deviant towards ⥽.

Remark. DC1, UC2 and UC4 are the only instances of wffs in the table at p. 317 which can be added to C1 as axioms without yielding the collapse of modalities. It can be shown that from the supposition that every other wff of the table at p. 317 is a thesis, the collapse of modalities is a straightforward consequence. The proof will be omitted since it simply rests on suitable choices of instances of A and B. For instance, in DC2 we can take B as $\neg$A, so to obtain $(\neg A \wedge (A \supset \neg A)) \equiv (\neg A \wedge (A \,\Box\!\!\rightarrow \neg A))$, hence $\neg A \equiv \Box \neg A$.

Let us turn now to C2 and to its characteristic axiom expressing the law of conditional excluded middle: $(A \,\Box\!\!\rightarrow B) \vee (A \,\Box\!\!\rightarrow \neg B)$(CEM). It is simple to derive from CEM the equivalence between $\Box\!\!\rightarrow$ and $\Diamond\!\!\Rightarrow$, namely the formula expressing the intermediate collapse (IC), i. e. $(A \,\Box\!\!\rightarrow B) \equiv (A \,\Diamond\!\!\Rightarrow B)$. The proof is simple from left to right since we have among the C2 theorems $\Diamond A \supset ((A \,\Box\!\!\rightarrow B) \supset (A \,\Diamond\!\!\rightarrow B))$, which by permutations of antecedents gives $(A \,\Box\!\!\rightarrow B) \supset (\Diamond A \supset (A \,\Diamond\!\!\rightarrow B))$, i. e. $(A \,\Box\!\!\rightarrow B) \supset (A \,\Diamond\!\!\Rightarrow B)$. In the converse direction: CEM equals $(A \,\Diamond\!\!\rightarrow B) \supset (A \,\Box\!\!\rightarrow B)$, and since we have also $\neg \Diamond A \supset (A \,\Box\!\!\rightarrow B)$, one further theorem is $\neg \Diamond A \vee (A \,\Box\!\!\rightarrow B)$, i. e. $(A \,\Diamond\!\!\Rightarrow B) \supset (A \,\Box\!\!\rightarrow B)$.

Having (IC) as a C2- thesis obviously yields a collapse of every explicit conditional containing $\Box\!\!\rightarrow$ on the corresponding explicit conditional containing $\Diamond\!\!\Rightarrow$, namely every intermediate collapse of explicit conditionals. On the other hand, subjoining CEM to C1 does not modify any result already reached in C1 about the deviance of $\Box\!\!\rightarrow$ toward $\supset$ and toward $\dashv$. From a semantic viewpoint CEM expresses the so-called "Stalnaker's Assumption", i. e. the view that for every possible world w_i there is only one world w_j such that $\{w_j\} = f(w_i)$. It is straightforward to check that the proof of T1 and T3 is unaffected by this semantic property of C2-models. To conclude, the following metatheorems concerning C2 are provable:

T4 *In C2* $\Box\!\!\rightarrow$ *is strongly deviant toward* $\supset$
T5 *In C2* $\Box\!\!\rightarrow$ *is weakly deviant toward* $\dashv$
T6 *In C2* $\Box\!\!\rightarrow$ *collapses on* $\Diamond\!\!\Rightarrow$

4. *The deviance of* $\Box\!\!\rightarrow$ *toward* $\dashv$ *in C2Pr.*

Looking at metatheorems T1-T6 it is natural to ask the following question: is it possible to find an extension of C2 which does not contain the collapse of modalities and in which $\Box\!\!\rightarrow$ turns out to be not weakly but *strongly* deviant toward $\dashv$? The question we are concerned with may be stated more specifically in the following way: is it possible to find a system preserving the properties of Stalnaker-Lewis conditionals in which $\Box\!\!\rightarrow$ collapses on $\Diamond\!\!\Rightarrow$, DC1, UC2, UC4 are theorems while the remaining statements of the table at p. 317 imply the collapse of modalities? The answer is in the affirmative. We take into consideration a new system, which we shall label as C2Pr, whose language is a probabilistic extension of the language of C2 obtained by adding to the language of C2 the language of standard real number theory and a sentential operator Pr which assigns to every sentence in the language a rational number in the interval [O, 1]. The axioms of C2Pr are the axioms of a minimal first order formulation of C2 extended with axioms for contingent identity (2), axioms for the theory of ordered fields and some new laws involving both Pr and non-probabilistic operators. Assuming the definition $\Box A =_{Def} \neg A \Box\!\!\rightarrow A$, the latter are in the first place the modal versions of Kolmogorov laws for Pr, i.e.

