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A LOGIC OF *BETTER*

LOU GOBLE

In "A Logic of *Good, Should* and *Would* – Parts I and II" ([1], [2]), I presented a logic for propositional operators 'it is good that...', 'it is bad that...', 'it ought to be that...', and 'it ought not to be that...', amongst others, as they are set within a framework of subjunctive concepts, such as the conditional 'if it were the case that...then it would be the case that...'. At the end of Part I of that work, I suggested that the approach taken there could be extended to apply to statements containing a binary connective 'it would be better if...than if...'. In this paper I follow up that suggestion to provide a logic for this connective 'better'. This not only extends the previous system, it also simplifies it, for the monadic operators 'good', 'bad', 'ought', 'ought not', etc. can be defined in terms of this single binary connective, as I will show below, whereas the converse is not true. 'Better' cannot be defined in terms of the monadic operators.

In Part 1 below, I present the logic of 'better' in a fairly informal way, defining truth conditions for statements built with this connective, and then looking at some of the chief principles which follow from that definition. In Part 2, I then develop the system more rigorously, defining a formal semantics and an axiomatic system which I prove consistent and complete with respect to the semantics.

PART I

In [1] I wrote '*G*P', '*B*P', '*I*P' for the monadic operators 'it is good that P', 'it is bad that P' and 'it is indifferent that P', and '*O*P', '*F*P', '*P*P' for 'it ought to be that P', 'it ought not to be that P' and 'it is all right that P', where P could be any proposition. 'Better' by contrast is binary. For it I will write 'P*B*Q', which may be read 'it would be better if P than if Q' or more precisely 'it would be better if it were the case that P than if it were the case that Q', but which I prefer to simplify to 'P would be better than Q' without worrying overmuch about the niceties of English grammar. Of course 'P*B*Q' represents a variety of locutions. For example, what one might say pedantically by 'it would be better if Smith were imprisoned than if he were hanged' might be said more colloquially by 'it would be better to imprison Smith than to hang him' or 'imprisoning Smith would be better than hanging him' or 'to imprison Smith would be better than to hang him'. In all of these, however, there is propositional content connected by 'better', and in all of these there seems to be a subjunctive aspect in the sense of what is being said. To bring this aspect out is a central goal of the present system.

The most straightforward way to interpret statements 'P would be better than Q' is to take them to be true just in case the possible situations in which P would be true are better than the possible situations in which Q would be true. Hence, as in [1], I shall assume we can speak of alternative possible situations or possible worlds, and that they can be ranked with respect to a relation of betterness, which I take to be at least transitive and asymmetric(1).

I also assume, as in [1], that we do not need to compare all situations in which P is true with all situations in which Q is true, but only those that may *reasonably* be called alternatives, that is, those P-worlds and Q-worlds which are closest to, or most similar to, the actual case, what I called in [1] the *available* P- (Q-) alternatives. It is this narrowing of the range of relevant alternatives which captures the subjunctive aspect of the connective; it is explicated through the underlying logic of subjunctives upon which the logic of 'better' is based(2).

Given that alternative situations or possible worlds may be ranked both with respect to their value and their availability, we may state truth conditions for statements 'P*B*Q' roughly thus:

(D.1) P*B*Q is true if and only if all available P-alternative situations are better than all available Q-alternative situations,

where, in order that this condition not be met vacuously, which would play havoc with the asymmetry of the connective, I shall assume henceforth that for P*B*Q to be true, there must be some available P-alternatives and some available Q-alternatives, that is, both P and Q must be possible. (In Section

(1) There may be many such rankings, of course. Each reflects a different way situations may be regarded as better or worse than others, and so corresponding to each would be a different interpretation of the connective 'better'.

(2) For this logic of subjunctives I draw freely from David Lewis's [3]. See also my [2].

II this will be made explicit.)

From (D.1) two principles, transitivity and asymmetry, generally regarded as a sine qua non of a notion of betterness, both follow immediately.

(P.1) $(PBQ \,\&\, QBR) \rightarrow PBQ$

(P.2) $PBQ \rightarrow \sim(QBP)$

(where the arrow represents logical implication and '$\sim$' represents negation in the usual way). From (P.2) comes irreflexivity, of course.

(P.3) $\sim(PBP)$

Nothing is better than itself.

Less obvious, perhaps, but widely accepted in standard systems of 'better', is the principle

(P.4) $(PBR \,\&\, QBR) \rightarrow (P \vee Q)BR$

(with '$\vee$' for truth-functional disjunction). This follows from (D.1) since any available $(P \vee Q)$-alternative must be either an available P-alternative or an available Q-alternative. Likewise

(P.5) $(PBQ \,\&\, PBR) \rightarrow PB(Q \vee R)$

is true, for the same reason.

Given the implicit restriction of (D.1),

(P.6) $PBQ \rightarrow (\Diamond P \,\&\, \Diamond Q)$

holds as well (with '$\Diamond$' for possibility). We do not, however, assume that

$$PBQ \rightarrow (\Diamond \sim P \,\&\, \Diamond \sim Q).$$

As remarked above, one of the advantages of introducing a connective 'better' is that it allows the wide variety of monadic deontic and axiological connectives to be defined. In [1] truth conditions for *O*P, *F*P, and *P*P were stipulated in terms of the relative values of available P- and (not-P)-alternatives. For example, *O*P was said to be true just in case all available P-alternatives are better than all available (not-P)-alternatives. That reading of 'ought' is reflected through the connective B by means of

(P.7a) $OP \leftrightarrow PB{\sim}P$ and similarly
(P.7b) $FP \leftrightarrow {\sim}PBP$
(P.7c) $PP \leftrightarrow {\sim}({\sim}PBP)$

(where the double arrow represents logical equivalence).

Since, as discussed in [1], the monadic operators GP, BP, and IP can be defined in terms of OP, FP, and PP, merely by adding a condition of truth on P, i.e.,

$$GP \leftrightarrow (P \,\&\, OP)$$
$$BP \leftrightarrow (P \,\&\, FP)$$
$$IP \leftrightarrow (P \,\&\, PP \,\&\, P{\sim}P),$$

they can, obviously, be defined in terms of 'better'. Thus:

(P.8a) $GP \leftrightarrow (P \,\&\, PB{\sim}P)$
(P.8b) $BP \leftrightarrow (P \,\&\, {\sim}PBP)$
(P.8c) $IP \leftrightarrow [P \,\&\, {\sim}(PB{\sim}P) \,\&\, {\sim}({\sim}PBP)]$

But GP, BP, and IP can also be regarded independently of OP, FP and PP (and then those might be introduced by definition in terms of them, as discussed in [1]), and so too they can be given more directly in terms of 'better' through

(P.9a) $GP \leftrightarrow \top B{\sim}P$
(P.9b) $BP \leftrightarrow {\sim}PB\top$
(P.9c) $IP \leftrightarrow [P \,\&\, {\sim}(\top B{\sim}P) \,\&\, {\sim}({\sim}PB\top)]$

where $\top$ is any truth-functional tautology. Since the only available $\top$-alternative to the actual situation is the actual situation itself, these forms with $\top B{\sim}P, {\sim}PB\top$, etc. compare available (not-P)-alternatives to the actual case, which is just what the operations GP and BP require.

