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# A BOOLEAN-VALUED VERSION OF GUPTA'S SEMANTICS

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In this paper we shall describe a Boolean-valued version of the intensional semantics of Gupta [2]. We believe that our presentation clarifies some points left obscure in the original work. Furthermore, we give a complete treatment of individuals with partial existence, thus extending the original semantics to cover this important case. The presentation is then followed by a discussion of the limitations of this semantics to handle some problems of philosophy of language. Our paper is a companion to [3] which deals with the semantics of Montague. These two papers thus motivate a new system of intensional logic, the logic of kinds, that may be found in [4] and [5].

## 1. *Introduction*

In the usual presentation of the semantics of quantified modal logic, common nouns and verb phrases (VP) which are translated as formulas with one argument (for instance, "run", "find an apple", "loves John" and "is red") are given the same interpretation. They belong to different grammatical and syntactical categories but the interpretation does not respect their grammatical and syntactical differences. Common nouns and verb phrases are interpreted as intensional properties $W \rightarrow 2^{E^W}$, where $W$ is the set of all possible worlds and $E$ is the set of individuals. We shall consider only count nouns in our presentation, for instance "dog" and "person". Gupta's study proceeds in just this way and only mentions casually other types of common nouns. The idea that it would be better to keep apart the semantical categories of common nouns and verb phrases comes mainly from the following remarks. Before going into this, however, let us say some words on the terminology that will be used.

Count nouns, for instance "dog", are translated as sorts, symbolized as *dog* and interpreted as kinds which are symbolized as dog. Verb phrases, like "run", are translated as formulas with one argument by *run(x)* and interpreted as predicates by $\| run \|$ $(g(x))$. We use sometimes the words property or unary relation instead of the word predicate. We shall use the expression *predicate of a kind* since, as will be explained shortly, all variables are sorted. If we use "run" which is translated as *run(x)*, we must know the sort of $x$ or, in other words, we must know to which kind the predicate is "applied".

The predicates come equipped with a principle of application. The principle of application for predicates of a kind specifies when a member of the kind has the predicate. This principle is not an epistemic criterion and is independent of our capacity to decide if the thing has or not the property. According to Gupta, kinds also come equipped with a principle of application. For example, if we consider the kind dog then the principle of application says what should count as a dog: it specifies, for instance, that a bitch with its puppies does not count as a dog, and that the legs and the tail of a dog do not count as a dog.

Furthermore, Gupta postulates that kinds come equipped with a principle of identity. For instance if we take the kind person, the principle of identity says when a person in a possible world is the same as another person in another possible world. This principle of identity is not an epistemic criterion but a metaphysical counterpart of it. We quote Gupta:

> *It is not the rule by which one determines, say, when an object is the same river as another object. It is rather the metaphysical counterpart of such an epistemic rule. The principle of identity for "river" is the rule in virtue of which an object at a time (and a world) is the same river as an object at another time (and a world).* [2, *page* 2]

The usual interpretation of count nouns such as Montague's takes into account only the principle of application and ignores the principle of identity. To illustrate this Gupta gives the following example. Let us consider a person who takes the plane twice in a week. This person is counted by the airlines companies as two passengers, even if these two passengers are the same person. In the usual semantics such as Montague's we cannot account for this fact. Passengers and persons are finally the same since all objects belong to a unique kind, namely the entities, and the only identity considered is the identity that comes with these entities. In order to take into account the principle of identity, which intuitively will be a different principle for horses, for dogs and for persons, Gupta, following Bressan [1], introduced different sorts in the language. The sorts and their interpretation, the kinds, permit us to understand more clearly some problems related to, for instance quantification. We shall consider sorted elements only in our presentation. Gupta, similarly to Montague, assumes a set of unsorted entities, elements, individuals. We believe that this assumption goes against the spirit of Gupta's own work.

## 2. *Interpretation of count nouns*

We describe here the interpretation of the count nouns with the help of the count noun "person". Gupta analyzes only kinds of the sort person, dog, passenger which are *separated* kinds. We shall define this notion later, but intuitively we say that a kind is separated if when two members of the kind happen to coincide in one possible world then they are the same member. We shall therefore construct those kinds which are separated as sets of individual concepts starting from individuals in their possible worlds or possible circumstances. Members of kinds have total existence in Gupta's approach and as a consequence, just as Montague, he is not able to distinguish between the real and the possible. We generalize this approach by constructing kinds such that their members have only partial existence.

