Logique & Analyse **127-128** (1989), 319-335

MONTAGUE'S SEMANTICS FOR INTENSIONAL LOGIC

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In this paper we shall describe a simplified version of the semantics of Montague for intensional logic as exposed in [6]. We hope that this new version is clearer than the original one and that will help to understand Montague's semantics, its achievements and its limitations. In particular, we discuss problems of identity, possibility and existence and we show some of its shortcomings to cope with these fundamental problems in the philosophy of language. This paper is a companion to [7], where a Boolean-valued version of Gupta's semantics [4] is explained and criticized. These two papers may be viewed both as an introduction and a motivation for a system of intensional logic that we believe free of the shortcomings of both Montague's and Gupta's semantics: the logic of kinds. The interested reader may consult [7] and [8].

1. *Introduction.*

The aim of Montague [9] is "to present in a rigourous way the syntax and the semantics of a fragment of a certain dialect of English". To achieve this goal, he translates first the English expressions of the fragment into a higher order intensional logic that uses the λ-calculus. Subsequently, he interprets this logic in the theory of sets. The language of this logic is a "rich" language which allows him to mirror a good deal of the semantics. This approach gives him the means to discuss problems at two different levels: problems whose solutions are achieved by looking at the syntactical side of the language and problems whose solution are achieved by looking at the semantical side of the language.

A basic idea of Montague is to impose on his logic a very fruitful constraint: if two expressions of the natural language belong to the same grammatical category, then their translations in intensional logic should belong to the same sort and their interpretations in sets should belong to the same kind (see for instance Dowty et al. [2, pages 260-262]).

In the context of his logic, Montague discusses traditional problems of philosophy of language: opacity versus transparency, the *de re* and *de dicto* readings of a sentence, etc. An opaque context is one for which substitutions of equals for equals result in different truth values. On the other hand, a context is transparent if it allows such substitutions without changing truth values. We discuss and illustrate these problems with some examples.

The constraint on translation can be understood as follows:

1. John overtakes Mary, therefore Mary walks slower than John (it is clear that some further assumptions have to be made for a "logical" deduction, for instance, "both Mary and John are walking", etc.).
2. John overtakes nobody, therefore nobody walks slower than John.

1. and 2. have the same grammatical form. In fact "Mary" and "nobody" belong to the same grammatical category: noun phrase (NP). Therefore their translation into intensional logic should belong to the same sort. If we interpret "Mary" as a person, then "nobody" and "Mary" belong to different kinds since "nobody" cannot be interpreted as a person. On the other hand if we think of "Mary" as a set of properties, then "nobody" can also be thought of as a set of properties, namely the properties that nobody has. In this way we can represent "Mary" and "nobody" as belonging to the same sort. Nevertheless they have different logical structure: "Mary" is translated as $Mary = \lambda\, PP(m)$ and interpreted as the set of properties that Mary has, whereas "nobody" is translated as $nobody = \lambda\, P\, \forall x\, \neg P(x)$ and interpreted as the set of properties that nobody has. Obviously 1. is valid and 2. is invalid.

Since Frege, logicians have translated "nobody" as a quantifier and "Mary" as a constant in the language of first order logic. With the introduction of higher order logic, Montague can translate "John", "John and Mary", "nobody", "the teacher of Plato", for instance, into expressions belonging to the same sort so that the phrases "John runs", "nobody runs", "the teacher of Plato runs" can all be analyzed in the same way. The possibility of forming NPs of the sort "John or Mary, but not Jane" and therefore of connecting both uses of "and" in NPs and "and" in sentences "John and Mary, but not Jane went to the market", "John went to the market, but it is not the case that Jane went to the market" was exploited by Keenan and Faltz [5]. We shall come back to this question later.

A different aspect of this discussion of logical form is found in the following example. From "John finds a thrush" we can certainly deduce "there are thrushes". On the other hand from "John seeks a unicorn" we cannot possibly deduce "there are unicorns". Once again, "find" and "seek" belong to the same grammatical category: verb phrase (VP), so their translations will be of the same sort and their interpretations will belong to the same kind, but their logic should be different. Nothing in the form of the verbs help to differentiate between the two. A solution will be reached only if the semantical part of the language is taken into account. Therefore Montague introduces the notion of "meaning postulates". This last example is quite intricate and brings about notions of transparency versus opacity, of *de re* readings versus *de dicto* readings.