K1. $0 \leq Pr(A) \leq 1$
K2. If $\vdash A = B$ then $Pr(A) = Pr(B)$
K3. $\neg \Diamond (A \wedge B) \supset Pr(A \vee B) = Pr(A) + Pr(B)$

To these we subjoin the following axioms:

K4. $\Diamond A \supset Pr(B/A) = \dfrac{Pr(A \wedge B)}{Pr(A)}$
K5. $\Box A \supset Pr(A) = 1$
K6. $Pr(A) = 1 \supset \Box A$
K7. $Pr(A \wedge C) \neq 0 \supset (Pr\, B/A \wedge C = Pr\, A \Box\!\!\rightarrow B/C)$ (3)

K7 is an object-language rendering of what is known as Generalized Stalnaker Thesis — henceforth GST — from which the simple Stalnaker's Thesis $Pr(A) \neq 0 \supset (Pr(B/A) = Pr(A \Box\!\!\rightarrow B))$ — henceforth ST — follows by substituting a logical truth to C. R. (Stalnaker has proved that the logical dependence also holds in the converse direction, since it is possible to derive GST from ST (see [13]).) According to the probabilistic notion of necessity,

(2) A restricted formulation of Leibniz's law is necessary in order to avoid a trivialization of modal operators by using an argument of the Quine-Follesdal kind (see Smokler [12]). In order to have a rigorous disproof of $A \supset \Box A$ in C2Pr, however, we need a semantic argument which will not be treated in this paper.

(3) Axioms K1, K2, K3 are found in Lewis [7]. The aim of K6 is of course to identify possibility with non-zero probability and necessity with maximal probability.

which has been axiomatized in C2Pr, it is appropriate to see GST as a computational device such as to guarantee that every C2-thesis has the maximum probability value, and in the light of K5 and K6, this amounts to saying that it is logically necessary ([4]). It is well known, however, that GST gives rise to the so-called "Lewis' Triviality Result". A modal version of this theorem is derivable in C2Pr in this form:

(TR) $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset Pr(B/A) = Pr(B)$ ([5])

This theorem opens a new problem which is the following. Is it possible, moving from (TR), to prove new theorems, not included in C2, which involve only truth-functional and conditional operators?

The answer is in the affirmative since we may prove the following two Critical Theses:

CT1: $\neg A \supset ((A \Box\!\!\rightarrow B) \supset \Box(A \supset B))$
CT2: $\neg B \supset ((A \Box\!\!\rightarrow B) \supset \Box(A \supset B))$

([4]) In Stalnaker [14] C2 is proved to be consistent and complete in respect of a probabilistic semantics defined by making an essential use of GST.

([5]) The proof of TR in C2Prt runs as follows:

(1) $Pr(A)=1 \supset \Box A$	K6
(2) $\neg\Box A \supset (Pr(A) \neq 1)$	(1)
(3) $\Diamond A \supset (Pr(\neg A) \neq 1)$	(2), $\neg A/A$, Def$\Diamond$
(4) $\Diamond A \supset (Pr(A) \neq 0)$	(3), $\vdash Pr(A)=0 \equiv Pr(\neg A)=1$
(5) $\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)$	Hypothesis
(6) $Pr(A \wedge B) \neq 0 \wedge Pr(A \wedge \neg B) \neq 0$	(5), (4)
(7) $Pr(A \Box\!\!\rightarrow B/B) = Pr(B/A \wedge B)$	GST, (6)
(8) $Pr(B/A \wedge B)=1$	(6), $Pr(A \wedge B) \neq 0$, ST
(9) $Pr(A \Box\!\!\rightarrow B/B)=1$	(7), (8)
(10) $Pr(A \Box\!\!\rightarrow B/\neg B)=0$	(6), $Pr(B/A\ \neg B)=0$, GST
(11) $Pr(A \Box\!\!\rightarrow B)=Pr(A \Box\!\!\rightarrow B/B).PrB + Pr(A \Box\!\!\rightarrow B/\neg B).\ Pr(\neg B)$	Probability Calculus
(12) $Pr(A \Box\!\!\rightarrow B)=1.Pr(B) + 0.Pr(\neg B)$	(9), (10), (11)
(13) $Pr(B/A)=Pr(B)$	(12), ST
(14) $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset Pr(B/A)=Pr(B)$	(5), (13)

The result follows by using Deduction Theorem, but it is possible to exhibit another proof without using this device.