Like the monadic operators just mentioned, 'better' is fully extensional, in the sense that

(P.10a) If $P \leftrightarrow Q$, then $PBR \leftrightarrow QBR$
(P.10b) If $Q \leftrightarrow R$, then $PBQ \leftrightarrow PBR$

will be true for any P, Q and R. Nevertheless, this connective does not distribute over entailment. That is, none of the principles

(i) If P → Q, then P*B*R → Q*B*R
(ii) If Q → R, then P*B*R → P*B*Q
(iii) If P → Q, then Q*B*R → P*B*R
(iv) If Q → R, then P*B*Q → P*B*R

hold in general under the conditions of (D.1). Nor should we expect them to, at least not on fairly ordinary readings of 'better'. For example, I think it would be a good thing if the United States and the Soviet Union would dismantle all their nuclear weapons, but better yet would be complete world peace. Let P be the proposition 'the United States and the Soviet Union dismantle all their nuclear weapons ', and let Q be 'there is complete world peace'. Then, I think, (1) Q*B*P is true. Let R be something innocuous, e.g. 'Fred takes Doris dancing on Saturday night'. (2) (Q& R)*B*P is also true, for the same reasons that (1) is (though (2) does not follow from (1)). What is not true is (3) R*B*P, that it would be better if Fred were to take Doris dancing than if the United States and the Soviet Union were to dismantle all their nuclear weapons. This is thus a counterexample to (i).

(We might imagine Fred one of the principle negotiators working out an agreement on nuclear disarmament. The negotiations have reached a critical and delicate stage; one more meeting would achieve the agreement everyone hopes for. Without that meeting all hope for a treaty is lost. The meeting is set for Saturday night. Fred, however, discovers it would be better to take Doris dancing than for the United States and the Soviet Union to dismantle their nuclear weapons. Accordingly he changes his suit and takes Doris dancing. Surely that would be a tragic mistake.)

The failure of (i), and one might argue similarly against (ii)-(iv), is analogous to the failure of

(v) If P → Q, then *O*P → *O*Q

for monadic deontic logic. In [1] I argued at length against this principle and some of its kindred; similar considerations apply to (i) - (iv). Of course, (v) is fundamental to standard systems of deontic logic. In much the same way, other accounts of 'better', e.g. Lewis [3] and van Fraassen [4], are committed to (i) and (ii) (though not (iii) and (iv), which cannot be coupled with (i) and (ii) without destroying the asymmetry of 'better'), and so the present system differs from theirs in much the same way that the deontic logic of [1] and [2] differs from standard deontic logic.

I should remark that, although they do not discuss their commitment to (i) and (ii), Lewis and van Fraassen are not confused about them, nor would they endorse Fred's fiasco described above. Lewis, for example, makes it quite clear that the sense of 'better' he is presenting is a sort of 'maximax' betterness, that, roughly speaking, a statement P*B*Q is true, on his account, just in case the best way to have P be true is better than the best way to have Q be true. (Cf. [3], p. 101.) Given this reading of 'better', (3) in the above example would indeed be true. Fred's taking Doris dancing would be better than the U.S.'s and USSR's dismantling their nuclear weapons, since the best situation in which Fred takes Doris dancing is one in which there is also complete world peace. Fred's mistake was to take (3) as a guide to action. As Lewis remarks, regarding his maximax interpretation of 'better', "This is not instrumental or intrinsic betterness of any familiar sort." (ibid.) Hence, it would seem, judgements of the sort P*B*Q are not to be used, as Fred tried to do, as part of ordinary practical reasoning. It is difficult to see, though, what other use they might have. (The same might be said of the sense of 'ought' derived from Lewis's notion of ' better' in the manner of (P.7a).)

In [1] and [2] I introduced two binary connectives, '⫽' and '△', into the underlying logic of subjunctives to express relations between sets of available alternative worlds. 'P ⫽ Q' says that all available P-alternatives (to a given situation) are available Q-alternatives. 'P △ Q' says that some available P-alternative is also an available Q-alternative. If all available P-alternatives are available Q-alternatives and all available Q-alternatives are better than all available R-alternatives, then all available P-alternatives must be better than all available R-alternatives. Hence,

(P.11) (P ⫽ Q & Q*B*R) → P*B*R

must be true, and likewise

(P.12) (R ⫽ Q & P*B*Q) → P*B*R.

Closely related to these we have

(P.13) (P » Q & Q » P) → (P*B*R ≡ Q*B*R)
(P.14) (Q » R & R » Q) → (P*B*Q ≡ P*B*R)

where '»' represents the (strong) subjunctive conditional 'if it were the case that...then it would be the case that...' as described in [2]. (This is Lewis's '□⇒', cf. [3] p. 25.)

If all available P-alternatives are better than all available Q-alternatives then there could not be any available P-alternative that was also an available Q-alternative, for it would then be better than itself, violating the asymmetry and irreflexivity of the relation ranking possible worlds by value. Hence,

(P.15) $\quad PBQ \rightarrow \sim(P \vartriangle Q)$

and as a special case

(P.16) $\quad PBQ \rightarrow \sim(P \,\&\, Q)$.

If P would be better than Q, then they cannot both be true.

It is thus apparent that 'better', as defined by (D.1), applies only to alternative states of affairs which are, in a sense, exclusive. Although P and Q need not absolutely exclude each other, we do have

(P.17) $\quad PBQ \rightarrow (P \gg \sim Q) \vee (Q \gg \sim P)$.

If P would be better than Q, then either if P were true, then Q would not be, or else if Q were true, then P would not be. That there is only disjunction on the right of (P.17), and not conjunction, is because one of P and Q might be properly more possible than the other, in which case the latter could be conjoined with the former in its (the latter's) available alternatives. It is in this sense that the two need not be absolutely exclusive. But whichever alternative is as possible, or more possible, than the other, that one must exclude the other. That is,

(P.18) $\quad (PBQ \,\&\, P \leq Q) \rightarrow P \gg \sim Q$
(P.19) $\quad (PBQ \,\&\, Q \leq P) \rightarrow Q \gg \sim P$

(writing '$P \leq Q$' for 'P is at least as possible as Q'. Cf. [2] and Lewis's [3] §2.5.)