These notions are dealt with by introducing into intensional logic "intensions" whose interpretations are functions. The idea is simply that some properties of functions depend on the whole graph of the functions in question. An example would be "f is increasing at 0". It is not enough to know the value $f(0)$ to decide whether f is increasing at 0 or not; we could have a function g which coincides with f at 0 but which is decreasing at 0. On the other hand, there are properties such as "f is positive at 0" for which knowledge of the value $f(0)$ suffices. Let us be more specific.

To solve the problems of transparency versus opacity and of differentiation between the two readings mentioned above, Montague interprets the VPs "very high" in the theory of higher order. Let us look at the sentence, "the temperature rises" (this is considered a paradigm by Montague). Let the interpretation of "the temperature" be a function $T \rightarrow R$, where T is the set of moments of time and R is the set of reals. One can ask at time t_0: "Is the temperature rising"? Since the answer is "yes" or "no", it is then natural to interpret "the temperature rises" as a function $T \rightarrow \{0, 1\}$ and consequently to interpret "rises" as a function $R^T \rightarrow \{0, 1\}^T$.

We now consider the classical example: "George IV wished to know whether Scott was the author of Waverley". Let $\phi(x)$ be the context "George IV wished to know whether x was the author of Waverley". Although ϕ *(Scott)* is true, and "Scott equals the author of Waverley" is also true, ϕ *(the author of Waverley)* is certainly false. The above remarks suggest that "Scott" and "the author of Waverley" should be interpreted as different functions which happen to coincide at our world. In other words, the notion of equality involved is not one of identity, but rather of coincidence and no logical problems arise from the fact that two different functions have different properties.

A context $\phi(x)$ is transparent if from $x = y$, *and* $\phi(x)$ one can conclude $\phi(y)$. A context is opaque if that conclusion cannot be drawn. We think that the existence of opaque contexts is a normal feature of everyday language.

Not only do opaque contexts occur in expressions like "wish to know", "believe", "necessary", but as Keenan and Faltz [5] have shown, expressions like "with Fred", "for Mary" introduce opaque contexts. For instance, it may well happen that the people who are working in a room are exactly those who are talking. Nevertheless, we cannot conclude that the people who are working with Fred are those who are talking with Fred. Contrary to what Keenan and Faltz believe, even the expression "in the park" can create opaque contexts, as the following example of M. Barr's indicates: "those who are doing research are those who are publishing" does not imply "those who are doing research in the park are those who are publishing in the park". This seems to indicate the ubiquity of opaque contexts in natural languages, independently of the occurrence of modal and epistemic operators. Following a well-established tradition we shall continue to use the symbol "=" for the coincidence relation, rather than the identity relation. Montague did not introduced a primitive symbol for the identity relation.

Montague's higher order logic provides a nice way to tackle the problem of descriptions. Descriptions are difficult to handle in first order logic and Russell's analysis can not be applied to all descriptions. Descriptions which occur in contexts where a *de re* reading (primary occurrence) and a *de dicto* reading (secondary occurrence) are possible can be analyzed à la Russell. But "Ponce de Leon was looking for the fountain of youth" seems to have only one possible reading, namely a *de re* reading: $\exists x$ *(x is a fountain of youth* $\wedge$ *Ponce de Leon was looking for x* $\wedge$ $\forall y$ *(y is a fountain of youth* $\rightarrow y = x$*))*. This reading makes the sentence false, even though Ponce de Leon was really looking for the fountain of youth! First order logic seems incapable of dealing with non-existent objects which are required to handle the logic of fables, fairy tales and literature in general. On the other hand, we shall see that in higher order logic we can obtain the two readings mentioned above by correlating, for instance, "the fountain of youth" with a set of properties that the fountain of youth has. This will be shown in detail for indefinite descriptions of the type "a unicorn", but obvious changes can be made to handle definite descriptions of the type mentioned above. This solution could offer the possibility of correlating Hamlet, Sherlock Holmes, etc. with a set of properties as Parsons [10] has done in his theory of fiction. For a critical discussion of Parsons's theory of fiction, see [6]

