CAN DEDUCTION BE JUSTIFIED?

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§1 *The justification of fundamental logical laws*

How can we be sure that our inferential practices are sound? Sceptics and naturalised epistemologists would join in claiming that no such certainty is to be had. In this paper I want to investigate the possibility that we *can* have good grounds for believing our deductive practices to be coherent.

When we formalise intuitively acceptable inferences, the question of their justification appears to bifurcate. A question of extrinsic correspondence now accompanies the question of intrinsic coherence. The "correspondence" question concerns the adequacy of our representation of intuitive practices. Can the introduction and elimination rules governing "$\supset$" really be thought of as capturing the meaning of the natural language "If ... then ..." connective? Do we (or should we) only consider "relevantly" related alternatives in disjunctive reasoning? ... and so forth.

These are questions I do not believe can be answered without constructing a meaning-theory for our language. I will not examine such "correspondence" questions in this paper, although I will, towards the end, comment very briefly upon Dummett's intriguing claim that there are strong apriori meaning-theoretic grounds for ruling out classical logic as the deductive theory for such meaning-theories.

The internal question of "coherence" is this – irrespective of whether a given logic or system of inference rules can be expanded into an adequate meaning-theory for our language, are the rules in that system themselves sound, do they preserve truth? It is this question which I will focus on in what follows.

In "The Justification of Deduction", Michael Dummett examines an "apparently convincing argument" for the thesis that fundamental logical laws cannot be justified at all.

The argument is that when we reach basic laws which are not accepted because they can be derived from more basic ones, the only possible justification for them would be a semantic rather than a proof-theoretic one. So we would have to show that such laws are truth-preserving. But:

> "in demonstrating soundness, we must use deductive argument and in so doing we either make use of those very forms of inference we're supposed to be justifying or we make use of the inference rules we'd already justified by reduction to our primitive inference rules ... *we should therefore either eventually be involved in circularity, or have embarked upon an infinite regress*." (1)

Dummett observes that there is then a puzzle concerning the status of soundness and completeness proofs for logical systems embodying such laws – for a soundness proof purports to show that these laws are in fact valid and the completeness proof that any valid inference may be effected by iterated applications of these laws. (2) So the puzzle is that we have very good candidates for justifying primitive inference rules – the soundness and completeness proofs for logical systems embodying these rules – "in the face of an apparently convincing argument that no such justification can exist". (3) Dummett sets out to show that this "apparently convincing argument" is flawed.

Dummett contrasts two sorts of argument. In a *suasive* argument, the epistemic direction must coincide with the consequential one: on the assumption that a person already believes the premises Γ, we wish to persuade him that the conclusion β is true in the argument from Γ to β. In an *explanatory* argument, the epistemic direction may run counter to the consequential one – we assume that our interlocutor agrees with us that the conclusion β is true and we seek to provide an explanation as to *why* it is true. (4)

The anti-justificationalist claims that attempts to justify fundamental logical laws either by semantic methods (e.g. soundness and completeness proofs) or proof-theoretic methods are either circular or lead to infinite regress. Dummett responds that the alleged circularity is not of the usual sort:

> "the validity of a particular form of inference is not taken as a premiss

(1) loc. cit. p. 292, italics mine.

(2) loc. cit. p. 291.

(3) loc, cit. p. 295.

(4) "It may well be that the only reason we have for believing the premisses is that they provide the best explanation of the truth of the conclusion" loc. cit. p. 296.

(5) loc. cit. p. 295.

for the semantic proof of its soundness – at the worst that form of inference is employed in the course of the proof". ([5])

Let us imagine that we are trying to justify Modus Ponens (MP). Dummett agrees that MP might be used in a proof of soundness for a system employing MP. This would be objectionable if we were trying to *persuade* someone that MP is valid. But, Dummett contends, we are not trying to persuade our imaginary interlocutor that primitive rules such as MP are valid – we are to suppose that he already accepts them as valid and seeks an *explanation* for their validity. So, if A presents B with an explanatory argument with "MP is justified" or, indeed, with "deductive reasoning is justified", as its conclusion, B can, acting as he is disposed to, acknowledge the explanation as cogent – i.e. as showing that the conclusion does indeed follow from the premises.