That P and Q must, to this extent, exclude one another seems an implicit prerequisite in many discussions of betterness or preferability, especially as this notion is used to explicate 'ought' within a utilitarian framework. This suggests, however, that 'better' in this sense, as given by (D.1), cannot, strictly speaking, be considered the comparative form of the positive 'good', at least as that was defined in [1] and [2] and which appears in (P.8) and (P.9) above. That sense of 'good', and 'bad' too, is too much tied to truth. *G*P and *B*P both imply P. Hence, because of (P.16), we cannot say that it is good that P and good that Q but better that P than that Q, as one would expect of a comparative form. Indeed, we cannot even say it is good that P and bad that Q, and so P must be better than Q. That we cannot certainly seems odd. The reason, however, is simply that in such a case P and Q are not appropriately exclusive alternatives, as required by 'better'.

To say that something is better than something else, whether 'better' be construed as a sentential connective, as it is here, or as a predicate of individuals, does not imply that either of the two is good. This is generally true of comparatives and their positive counterparts. To say that X is taller than Y is not to say that either is tall. What might be valid, however, is the inference from 'X is taller than Y' and 'Y is tall' to 'So, X is tall'. Similarly, if something is better than something else, and the latter is good, we might expect that the former must be good as well. As we have seen, however, the principle

(vi) $(\mathrm{P}B\mathrm{Q} \;\&\; G\mathrm{Q}) \rightarrow G\mathrm{P}$

cannot be true under (D.1) and the truth conditions for *G*P as given in [1] and [2]. Instead, we do have

(P.20) $(\mathrm{P}B\mathrm{Q} \;\&\; O\mathrm{Q}) \rightarrow O\mathrm{P}$

and similarly

(P.21) $(\mathrm{P}B\mathrm{Q} \;\&\; F\mathrm{P}) \rightarrow F\mathrm{Q}$

as well as the related

(P.22) $(\mathrm{P}B\mathrm{Q} \;\&\; G\mathrm{Q}) \rightarrow O\mathrm{P}$
(P.23) $(\mathrm{P}B\mathrm{Q} \;\&\; B\mathrm{P}) \rightarrow F\mathrm{Q}$

which are, perhaps, fair substitutes for (vi).

It is worth remarking that just as 'better' falls short of being a good comparative of 'good' on the present account, it also falls short on more orthodox treatments, such as that of David Lewis, mentioned earlier, although the reasons are somewhat different.

(P.16) does not hold on Lewis's account of 'better' and so one might expect *G*P, *G*Q and P*B*Q to be consistent, at least sometimes, and we might even expect (vi) to be valid. This is not the case, however. (Actually, since Lewis does not discuss the operator 'good', I must interpolate it into his account, which I do via the equivalence given above that $GP \leftrightarrow (P \,\&\, OP)$, where also *O*P is equivalent to $P\,B \sim P$.) But, since Lewis does not discuss 'good', it is more significant that (P.20) is valid on his account only because it is vacuous. Given Lewis's truth conditions for statements containing the operators *O* and *B*,

(vii) $OP \rightarrow \sim(QBP)$

will hold for any Q. That is because, in effect, *O*P says that P is the best among all possibilities, so nothing could be better than it. If we were thinking in terms of positive and comparative forms, we might say that Lewis's 'ought' constitutes the superlative. In any case, given (vii), *O*Q and P*B*Q are inconsistent and so, of course, are *O*P, *O*Q and P*B*Q (so (P.20) holds in that sense), and then so are *G*P, *G*Q and P*B*Q, just as they were on the present account because of (P.16). ((vii) does not hold under (D.1).) I do not know how to define a sense of 'better' which will stand to 'good', as presented in [1] and [2], as a genuine comparative to positive (3).

In [1] I mentioned a variation on (D.1) which might also be used to define truth conditions for statements 'P would be better than Q'. That was

(D.2) P*B*Q is true if and only if all available (not-P)-alternative situations are worse than all available (not-Q)-alternative situations; i.e. all available (not-Q)-alternatives are better than all available (not-P)-alternatives,

(where, as in (D.1), it is assumed that these sets of alternatives are not empty).

(3) 'Positive' and 'comparative' (and 'superlative') are terms of grammar; they apply to adjectives and sometimes adverbs, of natural languages. It may be a complete mistake to try to apply them to the categories of logical grammar, such as sentential connectives, picked out by systems like that developed here or other logics of 'good', of 'ought', of 'better', of 'best'.

(D.2) has the advantage over (D.1) of being closer to the form of the truth conditions that were originally given for statements *G*P, 'it is good that P', in [1]. Under (D.2) the principles (P.1) - (P.3) remain true, as we should require. (P.4) and (P.5), however, must be changed to

(P.4′) $(\mathrm{P}B\mathrm{R} \ \& \ \mathrm{Q}B\mathrm{R}) \rightarrow (\mathrm{P} \ \& \ \mathrm{Q})B\mathrm{R}$
(P.5′) $(\mathrm{P}B\mathrm{Q} \ \& \ \mathrm{P}B\mathrm{R}) \rightarrow \mathrm{P}B(\mathrm{Q} \ \& \ \mathrm{R})$

and (P.6) becomes

(P.6′) $\mathrm{P}B\mathrm{Q} \rightarrow (\Diamond \sim\mathrm{P} \ \& \ \Diamond \sim\mathrm{Q})$.

(P.7), the equivalence of *O*P and P*B*∼P, etc. continues to hold under (D.2), and so does (P.8), but (P.9) must be replaced by

(P.9a′) $G\mathrm{P} \leftrightarrow \mathrm{P}B\bot$
(P.9b′) $B\mathrm{P} \leftrightarrow \bot B\mathrm{P}$
(P.9c′) $I\mathrm{P} \leftrightarrow [\mathrm{P} \ \& \ \sim(\mathrm{P}B\bot) \ \& \ \sim(\bot B\mathrm{P})]$

where ⊥ is any truth-functional contradiction.