Montague's logical system is in fact a modal higher order theory, namely a higher order theory with two modal operators: an operator of necessity $\Box$, read as "it is necessary that" and an operator of possibility $\Diamond$, read as "it is possible that". In this context, we now mention another aspect of descriptions that played an important role in the discussions of Quine [11] and others on quantification and modality. Let us suppose that we are speaking about horse races. Let

$$a = \textit{the winner of the second race}$$

and the context

$$\phi(x) = \Box x \textit{ wins the second race.}$$

Obviously $\phi(a)$ is valid but strangely enough this does not imply that $\exists x\phi(x)$. On the other hand, if we take

$$a = \textit{Lucky Strike},$$

then for every context $\psi(x)$, the validity of $\psi(a)$ implies that $\exists x\psi(x)$ is valid. This indicates that the equality considered has to be handled with care, as we mentioned at the beginning of this section, and that the interaction between modal operators and quantifiers is intricate. Furthermore constants and descriptions do not have the same logical form, so ϕ*(the winner of the second race)* is quite different from ϕ*(Lucky Strike)*. We shall return to descriptions later on.

2. *The language of modal higher order theory and its interpretation*

In this section we introduce the language of modal higher order theory of Montague's intensional logic and interpret it in sets. We define *sorts* and *terms* by recursion as follows:

Sorts

1. U is a *basic sort*
2. Ω is a *sort*
3. If X and Y are *sorts*, so is Y^X
4. Nothing else is a sort.

Terms of a given sort are defined by recursion as follows (where $t{:}X$ is an abbreviation for "t is a term of sort X"):

1. *Basic constant terms* $c \in Con_X$ are *terms* of sort X, for instance, *John* $\in Con_{\Omega^{(\Omega^U)}}$, $j \in Con_U$

2. If $\alpha \in Var_X$, then α is a *term* of sort X, where Var_X is a countable set, for each sort X

3. If $\alpha \in Var_X$ and $t{:}Y$, then $\lambda\alpha t{:}Y^X$

4. If $t{:}Y^X$ and $s{:}X$ then $t(s){:}Y$

5. $\top$ and $\bot$ are *terms* of sort Ω

6. If $t{:}X$ and $s{:}X$, then $t=s{:}\Omega$

7. If $\phi{:}\Omega$ and $\psi{:}\Omega$, then $\phi \,\Delta\, \psi{:}\Omega$, where $\Delta \in \{\wedge, \vee, \rightarrow\}$

8. If ϕ is a *term* of sort Ω, then so are $\forall\alpha\phi$ and $\exists\alpha\phi$

9. If ϕ is a *term* of sort Ω, then so are $\Box\phi$ and $\Diamond\phi$

10. Nothing else is a *term*.

Montague introduces in his language the tense operators F that can be thought of as "it will be the case that ϕ" and P that can be thought of as "it has been the case that ϕ". We shall not introduce them although it could be done straightforwardly. If ϕ is a term of sort Ω, we let $\neg\phi \equiv \phi \rightarrow \bot$. The *formulas* are by definition the terms of sort Ω. The connectives, quantifiers and modal operators are all understood in the usual way. The expression $t(s)$ is understood as denoting the value of the function denoted by t for the argument denoted by s. If α is a variable of sort X, $\lambda\alpha t$ is understood as denoting that function from the objects of sort X which takes as value for any such object a the object denoted by t when α is understood as denoting a. We now interpret this language in sets by choosing:

1. An arbitrary non empty set W that can be thought of as the set of possible worlds.

2. An arbitrary non empty set E that can be thought of as the set of individuals or entities.

3. A function m which interprets the basic constants: $Con_X \xrightarrow{m} \|X\|$, where $\|X\|$ is defined by recursion as follows: $\|\Omega\| = 2^W$ where

$$2=\{0, 1\}, \ \|U\| = E^W \text{ and } \|X^Y\| = \|X\|^{|Y|}.$$

An interpretation is a triple $[W, E, m]$. We define, for every term $t{:}X$ and every $g{:}Var_X \to \|X\|$, $\|t\|_g \in \|X\|$ which is the interpretation of t under the assignment g as follows:

