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MODAL SEMANTICS

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Modal Predicate Logic is often thought to be a mess, but formalising the underlying predicate logic by means of the epsilon calculus offers some hope of ordering it. For the epsilon calculus is a neater form of predicate calculus which includes, within itself, terms for individuals. There is some difficulty understanding modal epsilon calculi, however. One was first described explicitly in Melvin Fitting's [1] (see also [2]); a significantly different one was sketched in Richard Routley et al's [6] (see also my [8]). But the expected formalisation of the latter has not appeared, in part because of the singular difficulty in understanding it. It involves a considerable review of the basics in modal semantics. That review is what I aim to provide here, showing first the fallacy involved in treating modal epsilon calculi as in [1] and [2], then going back to assess the material in Robert Stalnaker and Richmond Thomason's [7] and [9], on which this formalisation was based. This, rather neatly, also enables me to end by establishing what formalisation there can be for the semantics of [6] and [8].

1

The idea in the modal epsilon calculi of [6] and [8] is simply to add intensional operators to a normal first-order language, introducing epsilon terms for all the resulting predicates, whose distribution is then governed by the characteristic epsilon axiom

$$Py \supset P\epsilon xPx,$$

(where y is free for x in Py). The quantifiers can then be introduced by definition: $(\exists x)Px$ is $P\epsilon xPx$. For full details of the extensional predicate calculus augmented in this way, to get the traditional, unmodalised epsilon calculus, see [4]. The key semantical fact about epsilon terms, of course, is that 'ϵxPx' picks its referent from amongst the P's, if there are any, but draws arbitrarily from the universe, if not.

The Barcan Formulae are straightforwardly valid in the proposed system, but the central formula valid in the system of [6] and [8], though, as we shall see, unprovable in that of [1] and [2], is

$$L(\exists x)Px \supset (\exists x)LPx.$$

This formula is valid (see [8] p.215), simply because $(\exists x)Px$ is logically equivalent to $P\epsilon xPx$, and ϵxPx is an individual term. Hence the antecedent is equivalent to $LP\epsilon xPx$, from whence, by existential generalisation, we get the consequent. More complexly (see [6] p.307), the formula is derivable from the main predicate calculus thesis which justifies the introduction of epsilon terms, namely

$$(\exists y)((\exists x)Px \supset Py).$$

Hughes and Cresswell argue against the exportation of the quantifier in the given modal formula (see [3] p.144), but their counterexample is not compelling (see [8] p.216, and [6] p.307.) However, the way to the appropriate semantics was obscured in [6], for there (see p311) the fact that what descriptions are about may vary from world to world seemed to cast into doubt whether the evaluation of epsilon terms could be world invariant. We shall see, in the end, that their 'evaluation' (if that is still the right word) is indeed world invariant, but in [6] it seemed that it had to be world relative, which would have lead to a system like [1], and [2].

Fitting put the crucial point involved in seeing things his way like this ([1] p.103):

> If $X(x)$ is a formula with one free variable, x, classically ϵxX is intended to be the name of a constant such that, if $(\exists x)X(x)$ is true, $X(\epsilon xX)$ is true. But in a Kripke S4 model, $(\exists x)X(x)$ may be true in two possible worlds but yet there may be no single constant, c, such that $X(c)$ is true in both worlds. Thus ϵxX cannot be thought of as a constant in an ϵ-calculus S4. Instead we treat ϵxX as a function defined on the collection of possible worlds, and such that, if $(\exists x)X(x)$ is true in some possible world, the value of ϵxX at that world is a constant, c, such that $X(c)$ is true.

But, given that Fitting confusedly (though quite normally) sometimes uses 'ϵxX' for itself, and sometimes its own name, there is a fallacy here he is not clearly not committing. As we shall see, that fallacy, and the confusion of use and mention, compound into the central defect at the base of classical modal semantics. The point involved in the fallacy is quite elementary, but it is perhaps easily overlooked, since, as a result, almost a 'gestalt switch' is needed to get out of thinking in classical semantical terms and into the right mental frame to see the matter correctly. For, remembering that identities are necessary while non-identities (in general) are not (see [3] p. 190), suppose, for instance, that a, b, and ϵxPx are, in fact, i.e. in the actual world, distinct. It still might be the case that, in world α $\epsilon xPx=a$, while in world β $\epsilon xPx=b$ (even, indeed, it might be that $\epsilon xPx=a=b$). Suppose further, though, that, in world α, Pa, while, in world β, Pb, then, despite the fact that it is not truly the case that a=b, it is still the case that there is an object, namely ϵxPx, such that in both α and β it is P. Generalising, it might still be the case that there is an x which in all worlds is P, i.e. $(\exists x)LPx$, although in different worlds that thing is identical with things which are different in fact.