(D.2) preserves extensionality, (P.10), as one would expect, and the invalid principles (i) - (iv) remain invalid, but (P.11) and (P.12) now appear as

(P.11′) $(\sim\mathrm{P} \checkmark\!\!/ \sim\mathrm{Q} \ \& \ \mathrm{Q}B\mathrm{R}) \rightarrow \mathrm{P}B\mathrm{R}$
(P.12′) $(\sim\mathrm{R} \checkmark\!\!/ \sim\mathrm{Q} \ \& \ \mathrm{P}B\mathrm{Q}) \rightarrow \mathrm{P}B\mathrm{R}$

which are reminiscent of the principle (P.18) of [1] for *G*P. (P.13) and (P.14) above now become

(P.13′) $(\sim\mathrm{P} \gg \sim\mathrm{Q} \ \& \ \sim\mathrm{Q} \gg \sim\mathrm{P}) \rightarrow (\mathrm{P}B\mathrm{R} \equiv \mathrm{Q}B\mathrm{R})$
(P.14′) $(\sim\mathrm{Q} \gg \sim\mathrm{R} \ \& \ \sim\mathrm{R} \gg \sim\mathrm{Q}) \rightarrow (\mathrm{P}B\mathrm{Q} \equiv \mathrm{P}B\mathrm{R})$

Similarly, instead of (P.15) we have

(P.15′) $\mathrm{P}B\mathrm{Q} \rightarrow \sim(\sim\mathrm{P} \vartriangle \sim\mathrm{Q})$

which has

(P.16′) $PBQ \rightarrow P \vee Q$

as a special case.

Thus, where (D.1) says that of any two states of affairs if one is better than the other, at most one will be the case, (D.2), by contrast, says at least one will be so. It allows that both might be, and so it can apply when the alternatives are not exclusive. Hence, under (D.2) GP, GQ and PBQ could be consistent. Indeed, we even have the full principle that if P is good and Q is bad, then P is better than Q,

(P.21′) $(GP \,\&\, BQ) \rightarrow PBQ$,

which seems good to have. Moreover, (vi), which was rejected under (D.1), becomes true by (D.2). In this respect 'better' as given by this definition seems a better comparative for 'good' than that given by (D.1).

Nevertheless, (D.2) offers little real advantage over (D.1). For just as (D.1) yields the contrary of (P.21′), *viz.*

$$(GP \,\&\, BQ) \rightarrow \sim(PBQ)$$

simply because P and Q could not then be exclusive alternatives, so (D.2) yields

$$(B{\sim}P \,\&\, G{\sim}Q) \rightarrow \sim(PBQ)$$

which seems just as odd. Similarly, although (D.2) does give (vi) which (D.1) rejected, it in its turn rejects

(viii) $(PBQ \,\&\, G{\sim}P) \rightarrow G{\sim}Q$

which (D.1) validates. Furthermore, just as with (D.1) we cannot compare P and Q to say that one is better than the other when both are good, because then both are true, so under (D.2) we cannot compare them when both are false. It might be good that P is false and good that Q is false, yet we cannot say that P would be better than Q.

Indeed, (D.2) is merely the dual of (D.1). It seems a little closer to the spirit of the truth conditions given for statements 'it is good that P' in [1], but (D.1) seems simpler, easier to understand and to work with (though from a formal point of view this difference is insignificant). Hence, in what follows I shall follow the pattern of (D.1) instead of (D.2). All of the results obtained there, however, may be easily adapted to fit (D.2) if that is the form one prefers.

PART II

In this section I put the ideas introduced above on a more rigorous basis, defining a formal logic of 'better', *LB*, both semantically and axiomatically. I then prove the axiomatic system consistent and complete with respect to the semantics. (These results draw heavily on those of [2].)

Let L be a propositional language presumed adequate for expressing subjunctive conditionals. More precisely, let it be the same as the language L of [2], containing

(i) indefinitely many simple statements: p, q, r, ..., with the usual truth-functional connectives, and

(ii) subjunctive connectives, with A $\leq$ B, A » B, A ✓ B, A $\vartriangle$ B, and $\Diamond$A in particular, to be read 'A is at least as possible as B', 'if A were the case, then B would be the case', 'A is included in B', 'A overlaps B' and 'possibly, A' respectively, where A and B may be any statements of L. (Any of $\leq$, », ✓ might be taken as primitive and the other subjunctive connectives defined in terms of it, as shown in [2].)

For LB, add

(iii) the binary connective *B*, whereby A*B*B is well formed whenever A and B are. It is read 'it would be better if A were the case than if B were the case'.

To interpret this language, use exactly the same model-frames as for the logic of 'good', LG, in [2]. That is, a B-model-frame is a quintuple

$$b = \langle \underset{\sim}{W}, w_0, \leq, a, Bt \rangle$$

where $\underset{\sim}{W}$ is a non-empty set of so-called possible worlds and $w_0 \in \underset{\sim}{W}$ is the actual world. $\leq$ is a triadic relation of comparative accessibility of

$\underset{\sim}{W}$; it is transitive, strongly connected and strictly minimal. *a* is a function defining for each member of $\underset{\sim}{W}$ a range of possibility, i.e. those members of $\underset{\sim}{W}$ which are accessible to it. See [2] for details and conditions on these elements.

Bt is a triadic relation defining a ranking on members of $\underset{\sim}{W}$ relative to each member of $\underset{\sim}{W}$. That is '$w_j \; Bt_{w_i} w_k$ ' says that w_j is better, from the point of view of w_i, than w_k, for all $w_i, w_j, w_k \in \underset{\sim}{W}$. This is what in [1] and [2] I called local evaluation. Since for each $w_i \in \underset{\sim}{W}$, Bt_{w_i} is supposed to represent a relation of betterness, it must be both transitive and asymmetric. It must meet the conditions

(i) if $w_j \; Bt_{w_i} \; w_k$ and $w_k \; Bt_{w_i} \; w_m$, then $w_j \; Bt_{w_i} w_m$
(ii) if $w_j \; Bt_{w_i} \; w_k$, then not-($w_k \; Bt_{w_i} \; w_j$).

Given a B-model frame, $b = \langle \underset{\sim}{W}, w_0, \leq, a, Bt \rangle$, let v be an assignment function for *b* which defines truth-values, T or F, for every simple statement p for every $w_i \in \underset{\sim}{W}$:

1) $v(p, w_i) \in \{T, F\}$.

An evaluation function I based on v for *b* is then defined in the usual way for all statements A of LB. See [2] for details concerning the interpretation of statements containing subjunctive connectives. It will be useful, however, to repeat here the key notion of *available A-alternatives*, which is fundamental for interpreting both subjunctives and statements containing 'better'. For any statement A, the available A-alternative possible worlds (from a given world) are those members of $\underset{\sim}{W}$ in which A is true and which are accessible to the given world and which are as accessible to the given world as any others in which A is true. More precisely, for an assignment function v and any statement A and any $w_i, w_j \in \underset{\sim}{W}$, w_j is an available A-alternative for w_i on v, or for short:

> w_j A-Alt$_v w_i$ if an only if $w_j \in a(w_i)$ and $I_v(A, w_j) = T$ and for all $w_k \in \underset{\sim}{W}$ such that $I_v(A, w_k) = T$, $w_j \leq_{w_i} w_k$.