1. If $c \in Con_X$, then $\|c\|_g = m(c) \in \|X\|$
2. If $\alpha \in Var_X$, then $\|\alpha\|_g = g(\alpha) \in \|X\|$
3. If $t{:}Y$ and $\alpha \in Var_X$, then $\|\lambda\alpha t\|_g : \|X\| \to \|Y\|$ is defined by $\|\lambda\alpha t\|_g(a) = \|t\|_{g(a/\alpha)}$ where $g(a/\alpha)(\beta) = \begin{cases} g(\beta) & \text{if } \alpha \neq \beta \\ a & \text{if } \alpha = \beta \end{cases}$
4. If $\|t\|_g \in \|Y\|^{\|X\|}$ and $\|s\|_g \in \|X\|$, then $\|t(s)\|_g = \|t\|_g(\|s\|_g) \in \|Y\|$

We shall interpret $\top$ and $\bot$ after introducing the forcing relation. In order to interpret an equality between two terms, we first define by recursion on sorts and for every sort X a set $|X|$ and a "canonical" map

$$can_X : \|X\| \to |X|^W$$

as follows: $|\Omega| = 2$, $|U| = E$, $|Y^X| = |Y|^{(|X|^W)}$, $can_\Omega = Id$, $can_U = Id$, $can_{Y^X}(\phi) = \lambda w \lambda \alpha\, can_Y \phi(can_X^{-1}(\alpha))(w)$ where $\phi \in \|Y^X\|$.

We shall make many abuses of language. For instance, we shall use α as a variable of the language ($\alpha \in Var_X$) and then as a variable in the interpretation of the language ($\alpha : W \to |X|$).

Proposition 2.1 For every sort X, $can_X : \|X\| \simeq |X|^W$

Proof. It is clear that can_Ω and can_U are bijections and if can_X and can_Y are bijections so then is can_{Y^X}. □

With the help of this proposition we interpret the equality between two terms. If $\|t\|_g \in \|X\|$ and $\|s\|_g \in \|X\|$, then $\|t=s\|_g(w)=1$ if and only if $can_X(\|t\|_g)(w) = can_X(\|s\|_g)(w)$.

To interpret terms of sort Ω we introduce the forcing notation that will be used throughout this paper. Let $\phi : \Omega$, then $w_0 \Vdash \phi[g]$ iff $\|\phi\|_g (w_0) = 1$.

5. $w_0 \Vdash \top [g]$ *always* and $w_0 \Vdash \bot [g]$ *never*

6. $w_0 \Vdash t = s[g]$ iff $\|t = s\|_g (w_0) = 1$

7. We define $\Vdash$ for $\wedge$, $\vee$, and $\rightarrow$
   1. $w_0 \Vdash \phi \wedge \psi [g]$ iff $w_0 \Vdash \phi [g]$ and $w_0 \Vdash \psi [g]$,
   2. $w_0 \Vdash \phi \vee \psi [g]$ iff $w_0 \Vdash \phi [g]$ or $w_0 \Vdash \psi [g]$,
   3. $w_0 \Vdash \phi \rightarrow \psi [g]$ iff $w_0 \Vdash \phi [g]$ implies that $w_0 \Vdash \psi [g]$
8. We define $\Vdash$ for $\exists$ and $\forall$
   1. $w_0 \Vdash \exists\alpha\, \phi [g]$ iff $\exists a \in \|X\|\ w_0 \Vdash \phi [g(a/\alpha)]$,
   2. $w_0 \Vdash \forall\alpha\, \phi [g]$ iff $\forall a \in \|X\|\ w_0 \Vdash \phi [g(a/\alpha)]$
9. We define $\Vdash$ for $\Box$ and $\Diamond$
   1. $w_0 \Vdash \Box\phi[g]$ iff $\forall w \in W\ w \Vdash \phi [g]$,
   2. $w_0 \Vdash \Diamond\phi[g]$ iff $\exists w \in W\ w \Vdash \phi [g]$

We remark that $w_0 \Vdash \neg\phi[g]$ iff $w_0 \Vdash \phi \rightarrow \bot [g]$ is the same as $w_0 \nVdash \phi[g]$.

