INTENSIONAL IDENTITIES

B.H. Slater

In a recent article ('A New Puzzle About Intentional Identity', *Journal of Philosophical Logic* 15, 1986) Walter Edelberg has found difficulties in certain traditional treatments of Intentional Identity in statements containing psychological verbs. But he has also demonstrated that a further difficulty would arise if we tried to take another way out. I want first to show, in this paper, that Edelberg's problems result from using an inadequate theory of descriptions, and that an alternative theory of descriptions, available in the literature, facilitates a better expression for Intentional Identity in the contexts Edelberg is concerned with (hereafter 'Psychic Logic'). But this theory of descriptions needs more articulation than is commonly provided, and so I also want to show how it is to be developed and applied within Modal Logic. This will lead to a considerable simplification in Modal Logic, and a better understanding of its nature.

Specifically, Edelberg considers

Hob thinks a witch blighted Bob's mare, and Nob thinks she killed Cob's sow (1)

and objects (p. 3) to the formalization

$$(Ex)(Wx . Bh(Bx) . Bn(Cx)) \quad (2)$$

since it does not represent the 'de dicto' uses of (1). But he also objects (p. 4) to the Russellian formulations

$$Bh(Ex)(Wx . Bx) . Bn(Ex)(Wx . Bx . (y)((Wy . By) \rightarrow y=x) . Cx) \quad (6)$$

and $$Bh(Ex)(Wx . Bx) . Bn(Ex)(Bh(Wx . Bx) . (y)(Bh(Wy . By) \rightarrow y=x) . Cx) \quad (7)$$

since it may be the case (see his 'example 2', p. 2) that Nob knows nothing either about Bob's mare or about Hob. Edelberg therefore considers it plausible (p. 7) that (1), in its 'de dicto' use, should be formalized

$$(Ex)(Bh(Wx . Bx) . Bn(Cx)) \quad (13)$$

where the quantifier is interpreted substitutionally rather than with existential import. However he finds a further difficulty with this other style of analysis. For (p. 13)

Detective A believes someone murdered Smith and
Detective B believes he murdered Jones (19)

does not entail

Detective B believes someone murdered Jones and
Detective A believes he murdered Smith (20)

since Detective B might identify the two murderers while Detective A does not, and so, against (20), believe the murderer of Jones did not murder Smith. But, given merely the symmetry of conjunction (p. 14)

$$(Ex)(Ba(Sx) \,.\, Bb(Jx)) \quad (21)$$

is equivalent to

$$(Ex)(Bb(Jx) \,.\, Ba(Sx)) \quad (22)$$

so even substitutional quantification will not capture Intentional Identity, in this case.

The alternative theory of descriptions which I will show resolves Edelberg's puzzles in Psychic Logic is linked to that given in G.E. Hughes and M.J. Cresswell's *An Introduction to Modal Logic* (Methuen, 1968, p. 203). Hughes and Cresswell there introduce definite descriptions by means of axioms like

$$(E!x)\varphi x \rightarrow \varphi((ix)\varphi x)$$

and so take '$(ix)\varphi x$' to be the φ if there is exactly one thing which is φ, and otherwise an arbitrary member of the domain of individuals. This treatment of definite descriptions differs from Russell's (*Ibid.*, p. 207) in that, in the latter all expressions involving a descriptive term are abbreviations of quantificational formulae in which individual terms do not occur at all. Thus, with Russell

$$\Psi((ix)\varphi x) \leftrightarrow (Ex)(\varphi x \,.\, (y)(\varphi y \rightarrow y = x) \,.\, \Psi x)$$

However, in the case where there is not exactly one thing which is φ, on the alternative theory '$\Psi((ix)\varphi x)$' is not automatically false as with Russell, but depends upon the choice of referent for '$(ix)\varphi x$'.

This alternative style of analysis for definite descriptions was not originated by Hughes and Cresswell. They trace their version to J.B. Rosser (*Logic for Mathematicians* New York 1953, p. 185), but, in embryonic form it was also Frege's formal account: although commonly thought of as a pre-suppositionist with respect to individual terms, Frege, in his symbolic work, took 'non-denoting' terms to have, not no reference, but, as above, an arbitrary one (see, for instance A.N. Prior, *Time and Modality* Oxford 1957, pp. 61, 71). Also the analysis is available from general principles within Hilbert's ϵ-calculus, for if we take the Hilbertian axiom