Now, only if $(\exists!x)Px$ is the case is ϵxPx (sic) a 'constant' in one sense, i.e. an object identifiable by means of its properties. But, even when the existence and uniqueness of a P are not guaranteed, 'ϵxPx' (sic) is still a 'constant' in the only sense needed in logic: it is an individual term even if its reference is indeterminate. Indeed the object referred to (obtainable, note, just by dropping the quotes) still has an identity then, even if it is a fiction, i.e. has no determinate contingent properties. For, at least, we know bare facts about it like $(\exists y)(y=\epsilon xPx)$, and so, for instance, we know $\epsilon xPx=\epsilon y(y=\epsilon xPx)$. Hence there is no formal objection to ϵxPx taking the place of c above, and so, given $L(\exists x)Px$, and hence $LP\epsilon xPx$, the fact that ϵxPx might have different identities in different worlds is no bar to deriving $(\exists x)LPx$. Any *actual* identity will be maintained through all possible worlds, as above, but differing further identities, in different worlds, may be available, as well — even when ϵxPx is a 'constant' in the full sense. None of this, however, prevents the exportation of the quantifier.

Clearly, therefore, the exportation of the quantifier in our central formula does not thereby export any identity save any actual one. And not only Fitting, but, as we shall see, a whole tradition within modal logic seems to have lost sight of this fact. To be the same thing in another world an object need only have its actual identity, indeed it is just that which allows the 'quantifying in'. More well known, of course, (though still often a locus for misunderstanding) is the fact that exportation of the quantifier in such forms as the above does not thereby export any general property. Thus it does not follow without some specification of 'L', for instance, that we have, say

$$L(\exists x)(Px.Qx) \supset (\exists x)(Px.LQx).$$

This would follow, however, if Lp entailed p. Indeed, in that case, if 'P' is individuating, i.e. contains a uniqueness clause, we have a pair of well-known expressions, symbolisable using the varying scopes of Russellian iota terms. On the left we have a 'secondary' form with smallest scope, on the right we have a 'primary' form, with widest scope, and there is an entailment between them. It will be of some importance in what follows that we have clear just how such pairs of forms are to be read. We should read them as their quantificational expression requires:

> Necessarily there is one and only one President of the U.S. and he is a citizen of the U.S.
>
> There is one and only one President of the U.S. and he is necessarily a citizen of the U.S.

Russell, of course, would have preferred to read the two forms as both about the subject 'the President of the U.S.'. But he lacked the 'logically proper name' to make his formula '$Q\iota xPx$' an elementary proposition. Stalnaker and Thomason, we shall now see, were amongst the first to provide this.

2

Fitting, in fact, to formulate his version of a modal epsilon calculus, worked with the previous semantics for a variety of epsilon term used by Stalnaker and Thomason in [7] and [9]. We must now look at some of the details of Stalnaker and Thomason's semantics, before using it to approach the overall issue of modifying more appropriately the classical one. For, while these two writers used an iota term in their theory expressly without the traditional Russellian existential and uniqueness implications, capturing thereby a form of 'logically proper name', their treatment was confused in certain ways, and therefore still obscured what the proper semantics of such expressions is in modal contexts. As we saw with the point about the identity of

ϵxPx before, the required semantics is one which allows for 'quantifying in', and therefore transparency in modal contexts, and so it allows a form of 'direct reference' which needs no intermediary. What the 'rigid designator' 'ϵxPx' means in any world is not given by its (contingent) identities in that world, since, expressly because of the transparency, its meaning is world invariant. Maybe this account of the meaning of certain terms is not a 'semantics' in the traditional sense, but a 'semantics' of this kind is what we are aiming for, and Stalnaker and Thomason approached it, even if they could not properly grasp its immediacy.

Stalnaker and Thomason said ([9] p.363):

> In contrast with the Russellian analysis, definite descriptions are treated as genuine singular terms; but in general, they will not be substance terms [i.e. carry the implication of a property.] An expression like ιxP(x) is assigned a referent which may vary from world to world. If in a given world there is a unique existing individual which has the property corresponding to P, this individual is the referent of ιxP(x); otherwise, ιxP(x) refers to an arbitrarily chosen individual which does not exist in that world.