We define the notion of *validity* for this interpretation: if $M = [W, E, m]$, then $\theta(\alpha_1, \alpha_2, \ldots)$ is valid in M symbolized as $M \models \theta(\alpha_1, \alpha_2, \ldots)$

$$\text{iff } \forall w \in W \forall g\, w \Vdash \theta(\alpha_1, \alpha_2, \ldots)[g]$$

$$\text{iff } \forall w \in W \forall g\, \|\theta(\alpha_1, \alpha_2, \ldots)\|_g (w) = 1.$$

We remark that

$$M \models \theta(\alpha_1, \alpha_2, \ldots) \text{ iff } M \models \Box\forall\alpha_1, \alpha_2, \ldots\theta(\alpha_1, \alpha_2, \ldots)$$

$$\textit{iff } M \models \forall\alpha_1, \alpha_2, \ldots\ \Box\theta(\alpha_1, \alpha_2, \ldots).$$

3. *Description of a fragment of English and its translation into modal higher order theory*

For the purpose of this exposition we shall restrict ourselves to a fragment of the fragment studied by Montague. Furthermore our translation takes into account the simplifications that we have introduced in the modal higher order theory just described; for instance we have not introduced symbols for intension (^) and extension (ˇ) as Montague does in his intensional logic.

The fragment studied will contain basic expressions (*B*) belonging to the following categories: intransitive verbs (*IV*), common nouns (*CN*), names and pronouns (*T*), transitive verbs (*TV*). In 1, we have presented a motivation for the requirement that basic expressions like "rise" should be of sort Ω^U in the same way we could motivate the sort of the other categories introduced. Let us describe the fragment.

B_{IV}	=	{*run, rise*}	of sort Ω^U
B_{CN}	=	{*man, unicorn*}	of sort Ω^U
B_T	=	{*John, Mary, he*$_0$*, he*$_1$*, he*$_2$*, ...*}	of sort Ω^{Ω^U}
B_{TV}	=	{*seek, find, be*}	of sort $(\Omega^U)^{(\Omega^{(\Omega^U)})}$
$B_{IV/S}$	=	{*believe that*}	of sort $(\Omega^U)^{\Omega}$

A basic expression of the fragment is understood as a member of

$$\bigcup_{A \in \mathrm{Cat}} \mathrm{B}_{\mathrm{A}}.$$

P_A will be the set of composite expressions of the category A. P_S is understood as containing all the statements of the fragment, which are of course of sort Ω . We remark that there are no basic expressions of sort Ω corresponding to the sort t in Montague, and of sort U corresponding to the sort e in Montague. The sets P_A are the smallest sets satisfying the syntactical rules and the corresponding translation rules given by Montague. We will write $T(\varsigma)$ for the translation of ς. If ς is in B_A then $T(\varsigma)=\varsigma$ except for the members of B_T and *be*. The translations for the members of B_T and *be* will be given if needed in the examples.

We present some examples which tacitly use Montague's rules of syntactical derivation and translation to give an idea of what is involved. The last example presented will contrast the *de re* and the *de dicto* readings of the same statement.

1. "John runs". This example allows ambiguous syntactical derivations but unambiguous translations. "John runs" will be interpreted the same way no matter what syntactical analysis is used.

(a) John runs
↙ ↘
John run

(b) John runs
↙ ↘
John he_0 runs
↙ ↘
he_0 run

(a) T(run) : run
T(John) : $\lambda PP(j)$
T(John runs) : $(\lambda PP(j))\ (run)$
$= (run)\ (j)$ by λ - conversion

(b) T(he_0) : $\lambda PP(x_0)$
T(run) : run
T(he_0 runs) : $run(x_0)$
T(John runs) : $\lambda PP(j)\ (\lambda x_0(run(x_0)))$
$= (\lambda x_0(run(x_0)))(j) = run(j)$