We consider the family $(|person|_w)_{w \in W}$ where intuitively $|person|_w$ is the set of persons that exist in the possible world $w$ and is given by the principle of application. We assume that the principle of identity of person gives an equivalence relation $=_{person}$ on

$$\bigcup_{w \in W} |person|_w = \{(p, w) : p \in |person|_w\}$$

such that

$$(p_1, w_1) =_{person} (p_2, w_2) \wedge w_1 = w_2 \rightarrow p_1 = p_2$$

We now define the notion of individual concept associated with the count noun "person". Let $p \in |person|_w$ and let us define a partial function on $W$, $\alpha^{person}_{<p,w>}$ whose domain is given by

$$w' \in dom\alpha^{person}_{<p,w>} \leftrightarrow \exists p' \in |person|_w \ (\alpha^{person}_{<p,w>} (w), w) =_{person} (p', w')$$

and whose value at $w'$ is $\alpha^{person}_{<p,w>} (w') = p'$. We remark that $p'$ is unique for a given $w'$, by our assumption on $=_{person}$. We shall use the following notation and definition:

$$\|person\|(w) = \{\alpha^{person}_{<p,w>} : p \in |person|_w\}$$

Finally we define the interpretation of "person" as

$$\|person\| = \bigcup_{w\in W} \|person\|(w)$$

In general if $K$ is a count noun we define

$$\|K\|(w) = \{\alpha^{K}_{<k,w>} : k \in |K|_w\}$$

and the interpretation of $K$ as

$$\|K\| = \bigcup_{w\in W} \|K\|(w).$$

So far, we have succeeded in interpreting "person" without the help of unsorted individuals. We have also been able to deduce the notion of an individual concept attached to a particular member of a kind from the principle of identity and application from the notion of an individual-at-a-world/situation. We can imagine that the individual concept attached to a particular person at $w$ describes the trajectory or the history of that particular person through times and possible worlds or possible circumstances.

*Proposition 2.1* $\|person\|(w) \simeq |person|_w$

*Proof.* We define the isomorphism by

$$ev_w(\alpha^{person}_{<p,w>}) = \alpha^{person}_{<p,w>}(w) = p$$

where $p \in |person|_w$.

(a) $ev_w$ is surjective by construction, since we have constructed an individual concept attached to each $p \in |person|_w$ such that $\alpha^{person}_{<p,w>}(w) = p$.

(b) $ev_w$ is injective, that is

$$ev_w(\alpha^{person}_{<p,w>}) = ev_w(\beta^{person}_{<p,w>}) \Rightarrow \alpha^{person}_{<p,w>} = \beta^{person}_{<p,w>}.$$

Let us assume the hypothesis:

$$\alpha^{person}_{<p,w>}(w) = \beta^{person}_{<p,w>}(w).$$

Let $\alpha^{person}_{<p,w>}(w') = p'$ and $\beta^{person}_{<p,w>}(w') = q'$ where $w'$ is in their domain. By the construction of individual concepts we obtain

$$(\alpha^{person}_{<p,w>}(w), w) =_{person} (p', w'), (\beta^{person}_{<p,w>}(w), w) =_{person} (q', w')$$

which implies that $q' = p'$. Hence $\alpha^{person}_{<p,w>} = \beta^{person}_{<p,w>}$. More generally we have $\|K\| \overset{ev_w}{\simeq} |K|_w$ where $K$ is a count noun. □

*Proposition 2.2* $(p_1, w_1) =_{person} (p_2, w_2)$ if and only if

$$\alpha^{person}_{<p_1,w_1>} = \alpha^{person}_{<p_2,w_2>}$$

if and only if

$$\exists\alpha \in \|person\|(w_1) \cap \|person\|(w_2)$$

such that

$$\alpha(w_1) = p_1 \text{ and } \alpha(w_2) = p_2. \ \square$$

*Corollary 2.3* $\coprod_{w \in W} |\text{person}|_w / =_{\text{person}} \overset{\Phi}{\simeq} \|person\| = \bigcup_{w \in W} \|person\|(w)$

*Proof.* Let $\Phi$ be $[(p, w)] \mapsto \alpha^{person}_{<p,w>}$. $\Phi$ is well defined and is injective by the previous proposition. Let $\alpha \in \|person\|(w)$, by definition

$$\alpha = \alpha^{person}_{<p,w>} = \Phi[(p, w)].$$

So $\Phi$ is surjective. □

Let us analyze the problem mentioned previously: the problem of passenger versus person. How will their different principles of identity affect the relations between $|passenger|_w$, $|person|_w$, $\|passenger\|(w)$ and $\|person\|(w)$ ?

