ON SOME LOGICALLY EQUIVALENT PROPOSITIONS

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In the concluding essay of *Logic Matters* ([1]: pp. 318ff.) Peter Geach contends that the propositions

(1) God providentially governs the world

and

(2) The world is providentially governed by God

are "clearly" logically equivalent. He construes Thomistic theory as regarding both as true, though he acknowledges that for this theory it is only (2) that predicates a real relation. He thus counts as "muddled" an interpretation of this theory which would construe it as asserting that (1) is false and that (2) is true.

In another essay in the same volume (pp. 70-74) Professor Geach asks us to consider the proposition

(3) Every cat detests every dog

as an instance of "Every A is F." The schematic letters here can receive two legitimate interpretations:

[A]: A = cat; ____is F = ____detests every dog
[B]: A = dog; is F = every cat detests____.

Under [A] the contrary form "No A is F" is instantiated as

(4) No cat detests every dog,

whereas under [B] we get

(5) (There is) no dog every cat detests

as the contrary. According to Geach (4) and (5) are not logically equivalent. However, he does regard

(6) Not every cat detests every dog

and

(7) (It is) not *every* dog (that) every cat detests

(Geach's parentheses and italics) as logically equivalent propositions. These are the respective instantiations under [A] and [B] of "Not every A is F," the schema contradictory to "Every A is F." This equivalence, he suggests, is not obvious at first glance, but its existence generates two important results. The first is that no proposition has two non-equivalent contradictories: that though a proposition may well have more than one contrary in the square of opposition sense of the word, we may speak of *the* contradictory of a proposition. Secondly, whereas contrariety can be treated as an operator only upon predicables, contradictory negation may be thought of either as operating upon entire propositions or as operating upon predicables.

This paper will argue, first of all, that since there are two distinct senses in which (1) and (2) are logically equivalent, it is not necessarily a muddle to claim that while (1) is false (2) is true. Section II will suggest that though there are senses under interpretations [A] and [B] in which (6) and (7) are logically equivalent, they cannot be logically equivalent in the manner in which Geach insists. The nature of the equivalence of active and passive voice propositions precludes this with the result that a proposition can have non-equivalent contradictories. These results are attained by regarding Geach's portrayal of the difference between contrary and contradictory negation as at best only an inchoate recognition of the dual function of "not" as the negation of a predicate and as part of a complex proposition-forming functor for a two term argument. The equivalence of propositions by obversion displays this dual function. For example, the particular affirmative form "Some S (not P)" with its negated predicate is the obverse of the particular negative form "Some S not P" with its use of "Some ____ not ____" as a proposition-forming functor for a two term argument. Talk of contradictory denial as a proposition-forming operation upon an entire proposition will accordingly emerge as misleading and otiose.

I

With brackets placed around the term or phrase designated as the subject of the proposition, with parentheses surrounding that designated as

its predicate, and with its unsurrounded main "quantifier" or syntactical device (if it not be suppressed) in italics, we can parse (1) either as

(1.1) [God] (providentially governs the world)

or as

(1.2) (God providentially governs) *the* [world]. ([1])

Through the simply convertible necessary truth that

(8) Whatever providentially governs the world the world is providentially governed by

parsed as

(8.1) *Whatever* [providentially governs the world] (the world is providentially governed by)

serving as the major premiss, (1.1) entails

(2.1) (The world is providentially governed by)[God].

In turn (2.1) entails (1.1) through the suppressed major:

(8.2) *Whatever* [the world is providentially governed by] (providentially governs the world).

Thus, (1.1) is logically equivalent to (2.1).

Through the simply convertible necessary affirmative that

(9) Whatever God providentially governs is providentially governed by God

parsed as

(9.1) *Whatever* [God providentially governs] (is providentially governed by God),

(1.2) entails

(2.2) *The* [world] (is providentially governed by God).

([1]) This way of parsing is not Geach's, but in ([2]: pp. 35, 201-02, 209) he does evince some sympathy for it as it cannot be combined with a doctrine of distribution.

(2.2) entails (1.2) through

(9.2) *Whatever* [is providentially governed by God]
(God providentially governs).

Thus, (1.2) is logically equivalent to (2.2), and not to either (1.1) or (2.1). That (1.2) cannot be established as logically equivalent to either (1.1) or (2.1) becomes obvious once it is noticed that any attempt to argue for such an equivalence via a suppressed necessarily true premiss must commit the fallacy of four terms.

The Thomistic theory mentioned by Geach would apparently regard both (1.1) and (2.1) with their predications of real relations in God as false, while it would affirm the truth of both (1.2) and (2.2). However, since (1.1) and (2.2) are the respective *prima facie* grammatical parsings of (1) and (2), those who assert that (1) is false and that (2) is true are not necessarily muddled in their exposition of this theory. They are somewhat obscurely noting that though those propositions which are indeed logically equivalent in their active and passive voices have different grammatical subjects, they have the same logical subjects.

II

Geach's exposition of the case for the logical equivalence of (6) and (7) makes it clear that though (6) can be parsed in our mode in such diverse ways as:

(6.1) (Not every cat detests) *every* [dog]

(6.2) (Not *every* [cat] detests every dog)

(6.3) *Not* (every cat detests) *every* [dog]

(6.4) *Not every* [cat] (detests every dog),

it is only the particular negative (6.4) which for him captures interpretation [A]. Similarly, though (7) can receive such diverse parsings as:

(7.1) (It is not *every* [dog] that every cat detests)

(7.2) (It is not every dog that *every* [cat] detests)

(7.3) (It is *not* every dog that *every* [cat] detests)

(7.4) (It is *not every* [dog] that every cat detests),

it is only the particular negative (7.4), which might also be rendered as

(7.41) (Every cat detests) *not every* [dog]

or in normal form as

(7.42) *Not every* [dog] (every cat detests),

that for him instantiates interpretation [B].