Stalnaker and Thomason's ιxP(x) (sic), therefore, resembles $\epsilon x(Px.(\forall y)(Py \supset y=x))$, with the proviso that *this object* exists in every world, even though it needn't have its inscribed *properties* there. For, of course, while

$$(\exists z)(z=\epsilon x(Px.(\forall y)(Py \supset y=x)))$$

is a necessary truth, the very definition of epsilon terms allows that, for instance,

$$P\epsilon x(Px.(\forall y)(Py \supset y=x))$$

is not necessarily true. Note also that, since, unlike with Russellian iota terms, there are no scope distinctions with epsilon terms, the general modal form

$$LQ\epsilon x(Px.(\forall y)(Py \supset y=x))$$

does not have a mate with the modality in another place.

Using their kind of term, however, Stalnaker and Thomason do try to separate *two* expressions of this general form. Thus they try to distinguish ([7] p.203)

Necessarily, the President of the U.S. is a citizen of the U.S.

The President of the U.S. is necessarily a citizen of the U.S.

where, of course, unlike in the Russellian case at the end of the last section, these do not have variously quantified, but only individual term subjects. They try to distinguish two such expressions, formally, using a modified version of lambda abstraction which we shall look at in a moment. But it is more important to see first that their glosses on what they are doing show their intended distinction is really a quite different one. Several things must be noted. First, these two new linguistic forms are, in fact, the same, given a properly referential, i.e. epsilon term replaces 'the President of the U.S.', since there are no scope distinctions with such terms, as before. If there is, in fact, a distinction, therefore, it must lie elsewhere. But the former of these linguistic forms Stalnaker and Thomason gloss as ([7] p.203) 'It is necessarily the case that any President of the U.S. is a citizen of the U.S.', and this is indeed not directly referential. It has the form

$$L(\forall x)(Px \supset Qx),$$

which certainly is expressible using epsilon terms, viz

$$L(Pd \supset Qd),$$

where $d = \epsilon x \neg(Px \supset Qx)$, but that shows clearly it is not of the form

$$LQ\epsilon y(Py.(\forall x)(Px \supset y=x)).$$

In addition, the first of these expressions is necessarily true, given the requirements for being President of the U.S., but the second expression, in the same material case, is not necessarily true. Clearly, I judge, it is these two forms which Stalnaker and Thomason are trying to separate in their discussion.

In a later gloss Stalnaker and Thomason affirm their distinction in a parallel pair of cases. They say ([7] p.205): 'Necessarily, the President of the U.S. is a president of the U.S., but it is not the case that the man who is President of the U.S. is necessarily a President of the U.S.'. This reinforces the present judgement about what they had in mind in the first kind of case. It is indeed a modalised universal conditional. For certainly again

$$L(\forall x)((Px.(\forall y)(Py \supset y=x)) \supset Px)$$

is necessarily true, while

$$LP\epsilon x(Px.(\forall y)(Py \supset y=x))$$

is not. Indeed, since Stalnaker and Thomason grant that, if when there is no unique existing P in a world, their iota term refers to an arbitrarily chosen individual, it is not clear how they can think that their own formalisation of the first case, namely

$$LP\iota xPx$$

is necessarily true. An arbitrarily chosen individual is not necessarily President of the U.S.

It is not certain, therefore, that the semantics these two authors provide to separate two referential forms is really serving that stated purpose. Stalnaker and Thomason distinguish $LQ\iota xPx$, i.e.

$$L(\lambda xQx(\iota xPx)),$$

in the first kind of case from,

$$\lambda xLQx(\iota xPx),$$

in the second kind of case, by treating 'ιxPx' as a functional term whose reference varies from world to world. But such a distinction not only is not needed, for the cases in hand, it would depart from the usual definitional relation between $\lambda xQx(\iota xPx)$ and $Q\iota xPx$. Indeed, given that relation is necessary, then both $L(\lambda xQx(\iota xPx))$ and $\lambda xLQx(\iota xPx)$ reduce to $LQ(\iota xPx)$, which is yet another way of showing there are not two forms of the referential expression.