First of all we remark that at a world $w$, $|passenger|_w \subseteq |person|_w$. This expresses the fact that a passenger at $w$ is a person at $w$, that each (human) passenger is a person. We also remark that

$$(pa_1, w_1) =_{passenger} (pa_2, w_2) \Rightarrow (pa_1, w_1) =_{person} (pa_2, w_2),$$

where *pa* is an abbreviation for "passenger". Now let us compare $\alpha^{person}_{<pa,\,w>}$ and $\alpha^{passenger}_{<pa,w>}$. By the principle of application of what a passenger is it is clear that

$$dom(\alpha^{passenger}_{<pa,w>}) \subseteq dom(\alpha^{person}_{<pa,w>}),$$

and if $w' \in dom(\alpha^{passenger}_{<pa,w>})$ then

$$w' \in dom(\alpha^{person}_{<pa,w>}) \text{ and } \alpha^{passenger}_{<pa,w>}(w') = \alpha^{person}_{<pa,w>}(w').$$

Thus

$$\alpha^{passenger}_{<pa,w>} = \alpha^{person}_{<pa,w>} \upharpoonright dom(\alpha^{passenger}_{<pa,w>}),$$

where $\upharpoonright$ stands for the restriction of a function.

Let us consider the extension $u$ of $\|passenger\|(w)$ into $\|person\|(w)$. This extension is injective; to each passenger we associate the "underlying person who is that passenger".

$$\|passenger\|(w) \overset{u}{\hookrightarrow} \|person\|(w)$$

$$\alpha^{passenger}_{<pa,w>} \longmapsto \alpha^{person}_{<pa,w>}$$

where $pa \in |passenger|_w \subset |person|_w$. The function $u$ gives rise to a new function

$$U\colon \|passenger\| \to \|person\|$$

which need not be a monomorphism

$$\|passenger\| = \bigcup_{w \in W} \|passenger\|(w),\ \|person\| = \bigcup_{w \in W} \|person\|(w)$$

and the principles of identity are different. The same person who travels twice can then be counted as two different passengers because, so to speak, each one of these passengers has his own domain of existence...on the same person!

### 3. *The language of a first order many sorted logic and its interpretation*

Following our resolution to consider only sorted individuals, we now describe a first order language which will be many sorted, will have a sort operator, a designator operator and two modal operators. The non-logical symbols will fall under the following three heads.

1. The constant sorts: with each "atomic" count noun (for instance, "person", "passenger" and "dog") we associate a constant sort.
2. The n-ary sorted relation symbols: a relational symbol $R$ comes equipped with a natural number n : the number of places of $R$; $R$ is called "n-ary relation symbol"; $R$ comes also equipped with an assignment of a constant sort $K_i$ $i = 1,\dots, n$; $K_i$ is the constant sort of the $i^{th}$ place of $R$. The operative effect of this assignment will be that only variables of the right sort can occupy a given place when forming formulas using $R$. We write $R \subset K_1 \times \dots \times K_n$ to indicate the sorting of $R$.
3. The n-ary sorted operation symbols: an operation symbol $F$ comes equipped with a natural number $n$ which indicates the number of places of $F$; $F$ is called an n-ary operation symbol; $F$ comes equipped with constant sort $K_i$ $i = 1,\dots, n$, $K_i$ being called the constant sort of the $i^{th}$ place of $F$, and also with an additional constant sort $K$, called the sort of the values of $F$. We write $F : K_1 \times \dots \times K_n \rightarrow K$.

We remark that 2. and 3. have been taken from Makkai and Reyes [6].

Let us give an example of an $R$ symbol, the binary relation symbol *is taller than*, and let us suppose that the constant sorts are *dog* and *cat*. We have then that *is taller than* $\subseteq$ *dog* $\times$ *cat*. The function $u$ described before, which could be thought of as the function "underlying" is an example of an operation symbol.

For each sort $K$ we have an infinite set of variables of sort $K$ and a set of constants of sort $K$; we denote the variables and constants respectively by $x^K, y^K,\dots,$ and $c^K, d^K,\dots$. We will not write the indices when the context of use is clear. The logical symbols are: $\neg, \rightarrow, \leftrightarrow, \vee, \wedge, \forall, \exists, \Diamond, \Box$. We define by simultaneous recursion the notion of term relative to a sort, the notions of sort, subsort and formula.