I write '$[A]_v w_i$' for the set $\{w_j : w_j$ A-Alt$_v w_i\}$.

Given this, the evaluation function can now be defined for statements A*B*B in LB as follows:

$I_v(ABB, w_i) = T$ if and only if $[A]_v w_i \neq \wedge$ and $[B]_v w_i \neq \wedge$ and for all $w_j \in [A]_v w_i$ and all $w_k \in [B]_v w_i$, $w_j\ Bt_{w_i}\ w_k$

which corresponds to (D.1) of Section I above.

A B-model-frame, $b = \langle \underset{\sim}{W}, w_0, \leq, a, Bt \rangle$, is *proper* (with respect to an assignment function v) just in case any statement A is possible (at a given world) if and only if it has an available alternative (to that world). I.e.

$Prop_v(b)$ if and only if for any A and any $w_i \in \underset{\sim}{W}$, $[A]_v w_i \neq \wedge$ if and only if there is a $w_j \in \underset{\sim}{W}$ such that $w_j \in a(w_i)$ and $I_v(A, w_j) = T$.

A statement A of LB is *valid* – $\Vdash$ A – if and only if for every $b = \langle \underset{\sim}{W}, w_0, \leq, a, Bt \rangle$ and every assignment function v such that $Prop_v(b)$, $I_v(A, w_0) = T$.

As in [2], this definition of validity, with its restriction to B-model-frames which are proper, is tantamount to adopting what Lewis calls the Limit Assumption in his [3]. One can avoid being committed to this assumption by using an alternative evaluation function, J, whereby, for example,

$J_v (A \leq B, w_i) = T$ if and only if for every $w_k \in a(w_i)$ such that $J_v(B, w_k) = T$, there is a w_j such that $J_v(A, w_j) = T$ and $w_j \leq_{w_i} w_k$,

and

$J_v(ABB, w_i) = T$ if and only if there is a $w_j \in a(w_i)$ and a $w_k \in a(w_i)$ such that $J_v(A, w_j) = T$ and $J_v(B, w_k) = T$ and for all w_m and w_n such that $J_v(A, w_m) = T$ and $w_m \leq_{w_i} w_j$ and $J_v(B, w_n) = T$ and $w_n \leq_{w_i} w_k$, $w_m\ Bt_{w_i}\ w_n$.

One could then stipulate that A is valid if and only if $J_v(A, w_0) = T$ for every B-model-frame $b = \langle \underset{\sim}{W}, w_0, \leq, a, Bt \rangle$ and every assignment function v on b, and thus avoid all reference to propriety. Because these two interpretations, with I and with J, make valid exactly the same statements (though they may differ with respect to truth for some statements), I will leave it open which function is being used, except that in the proof of completeness below I will work with the function I because it is easier.

Given these definitions, the principles (P.1) - (P.6) and (P.10) - (P.16) of Part 1 are all valid (as would be the others if LB contained the operators

O and *G*, etc.). That is

THEOREM I – For all statements A, B, and C of LB

(1) $\Vdash (\mathrm{A}\mathit{B}\mathrm{B} \;\&\; \mathrm{B}\mathit{B}\mathrm{C}) \supset \mathrm{A}\mathit{B}\mathrm{C}$
(2) $\Vdash \mathrm{A}\mathit{B}\mathrm{B} \supset {\sim}(\mathrm{B}\mathit{B}\mathrm{A})$
(3) $\Vdash {\sim}(\mathrm{A}\mathit{B}\mathrm{A})$
(4) $\Vdash (\mathrm{A}\mathit{B}\mathrm{C} \;\&\; \mathrm{B}\mathit{B}\mathrm{C}) \supset (\mathrm{A} \vee \mathrm{B})\mathit{B}\mathrm{C}$
(5) $\Vdash (\mathrm{A}\mathit{B}\mathrm{B} \;\&\; \mathrm{A}\mathit{B}\mathrm{C}) \supset \mathrm{A}\mathit{B}(\mathrm{B} \vee \mathrm{C})$
(6) $\Vdash \mathrm{A}\mathit{B}\mathrm{B} \supset (\Diamond \mathrm{A} \;\&\; \Diamond \mathrm{B})$
(7) $\Vdash (\mathrm{A} \checkmark\!\!/\, \mathrm{B} \;\&\; \mathrm{B}\mathit{B}\mathrm{C}) \supset \mathrm{A}\mathit{B}\mathrm{C}$
(8) $\Vdash (\mathrm{C} \checkmark\!\!/\, \mathrm{B} \;\&\; \mathrm{A}\mathit{B}\mathrm{B}) \supset \mathrm{A}\mathit{B}\mathrm{C}$
(9) $\Vdash (\mathrm{A} \gg \mathrm{B} \;\&\; \mathrm{B} \gg \mathrm{A}) \supset (\mathrm{A}\mathit{B}\mathrm{C} \equiv \mathrm{B}\mathit{B}\mathrm{C})$
(10) $\Vdash (\mathrm{B} \gg \mathrm{C} \;\&\; \mathrm{C} \gg \mathrm{B}) \supset (\mathrm{A}\mathit{B}\mathrm{B} \equiv \mathrm{A}\mathit{B}\mathrm{C})$
(11) $\Vdash \mathrm{A}\mathit{B}\mathrm{B} \supset {\sim}(\mathrm{A} \vartriangle \mathrm{B})$
(12) $\Vdash \mathrm{A}\mathit{B}\mathrm{B} \supset {\sim}(\mathrm{A} \;\&\; \mathrm{B})$
(13) If $\Vdash \mathrm{A} \equiv \mathrm{B}$ and $\Vdash \mathrm{C} \equiv \mathrm{D}$, then $\Vdash \mathrm{A}\mathit{B}\mathrm{C} \equiv \mathrm{B}\mathit{B}\mathrm{D}$.