$$(Ex)\varphi x \rightarrow \varphi(\epsilon x \varphi x)$$

(see A.C. Leisenring, *Mathematical Logic and Hilbert's ϵ-symbol*, London, 1969, p. 40) we can get Hughes and Cresswell's axiom by taking 'φx' as '$\varphi x.(y)(\varphi y \rightarrow y=x)$' and then '$\epsilon x(\varphi x.(y)(\varphi y \rightarrow y=x))$' as '$(ix)\varphi x$'. Hilbert himself was a pre-suppositionist and did not give a reading of his ϵ-terms except when the unique-existence condition held, in which case the ϵ-term above was equivalent to his pre-suppositional one (see G.K. Kneebone, *Mathematical Logic and The Foundations of Mathematics*, Von Nostrand, 1963, pp. 93, 102); but to deal with definite descriptions as one kind of ϵ-term is clearly an advance upon Hughes and Cresswell, since we then have access to a more comprehensive theory (see my 'E-type Pronouns and ϵ-terms' *Canadian Journal of Philosophy*, 1986, for an interpretation of general ϵ-terms as anaphora; also Michael McKinsey's 'Mental Anaphora' in *Synthese* 66, 1986, for further discussion of the relation between Intentional Identity and E-type pronouns, and my 'Hilbertian Reference' forthcoming in *Nous*, and 'Fictions' forthcoming in *The British Journal of Aesthetics*, for more on reference); the advantage over Frege, of course, is that he had no symbolism, of any kind, to articulate the inner structure of definite descriptions.

In the next section of this paper I shall use the ϵ-formulation of descriptions, given above, to resolve Edelberg's puzzles in Psychic Logic; but Hughes and Cresswell find there is a difficulty with their version of this kind of analysis (*op. cit.*, pp. 204-5) because they cannot see how it can apply, extensionally, in Modal Logic. In the third part of this paper, therefore, I shall extend the ϵ-term treatment of descriptions to give a better account of descriptions in Modal Logic, and this will enable me to clarify still further the notion of Intentional (or 'Intensional') Identity, which arises in both cases. The relation between Modal Logic and Psychic Logic is also of more general moment, since it bears on both psychological accounts of logic and logical accounts of psychology; and this will lead to a clearer view of Modal Logic in itself.

1

We must first have a working conception of Hilbert's ϵ-calculus: this is a simple extension of standard predicate logic in which names for individuals are automatically provided. Hilbert's operator 'ϵ' is most readily thought of as a choice function: if there are F's then 'ϵxFx' selects one from amongst the F's, if there are no F's then 'ϵxFx' selects one thing from the universe at large. These two features ensure that 'ϵxFx' invariably has a reference, but also that ϵxFx is an F if and only if there are F's. Hence we can say

$$F(\epsilon xFx) \leftrightarrow (Ex)Fx$$

giving $-F(\epsilon xFx) \leftrightarrow (x)\text{-}Fx$

and $F(\epsilon x\text{-}Fx) \leftrightarrow (x)Fx$

so the quantifiers become definable and need not be taken as primitive. To formalize an ϵ-calculus, therefore, we need only allow the formation of ϵ-terms for every predicate in the language, and add to standard propositional logic the single axiom scheme

$$Fy \rightarrow F(\epsilon xFx)$$

where 'y' does not contain a free variable bound in any well-formed part of 'F' in which 'x' freely occurs (i.e. if 'y' is free for 'x' in 'F', see Leisenring, *op. cit.*, p. 12). The quantifiers can then be introduced by definition, as above, and all of standard predicate logic emerges as part of the system, as was said.

I am not concerned here with alternative axiomatizations, or with details of the choice-function semantics of this calculus (for these see Leisenring *op. cit.*, pp. 39f, and 18f). What concerns me in this paper are the linguistic readings of ϵ-terms, and one common misapprehension, which must be put aside for a start, is that ϵ-terms formalize *indefinite descriptions*. Certainly there is something arbitrary about them, but ϵxFx needn't be an F (see above), for one thing, and 'G(ϵxFx)' with 'H(ϵxFx)' entails '(Ex)(Gx . Hx)' (because 'ϵxFx' is a term), so 'G(ϵxFx)' and 'H(ϵxFx)' cannot be read 'Some F is G', and 'Some F is H' – these indefinite expressions do not entail 'Some G is H'. Hence an ϵ-term cannot be read 'some F' or 'an F', and the indefinition in ϵ-terms must arise in some other way: in my article 'E-type Pronouns and ϵ-terms' mentioned above I defend the view that constant ϵ-terms are *demonstratives*: these, on any occasion of use, have a unique reference, so they are constant expressions, but arbitraryness arises because details of the reference are not, in general, given in the term itself, but instead by means of some extra-linguistic accompanyment, for instance the context, or a gesture. The case involving uniqueness is the exception to this rule, since the reference of '$\epsilon x(Kx.(y)(Ky \rightarrow y=x))$' is determined independently of the speaker, if there is one and only one K. Certainly if there is not one and only one K then the reference of this term is not determined, but indefinition of that kind is present will all ϵ-terms: if there are no F's then 'ϵxFx' selects at random from the universe at large.

The main difference between the present account of the unique case and Russell's arises because of this feature:

$$(Ex)(Kx.(y)(Ky \rightarrow y=x).Bx)$$

i.e. $Kh.(y)(Ky \rightarrow y=h).Bh$

where $h = \epsilon x(Kx.(y)(Ky \rightarrow y=x).Bx)$

entails but is not entailed by

$$Bk$$

where $k = \epsilon x(Kx.(y)(Ky \rightarrow y=x))$.