1. The constants and variables of sort $K$ are terms of sort $K$.
2. If $F$ is an operation symbol and if $t_1,\dots, t_n$ are terms of the right con-

stant sorts $K_1, \ldots, K_n$ than $F(t_1, \ldots, t_n)$ is a term of constant sort $K$.

3. If $R$ is a relation symbol and $t_1, \ldots, t_n$ are terms of the right constant sorts $K_1, \ldots, K_n$ then $R(t_1, \ldots, t_n)$ is a formula.
4. If $t$ and $s$ are terms of the same sort then $t = s$ is a formula. (More generally we could require that $t$ and $s$ belong to a sort and a subsort of that sort respectively.)
5. If $\phi$ and $\psi$ are formulas so are $\neg \phi$, $\Box\phi$, $\Diamond\phi$, $\phi \rightarrow \psi$, $\phi \leftrightarrow \psi$, $\phi \vee \psi$, $\phi \wedge \psi$.
6. If $\phi$ is a formula and $x^K$ a variable of sort $K$ then $\exists x^K \phi$ and $\forall x^K \phi$ are formulas
7. If $x^K$ is a variable of sort $K$ and $\phi$ a formula then $\{x^K : \phi\}$ is a sort called a subsort of $K$.
8. If $K$ is a sort, then $\dagger K$ is a term of sort $K$.

We give an example to illustrate 7.
Let $K = person$ and $\phi = sick\,(x^{person})$, $\{x^{person} : sick(x^{person})\}$ is the count noun "person who is sick" formed from the (atomic) count noun "person" and the formula $sick(x^{person})$. We could also form the following count noun: "person who is sick who is red" where $K = person\ who\ is\ sick$ and $\phi = red(x^{person\ who\ is\ sick})$, then we have

$$\{x^{person\ who\ is\ sick} : red(x^{person\ who\ is\ sick})\}.$$

We interpret the language by choosing an arbitrary non empty set $W$ thought of as the set of "possible worlds"; we associate to each sort $K$ a family $(|K|_w)_{w \in W}$ thought of as the $K$s which exist at $w$, and an equivalence relation $=_K$ on $\bigsqcup_{w \in W} |K|_w$ in terms of which we define

$$\|K\| = \bigcup_{w \in W} \|K\|(w).$$

Notice that $\|K\|(w)$ can be recovered from $\|K\|$ and $w$ as the set $\alpha \in \|K\|$ satisfying $w \in dom(\alpha)$. We remark that Montague has only one sort $U$ interpreted as $\|U\| = E^W$ where $E$ is thought of as the set of "possible individuals" rather than existing individuals. We introduce as many sorts as there exist count nouns.

When Gupta considers $K$ given with a principle of identity, he really studies objects of the form $(\|K\|, \delta)$ where $\delta : \|K\| \times \|K\| \rightarrow 2^W$ given by $\delta\,(\alpha, \beta) = \{w \in dom\,\alpha \cap dom\,\beta : \alpha(w) = \beta(w)\}$ for all $\alpha, \beta \in \|K\|$. Intuitively $\delta$ expresses the extent to which $\alpha$ and $\beta$ coincide. We say that

$(\|K\|, \delta)$ is separated if

$$\delta(\alpha, \beta) = \begin{cases} dom(\alpha) = \delta(\alpha,\alpha) & \text{if } \alpha = \beta \\ \varnothing & \text{if } \alpha \neq \beta \end{cases}$$

Since for separated $(\|K\|, \delta)$ $\delta$ is completely determined by $\epsilon(\alpha) = \delta(\alpha, \alpha)$, We write sometimes $(\|K\|, \epsilon)$ rather than $(\|K\|, \delta)$. We interpret count nouns as sets obtained this way. Such interpretation follows the intuition that if two persons coincide in one possible world then they coincide in all possible worlds belonging to their domain of existence. Notice however that this does not cover all common nouns: as already mentioned words like "water" and "temperature" which are not count nouns are not analyzed in this study. Gupta's semantics differs from Montague's semantics in that for each $w$, only the set of existing individuals is considered. For each $w$, Montague considers the set of all possible individuals $E$. In Gupta's semantics the $|K|_w$s may differ.