Now I agree with Dummett against his imagined protagonist that a deductive argument which has as its conclusion "MP is justified" or even "Deduction is justified" need not necessarily be circular. Indeed, I claim, more strongly than this, that the thesis that all such arguments must be circular is mistaken. But I think that Dummett's dialectical strategy against his opponent is misguided. The plain fact is that there *are* people who deny that the practice of deductive reasoning can be justified – namely, the anti-justificationalists; likewise, there are people who deny the general validity of particular fundamental logical laws – Carroll's Tortoise denied or at least refused to believe MP, Brouwer refused to assert LEM, Heraclitus even denied the Law of Non-Contradiction (LNC). So it appears that there are those who need to be persuaded that deductive reasoning is justified or that MP, say, is justified ... and for such ones, on Dummett's own admission, no explanatory argument is of any utility in convincing them.

What Dummett *ought* to provide is an argument that convinces *any rational person* that deduction can be justified, not one that seeks to either persuade one who denies this or explain to one who already believes it how this can be so.

Let me spell this out a little more clearly. Most agents correctly use deductive inferences or at least the more basic ones in a non-reflective fashion. What we are seeking to vindicate is the meta-belief that deduction can be justified. So I am assuming that our task is to convince a rational person, that is, one who can perform correct deductions in an non-reflective way, to accept the meta-belief that deduction is justified.

Dummett, on the other hand, thinks that the task is to explain to one who is already disposed to believe that deduction is justified, how it can be that it is: to attempt to persuade one who doubts this or, worse, one who explicitly denies this, by means of the doubted or disputed deductive method is somehow to beg the question against him. There is no straightforward circularity involved in a justification effected by a soundness proof, say, since (as already noted):

> "The validity of a particular form of inference is not a premiss for the semantic proof of its soundness; at worst, that form of inference is employed in the course of the proof".

However, if we tried to convince one sceptical of whether deduction was justified that certain forms of inference were truth-preserving by means of such a proof, there would be an objectionable circularity involved. ([6])

I would have thought that whether an argument is circular or not depends upon the nature of that argument and not on what its intended audience happens to believe. An argument might be dialectically useless against one who explicitly denied the soundness of the inferences it deploys, but this does not clearly make it circular in any objectionable sense. Similarly, whether an argument is a *valid* argument or not cannot depend upon what its audience believes either – for they might to a man disbelieve the conclusion of that argument and yet acknowledge it as a good argument for that (false) conclusion. ([7]).

Consider someone who either doubts whether deduction is justified or believes that it is without knowing why or denies that it is. What does he or she need to be persuaded of if they are to believe that deductive reasoning is sound? They need to be shown how it is that the premises of a deductive argument jointly necessitate the truth of the conclusion. They need to be convinced of a *general truth* – that it must be the case

([6]) "Now, clearly, a circularity of this form would be fatal if our task were to convince someone, who hesitates to accept inferences of this form, that it is in order to do so. But to conceive the problem of justification in this way is to misrepresent the position that we are in. Our problem is not to persuade anyone, not even ourselves, to employ deductive arguments: it is to find a satisfactory explanation of the role of such arguments in our use of language." (loc. cit. p. 296)

([7]) This would presumably give them an incentive to search for the premise(s) that were false in the argument.

that whenever the premises in an arguments are true the conclusion is also true. (8) Now it is perfectly conceivable that someone could be in doubt about this general relation between premises and conclusion in a correct deductive argument without its being the case that he cannot *recognise* valid arguments when confronted with them – indeed if this were his epistemic plight, it would be absurd to even contemplate trying to *convince* him by means of argument of anything.

The anti-justificationalist claims that once we so much as *use* Modus Ponens, say, in an argument purporting to establish that MP is valid, we have committed some sort of petitio and Dummett seems to think that this is indeed the case when we seek to persuade one who is not already predisposed to believe it, that deduction is justified, even though the circularity "is not of the usual sort".

I fail to see that there is *any* circularity at all in a deductive argument for the justification of deductive practice, or for the soundness of MP – for so long as the argument does not either explicitly or implicitly *appeal to* the validity of MP, it is simply beside the point, apropos the question of circularity, that it *uses* a MP inference.