In sum so far, we do not need to depart from the traditional abstraction principle to distinguish formally the different stated kinds of expression in this area. For, in the first place, there aren't two relevant forms with the referential subject $\epsilon y(Py.(\forall x)(Px \supset y=x))$, and, in the second place, other, distinct forms, which Stalnaker and Thomason clearly had in mind, have readily available formal expressions using just the devices of ordinary quantification theory. This means that not only did Fitting's modal epsilon calculus illicitly import functional terms where there are none, the original motivation for such terms in Stalnaker and Thomason's work was wanting, as well. By contrast, it is seeing that (constant) epsilon terms are not functions over different possible worlds which will enable us now to formalise not just 'logically proper names' but also 'rigid designators', and so obtain transparency, i.e. 'direct reference', without any intermediary. Indeed that will also mean we shall do without any 'semantics', in one traditional sense.

3

Remarkably, it is Stalnaker and Thomason's very own distinction which helps us to expose the fundamental move which must be made to take us away from the old modal semantics and towards the new one. That is not just because they may be guilty of committing the same fallacy we suspected was committed by Fitting, though it is important, first of all, to see their relation to that fallacy, as well. Thus, when attempting to falsify, on their understanding

$$L(\lambda x Px(\iota x Px)) \supset \lambda x LPx(\iota x Px),$$

they specifically take ([7] p.205, [9] p.366) $\lambda x Ax(t)$ to be true in world α in case the thing referred to by t (sic) in α satisfies A (sic) in α. This enables them, it may seem, to construct a falsifying model in which there are two worlds, α and β, each accessible to itself, with the latter accessible to the former, and two individuals 1, and 2. 1 is the only P in α, 2 is the only P in β. The left hand side of their conditional is then, surely, true in α because in every world accessible to α the only P there is indeed P; but the right hand side, surely, is false, because the only P in α is not P in β. Hence, it seems, the conditional is false.

Now one can follow this argument through, and many like it in classical modal semantics, and it still not dawn on one what fallacy is committed. Indeed, as before, a complete 'gestalt switch' is probably involved. For while 1 and 2 are distinct, in fact, i.e. in the actual world, they need not be distinct in β, so the case I have described provides no clear countermodel. Indeed, if α is the actual world, as it must be for the formula as stated, then any identity there between ιxPx and 1 must carry through to β, as we saw before, showing that 1=2 will certainly hold in β, as I have stated the case. But that will mean the right hand side is not false. Indeed, the conditional is thus shown to be not falsifiable in any model.

But my way of presenting the model is significantly different from Stalnaker and Thomason's. My way of presenting the model is plausible, and not clearly not intended, because of the persistent use/mention confusion in this tradition. But, on Stalnaker and Thomason's way of presenting the model, the conditional *is* falsifiable, which shows that different expressions are involved in mine and their cases. Moreover, somewhat miraculously, the difference between these two forms of speech is exactly what the falsity of the conditional in Stalnaker and Thomason's case is about. Indeed, the difference exposed, in Stalnaker and Thomason's way of presenting the model, *directly shows the difference between the two sorts of semantics*. For, as Stalnaker and Thomason would have it,

L(λxAx(t))

says that in every world the object denoted by 't' there has the property denoted by 'A' there. And that significantly is not to say that *t* there has the property *A* there (which is the way I put such matters.) For the words 't' and 'A' might be used differently in that other world, so that they do not relate to *t* and *A*. Certainly, by contrast, on Stalnaker and Thomason's understanding,

λxLAx(t)

says that the object denoted by 't' here, i.e. *t* has the property denoted by 'LA' here, i.e. *LA*, but that is expressly what modal logic should mostly be about. So the difference between the two forms, very remarkably, displays exactly the difference between the two ways of conceiving semantics.

However, if *this* is the distinction we need to make, then we do not need to break with the abstraction scheme to make it. As before, we can use ordinary quantification theory, though now with a new item. The contrast is between everything called 't' being called 'A' in every possible world, i.e.

L(∀x)(Cx't' ⊃ Cx'A')

and t being A in every possible world, i.e.

LAt.

The distinction is drawn not just by moving from a universal to a referential form of speech, but more crucially by moving from certain mentioned speech to the related used speech. *That* is the move we must make now, to get into the new semantics, see [5] chapter 7, and [8] chapter 2.