We now interpret the sorts, the sorted constants, the operation sorted symbols and the relation sorted symbols as follows:

1. Each constant sort $K$ is interpreted as a separated set $(\|K\|, \epsilon_K)$
2. Each constant $c$ of sort $K$ is interpreted as a function

   $$\|c\| : W \rightarrow \|K\|$$

   such that

   $$\|c\|(w) \in \|K\|(w)$$

3. Each sorted operation symbol $F : K_1 \times \ldots \times K_n \rightarrow K$ is interpreted as a map

   $$\|F\| : \|K_1\| \times \ldots \times \|K_n\| \rightarrow \|K\|$$

   satisfying the condition

   $$\epsilon_{K1}(\alpha_1) \cap \ldots \cap \epsilon_{Kn}(\alpha_n) \subseteq \epsilon_K(\|F\|(\alpha_1, \ldots, \alpha_n))$$

4. Each sorted relation symbol $R \subseteq K_1 \times \ldots \times K_n$ is interpreted as a map

$$\|R\| : \|K_1\| \times \ldots \times \|K_n\| \to 2^W$$

satisfying the condition

$$\|R\| (\alpha_1, \ldots, \alpha_n) \subseteq \epsilon_{Kn} (\alpha_1) \cap \ldots \cap \epsilon_{Kn} (\alpha_n)$$

We shall give an example of a constant, an operation symbol and a relation symbol.

To interpret a sorted constant such as "Socrates", as well as to interpret descriptions, we need a function

$$i^*_{person} : W \to \|person\|$$

such that $i^*_{person} (w) \in \|person\| (w)$ should be thought of as "the non-existent *person*". In terms of $i^*$ we define the interpretation of "Socrates" as the function

$$\|Socrates\| : W \to \|person\|$$

in the following way:

$$\|Socrates\| (w) = \begin{cases} Socrates \in \|person\| \text{ if } w \in dom(Socrates) \\ i^*_{person} (w) \text{ if } w \notin dom(Socrates) \end{cases}$$

We have seen how to construct $(\|person\|, \epsilon)$ and $(\|passenger\|, \epsilon)$. We interpret the operation sorted symbol

$$u : passenger \to person \text{ as } \|u\| : \|passenger\| \to \|person\|$$

where $\|u\| (\alpha^{passenger}_{<pa,w>}) = \alpha^{person}_{<pa,w>}$. Clearly, the condition

$$\epsilon_{passenger} (\alpha^{passenger}_{<pa,w>}) \subseteq \epsilon_{person} (\|u\| (\alpha^{passenger}_{<pa,w>}))$$

is satisfied.

The relation sorted symbol *sick* $\subseteq$ *person* is interpreted as follows

$$\|sick\| : \|person\| \to 2^W$$

which satisfies the condition

$$\|sick\|(\alpha^{person}_{<pa,w>}) \subseteq \epsilon_{person}(\alpha^{person}_{<p,w>}).$$

Intuitively, the person is sick at most as long as she (or he) exists. The binary relation sorted symbol *is taller than* $\subseteq$ *tree* $\times$ *flower* is interpreted as

$$\|is\ taller\ than\| : \|tree\| \times \|flower\| \rightarrow 2^W$$

which satisfies the condition

$$\|is\ taller\ than\|(\alpha, \beta) \subseteq \epsilon_{tree}(\alpha) \cap \epsilon_{flower}(\beta)$$

An assignment is a function $g : Var_K \rightarrow \|K\|$ for each $K$. For a given assignment, we interpret sorts, terms and formulas by recursion in the usual fashion.

Given an assignment, we define forcing as a relation between a possible world and a formula as follows:

$$w \Vdash \phi(x^{K_1}_1, \ldots, x^{K_n}_n)[g]$$

where $g(x^{K_i}_i) \in \|K_i\|(w)$ (equivalently $w \in dom(g(x^{K_i}_i)) = \epsilon(g(x^{K_i}_i))$

We give some examples to illustrate the interpretation of formulas.

1. $w \Vdash x^K = y^K\,[g]$ if and only if $w \in \delta(g(x^K), g(y^K))$ which is the case just when
$g(x^K)(w) = g(y^K)(w)$

2. $w \Vdash u(x^{passenger}) = y^{person}\,[g]$ iff $\|u\|(g(x^{passenger})(w) = g(y^{person})(w)$ since $(\|person\|, \delta)$ is separated we can conclude that

$$\|u\|(g(x^{passenger})) = g(y^{person})$$

3. If $R \subseteq K_1 \times \ldots \times K_n$ and $w \in \epsilon(g(x^{K_i}_i))$ for all $i$, then

$$w \Vdash R(x^{K_1}_1, \ldots, x^{K_n}_n)[g] \text{ iff } w \in \|R\|(g(x^{K_1}_1), \ldots, g(x^{K_n}_n))$$