Now (6) construed as the universal affirmative (6.1) is logically equivalent to (7) construed as the universal affirmative (7.1). This is evidenced by construction of the following two syllogisms in *Barbara* with the convertibly necessary truth

(10) It is not whatever not every cat detests that every cat detests serving as the major premiss:

[C]: (10.1) (It is not *whatever* [not every cat detests] that every cat detests)

(6.1) (Not every cat detests) *every* [dog]

(7.1) (It is not *every* [dog] that every cat detests)

[D]: (10.2) [It is not *whatever* (not every cat detests) that every cat detests]

(7.1) (It is not *every* [dog] that every cat detests)

(6.1) (Not every cat detests) *every* [dog] ([2])

(10.1) should be read as claiming that whatever not every cat detests is such that it isn't it that every cat detests, while (10.2) can be paraphrased as asserting that whatever it isn't that every cat detests is such that not every cat detests it.

Furthermore, (6.2) is logically equivalent to (7.2) for these universal affirmative propositions entail one another through the suitably parsed convertibly necessary truth

(11) It is not every dog that not whatever detests every dog detests serving as the major premiss in the following inferences:

([2]) Syllogisms with such gapped terms are used by Geach in ([2]: pp. 174-76) and by Sommers in ([3]: p. 222).

[E]: (11.1) (It is not every dog that [not *whatever* detests every dog] detests)

(6.2) (Not *every* [cat] detests every dog)

(7.2) (It is not every dog that *every* [cat] detests)

[F]: (11.2) [It is not every dog that (not *whatever* detests every dog) detests]

(7.2) (It is not every dog that *every* [cat] detests)

(6.2) (Not *every* [cat] detests every dog)

(11.1) claims that whatever is such that it does not detest every dog is such that it is not every dog that it detests, while (11.2) maintains that whatever is such that it isn't every dog that it detests does not detest every dog.

We must note that (6.1) can be presented in normal form as

(6.11) *Every* [dog] (not every cat detests).

By obversion it is clear that this is equivalent to the universal negative

(5.1) *No* [dog] (every cat detests),

which is the obvious parsing of (5), the contrary of (3) generated by Geach under [B]. Furthermore, the universal affirmative (6.2) is equivalent by obversion to

(4.1) *No* [cat] (detests every dog),

our parsing of the universal negative intended as the contrary of (3) under [A]. Geach's denial of the equivalence of (4) and (5) thus receives confirmation as we become aware that despite their surface grammar (6) and (7) can be understood as contraries of (3) rather than as contradictories. (5.1), (6.1) and (7.1) are logically equivalent contraries as are (4.1), (6.2) and (7.2), though, and this we take to be Geach's point, no one of the former group is equivalent to any of the latter.

These results will soon become momentous, but we must first show (*pace* Geach) that it is the particular negative (6.3), and not the particular negative (6.4), that is logically equivalent to the particular negative (7.4). This logical equivalence is demonstrated by using the suitably parsed necessary truth:

(12) It is whatever every cat detests that every cat detests as the major premiss in the following two syllogisms in *Baroco*:

[G]: (12.1) [It is *whatever* (every cat detests) that every cat detests]

(6.3) *Not* (every cat detests) *every* [dog]

(7.4) (It is *not every* [dog] that every cat detests)

[H]: (12.2) (It is *whatever* [every cat detests] that every cat detests)

(7.4) (It is *not every* [dog] that every cat detests)

(6.3) *Not* (every cat detests) *every* [dog]

Like (7.41) and (7.42), (6.3) is just another rendering of (7.4).

However, (6.4) is logically equivalent to (7.3). This is shown by the following two inferences in *Baroco* which employ the necessary truth

(13) It is every dog that whatever detests every dog detests as the major premiss:

[I]: (13.1) [It is every dog that *whatever* (detests every dog) detests]

(6.4) *Not every* [cat] (detests every dog)

(7.3) (It is *not* every dog that *every* [cat] detests)

[J]: (13.2) (It is every dog that *whatever* [detests every dog] detests)

(7.3) (It is *not* every dog that *every* [cat] detests)

(6.4) *Not every* [cat] (detests every dog)

How could Geach maintain that (6.4) and (7.4) are logically equivalent? Only by showing that the parsings of (3) as

(3.1) *Every* [cat] (detests every dog)

and as

(3.2) *Every* [dog] (every cat detests)

are logically equivalent. But such an effort is doomed to commit the fallacy of four terms and would only be undertaken by one who thought that there was just one sense in which "Every cat detests every dog" is logically equivalent to "Every dog is detested by every cat", a claim that our reflections on the dual nature of the equivalence of (1) with (2) have im-

plicitly disputed. Moreover, the logical equivalence of (3.1) and (3.2) would undermine the sense in which it has been correctly maintained that (4) and (5) are not logically equivalent as (4.1) and (5.1) now would entail one another. Geach's contention that contradictory denials are logically equivalent conflicts with his thesis that contrary negations are not so equivalent. We must come to the recognition that propositions can have more than one non-equivalent contradictory.

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REFERENCES

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