Susan Haack is one philosopher who sharply disagrees with my claim above. She argues that the very sort of argument which could be used to establish the soundness of MP could be used to establish the soundness of an intuitively invalid rule. Thus, consider the following justification of MP:

"Suppose that p is true and that p ⊃ q is true. By the truth table for '⊃', if p is true and p ⊃ q is true, then q is true also. So, q must be true too.".... (α) (9)

Now (α) is of the form of Modus Ponens, since it proceeds:

"Suppose A (that p is true and that p ⊃ q is true); if A then B (if p is true and p ⊃ q is true, then q is true); so B (q is true too) ... (α*)

Haack produces an argument using an inference rule she calls "Modus Morons", which proceeds:

"Suppose that q is true and that p ⊃ q is true. By the truth table for

(8) Here I am siding with Dummett in taking the semantic characterisation of validity to be the one relevant to discussing the justification of logical laws whose validity does not seem to stem from the fact that they are derivable from more fundamental laws.

(9) Susan HAACK discusses this argument at "The Justification of Deduction" *Mind*, 85, 1976, pp. 112-119, reprinted in Copi, I and Gould, J (eds.), *Contemporary Philosophical Logic* loc. cit. at p. 57.

'$\supset$' if q is true and p $\supset$ q is true, then p is true also. So, p must be true too." ... (β)

This argument also has the form of the inferential rule it supposedly justifies, viz.:

"Suppose D (that p $\supset$ q is true and q is true); if C then D (if p is true then p $\supset$ q is true and q is true). So, C (p must be true)." ... (β*)

Haack argues that it is useless to protest that this new MM-styled argument for MM is invalid because it uses an invalid inference rule whereas (α) above does justify MP because it uses a valid inference rule, for the task at hand is to justify our belief that MP is valid and MM not.

I agree with Haack that it is not sufficient for (α) to justify MP that it use a valid inference rule. But I disagree with her that we cannot know whether her MM-styled argument is valid or not before deciding on which inference rules are to count as valid. For, given that "$\supset$" has the same meaning in (β) as it has in (α), we can know that any argument using MM must be invalid in advance of having decided which forms of argument are to count as valid – we can know this because we can recognise that MM is *itself* invalid. ([10])

Pace Haack, what is objectionable about (α) is *not* that it has the form (α*) of a MP inference, but rather that it explicitly assumes in its premises that the inference rule MP *is* truth-preserving, and since this is precisely what has to be established, it *is* a straightforward petitio. So whilst the argument for the soundness of MP does indeed use a correct inference rule, it is debarred from justifying *any* conclusion just because it is circular.

I believe that Haack's discussion highlights the dangers of taking a purely syntactic approach (that is, one that ignores questions of meaning) to the question of justifying basic logical laws but the matter is less straightforward than my discussion has so far suggested, for it would seem that Haack has a powerful line of response to my criticisms.

Haack claims that any inference rule can be justified by using that inference rule in the metalanguage. Suppose that this is true. Then if we suspect or even know that some of our inference rules are incorrect, but do not know which, we would never be able to extricate ourselves from this predicament, since for any inference rules we put forward as incor-

([10]) Just take any concrete instances e.g. p="It is raining"; q="I am wet" – anyone who cannot simply see that the resulting inference can be counterexemplified is deductively blind.

rect, we can "justify" them by using them in the metalanguage in the manner indicated by Haack.

Dummett in his unpublished *William James Lectures* argues persuasively that Haack's crucial assumption here is not generally correct. ([11]) It is only in the special case of a "programmatic interpretation" – that style of semantic theory corresponding to a disquotational Tarskian truth-definition – that the assumption that metalanguage logic and object language logic *must* coincide is true. For other styles of semantic theory, it is just not true that the fact that a given law holds in the underlying logic of the metalanguage of a semantic theory entails that that law can be shown to also hold in the logic of the object language. For example, if we use Beth trees to give a semantic theory for intuitionistic logic, we can prove the soundness and completeness of that logic with respect to that semantic theory even if we assume a metalanguage obeying a classical logic. Looked at from the classical standpoint, there is no risk of those intuitionistically invalid laws of the metalanguage infecting the logic of the object language since for each intuitionistically invalid classical law we will be able to prove its invalidity in the object language by means of a "weak counterexample" (a counterexample in which though the premises are true, the conclusion fails to be true, although without actually being false). So our ability to justify or refute logical laws by appeal to a semantic theory is not impaired by adopting in the metalanguage a logic that is *stronger* than that which we take to hold in the object language. ([12])

So it would seem, if Dummett's claims are correct, that if we were to find ourselves in the situation above of suspecting or even knowing that

([11]) Cf *WJL* 6, 7, 8 and in particular *WJL* 7:34-35.