(Demonstration of these is straightforward and left to the reader.) In addition some other principles, easily proved valid, are:

(14) $\Vdash (\mathrm{A} \vee \mathrm{B})\mathit{B}\mathrm{C} \supset (\mathrm{A}\mathit{B}\mathrm{C} \vee \mathrm{B}\mathit{B}\mathrm{C})$
(15) $\Vdash \mathrm{A}\mathit{B}(\mathrm{B} \vee \mathrm{C}) \supset (\mathrm{A}\mathit{B}\mathrm{B} \vee \mathrm{A}\mathit{B}\mathrm{C})$

Comparable distribution principles for conjunction require a condition of overlap, namely

(16) $\Vdash (\mathrm{A} \;\&\; \mathrm{B})\mathit{B}\mathrm{C} \supset [\mathrm{A} \vartriangle \mathrm{B} \supset (\mathrm{A}\mathit{B}\mathrm{C} \;\&\; \mathrm{B}\mathit{B}\mathrm{C})]$
(17) $\Vdash \mathrm{A}\mathit{B}(\mathrm{B} \;\&\; \mathrm{C}) \supset [\mathrm{B} \vartriangle \mathrm{C} \supset (\mathrm{A}\mathit{B}\mathrm{B} \;\&\; \mathrm{A}\mathit{B}\mathrm{C})]$

Likewise, we have

(18) $\Vdash (\mathrm{A} \vartriangle \mathrm{B} \;\&\; \mathrm{A}\mathit{B}\mathrm{C}) \supset (\mathrm{A} \;\&\; \mathrm{B})\mathit{B}\mathrm{C}$
(19) $\Vdash (\mathrm{B} \vartriangle \mathrm{C} \;\&\; \mathrm{A}\mathit{B}\mathrm{B}) \supset \mathrm{A}\mathit{B}(\mathrm{B} \;\&\; \mathrm{C})$
(20) $\Vdash (\mathrm{A}\mathit{B}\mathrm{B} \;\&\; \mathrm{B} \vartriangle \mathrm{C} \;\&\; \mathrm{C}\mathit{B}\mathrm{D}) \supset \mathrm{A}\mathit{B}\mathrm{D}$.

The converses of (4) and (5) are also valid, under a condition of equipossibility. That is, writing ‘$A \approx B$’ for ‘$A \leq B \,\&\, B \leq A$’, we have

(21) $\Vdash A \approx B \supset [(A \vee B)BC \supset (ABC \,\&\, BBC)]$
(22) $\Vdash B \approx C \supset [AB(B \vee C) \supset (ABB \,\&\, ABC)]$.

In addition, of course, all the principles regarding subjunctives which were given as Theorems VII, VIII and IX in [2] remain valid. These include all statements derivable from the following axiom schemata and rules, which comprise the logic *L* (which is equivalent to Lewis's VC of [3]):

(R.0) If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$
(R.1) If $\vdash A \supset (B_1 \vee \ldots \vee B_n)$, then $\vdash B_1 \leq A \vee \ldots \vee B_n \leq A$

(A.0) All truth-functional tautologies of statements of LB
(A.1) $\vdash (A \leq B \,\&\, B \leq C) \supset (A \leq C)$
(A.2) $\vdash (A \leq B) \vee (B \leq A)$
(A.3) $\vdash (A \leq B) \supset (B \supset A)$

For the calculus of ‘better’, *LB*, add:

(B.1) $\vdash (ABB \,\&\, BBC) \supset ABC$
(B.2) $\vdash ABB \supset {\sim}(BBA)$
(B.3) $\vdash ABB \supset (\Diamond A \,\&\, \Diamond B)$
(B.4a) $\vdash (ABC \,\&\, BBC) \supset (A \vee B)BC$
(B.4b) $\vdash (ABB \,\&\, ABC) \supset AB(B \vee C)$
(B.5a) $\vdash (A \checkmark\!\!/ B \,\&\, BBC) \supset ABC$
(B.5b) $\vdash (C \checkmark\!\!/ B \,\&\, ABB) \supset ABC$.

(If (B.5) seems too unfamiliar, it may be replaced by

(B.5a′) $\vdash (A \gg B \,\&\, B \gg A) \supset (ABC \equiv BBC)$
(B.5b′) $\vdash (B \gg C \,\&\, C \gg B) \supset (ABB \equiv ABC)$

and a strengthening of (B.4) to

(B.4a′) $\vdash A \approx B \supset [(ABC \,\&\, BBC) \equiv (A \vee B)BC]$
(B.4b′) $\vdash B \approx C \supset [(ABB \,\&\, ABC) \equiv AB(B \vee C)]$

The two axiomatizations are equivalent.)

Since all the axioms of *L* are valid (Theorem I) and the rules preserve validity (easily shown), *LB* is semantically consistent. All theorems are valid.

THEOREM II – For all A of LB, if $\vdash$ A, then $\Vdash$ A.

To establish semantical completeness, that every valid statement is provable in *LB*, I will appropriate the results of [2] wholesale, without bothering to demonstrate them again here in order to concentrate on what the new logic of 'better' requires.

In order to show that a given non-theorem, A, of *LB* is not valid, let

$$b_A = \langle \underset{\sim}{L}, L_0, \leq, a, Bt \rangle$$

be a quintuple where $\underset{\sim}{L}$ is the set of all maximal consistent extensions of *LB*; L_0 is a particular member of $\underset{\sim}{L}$ containing $\sim$ A, and $\leq$ and *a* are defined exactly as in [2]. These four elements suffice to establish the completeness of the basic logic *L* given by (R.0), (R.1) and (A.0) - (A.3).

Before defining *Bt* for b_A, and then proving the key lemma to the completeness theorem for *LB*, namely

LEMMA 11 – For all A and all $L_i \in \underset{\sim}{L}$, A $\in L_i$ if and only if $I_v(A, L_i) = T$, it will be useful to restate some preliminary lemmas from [2]. (Their numbers here refer to that work.)

LEMMA 7 – For all $L_i \in \underset{\sim}{L}$, $\Diamond B \in L_i$ if and only if $(B)\, L_i \neq \wedge$,

LEMMA 8 – For all $L_i \in \underset{\sim}{L}$, $B \leq C \in L_i$ if and only if for every $L_j \in (C)L_i$ there is an $L_k \in (B)\, L_i$ such that $L_k \leq_{L_i} L_j$

and the corollary to Lemma 8,

COR. 1 – For all $L_i \in \underset{\sim}{L}$, $B \vartriangle C \in L_i$ if and only if $(B)\, L_i \cap (C)\, L_i \neq \wedge$,

where $(B)\, L_i$ is a syntactical counterpart to $[B]_v L_i$, that is,

$(B)L_i = \{ L_j : L_j \in a(L_i)$ and $B \in L_j$ and for every $L_k \in \underset{\sim}{L}$, if $B \in L_k$, then $L_j \leq_{L_{L_i}} L_k \}$.

Under the inductive hypothesis for Lemma 11, it is easy to show that

SUB-LEMMA – (B) L_i = $[B]_v L_i$

for the stipulated assignment function v, which is defined for b_A by

$v(p, L_i) = T$ if and only if $p \in L_i$.

And once Lemma 11 has been established, it is also easy to show

LEMMA 12 – $Prop_v(b_A)$

for this v.