4. $w \Vdash \neg\phi(x^{K_1}_1, \ldots, x^{K_n}_n)[g]$ iff $w \nVdash \phi(x^{K_1}_1, \ldots, x^{K_n}_n)[g]$

5. $w \Vdash \exists x^K\, \phi(x^K)[g]$ iff $\exists\alpha \in \|K\|(w)\ w \Vdash \phi(x^K)[g(\alpha/x^K)]$

6. $w \Vdash \Box\phi(x_1^{K_1}, \ldots, x_n^{K_n})[g]$ iff

$$\forall w' \in \bigcap_{x_i^{K_i} \in FV(\phi)} \epsilon(g(x_i^{K_i}))\ w' \Vdash \phi(x_1^{K_1}, \ldots, x_n^{K_n})[g].$$

The restriction of the quantifier to the intersection means that we do not look at all the possible worlds but rather at those that belong to the common domain of existence of the $g(x_i^{K_i})$.

*Proposition 3.4* $x^K = y^K \rightarrow \Box(x^K = y^K)$

*Proof.* $w \Vdash ((x^K = y^K) \rightarrow \Box(x^K = y^K))[g]$ if and only if

$$w \Vdash (x^K = y^K)[g] \Rightarrow w \Vdash \Box(x^K = y^K)[g]$$

if and only if

$$g(x^K)(w) = g(y^K)(w) \Rightarrow \forall w' \in \epsilon(g(x^K)) \cap \epsilon(g(y^K))\ w' \Vdash x^K = y^K[g]$$

Since $(\|K\|, \delta)$ is separated,

$$g(x^K)(w) = g(y^K)(w) \text{ iff } g(x^K) = g(y^K)$$

implies that $g(x^K)(w') = g(y^K)(w')$. □

We now show that one half of the Barcan rule is satisfied, namely

*1.* $\Box\forall x\phi(x) \rightarrow \forall x\Box\phi(x)$.

Then, we construct a model showing that the other half of the Barcan rule, namely

2. $\forall x\Box\phi(x) \rightarrow \Box\forall x\phi(x)$

is not satisfied.

1. Let $M$ be a model and $w$ a possible world. We must show that

$$w \Vdash \Box\forall x^K\ \phi(x^K)[g] \Rightarrow w \Vdash \forall x^K\ \Box\phi(x^K)[g]$$

Let $\alpha \in \|K\|(w)$ and $w' \in \epsilon(\alpha)$ then we must show that $w' \Vdash \phi(x^K)$

$[g(\alpha/x^K)]$. But by hypothesis, $w' \Vdash \forall x^K\ \phi(x^K)[g]$ since $w' \in \epsilon(g(x^K))$. We notice that

$$w' \in \epsilon(g(x^K)) = \epsilon(\alpha) \Rightarrow \alpha \in \|K\|(w').$$

In particular, then $w' \Vdash \phi(x^K)[g(\alpha/x^K)]$.

2. Let $W = \{0, 1\}$ and $|A|_0 = \{p_0\}$ and $|A|_1 = \{c_1, p_1\}$ we define $(\|A\|, \delta)$ where $\|A\| = \{c, p\}$, $p(0) = p_0$, $p(1) = p_1$, $c(1) = c_1$, it is clear that $(\|A\|, \delta)$ is separated. We see that $\|A\|(0) = \{p\}$ and that $\|A\|(1) = \{c, p\}$. We consider the following predicate:

$$\|P\| : \|A\| \rightarrow 2^{\{0, 1\}}$$

where

$$\|P\|(c) = \varnothing \subseteq \epsilon(c) = \{1\} \text{ and } \|P\|(p) = \{0, 1\} \subseteq \epsilon(p) = \{0, 1\}$$

.

Now let us prove that

$$0 \nVdash (\forall x^A\ \Box P(x^A) \rightarrow \Box \forall x^A\ P(x^A))[g]$$

or equivalently that

$$0 \Vdash \forall x^A\ \Box P(x^A)[g] \not\Rightarrow 0 \Vdash \Box \forall x^A\ P(x^A)[g]$$

or that

$$0 \Vdash \forall x^A\ \Box P(x^A)[g] \text{ and } 0 \nVdash \Box \forall x^A\ P(x^A)[g].$$

Let us analyze the two members of this conjunction.