The proof of CT1 and CT2 is given in the Appendix. From it turns out also that CT2 is a simple corollary of CT1. But, as we already know, CT1 and CT2 are equivalent to UC2 and UC4, and it has been previously proved that they are not C2-theorems. Thus it turns out that C2Pr has the property we were looking for: in it $\Box\!\!\rightarrow$ it is strongly deviant toward $⥽$.

Since we already proved that UC2 and UC4 are refuted in some model having more than two possible worlds, we may conclude that C2Pr is characterized by models having two possible worlds at most. If a proposition is conceived as a set of possible worlds, this means that the only propositions are $\varnothing$, $\{w_i\}$, $\{w_j\}$, $\{w_i, w_j\}$. This conclusion is in agreement with the one D. K. Lewis draws from his Triviality Result ([6]).

The results concerning C2Pr may be summarized as follows:

T7. *In C2Pr* $\Box\!\!\rightarrow$ *is strongly deviant toward* $\supset$
T8. *In C2Pr* $\Box\!\!\rightarrow$ *is strongly deviant toward* $⥽$
T9. *In C2Pr* $\Box\!\!\rightarrow$ *collapses on* $\Diamond\!\!\Rightarrow$

In C2Pr, DC1, UC1, UC2 are theorems, while nothing change in the already mentioned proof that the remaining wffs of the table of p. 317, added as axioms, yield the collapse of modalities.

5. *Stalnaker-Lewis conditionals as holistic conditionals.*

It is useful to recall that the theorems which in C1, C2, C2Pr respectively allow us to prove the collapses mentioned in the receding sections are the following:

CF: $(A \land B) \supset (A \Box\!\!\rightarrow B)$
CEM: $(A \Box\!\!\rightarrow B) \lor (A \Box\!\!\rightarrow \neg B)$
CT1: $\neg A \supset ((A \Box\!\!\rightarrow B) \supset (A ⥽ B))$

and it is straightforward to see that they are essential to the relevant conclusions. The intuitive meaning of each one of these wffs may be seen to be an expression of an holistic view of the relations holding among propositions

([6]) In his words: "If an interpreted language cannot provide three possible but pairwise incompatible sentences, then we may call it a *trivial language*. We have proved this theorem: any *language having a universal probability conditional is a trivial language* ([7], p. 300).

of suitably defined classes. If we agree with the idea that conditional implication is a kind of necessary implication, CF asserts that any true proposition has a necessary dependence on any other true proposition. CEM yields what we called Intermediate Collapse (of $\Box\!\!\rightarrow$ on $\Diamond\!\!\Rightarrow$) and asserts that if any proposition A is (non-vacuously) cotenable with another proposition B, B has a necessary dependence on A. Bas van Fraassen in [16] sees Stalnaker's logic as a logic suitable to "hidden variable theories" ("Stalnaker models are Lewis models with hidden variables." [16], p. 176): and since $\Box\!\!\rightarrow$ has no temporal connotation, a possible interpretation of van Fraassen's result is that Stalnaker's logic presupposes an holistic conception of events which are described as cotenable.