Before that, however, we must define *Bt* for b_A. This can be done more directly than in [2]. For all $L_i, L_j, L_k \in \underset{\sim}{L}$, let

$L_j\ Bt_{L_{L_i}}\ L_k$ if and only if there are statements B and C such that $L_j \in (B)\ L_i$ and $L_k^{\ i} \in (C)L_i$ and $BBC \in L_i$.

To prove that this *Bt* is transitive and asymmetric, it is helpful to know that the following are all theorems of *LB*:

(TB.1) $\vdash$ (B*B*C & C Δ D) $\supset$ B*B*(C & D)
(TB.2) $\vdash$ (B*B*C & B Δ D) $\supset$ (B & D)*B*C
(TB.3) $\vdash$ B*B*C $\supset$ ~(B Δ C)
(TB.4) $\vdash$ (B*B*C & C Δ D & D*B*E) $\supset$ B*B*E.

These are easily demonstrated, using especially the axioms (B.5).

We now prove

LEMMA 9a – For all $L_i \in \underset{\sim}{L}$, Bt_{L_i}i is transitive.

Suppose $L_j\ Bt_{L_i} L_k$ and $L_k\ Bt_{L_i} L_m$, and let B, C, D, and E be statements such that $L_j \in (B)\ L_i$, $L_k \in (C)\ L_i$ and B*B*C $\in L_i$, and $L_k \in (D)\ L_i$ and $L_m \in (E)\ L_i$ and D*B*E $\in L_i$. By Cor. 1, C Δ D $\in L_i$, so by (TB.4), B*B*E $\in L_i$. Hence, $L_j\ Bt_{L_i} L_m$.

LEMMA 9b – For all $L_i \in \underset{\sim}{L}$, Bt_{L_i} is asymmetric.

Suppose not, suppose that for some L_j, $L_k \in \underset{\sim}{L}$, $L_j Bt_{L_i} L_k$ and $L_k Bt_{L_i} L_j$. Then for some B and C, $L_j \in$ (B) L_i and $L_k \in$ (C) L_i and B*B*C $\in L_i$ and also for some D and some E, $L_j \in$ (D) L_i and $L_k \in$ (E) L_i and E*B*D $\in L_i$. By Cor. 1, B $\triangle$ D $\in L_i$ and C $\triangle$ E $\in L_i$. Since B*B*C $\in L_i$ and E*B*D $\in L_i$, B*B*D $\in L_i$, by (TB.4), but by (TB.3) that means $\sim$(B $\triangle$ D) $\in L_i$, contrary to the consistency of L_i.

By the two parts of Lemma 9 it follows that *Bt* is an appropriate relation for a B-model-frame. Given the results of [2], we may then say

LEMMA 10 – b_A is a B-model-frame.

Given the results of [2], it remains only to show

LEMMA 11 – For any $L_i \in \underset{\sim}{L}$, B*B*C $\in L_i$ if and only if I_v(B*B*C,L_i)=T

under the inductive hypothesis for the full lemma, that for any D shorter than B*B*C and any $L_j \in \underset{\sim}{L}$, D $\in L_j$ if and only if I_v(D, L_j) = T.

(a) Suppose B*B*C $\in L_i$. By axiom (B.3), $\diamond$B $\in L_i$ and $\diamond$C $\in L_i$. Hence, by Lemma 7 and Sub-lemma, $[B]_v L_i \neq \wedge$ and $[C]_v L_i \neq \wedge$. Let L_j be any member of $[B]_v L_i$ and let L_k be any member of $[C]_v L_i$. By Sub-lemma, $L_j \in$ (B) L_i and $L_k \in$ (C) L_i. Since B*B*C $\in L_i$, it follows immediately that $L_j\ Bt_{L_i} L_k$, and hence that I_v(B*B*C, L_i) = T.

To prove the converse is harder.

(b) Suppose I_v(B*B*C, L_i) = T. Then $[B]_v L_i \neq \wedge$ and $[C]_v L_i \neq \wedge$, and so both $\diamond$B $\in L_i$ and $\diamond$C $\in L_i$, by Sub-lemma and Lemma 7. Now, for reductio, suppose that B*B*C $\notin L_i$. On this supposition we can then define two sets, L^B and L^C, such that $L^B \in [B]_v L_i$, $L^C \in [C]_v L_i$ and not-($L^B\ Bt_{L_i}$ L^C). That will establish that I_v(B*B*C, L_i) $\neq$ T, contrary to the primary assumption for this case. (Theorem numbers below, such as T.22, etc. refer to [2].)

Define L^B first. Let X^B = { D: B < $\sim$D $\in L_i$} and Y^B = { E: B $\triangle$ $\sim$E $\in L_i$ & $\sim$E*B*C $\in L_i$}. $Z^B = X^B \cup Y^B \cup$ {B} is consistent. For suppose it were not, then there would be $D_1, \ldots, D_n \in X^B$ and $E_1, \ldots, E_m \in Y^B$ such that $D_1, \ldots, D_n, E_1, \ldots, E_m \vdash \sim$B, whence $\vdash$ [(D_1 & ... & D_n) & B] $\supset \sim$(E_1 & ... & E_m). By the logic of subjunctives (T.22), this yields $\vdash$ [(B » D_1 &

... & $B \gg D_n$) & $B \gg B] \supset [B \gg \sim(E_1$ & ... & $E_m)]$. For each $D_i \in X^B$, $B < \sim D_i \in L_i$, hence $B \gg D_i \in L_i$ (by T.18). Also $\Diamond B \in L_i$, so $B \gg B \in L_i$ (T.13). It follows therefore that (i) $B \gg \sim(E_1$ & ... & $E_m) \in L_i$. Since each $E_j \in Y^B$, $B \vartriangle \sim E_1 \in L_i$ and ... and $B \vartriangle \sim E_m \in L_i$. Therefore, $B \vartriangle (\sim E_1 \vee \ldots \vee \sim E_m) \in L_i$ (T.20), so (ii) $B \vartriangle \sim(E_1$ & ... & $E_m) \in L_i$. (i) and (ii) together yield (iii) $B \checkmark \sim(E_1$ & ... & $E_m) \in L_i$. Also, since each $E_j \in Y^B$, $\sim E_1 \, BC \in L_i$ and ... and $\sim E_m \, BC \in L_i$. As a result, $(\sim E_1 \vee \ldots \vee \sim E_m) \, BC \in L_i$, by axiom (B.4a); that is, $\sim(E_1$ & ... & $E_m) \, BC \in L_i$. This and (iii) yield $BBC \in L_i$ by axiom (B.5a), contrary to the supposition made above. Hence Z^B must be consistent. Let L^B be a maximal consistent extension of Z^B.