([12]) One interesting question here is whether the converse holds: is it possible to establish the validity of all the inference rules and logical laws used in the object language where the object language uses a stronger logic than that of the metalanguage? We are strongly inclined to think that this must be impossible, but perhaps we could prove the validity of some classical law using only intuitionistically valid forms of argument if we were to proceed from the premise that every sentence must be either true or false. This may seem like mere subterfuge since we will be able to convert that particular proof into one containing LEM and the other inference rules deployed in the proof. But for this to hold quite generally, it would always have to be possible to produce a logical law corresponding to every semantic principle in the way that LEM corresponds to Bivalence and, as far as I know, this has not been demonstrated.

some or our inference rules and logical laws were unsound without knowing which, it might be possible to discover which ones were to blame by surrendering the use of a "programmatic" semantic theory in favour of some other semantic theory in which the metalanguage logic could not be assumed to coincide with the object language logic. If Dummett is right then Haack's counter-objection fails: we *cannot* guarantee for non-programmatic semantic theories that an inference rule will be valid in the logic of the object language just because it holds in the logic of the metalanguage.

A suasive deductive argument ought not to be judged circular, then, simply because it uses deductive inferences – for even if the anti-justificationalist denies that such an argument could possibly succeed, he will be forced to revise his view if he is confronted with an argument to that conclusion proceeding from premises he accepts for which he cannot conceive of how the premises could be true without the conclusion being true. This is just what it is to recognise the argument as correct. In the case of soundness proofs for MP, there will be no undischarged assumptions involved since the soundness of MP, as well as that of the other introduction and elimination rules for the logical constants, is a theorem.

Thus if we were able to produce a convincing non-circular argument with "Modus Ponens is justified" or "deductive reasoning is justified" as its conclusion, it would be spurious to object that that argument used the very law it sought to justify and was a fortiori question-begging – it would be spurious because MP would not occur as a premise explicit or suppressed anywhere in that argument: thus in seeking to persuade a rational person of its cogency, we are not asking him to make any judgement about the general validity of MP, we are simply asking him to decide at each stage whether a particular inference is valid. He may believe with Carroll's Tortoise that one is only permitted to infer that q from the premises $p \supset q$ if one accepts the further premise $p \supset (p \supset (p \supset q))$ and so forth, but if we present him with an apparently cogent argument from premises he accepts with "MP is justified" as conclusion, he will either have to admit that his former theory about the general relation between the premises and the conclusion in a MP inference was wrong or seek to discredit the truth of one or more premises on the grounds that the conclusion is definitely false. He cannot plead that we have begged the question against him by assuming the truth of the general logical law of MP, for this is just not true – we do not accept the conclusion of the argument (that MP is justified) because we have accepted the argument and also implicitly agreed to accept MP as a rule of inference, we accept the conclusion of the argument because we've accepted the argument as valid ([13]). If someone claims that we need to include as an extra premise in the argument one expressing the validity of MP, we should respond as Achilles should have responded to the Tortoise: "the inference is valid as it stands and does not need supplementation with a premise that purportedly licenses the inference". This is one thing Carroll's Tortoise ought to have taught us. ([14]) So I disagree with Dummett's claim that no suasive argument can be advanced for the thesis that deduction can be justified.

§2: *Deduction, observation and conservativeness*

Suppose our more primitive forebears had used a very simple language L_0 consisting of unanalysed observational sentences for which a derivability relation "$\vdash_0$" could be defined on those sentences. Suppose that L_0 contained no logically complex sentences. Consider the project of introducing such logical complexity into their language. In order to avoid inconsistency, we should require that the extended language L_1 together with the new derivability relation "$\vdash_1$", formed by adding logical constants and observationally undecidable theoretical predicates to the original language, be *conservative* with respect to deducibility – that is, no atomic observational sentence should be deducible from any specified subset of atomic observational sentences in L_0 after the addition of those constants if it wasn't already deducible before. Deduction should conserve observationality.