As regards CT1, it must be noticed that it implies via CEM

$$\text{CT1'}\quad \neg A \supset (\neg(A \Box\!\!\rightarrow \neg B) \supset (A \rightarrow B))$$

which asserts that every proposition B cotenable with a false statement A is strictly implied by it. Notice that moving from CT1' we can perform the following steps:

(1) $\neg A \supset (\neg(A \Box\!\!\rightarrow \neg B) \supset (A \rightarrow B))$	CT1'
(2) $\neg A \supset (\Diamond(A \wedge \neg B) \supset (A \Box\!\!\rightarrow \neg B))$	(1), T
(3) $\neg A \supset (\Diamond(A \wedge \neg B) \supset (A \rightarrow \neg B))$	(2), CT1
(4) $(\neg A \wedge \Diamond(A \wedge B)) \supset (A \rightarrow B)$	(3), PC

(4) may be read by saying that every B consistent with any false A is logically implied by it. CT2 gives also rise to a meaningful result via the following proof:

(1) $\neg B \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	CT2
(2) $\neg B \supset (\Diamond(A \wedge \neg B) \supset \neg(A \Box\!\!\rightarrow B))$	(1), PC
(3) $B \supset (\Diamond(A \wedge B) \supset (A \Box\!\!\rightarrow B))$	(2), CEM

(3) may be read by saying that every A consistent with any true B conditionally implies it. We cannot have a theorem analogous to (3) having $\rightarrow$ in place of $\Box\!\!\rightarrow$ since instantiating A by any truth-functional tautology would lead to $(B \wedge \Diamond B) \supset \Diamond B$, hence to the collapse of modalities.

We may conjecture that row (4) of the first proof and row (3) of the second proof, even if they do not imply the collapse of modalities, together state the strongest approximation to hegelian holism which is expressible in a logical calculus preserving modal distinctions.

The three Stalnaker-Lewis systems discussed in the preceding pages are interesting contributions to the formalization of reasoning moving from holistic presupposition. It is clear however that anyone not willing to committ himself to such presuppositions can accept neither the three formulas CF, CEM, CT1 nor the approximations to collapse which are provable by their means. The anti-holist has then to choose between working with weakened axiomatic bases of the Stalnaker-Lewis family or turning to an alternative conception of conditionals such as to avoid any trivialization of both explicit and non-explicit conditionals (7).

ACKNOWLEDGEMENTS

The author wishes to thank Peter Gärdenfors and Wlodzimierz Rabinowicz for useful comments to a preceding version of the present paper.

APPENDIX

To reach theorems CT1 and CT2 we first prove:

L1 $((A \rightarrow B) \vee (A \rightarrow \neg B)) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$

in the following way:

(1) $\neg(A \Box\!\!\rightarrow B) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	$\vdash \neg A \supset (A \supset B)$
(2) $(A \Box\!\!\rightarrow \neg B) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	(1), $\Diamond A \supset ((A \Box\!\!\rightarrow \neg B) \supset \neg(A \Box\!\!\rightarrow B))$, $\neg \Diamond A \supset (A \rightarrow B)$
(3) $(A \rightarrow \neg B) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	(2), $\vdash (A \rightarrow B) \supset (A \Box\!\!\rightarrow B)$
(4) $(A \rightarrow B) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	$\vdash A \supset (B \supset A)$
(5) $((A \rightarrow \neg B) \vee (A \rightarrow B)) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$	(3), (4)

(7) It is to be noted that both CF and CEM are avoided in some of the more recent investigations on conditionals: see for instance Kvart [5] and Nute [8]. For an alternative anti-holistic account of conditionals see Pizzi [10].

We also need the following theorems (L2 and L3) :

L2 $((A \Box\!\!\rightarrow B) \wedge \neg(A \rightarrowtail B)) \supset Pr(A \Box\!\!\rightarrow B) = Pr(B)$

which may be proved in the following way:

(1) $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset Pr(A \Box\!\!\rightarrow B) = Pr\,B$ Tr, ST, $\vdash \Diamond(A \wedge B) \supset \Diamond A$
(2) $((A \rightarrowtail \neg B) \vee (A \rightarrowtail B)) \vee Pr(A \Box\!\!\rightarrow B) = Pr\,B$ (1), T
(3) $((P \vee Q) \wedge (P \supset R)) \supset (R \vee Q)$ PC
(4) $((A \Box\!\!\rightarrow B) \vee \neg(A \rightarrowtail B)) \vee Pr(A \Box\!\!\rightarrow B) = Pr\,B$ (2), (3), L1
$(A \rightarrowtail \neg B) \supset (\Diamond A \supset \neg(A \rightarrowtail B)), \neg \Diamond A \supset (A \Box\!\!\rightarrow B)$