That X^B is included in L^B is sufficient to prove that $L^B \in (B) \, L_i$ (see the proof of Lemma 7 in [2]) and hence, by Sub-lemma, $L^B \in [B]_v L_i$. It is also a fact that for every D such that $L^B \in (D) \, L_i$, $DBC \notin L_i$. For if not, if $DBC \in L_i$ and $L^B \in (D) \, L_i$, then by Cor.1, $B \vartriangle D \in L_i$, so $\sim D \in Y^B$ and so $\sim D \in L^B$, contrary to the consistency of L^B (given that $L^B \in (D) \, L_i$ implies $D \in L^B$).

Next define L^C. Let $X^C = \{E: C < \sim E \in L_i\}$, $Y^C = \{F: C \vartriangle \sim F \in L_i$ and there is a G such that $L^B \in (G) \, L_i$ & $GB \sim F \in L_i\}$. Let $Z^C = X^C \cup Y^C \cup \{C\}$. That Z^C is consistent is shown as for Z^B down to the point where (iii) $C \checkmark \sim(F_1$ & ... & $F_m) \in L_i$, where each $F_j \in Y^C$. For each such F_j there is a G_j such that $L^B \in (G_j) \, L_i$ and $G_j B \sim F_j \in L_i$. Let these be $G_1, \ldots, G_m$, so that $L^B \in (G_1) \, L_i$ and $G_1 B \sim F_1 \in L_i$ and ... and $L^B \in (G_m) \, L_i$ and $G_m B \sim F_m \in L_i$. Since $L^B \in (G_1) \, L_i$ and ... and $L^B \in (G_m) \, L_i$, it follows that for all of these statements G_i, G_j, etc., $G_i \approx G_j \in L_i$. Hence, $[(G_1$ & ... & $G_m) \checkmark G_1] \in L_i$ and ... and $[(G_1$ & ... & $G_m) \checkmark G_m] \in L_i$. So, by axiom (B.5a), we have $(G_1$ & ... & $G_m) \, B \sim F_1 \in L_i$ and ... and $(G_1$ & ... & $G^m) B \sim F_m \in L_i$, which yields $(G_1$ & ... & $G_m) B(\sim F_1 \vee \ldots \vee \sim F_m) \in L_i$ by repeated applications of (B.4b). So $(G_1$ & ... & $G_m) \, B \sim(F_1$ & ... & $F_m) \in L_i$. But this gives us (iv) $(G_1$ & ... & $G_m) BC$ by axiom (B.5b) and (iii) above. Since $(G_1$ & ... & $G_m) \in L^B$ and $L^B \in (G_j) \, L_i$ for each of these statements G_j $(1 \leq j \leq m)$, $L^B \in (G_1$ & ... & $G_m) \, L_i$. Moreover, $L^B \in (B) L_i$ as established above. Hence $B \vartriangle (G_1$ & ... & $G_m) \in L_i$, by Cor. 1. But that with (iv) means that $\sim(G_1$ & ... & $G_m) \in Y^B$, and so $\sim(G_1$ & ... & $G_m) \in L^B$, contrary to the consistency of that set. Therefore Z^C must be consistent.

Let L^C be any maximal consistent extension of Z^C. The inclusion of X^C in L^C guarantees that $L^C \in (C) L_i$, as shown above with L^B. Hence $L^C \in$

$[c]_v L_i$, by the Sub-lemma.

We now show that not-($L^B Bt_{L_i} L^C$). Suppose the contrary, and let D and E be statements such that $L^B \in (D)L_i$ and $L^C \in (E)L_i$ and $DBE \in L_i$. Since $L^B \in (B)L_i$ and $L^C \in (C)L_i$, $B \Delta D \in L_i$ and $C \Delta E \in L_i$ by Cor. 1. Hence $C \Delta \sim\sim E \in L_i$ and $DB\sim\sim E \in L_i$ by double negation. Thus, there is a G (namely D) such that $L^B \in (G)L_i$ and $GB\sim\sim E \in L_i$. Hence $\sim E \in Y^C$, and so $\sim E \in L^C$, contrary to the fact that $E \in L^C$ and L^C is consistent.

This suffices to show that if $BBC \notin L_i$, then $I_v(BBC, L_i) \neq T$. This, combined with the previous results and the results of [2], completes the proof of the semantical completeness of *LB*. By Lemma 10, b_A is a B-model-frame, and by Lemma 11, $I_v(\sim A, L_0) = T$, so $I_v(A, L_0) \neq T$. Moreover, b_A is proper with respect to v (Lemma 12). Therefore, the non-theorem A is not valid. Hence,

THEOREM III – For any statement A of LB, if $\Vdash A$, then $\vdash A$.

The language LB, as presented so far, does not contain the monadic operators *G*, *B*, *I* and *O*, *F*, *P* of the logic of 'good' and 'ought' of [2]. These may, however, be introduced easily, either by definitions corresponding to (P.7) and (P.9) of Part I above, or as primitives in their own right. Taking the latter course, we should then stipulate truth conditions for statements containing these operators following the rules of [2], p. 257. For example,

$I_v(GA, w_i) = T$ if and only if $I_v(A, w_i) = T$ and $[\sim A]_v w_i \neq \Lambda$, and for all $w_j \in [\sim A]_v w_i$, $w_i Bt_{w_i} w_j$;

$I_v(BA, w_i) = T$ if and only if $I_v(A, w_i) = T$ and $[\sim A]_v w_i \neq \Lambda$, and for all $w_j \in [\sim A]_v w_i$, $w_j Bt_{w_i} w_i$;

$I_v(OA, w_i) = T$ if and only if $[A]_v w_i \neq \Lambda$ and $[\sim A]_v w_i \neq \Lambda$, and for all $w_j \in [A]_v w_i$ and for all $w_k \in [\sim A]_v w_i$, $w_j Bt_{w_i} w_k$.

The axiomatic system L*B* may then be extended to include these operators merely by adding the axiom schemata

(C.1a) $\vdash GA \equiv \top B \sim A$
(C.1b) $\vdash BA \equiv \sim AB\top$

(C.1c) $\vdash OA \equiv AB{\sim}A$,

where $\top$ is any truth-functional tautology. (The other monadic operators should then be included by definitions.) The postulates of *LG* of [2] are then easily shown to be derivable in this extended logic of 'good', 'ought ' and 'better', *LGOB*. Moreover, combining the results of [2] with those established here, we know that every statement provable in *LGOB* is valid, and every valid statement is provable in *LGOB*.

Department of Philosophy
Willamette University
Salem, Oregon 97301 USA

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