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EQUIVALENCE AND DUALITY IN THE THEORY OF THE SYLLOGISM

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This brief note is in effect a retraction of Johnstone (1982), published in *Logique et Analyse*. It is offered in the belief that if a journal of formal logic publishes an article based on a mistaken assumption, it may reasonably later publish the correction of the mistake, provided that either the mistake itself or its correction is sufficiently interesting. Usually mistakes in print are corrected by persons other than the perpetrators of the mistakes; but in this case, the persons are the same.

I will not begin by summarizing the 1982 article I want to discuss. That article is so short as in itself to constitute a summary. The idea I had in mind in it will soon enough emerge in the course of the present discussion.

The crucial concept that I neglected in 1982 was the concept of the equivalence of two sets of rules or axioms. I now introduce this concept by saying that two such sets S1, S2, are equivalent when and only when every axiom of S2 can be deduced as a theorem from S1, and every axiom of S1 can be deduced as a theorem from S2.

The following two sets of axioms (in this case usually called "rules") for the syllogism on the hypothetical interpretation of universal propositions are in this sense equivalent, as I in effect showed in Johnstone (1953).(In my present statement of the axioms of both sets, I have made some very minor changes in wording.)

FIRST SET

H1. Not both premises can be negative.
H2. If one premise is negative, the conclusion is negative.
H3. If both premises are universal, the conclusion is universal.
H4. The middle term must be distributed at least once.
H5. Any term distributed in the conclusion must be distributed in the premise in which it occurs.

SECOND SET

H′1. Not both premises can be particular.
H′2. If one premise is particular, the conclusion is particular.
H′3. If both premises are affirmative, the conclusion is affirmative.
H′4. The middle term must be undistributed at least once.
H′5. Any term undistributed in the conclusion must be undistributed in the premise in which it occurs.

Notice a certain relationship between the First Set and the Second Set of axioms (rules) written down above. Many equivalent pairs of sets bear this relationship to one another. The relationship I have in mind can in the present case be spelled out extensionally in the following manner:

> If, in the First Set, we replace "negative " by "particular" in H1 and H2,"universal" by "affirmative" in H3, and "distributed" by "undistributed" in H4 and H5, we obtain the Second Set. The procedure we have used to derive this Set from the First Set is that of replacing some of the words in the First Set by what I am calling their "duals." Thus "negative" and "particular" are duals, as are "affirmative" and "universal," and "distributed" and "undistributed." Of course, we can return to the First Set by putting back the original words in place of their duals. There is no reason not to call these original words "duals" of their duals.

Outside the theory of the syllogism, there are certainly other illustrations of duals in this sense, conforming to the same principle. All that seems to be required for the existence of duals in this sense is that in an equivalent pair of sets of axioms there be the same number of axioms in each set, and that one can obtain one of the two sets by replacing certain words in the other set. It is likely, for example, that there is at least one set of axioms for Boolean Algebra that can be transformed into an equivalent set by making a simple word-for-word translation of the type indicated above. If so, the words replaced and the words that replace them count as duals.

Just as any of H′1,...,H′5 is a theorem deducible from Set One, so any of H1,...,H5 is a theorem deducible from Set Two; this is what it means to call the two sets equivalent. In addition to the words already mentioned as examples of duality in the context of rules for the hypothetical interpretation, the deductions of the theorems in question in either direction involve several other pairs of dual words as well; e.g., "subject," and "predicate," and "E-proposition," and "I-proposition." To see how such words function in the deduction of, say, H'*n* from Set One, a number of older logic treatises and textbooks, such as Keynes (1894) and Cohen & Nagel (1934), may be consulted; but to see how H*n* can be deduced from Set Two, consult Johnstone (1953). We see then that the deduction of H*n* from Set Two requires only the replacement of duals for duals in the deduction of H'*n* from Set One.

By completing our lexicon of duals (a task which, as I have already indicated, includes determining the duals of the canonical propositions A, I, O, and E), we find it exactly as easy to deduce H'*n* from Set One as it is to deduce H*n* from Set Two. So it makes sense to say that the deductions themselves are duals one of the other. We must bear in mind, however, that we are now using "duals" in a new and different sense, since it is not now *words* that we are calling duals one of the other. And without stretching the word "dual" too far, although using it in a *third* sense, we can find a clear meaning for "Set One is the dual of Set Two." Two sets of axioms or rules are duals one of the other when and only when (1) the sets are equivalent and (2) each set is formed from the other by replacing duals by duals throughout. (It is, of course, the requirement of equivalence that is met by all three of the meanings of "dual" that I have permitted myself to use. It is, as we shall see, when we are working with sets of axioms that are *not* equivalent that duality cannot be properly appealed to.)

The question "What is the dual of the Hypothetical Interpretation?" is also askable and answerable. If the Hypothetical Interpretation is simply Set One, or else simply Set Two, then, since Set One is the dual of Set Two, the Hypothetical interpretation must be its own dual.

But in Johnstone (1982) I am afraid I stretched the word "dual" beyond its breaking-point by applying it to what I claimed was the dual of the Existential Interpretation. This Interpretation can be formed from the Hypothetical Interpretation by replacing H3 in Set One ("If both premises are universal, the conclusion is universal") by H'3 from Set Two ("If both premises are affirmative, the conclusion is affirmative"). (The fact that this resulting Set is sufficient to generate the Existential Interpretation was apparently unknown to Keynes or Cohen & Nagel; I think I was the first to publish it, in Johnstone (1953). But this is beside the point.)

Consider the Set for the Existential Interpretation that I have just given directions for forming. In 1982, I was tempted to suppose that this Set has a dual. H1, H2, H4, and H5, all of which this Existential set shares with Set One, have duals; namely, H'1, H'2, H'4, and H'5. H'3, in the Existential Set, has H3 as its dual. May we not then assume that the Set consisting of H1, H2, H'3, H4, and H5 is the dual of the Set consisting of H'1, H'2, H3, H'4, and H'5? If so, it would seem that we have succeeded in formulating the dual of the Existential Interpretation.

This alleged dual interested me because it seemed to formulate an ontolgy radically different from that of the Existential Interpretation. In (1982) I outlined this ontology and drew an Euler Diagram representing it. What I failed to see was that two Sets formulating different ontologies cannot be duals, since they are not equivalent. (I assume that it would be a trivial task to show that the sense of "equivalence" required here is itself equivalent with the sense in which two sets of axioms or rules can be equivalent.) Hence the words of one of the Sets cannot be duals of the corresponding words of the other Set.

One need not appeal to ontology in order to make the same point. If we list the syllogistic moods validated by each of the sets, we shall find nine moods (e.g., AAI in the First Figure) valid on the Existential Interpretation but not on its alleged dual, and nine moods (e.g., AAE in the First Figure) valid on the latter but not on the Existential Interpretation. This shows conclusively that the two Sets are not equivalent. In fact their relation to one another is that of contraries; not both can be true at the same time.

It was the kindly comments of a referee for *Logique et Analyse* regarding what I intended to be a sequel to (1982), that brought me to see my mistake. I was simply abusing the notion of duality by failing to see that equivalence was essential to it.

I am also indebted to Dale Jacquette.

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