Dummett illustrates how an indirect deductive verification of a statement is conservative with respect to its direct observational verification. He emphasises that one who makes an observation selectively discerns certain patterns in his sensory information (and overlooks others). Suppose that Klaus is watching his friend Fritz taking his daily constitutional

([13]) Cf LEAR, J. *Aristotle and Logical Theory* ch. 6.

([14]) In order to forestall confusion, I am not claiming that a soundness proof for a formal system suffices to justify the use of the inference rules deployed in that system.

around the bridges at Konigsberg. It seems possible that Klaus could observe that Fritz had crossed every one of the Konigsberg bridges without observing that, as indeed he must have done, Fritz had crossed at least one twice ([15]): had Klaus been more observant, he could have detected this pattern *directly* ([16]).

But suppose Klaus is familiar with Euler's proof. Then he knows that it is a consequence of the proof that if Fritz crossed every bridge on a certain occasion, he must have crossed at least one twice. So, given his observation that Fritz has crossed every bridge, he can *deduce* that Fritz must have crossed at least one twice.

Let p= the proposition that Fritz has crossed every bridge; q= the proposition that Fritz has crossed at least one bridge twice. The inference p ∴ q then comprises an indirect means of establishing q. For the direct means of establishing q is to simply observe that it is so when it is – and Klaus could have observed this had he been more attentive.

But how can Klaus know that his indirect verification of q is correct? Dummett's suggestion is that an indirect verification of A is correct just when we either have in our possession an EP for obtaining a direct verification for A or we recognise that had we but had a sufficiently detailed set of observations, we would have obtained an EP for acquiring a direct verification of A. And it seems that Klaus *has* such an EP available to him – it is, to express it *very* roughly, this:

(1) Take the sequence of observations which verify the premise "A crossed every bridge".
(2) Feed these observations into the general procedure implicit in Euler's proof.
(3) Euler's proof will then restructure the original sequence of observations as a sequence directly verifying the conclusion.

In this manner, the indirect inference p ∴ q is to be vindicated as valid. We can set this out more explicitly as follows: Let us represent the proposition that Fritz crossed every bridge, designated previously by p, as ∅(a); the conclusion that Fritz crossed some bridge twice previously

([15]) Indeed, it seems possible that this could be the content of Fritz's self-observation.

([16]) "In a given case we may have verified the premises without having noticed or recorded the route in detail". loc. cit. p. 314.

designated by q as $\Psi(a)$. Then the canonical derivation of q from p via Euler's proof can be represented thus:

$$\begin{array}{l} \nabla_p \\ \emptyset(a) \\ . \\ . \\ . \quad \text{(Euler's proof)} \\ \Psi(a) \end{array}$$

We now have all the materials necessary for finding a direct verification of the conclusion q ($= \Psi(a)$); the Euler-based derivation is not by itself such a proof – what it does do is provide an EP for transforming direct verifications of the premises into a direct verification of the conclusion:

(1) We have a direct verification for p:
∇_p = The observation that Fritz crossed every bridge.
(2) An indiret verification consists in a direct verification of the premises together with the derivation of the conclusion.
(3) We have a derivation of q and a direct verification of the premise p.
(4) So we can obtain a direct verification of the conclusion q by transforming the observational evidence for p, in accord with the steps of the derivation utilising Euler's proof, into a direct verification of q. ([17])

We should note, as Dummett himself does, that the indirect verification can only actually yield a direct verification if implemented at the time of observation. Later on, Klaus might not recall whether Fritz really had crossed every bridge – just the information that would be required as input to the reorganisation of that information effected by Euler's proof. Further, if this could be the content of Klaus's memory, it seems that it could also be the content of Klaus's contemporaneous observation of Fritz's walk – if so, the same result would hold: the observation would not provide information detailed enough to serve as input to the Euler procedure.

Dummett's point is that the observational evidence, ∇_p, which verifies the assertion that Fritz crossed every bridge also verifies the assertion that he crossed some bridge twice. Insofar as we use deductive arguments in the context of making observations, then, we can be assured of their justifiability.

([17]) Cf "Justification of Deduction" p. 308; *WJL* 6.

Dummett thinks that our classical inferential practices in mathematics and science should be revised, primarily because the classical notion of truth in terms of which the relation of logical consequence between sentences is to be characterised is a verification-transcendent concept which he argues to be both unlearnable and incommunicable. I cannot examine his arguments for that conclusion here, but I do want to comment briefly on his worry that classical reasoning might non-conservatively extend the derivability relation holding between sentences in some decidable fragment of a natural language.