(a) $0 \Vdash \forall x^A\ \Box P(x^A)[g]$ if and only if

$$\forall \alpha \in \|A\|(0)\ 0 \Vdash \Box P(x^A)[g(\alpha/x^A)]$$

if and only if

$$\forall \alpha \in \|A\|(0)\ \forall w \in \epsilon(g(x^A))\ w \Vdash P(x^A)[g(\alpha/x^A)].$$

But since there is only $p$ at $\|A\|(0)$, we have

$$\forall w \in \epsilon(p) = \{0, 1\}\ w \Vdash P(x^A)[g(p/x^A)].$$

Hence, we obtain $0 \Vdash P(x^A)[g(p/x^A)]$ and $1 \Vdash P(x^A)[g(p/x^A)]$. We conclude that $\|P\|(p) = \{0, 1\}$ which is true by the definition of $P$.

(b) $0 \nVdash \Box \forall x^A\ P(x^A)[g]$, let us suppose that $0 \Vdash \Box \forall x^A\ P(x^A)[g]$. We then have that

$$\forall w\ w \Vdash \forall x^A\ P(x^A)[g]$$

if and only if

$$\forall w\ \forall \alpha \in \|A\|(w)\ w \Vdash P(x^A)[g(\alpha/x^A)].$$

There are two cases to consider.

1. $w = 0, p \in \|A\|(0)\ 0 \Vdash P(x^A)[g(p/x^A)]$ hence $0 \in \|P\|(p)$.
2. $w = 1, \{p, c\} = \|A\|(1)\ 1 \Vdash P(x^A)[g(p/x^A)]$ hence $1 \in \|P\|(p)$ but $1 \nVdash P(x^A)[g(c/x^A)]$ hence $1 \notin \|P\|(c) = \varnothing$ and we conclude that

$$0 \nVdash \Box \forall x^A\ P(x^A)[g].$$

We can imagine that $A$ translates "animals on Joe's farm" and $P$ translates "is a pig". We can read this affirmation as follows: "Every animal on Joe's farm is necessarily a pig" does not imply that "necessarily every animal on Joe's farm is a pig". Gupta [2, page 43] explains this as follows:

> *For although it may be true that "Every animal on Joe's farm is essentially a pig", for on Joe's farm there are only pigs, it does not follow that "It is necessary that every animal on Joe's farm is a pig". Joe might also have grown chickens on his farm.*

Gupta says that the two halves of the Barcan rule are false in his system. To show that 1. is false he gives the following example: "It is necessary that every bachelor is unmarried" is true, but "Every bachelor is necessarily unmarried" is false. We believe that the trouble with this example and with similar examples found in the literature is the following: since variables are sorted, the formalization of these sentences in our language is not uniquely determined. For instance, if "It is necessary that every bachelor is unmarried" is formalized as $\Box\forall x\ unmarried(x)$, where "x" is a variable of sort *bachelor* and "Every bachelor is necessarily unmarried" is formalized as $\forall x\ \Box unmarried(x)$, then our argument that 1. is true implies that the purported counterexample of Gupta is not a genuine one. On the other hand, we might formalize "It is necessary that every bachelor is unmarried" as $\Box\forall x\ bachelor(x) \rightarrow unmarried(x)$, where "x" is a variable of sort *man*. If furthermore, "Every bachelor is necessarily unmarried" is formalized as $\forall x\ bachelor(x) \rightarrow \Box unmarried(x)$, then Gupta's counterexample is correct and the argument that the first sentence implies the second is an example of what Mates [4, page 117] calls the "fallacy of the slipped modal operator". This example shows that formalization of sentences of natural languages has to be handled carefully.

We remark that any formula $\phi(x_1^{K_1},\ldots, x_n^{K_n})$ gives rise to a map

$$\|\phi(x_1^{K_1},\ldots, x_n^{K_n})\| : \|K_1\| \times \ldots \times \|K_n\| \rightarrow 2^W$$

defined by

$$\|\phi(x_1^{K_1},\ldots, x_n^{K_n})\|(\alpha_1,\ldots, \alpha_n) =$$

$$\{w \in \bigcap_{i=1,\ldots,n} domg(x_i^{K_i}) : w \Vdash \phi(x_1^{K_1},\ldots, x_n^{K_n})[g(\alpha_i/x_i^{K_i})]\}.$$

This allows us to define the interpretation of a subsort $\{x^K : \phi\}$ as $(\|K\|, \delta_\phi)$ where $\delta_\phi : \|K\| \times \|K\| \rightarrow 2^W$ and $\delta_\phi(\alpha, \beta) = \|\phi(x)\|(\alpha) \cap \delta(\alpha, \beta)$. Let us, for example, consider the sort *person* and the unary formula *x is sick*. We are restricting the domain of existence of a person to the extent that the person is sick (her domain of sickness, so to speak) to obtain the sort *person who is sick*.