Let us call for sake of simplicity C the wff $(A \Box\!\!\rightarrow B) \supset (A \rightarrowtail B)$ and let us notice that L1 equals

L1' $((A \Box\!\!\rightarrow B) \wedge \neg(A \rightarrowtail B)) \supset (\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B))$:

Then we have:

L3 $\neg C \supset (PrB = Pr(A \wedge B))$

The proof is the following:

(1) $(\Diamond(A \wedge (A \wedge B)) \wedge \Diamond(A \wedge \neg(A \wedge B)) \supset Pr(A \Box\!\!\rightarrow (A \wedge B)) = Pr(A \wedge B)$ row(1) of prec. proof, $A \wedge B\,/B$
(2) $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge (A \wedge \neg B)) \supset Pr(A \Box\!\!\rightarrow (A \wedge B)) = Pr(A \wedge B)$ (1), $\vdash \Diamond(A \wedge \neg B) \supset \Diamond \neg(A \wedge B)$
(3) $(A \Box\!\!\rightarrow (A \wedge B)) \equiv (A \Box\!\!\rightarrow B)$ C2
(4) $((\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset Pr(A \Box\!\!\rightarrow B) = Pr(A \wedge B)$ (2), (3)
(5) $((A \Box\!\!\rightarrow B) \wedge \neg(A \rightarrowtail B)) \supset PrB = Pr(A \wedge B)$ L1', L2, (4), PC, TR

Thus we are able to prove the two lemmas L4 and L5:

L4 $C \vee (B \supset A)$
(1) $PrB \neq 0 \supset (PrB = Pr(A \wedge B)) \supset PrB/PrB = Pr(A \wedge B)/PrB$ $x \neq 0 \supset (x = y \supset y/x = x/x)$
(2) $\Diamond B \supset (Pr(B) = Pr(A \wedge B) \supset Pr(B \Box\!\!\rightarrow A) = 1)$ (1), K4, K7 (ST)

(3) $\Diamond B \supset (Pr(B)=Pr(A \wedge B) \supset (B \supset A))$ (2), $\vdash \Box A \supset A$
(4) $\neg \Diamond B \supset (Pr(B)=Pr(A \wedge B) \supset (B \supset A))$ $\neg \Diamond B \supset (B \supset A)$
(5) $\neg C \supset (B \supset A)$ (4), L3

L5 $\neg(\neg A \supset \neg \Diamond A) \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$
(1) $C \vee \neg B \vee A$ L4
(2) $C \vee \neg B \vee \Box B \vee \neg \Diamond A \vee A$ (1)
(3) $C \vee (B \supset \Box B) \vee (\neg A \supset \neg \Diamond A)$ (2)
(4)$((A \Box\!\!\rightarrow (A \Box\!\!\rightarrow B)) \supset (A \rightarrow (A \Box\!\!\rightarrow B))) \vee ((A \Box\!\!\rightarrow B) \supset$
$\Box(A \Box\!\!\rightarrow B)) \vee (\neg A \supset \neg \Diamond A)$ (3), $A \Box\!\!\rightarrow B/B$
(5)$((A \Box\!\!\rightarrow B) \supset (A \rightarrow B)) \vee ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B)) \vee$
$(\neg A \supset \neg \Diamond A)$ (4), C2
(6)$((A \Box\!\!\rightarrow B) \supset (A \rightarrow B)) \vee (\neg A \supset \neg \Diamond A)$ (5), $AvA \equiv A$

Hence T1 simply follows by the following steps:

(1) $\Diamond A \supset (\neg A \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B)))$ L5, PC
(2) $\neg \Diamond A \supset (\neg A \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$ $\neg \Diamond A \supset (A \rightarrow B)$
(3) $\neg A \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$ (1), (2)

From T1 and L4 T2 is reached in the following way:

(1) $\neg A \supset C$ T1
(2) $(\neg A \wedge \neg B) \supset C$ (1), PC
(3) $(\neg B \wedge (\neg B \supset \neg A)) \supset C$ (2), PC
(4) $(\neg B \wedge (A \supset B) \wedge (A \Box\!\!\rightarrow B)) \supset (A \rightarrow B)$ (3), PC
(5) $\neg B \supset ((A \Box\!\!\rightarrow B) \supset (A \rightarrow B))$ (4), A4

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