4

The former type of expression, *about words*, certainly has a place in modal logic, but it is not a central place, and it is not the pattern on which all modal expressions are to be understood, as classical modal semantics presumes. Whether, for instance, Plato might not have been a philosopher is not a matter of how the word 'Plato' might have been used. It is not a matter of the word possibly having another referent, but expressly, for a start, of the word having the same referent. The classical semantics, indeed, would improperly validate 'Plato was called 'Plato'' on account of this mistake. It is not necessary that Plato was called 'Plato', although the associated fact that any person called 'Plato' in any world is called 'Plato' in that world in clearly necessary. More generally, there would be no evaluating certain worlds as properly counterfactual on such a semantics. The fact that 'Plato is not a philosopher' expressed a truth in a certain other world would not necessarily make it false there that Plato was a philosopher, since, as before, 'Plato' need not refer to Plato there. Even 'philosopher' might not have its usual sense. Indeed, on this account, even in this world someone telling a story saying 'The President of the U.S. is not a citizen of the U.S.' is only referring to and saying something private, and cannot be held up for libel.

Modal expressions involving the mention of words in the above way only approximate to a pattern for modal expressions generally if we take the language not to vary over possible worlds. This means, for instance, that the sense and entailments (though not necessarily the denotation) of general terms remains the same, while the reference and identity (though not necessarily the connotations) of particular terms remains the same. The latter point is a version of the fact about actual identities mentioned many times before; the former point needs to be slightly qualified to allow for the epsilon axiom, as we shall see in a moment.

But even if we allow this world-invariant use for 'mention' forms, we must still beware of the particular fallacy we noted before. For classical modal semantics also standardly presumes that all identity and non-identity relations in the domain, i.e. relevant part of the actual world, automatically hold in all other worlds — quite contrary to theses provable within modal logic itself. Certainly *identities* are in this class, but, as we saw before, *non-identities* are not in general so, and therefore in other possible worlds things actually distinct in the domain may fuse. When this point is allowed for in the semantics it is, in fact, no longer pertinent to say what member of the domain a certain term refers to in another possible world. What we must assemble instead, to parallel that part of classical semantics, are simply a series of remarks giving the identity and non-identity relations within each world (including the actual one). But those remarks are just part of the world-stories. The terms used are the items which automatically link each world with the others, i.e. identities across worlds are shown in Tractarian fashion by means of identity of sign. That is why (c.f. section one) the object referred to by 'ϵxPx' is obtained not by adding some further semantical gloss, but simply by dropping the quotes around it, and so using *it* to refer to the object in question. The object in question in that case was simply ϵxPx, whatever its further identities, or properties.

That ensures transparency, as before, but it also allows, of course, a non-functional understanding of referring expressions. On that matter one must be careful, though, not to confuse other worlds with, say, other times or other places. 'The President of the U.S. at time t', for instance, is a functional expression, for there is indeed no individual entity 'the President of the U.S.' present whenever there is a President of the U.S. at some time. The only entity that is present at each time is the property of being a (sole) President of the U.S. But describing another world α by saying 'The President of the U.S. now is not a citizen of the U.S.' can be easily misconstrued, if the expression is modelled on the temporal or spatial case, and taken to be about some different referential subject 'the President of the U.S., now, in world α'. For the expression is about an alternative world not in the sense of being objectively true somewhere else, but expressly by being objectively false here, i.e. false of the actual President of the U.S., now. It is thus not a conception of some other 'world', but a misconception of this world, a variant 'world-story'. Hence (given its subject is temporally definite) it needs no index.

The idea behind the contrary accounts of modal semantics is thus inextricably linked with a form of modal realism, and what that must be replaced by is a modal conceptualism, where the only reality is the content of a set of world-stories. What these world-stories say may be true or false, but that depends upon their relation to the one and only true story, and hence actuality itself. To say a world-story is true about its world is to make a relative (though tautologous) judgement, not an objective (and contingent) one. The interpretation of the terms, individual and predicative, in each of these stories thus needs no world-relative semantical gloss, as though giving a spurious 'truth-condition' for whether something is true in that world. What goes on in any merely-possible world is a matter just of logic and stipulation, and so there are no grounds for it. There are grounds only in the one case of the actual world, i.e. true world-story. And the actual world is the only world there is.