2. We contrast the two statements: "John is Mary" and "John is a man".

(a) John is Mary
↙ ↘
John be Mary
↙ ↘
be Mary

(b) John is a man
↙ ↘
John is a man
↙ ↘
be a man

(a) T(*be*) : $\lambda\wp\lambda x\wp(\lambda y(x=y))$
T(Mary) : $\lambda PP(m)$
T(be Mary) : $\lambda\wp\lambda x\wp(\lambda y(x=y))(\lambda PP(m))$
$= \lambda x(\lambda PP(m))(\lambda y(x=y)$
$= \lambda x(\lambda y(x=y))(m)$
$= \lambda x(x=m)$
T(John) : $\lambda PP(j)$
T(John is Mary) : $\lambda PP(j)(\lambda x(x=m))$
$= (\lambda x(x=m))(j)$
$j=m$

(b) T(a man) : $\lambda Q(\exists x)(man(x) \wedge Q(x)))$
T(be) : $\lambda\wp\lambda z\wp(\lambda(z=y)))$
T(is a man) : $\lambda z(\lambda Q(\exists x(man(x) \wedge Q(x))))(\lambda y(z=y))$

$$= \lambda z(\exists x(man(x) \wedge (\lambda y(z=y))(x))$$
$$= \lambda z(\exists x(man(x) \wedge (z=x))$$

T(John is a man) : $\lambda PP(j)(\lambda z(\exists x(man(x) \wedge (z=x)))$
$$= \lambda z(\exists x(man(x) \wedge (z=x))(j)$$
$$= \exists x(man(x) \wedge =x)$$

3. We now analyze the two possible readings of the statement: "John seeks a unicorn". These two readings have different interpretations as we will see.

(a) John seeks a unicorn (*de dicto*)

↙ ↘

John   seeks a unicorn

↙ ↘

seek   a unicorn

↙ ↘

a   unicorn

T(unicorn)	: *unicorn*
T(a unicorn)	: $\lambda Q\exists x(unicorn(x) \wedge Q(x))$
T(seek)	: *seek*
T(seek a unicorn)	: $seek\ (\lambda Q\exists x(unicorn(x) \wedge Q(x)))$
T(John)	: $\lambda PP(j)$
T(John seeks a unicorn)	: $\lambda PP(j)(seek(\lambda Q\exists x(unicorn(x)\lambda Q(x))))$
	$= seek\ (\lambda Q\exists x(unicorn(x) \wedge Q(x))))(j)$

And finally, we interpret $seek\ (\lambda Q(\exists x(unicorn(x) \wedge Q(x))))(j)$ *as*

$$(m(seek)(\| \lambda Q\exists x(unicorn(x) \wedge Q(x)) \|_g))(m(j)) \in \| \Omega \|$$

where $m(seek) :\ \| \Omega \|^{(\|\Omega\|^{|U|})} \rightarrow \| \Omega \|^{|U|}$,

$$\| \lambda Q\exists x(unicorn(x) \wedge Q(x)) \|_g :\ \| \Omega \|^{|U|} \rightarrow \| \Omega \|$$

and $m(j) \in \| U \|$.

(b) John seeks a unicorn (*de re*)

↙ ↘

a unicorn John seeks him_0

↙ ↘ ↙ ↘

a unicorn John seek him_0

↙ ↘

seek he_0

T(he_0)	: $\lambda QQ(x_0)$
T(*seek*)	: $seek$
T(seek him_0)	: $seek(\lambda QQ(x_0)$
T(John seeks him_0)	: $\lambda PP(j)(seek(\lambda QQ(x_0)$
	$= seek(\lambda QQ(x_0))(j)$
T(a unicorn)	: $\lambda P(\exists x(unicorn(x) \wedge P(x)))$
T(J. seeks a unicorn)	: $\lambda P(\exists x(unicorn)(x) \wedge P(x)))(\lambda x_0 seek(\lambda QQ(x_0))(j))$
	$= \exists x(unicorn(x) \wedge (\lambda x_0 seek(\lambda QQ(x_0))(j))(x))$
	$= \exists x(unicorn(x) \wedge seek(\lambda QQ(x))(j))$

We interpret $\exists x(unicorn(x) \wedge seek(\lambda QQ(x))(j))$ as

$$\| \exists x(unicorn(x) \wedge seek(\lambda QQ(x))(j)) \|_g$$

4. *Transparency and de re.*

In the last section, we derived the statement "John seeks a unicorn" in two different ways. We obtained two different translations of it, one corresponding to the *de re* reading and the other corresponding to the *de dicto* reading. These different readings can be paraphrased as follows: when John is seeking a unicorn he might (*de re*) or might not (*de dicto*) be seeking a particular unicorn. So natural language has been disambiguated by allowing for two different derivations and their corresponding different translations.