Consider our primitive observational language L_0 again whose sentences are all decidable by observation. Then if upon adding to L_0 observationally undecidable theoretical sentences containing logical expressions whose introduction and elimination rules were classical we found that we were able to deduce observational sentences not previously deducible, Dummett thinks that this would be a ground for revising those classical rules.

But as John P. Burgess has argued ([18]), it would equally, and surely more plausibly, be a ground for revising *the theory* rather than its logic and this because as Quine has persistently reminded us, it is rational to prefer to make revisions in our total theory which disturb the least number of well-entrenched and successful practices as possible. If we do have hold of a notion of truth which transcends the possibility of verification (as I would argue that we do), then the classical notion of logical consequence can be vindicated against its intuitionistic and other competitors.

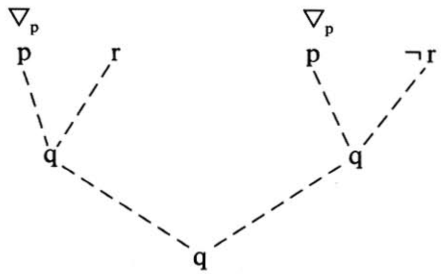
Moreover Gentzen proved, using only the resources of an intuitionistic metatheory that a Cut-Elimination theorem held for classical sequent calculi, according to which any sequence $p_1, p_2, \ldots, p_n \vdash q$ which has a derivation at all has a derivation in which no symbols occur which do not already occur in the sequence already. So given that we have a Realist conception of verification-transcendent truth for an atomic fragment, the addition of classical operators to that fragment will in fact conservatively extend that fragment.

If we do have a coherent conception of verification-transcendent truth, then some analogue of Dummett's procedure for justifying indirect verifications through deduction ought to be available to us in those circumstances where our observational evidence is less than conclusive and

([18])"Dummett's Case for Intuitionism" *History and Philosophy of Logic*, 1984.

where we employ distinctively classical inference rules such as LEM to deduce consequences that are, if not decidable by observations as in the Konigsberg case, at least highly confirmed by them. So instead of an observational warrant ∇_p being restricted to that small subclass of observations which are conclusively verifiable, we could require that it confer a sufficiently high degree of probability on a given empirical statement and instead of our inference rules being limited to constructivistic ones, we could include non-constructivistic ones such as Double Negation Elimination (DNE) or Non-Constructive Dilemma (NCD). For a Realist (in Dummett's sense of Realism), to possess an observational warrant ∇_p for asserting empirical p is to possess evidence strongly confirming p. In order to show that an application of a classical inference rule such as NCD was justified in a given empirical context in which a statement q was deduced using NCD from a statement p that was highly confirmed by observation, the Realist would have to show that the probability of q was at least as high as that of p.

Now suppose we have a deduction of q from p, using NCD:



In this deduction, p has a probabilistic warrant for its assertion in ∇_p but neither r nor $\neg r$ has such a warrant. We would need to show that we have, by using NCD, a warrant for asserting q, given that we have such a warrant for p.

Now in classical probability calculi, $p(r) + p(\neg r) = 1$ and classical inferences preserve degree of probability. Assuming the probability of p is 0.9 say, we would need to show that the probability of q is at least as high. This is not difficult to show classically. We would need the joint probability of p and r and the joint probability of p and $\neg r$. We would then have to show that the sum of the probabilities of q conditional on p and r together with q conditional on p and $\neg r$ was at least 0.9. This is again straightforward classically. Given such a demonstration, we would have shown that from the Realist viewpoint there is a (probabilistic) warrant, ∇_q for asserting q and thus that the use of NCD was justified in such instances.

Establishing against Dummett that Realism as the belief in verification-transcendent truth with its attendant commitment to classical modes of reasoning, is defensible is a very difficult project. I have not attempted this here. I have only tried to show that the arguments purporting to prove that there is something objectionably circular about deductive justifications of deductive practice are unconvincing and that the use of classical modes of reasoning in the context of making observations is as justifiable as the use of non-classical principles. Indeed, given that these classical principles so well codify the inferences of an immensely successful mathematical and scientific theory, we have good reason, some would say the best possible reason, for taking those principles to be justified.

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