Specifically, therefore, and in summary, instead of saying that 'ιxPx' (sic) refers to 1 in α we now say that $\iota xPx = 1$ there. Stalnaker and Thomason (adopting, for the moment, their confused use/mention usage, along with Fitting's) missed the appropriate modal semantics expressly by not taking the reference of ιxPx to 1 to be in the form of a straight identity. It is thus generally a 'use' conception of semantics (part of the common confused form of expression), which we must adopt if we are to understand epsilon terms in modal logic. If it emasculates 'semantics' and leaves nothing worth the name, so much the worse for 'semantics'. But 'ιxPx' (sic) is only capable of direct reference when nothing further need be given to obtain its meaning: identities between ιxPx (sic) and other things may still be given, but as with c (sic) in the Fitting case, they are not essential, in general, for meaning. A complaint here might be that all this confuses meta and object languages, and indeed semantic closure is a consequence. But we must learn how easy it is to live with that (see again [5] chapter 7 and [8] chapter 2.) Living with semantic closure, however, expressly means taking more care with the use/mention distinction. This care enables us not only to separate the old semantics from the new one clearly, but also to object to the common, informal use of terms for their names, as with ϵxX (sic) in Fitting's case, and ιxP(x) (sic) in the case of Stalnaker and Thomason.

The main thing we must remember in connection with the world-invariant meaning of individual terms is that the actual world provides whatever reference they may have. For, as before, but yet again, it is actual identities which are carried through as a matter of necessity. That does not apply to ordinary properties, of course, although their denotation is limited by the epsilon axiom, and so is not entirely arbitrary, as in the classical manner.

The objects in the new semantics are not bare monads which can be alternatively conceived just anyway. What properties an object has may depend on what properties are elsewhere in the group, since y cannot be P unless $\epsilon x P x$ is. Of course, $\epsilon x P x$ itself is not limited by that fact. It is the leader.

That means, formally, that every possible world simply obeys the same logical laws as the actual one. We have, for each world,

not-($V(p, \alpha)=1$) iff $V(\neg p, \alpha)=1$,
$V(p \vee q, \alpha)=1$ iff $V(p, \alpha)=1$ or $V(q, \alpha)=1$,
If $V(Py, \alpha)=1$ then $V(P\epsilon x P x, \alpha)=1$, for every x which y is free for in Py,
$V(a=b, \alpha)=1$ iff for all P $V(Pa, \alpha)=1$ iff $V(Pb, \alpha)=1$,
$V(Lp, \alpha)=1$ iff for all β $R\alpha\beta$ only if $V(p, \beta)=1$.

But here '$V(p, \alpha)=1$' means that it is true in α that p, not that 'p' is true in α. And one world-story, say ω, is objectively true, i.e.

$V(p, \omega)=1$ iff p,

which means we can get (not as axioms, but as consequences),

any substitution instance of a propositional tautology,
any appropriate substitution instance of '$Py \supset P\epsilon x P x$',
The standard Leibniz' Law: $a=b$ iff for all P, Pa iff Pb,
The definition of actual necessity:
 Lp iff for all β, $R\omega\beta$ only if $V(p, \beta)=1$.

By specifying the relation R, other definitions may be obtained, and in particular, if $(\forall\beta)R\omega\beta$ then we get logical necessity, i.e. validity. The former laws are then all there is to the general 'semantics' of the latter 'logic', although logic proper now includes its own semantics, as befits the accommodation of intensions.

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REFERENCES AND NOTES

[1] M.Fitting 'An ϵ-Calculus System for First Order S4' *Conference on Mathematical Logic*, Bedford College 1970 Springer Verlag 1972. The fallacy Fitting does not clearly not commit here is taking it that, if ϵxPx, in different worlds, is identical with things which are different in fact, then it cannot be a constant, and so 'ϵxPx' (sic) must be a functional term. Added to this is a conflation of 'ϵxPx' with ϵxPx (sic).

[2] M.Fitting 'A Modal Logic ϵ-Calculus' *Notre Dame Journal of Formal Logic* XVI,1, 1975. Fitting continues his use-mention conflations here, and certainly these are often of no consequence, but in the context of a conclusion about Prior's theory of indirect speech, as in this paper, it is essential to erase such confusions.

[3] G.E.Hughes and M.J.Cresswell, *Introduction to Modal Logic* Methuen 1968. In addition to the standard theorems about identity and non-identity used in the text, Hughes and Cresswell also consider systems of 'contingent identities' which are not relevant here. Indeed, use of the epsilon calculus suggests these are improperly called 'identities'. Thus while Russell might write '$x=\iota yPy$', and this is contingent, it is so expressly because it is shorthand for '$Px.(\forall y)(Py \supset y=x)$', which does not have the form of an identity. '$x=\epsilon yPy$' is not reducible in this way, leaving all proper identities necessary.