We remark that different derivations do not always lead to different translations as we have seen in the first example of the last section. If we consider the statement "John finds a unicorn" and we apply the same rules, we obtain two different translations:

1. $find(\lambda Q(\exists x(unicorn(x) \wedge Q(x))))(j)$

2. $\exists x\ (unicorn(x) \wedge find(\lambda Q\ Q(x))(j))$.

Unlike the case of "seek" there seems to be no ambiguity here. If John finds a unicorn, there must exist a particular unicorn that he finds. So there should be only one reading. Since the system was built to permit two readings it seems that there are no straightforward ways to stop the analysis that leads to 1. To solve this problem, Montague restricts the interpretation: he accepts only models where 1. and 2. are equivalent, namely he postulates in the language of the intensional logic that 1. and 2. are equivalent. Montague recognises many contexts where there exist such phenomena and he introduces as many postulates as there are expressions presenting this phenomenon. We call these postulates *transparency postulates*. The reasons for that name will become clear when we write some of these postulates.

We prefer not to use the name *meaning postulates* despite the fact that Dowty et al., for instance, have used it. Carnap [1] in 1947 introduced meaning postulates to deal with analytically true sentences, sentences which were true in virtue of the meaning of the words, but which could not be analyzed as logically true: true as a consequence of their syntactical form. To analyze "all bachelors are unmarried", Carnap's meaning postulate is

$$\forall x\ (B(x) \rightarrow \neg M(x)),$$

where B stands for bachelors and M for married. A model would then be admissible only if that sentence is true in the model. In other words we restrict the models to the ones that make that sentence true. Montague does not consider postulates that relate the meaning of two words, apart from one exception when he analyzes "seek" as "try to find", but rather he considers postulates that articulate the logic of the expressions considered as, for example, in the case of "find".

We shall reformulate some of Montague's transparency postulates. In Montague's terminology the elements of E^W are called "individual concepts", the elements of 2^W are called "propositions" and the elements of $(2^W)^{(E^W)}$ are called "intensional properties". Since we choose a fragment of the fragment studied by Montague, we shall mention only the postulates relevant to our fragment.

[TP1] $m(\alpha) : W \rightarrow E$ is a constant function. In other words $\exists e \in E$ such that $\forall w \in W\ m(\alpha)(w)=e$, where α is j or m.

[TP2] $\forall w \in W\ (m(\delta)(a))(w)=1 \Rightarrow \exists e \in E\ \forall w' \in W\ a(w')=e$, where δ is *man*, or *unicorn*

[TP3] All members of B_{IV} except "rise" are transparent. Namely,

$$x=y \rightarrow (\delta\ (x) \leftrightarrow \delta\ (y)),$$

where δ is *run*.

[TP4] 1. $\delta\ (\lambda\ PP(x))(y)$ is transparent both in x and in y namely that $x=x' \wedge y=y' \rightarrow (\delta\ (\lambda\ PP(x))(y) \leftrightarrow \delta\ (\lambda\ PP(x'))(y'))$, where δ is *find*.
2. $\delta\ (\wp)(x) \leftrightarrow \wp\ (\lambda\ y\ \delta\ (\lambda\ PP(y)(x)))$, where δ is *find*

[TP5] $seek\ (\lambda\ PP(x))(y)$ is transparent in y.

[TP6] $believe\ that\ (\sigma)(y)$ is transparent in y.

We remark that TP1 guarantees that names are rigid designators in contrast to descriptions: the denotation of "j" is the constant individual concept which picks the same individual (namely John) in each possible world. We cannot, in this language, give a syntactical version of TP1 and TP2 since we have no means of reaching the elements of E. We cannot, for example, write $\exists e \in E$ *such that ...*

From TP4 we can deduce the following corollaries that we apply to the statement "John finds a unicorn"

1. $find(\wp)(j) \wedge \wp = \wp' \rightarrow find\ (\wp')(j)$. We remark that $\wp = \wp'$ implies that $\forall P\ \wp\ (P)=\wp'\ (P)$, hence

$$find(\wp)(j) \leftrightarrow \wp(\lambda y\ find\ \lambda PPy)(j) = \wp'(\lambda\ y\ find\ \lambda PPy)(j) \leftrightarrow find(\wp')(j)$$

2. $find(\lambda Q \exists x(unicorn(x) \wedge Q(x)))(j) \leftrightarrow \exists x\ (unicorn(x) \wedge find\ \lambda\ PP(x)(j))$.