The interpretation of the description $\dagger K$ is the function

$$\|\dagger K\| : W \rightarrow \|K\|$$

defined by

$$\|\dagger K\|(w) = \begin{cases} \alpha & \text{if } \|K\|(w) = \{\alpha\} \\ i^*(w) & \text{if } \|K\|(w) \text{ is not a singleton.} \end{cases}$$

Notice that $\|\dagger K\|(w) \in \|K\|(w)$ for all $w$. In terms of descriptions we can think of the interpretation of "Socrates" already given as being the interpretation of the description "the person who is Socrates".

## 4. *Criticisms and conclusion*

Gupta works with first order logic only. In another paper [3], we have pointed out the necessity of having a higher order logic, and we shall not elaborate on that point here. Furthermore, Gupta deals only with necessarily existent objects. Usual kinds such as person and dog have members that come in and go out of existence. In our presentation, we have eliminated this limitation by introducing the coincidence relation and the predicate of existence.

Another difficulty with Gupta's approach is his uncontrolled notion of "possible K", where K is a kind. This notion is needed to define modal constancy of the interpretations of some count nouns. For instance, person is modally constant if and only if possible persons are persons. In our view, this notion of "possible K" is not cogent, as we shall argue presently. In our view, every kind should be modally constant by definition, so to speak, and "possibility" and "necessity" may be applied only to predicates of kinds and not to kinds themselves. This way of considering "possibility" (and "necessity") agrees with the grammar of these notions. Suppose that we find an archeological site with skeletons of some anthropoids. If we are asked whether some are hominoids, we could naturally reply that "three of these skeletons are possibly hominoid skeletons" or "it is possible that three of these skeletons are hominoid skeletons", but we would not say "there are three possible hominoid skeletons". Similarly, we do not say that Mr. and Ms. X have twelve possible children, but rather that it is possible for Mr. and Ms. X to have twelve children. Independently of grammar, we question the cogency of this notion of "possible K" on the basis that neither the principle of application nor the principle of identity is well defined for them. In other words, we deny that "possible K" is a kind. As regards the principle of application, does an apple jelly count as a possible apple? Does a piece of junk of metal count as a possible car? As regards the principle of identity, Quine has already asked the relevant question: How many fat men are there in the door?

We can recover some of the intuitions of Bressan and Gupta on modal constancy by applying this notion to predicates rather than to kinds, which are for us, as already mentioned, automatically modally constant. Thus we could say, for instance, that "to be an apple" is modally constant as a predicate of fruit, although presumably will not be modally constant as a predicate of ingredient in a recipe.

This analysis shows the importance of distinguishing sharply between kinds and predicates of kinds.

Another problem with Gupta's analysis of common nouns is his distinction between common nouns such as "man" on the one hand and "man born in Jerusalem" on the other. According to Gupta [2, page 35], common nouns such as "man", "number" and "river" express sorts that give essential properties of objects, whereas "man born in Jerusalem" does not express such a sort, because "being born in Jerusalem" is not an essential property of any man. This puts the notorious problem of untangling essential and not essential properties at the very foundation of his theory.

A further problem with Gupta's semantics is his notion of individual concepts. We think that individual concepts introduce irrelevant problems at the foundational level and should be avoided. Let us recall that in the usual semantics (Montague [8], Gupta [2], Scott [11]), kinds are envisaged as sets whose members are individual concepts which are total or partial functions defined over situations. The trouble with this view is simply this: what is the principle of identity for kinds? Clearly, identity of two functions means identity of their values, but where do the values live, say for the kind person? We seem to need a kind other than person with new principles of application and identity to receive the values of the individual concepts. But person was precisely the kind needed to provide a notion of identity for their members! Furthermore, this new kind should again be a set of individual concepts whose principle of identity should be defined in terms of a new kind, etc.

Finally, although we have used the terms "principle of application" and "principle of identity" to follow Gupta, we believe that these notions are problematic. In fact, according to Gupta himself,

> *Common nouns, like predicates, are true or false of objects. They divide all the objects in the world into two classes: those objects that fall under them, and those that do not. That is, common nouns, like predicates, supply a principle of application.*

Thus, unsorted entities seem to be required for the formulation of these principles, a requirement that goes against the spirit of Gupta's own work.

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