[4] A.C.Leisenring *Mathematical Logic and Hilbert's ϵ-Symbol*, London 1969. Note that epsilon terms are individual terms, and so are not indefinite descriptions as is sometimes supposed. Hilbert's reading of 'ϵxPx' was 'the first P'.

[5] A.N.Prior *Objects of Thought* Oxford, 1971. Prior's theory of indirect speech was also R.L.Goodstein's, see 'On the Formalisation of Indirect Discourse' *Journal of Symbolic Logic*, 1958. For the further viability of this approach see Charles Sayward's 'Prior's Theory of Truth' *Analysis* 1987.

[6] R.Routley, R.K.Meyer and L.Goddard, 'Choice and Descriptions in Enriched Intensional Languages — I' *Journal of Philosophical Logic* 3, 1974. Articles II and III in this series are to be found in *Problems in Logic and Ontology* eds E.Morscher, J.Czermak, and P.Weingartner, Graz 1977, see in particular pp185-6 for Routley's discussion of a 'rigid semantics' based on normal models with some relation to that proposed here. Routley there evaluates terms as themselves, writing '$v(\epsilon xPx)= \epsilon xPx$', for instance, in place of '$v('\epsilon xPx')='\epsilon xPx'$' (see note 2). The

Priorian semantics would be expressed instead by ‘$v(\text{'}\epsilon xPx\text{'})=\epsilon xPx$’, showing that terms have a value given by their use.

[7] R.C.Stalnaker and R.H.Thomason, ‘Abstraction in First-Order Modal Logic’ *Theoria* 34, 1968. Some care must be taken with the two-world model Stalnaker and Thomason here use to invalidate the conditional ‘$L(\lambda xPx(\iota xPx)) \supset \lambda xLPx(\iota xPx)$’, since they seem to think this model also validates the antecedent of their conditional, i.e. shows the assertion of ‘$\lambda xPx(\iota xPx)$’ to be necessarily true. Strictly, of course, all worlds must be considered to establish that.

[8] B.H.Slater *Prolegomena to Formal Logic* Gower 1989. All symbols in this book are taken to be abbreviations of natural language expressions, hence no ‘formal’ semantics is to be found there, in the classical sense. The present paper shows why no such can be provided, though not by showing say that only an ‘informal’ one is available: the ‘semantics’ of an expression is now just its natural interpretation — which can certainly be put symbolically, so long as the symbol is indeed an abbreviation. For further work on Prior and Hilbert, see also my ‘E-type Pronouns and ϵ-terms’ *Canadian Journal of Philosophy* 16.1, 1986, 27-38, ‘Prior's Analytic’ *Analysis* 46.2, 1986, 76-81, ‘Fictions’ *British Journal of Aesthetics* 27.2, 1987, 145-55, ‘Hilbertian Tense Logic’ *Philosophia* 17.4, 1987, 477-89, ‘Hilbertian Reference’ *Nous* 22, 1988, 283-97, ‘Contradiction and Freedom’ *Philosophy* 63.245, 1988, 317-330, ‘Intensional Identities’ *Logique et Analyse* 121-122, 1988, 93-107, ‘Prior and Cresswell on Indirect Speech’ *Australasian Journal of Philosophy*, 67.1, 1989, 25-36, ‘Subjunctives’ *Critica* XX 58, 1989, 97-106, ‘Excluding the Middle’ *Critica* XX 60, 1989, 55-71, ‘Consistent Vagueness’ *Nous* 23, 1989, 241-252, ‘Routley's Formulation of Transparency’ *History and Philosophy of Logic*, 1992, ‘The Epsilon Calculus and Its Applications’, *Grazer Philosophische Studien*, 1992, ‘Descriptive Opacity’ *Philosophical Studies* 1992.

[9] R.H.Thomason and R.C.Stalnaker ‘Modality and Reference’ *Nous* 2, 1968. What is ‘logically proper’ about Thomason and Stalnaker's iota term is simply that it is a complete symbol, unlike Russell's incomplete one. Their iota term, however, only resembles an epsilon term, as the text makes clear, and that is crucially not because epsilon terms are themselves incomplete symbols, say for indefinite descriptions (see note 4).