This corollary says that there is only one reading (*de re* reading) for "John finds a unicorn".

3. $find(\lambda Q \forall x(unicorn(x) \rightarrow Q(x)))(j) \leftrightarrow \forall x\ (unicorn(x) \rightarrow find\ \lambda\ PP(x)(j))$

4. $find(\wp \wedge \wp')(j) \leftrightarrow find(\wp)(j) \wedge find\ (\wp')(j)$

5. $find(\wp \vee \wp')(j) \leftrightarrow find\ \wp\ (j) \vee find\ \wp'(j)$

6. $find(\wp \vee \neg\wp)(j) \leftrightarrow find\ \wp\ (j) \wedge \neg find\ \wp(j)$

These clauses assure us that the interpretation of "find" is a Boolean homomorphism which preserves existential and universal quantifiers. This is closely connected to the meaning postulates of Keenan and Faltz. These authors, however, impose their conditions at the level of the models only; their models have to be of a very special kind given in terms of complete and atomic Boolean algebras. From these clauses, it should be clear that such restriction is not necessary to formulate their insights.

5. *Criticisms and conclusion*

We can summarize this section on Montague's semantics by saying that higher order logic is needed to provide a satisfactory treatment of opacity, descriptions and "formal grammar" (for instance the fact that "John but not Mary", "nobody" and "John" should belong to the same sort). However, all the sorts in the intensional logic of Montague are constructed from only one basic sort which is interpreted as the set of all the possible and actual entities needed to interpret the constants of that basic sort in the fragment under consideration. We shall point out and briefly describe some of the difficulties with this approach.

The first thing that we notice is that counting does not apply to heaps or conglomerates of objects. As was argued by Frege, the same conglomerate which makes up an army could be counted as 1 army, 6 divisions, 18 brigades or 500, 000 men. The same remark applies to all quantifiers. Although Montague never says so explicitly, he seems to be committed to "bare individuals" or objects, or things. Indeed, his "possible entities" may well be unicorns in one world, persons in another and minerals in still another possible world: the only link being the bare individual underlying the unicorn, the person and the mineral in the possible worlds under consideration. How can we make clear in such an approach what we are baptizing when we say "I baptize thee "John" in nomine..."? Are we baptizing the nose, the baby, the godfather with the baby, the baptismal robe, the set of molecules that constitute the body of the baby or what?

Another problem pointed out by Gupta in [4] is the following: How is it that airline companies count differently human passengers and persons? How can we explain this fact since in any given situation the human passenger coincides with the person? This problem is a modern version of the problem of heralds treated in the Middle Ages; see for instance Geach [3]. We mention a problem which was considered by Keenan and Faltz: How to account for the fact that the same individual may be tall ...as a pygmy, but not tall ...as a man? Finally, let us mention that Montague does not have any means to distinguish between the real and the possible. His set E of "entities" is considered by him as a set of "possible individuals":

> *If there are individuals that are only possible but not actual, E is to contain them; but this is an issue which would be unethical for me as a logician (or linguist or grammarian or semanticist, for that matter) to take a stand.* [*9, page 257*]

Of course, it is not for the logician to decide the question, but nothing forbids him from formulating it !

Some of these shortcomings have been overcome by A.Gupta in [4] as we shall explain in a companion to this paper (see [7]).

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ACKNOWLEDGEMENTS

This paper originated from a talk that one of us gave at the University of McGill in the winter of 1985, which were subsequently written up and worked out as a term paper by the other and became part of chapter 1 of [6]. We would like to thank J. Lambek and J. Macnamara for their comments. The research of the second author was partly supported by the Natural Sciences and Engineering Research Council of Canada and the Ministère de l'Education du Gouvernement du Québec.

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