For the latter does not imply the unique-existence condition in the former; but if that unique-existence condition does hold, then the ϵ-term 'k' satisfies '$Kx.(y)(Ky \rightarrow y=x)$', making the former entail '$h=k$', and hence 'Bk'. That gives the present account two prominent advantages over Russell's: it enables us to *distinguish* between 'there is one and only one king of France, and he is bald', and 'The king of France is bald'; and it enables us to isolate *atomic* propositions i.e. referential propositions with a singular subject, about an individual. For 'There is one and only one king of France, and he is bald' is now the *conjunction* of 'There is one and only one king of France' ($Kk.(y)(Ky \rightarrow y=k)$) and 'The king of France is bald' (Bk); and the latter is not quantified, but atomic.

It should be clear how these two features immediately resolve Edelberg's problems. For we now have available, in addition to (6) and (7) above

$$Bh(Ex)(Wx.Bx).BnC\epsilon x(Wx.Bx.(y)((Wy.By) \rightarrow y=x)) \quad (6')$$

and $$Bh(Ex)(Wx.Bx).BnC\epsilon x(Bh(Wx.Bx).(y)(Bh(Wy.By) \rightarrow y=x)) \quad (7')$$

also $$Bh(Ex)(Wx.Bx).BnC\epsilon x(Wx.Bx) \quad (6'')$$

and $$Bh(Ex)(Wx.Bx).BnC\epsilon xBh(Wx.Bx) \quad (7'')$$

(with (6″) the most plausible formalization of (1)), and in no case do we now have Nob believing the person who killed Cob's sow was, or was believed by Hob to be, a witch that blighted Bob's mare. But also, in all cases, we have Nob believing *an individual* killed Cob's sow, i.e. we have beliefs about something which can be *identified* with other objects of other beliefs, and so we have an expression for Intentional Identity. This means we need not go to (21) and (22) to try to get expressions which identify the objects of Detective A and Detective B's beliefs: instead we have

$$BaS(\epsilon xSx).BbJ(\epsilon xSx) \quad (21')$$

and $$BbJ(\epsilon xJx).BaS(\epsilon xJx) \quad (22')$$

and these are, as desired, not logically equivalent.

However, the benefits do not stop there, because Edelberg argues (p. 18) that the lack of symmetry between (19) and (20) reflects a special feature of the *intentionality* of the case, and just what that feature is now becomes more apparent. For, following Edelberg, we may consider a comparison of (19) and (20) with

Someone is in the park, and she is eating an apple (28)
Someone is eating an apple, and she is in the park (29)

where these extensional expressions are to be formalized, and shown to be generally non-equivalent, in the same manner as before, viz

$$P(\epsilon xPx).A(\epsilon xPx) \quad (28')$$

$$A(\epsilon xAx).P(\epsilon xAx). \quad (29')$$

But if $\epsilon xPx = \epsilon xAx$, this latter pair would become equivalent and Edelberg recognizes (p. 19) there would still be something different, in the intentional case, if 'the man who shot Smith' and 'the man who shot Jones' are the same 'thought object': for (20) might still be, in a certain sense, false (see above). Now if $\epsilon xSx = \epsilon xJx$ then (21′) and (22′) would, like (28′) and (29′), become equivalent, so we are forced into looking deeper for just where the difference between the intentional and the extensional cases lies. And if Detective A thinks that someone murdered Smith (S(ϵxSx)) but that the murderer of Jones did not murder Smith (-S(ϵxJx)), although the murder of Smith *is* the murderer of Jones, then the singular thing is that we have a case of *mistaken identity*, in which Detective A is confused about things, and believes the murderer of Smith (i.e. the murderer of Jones) both did murder Smith and also did not murder Smith, i.e. upon substituting 'ϵxSx' for 'ϵxJx' immediately above we get 'S(ϵxSx)' and '-S(ϵxSx)'. This fact about certain intentional contexts, (which renders them all transparent) is not commonly recognized, but there is a very simple proof of it, once we remember cases of mistaken identity. For if Detective A believes the murderer of Smith is not the murderer of Jones, and *is mistaken*, then it *must be* the murderer of Jones he thinks is not the murderer of Jones, i.e. 'the murderer of Smith' has its real reference in our expression for his beliefs. And this is the crucial case, for if there is no opacity *here*, amidst the ultimate confusion, there is no reason for it elsewhere. But if who Detective A thought was not the murderer of Jones was *not* the murderer of Smith, but some other object (*any* other object) then he would not be mistaken. Hence the case must be transparent, making 'BaS(ϵxSx)' and 'Ba-S(ϵxSx)' both true, and showing that 'BaS(ϵxSx)' does not rule out 'Ba-S(ϵxJx)' in connection with (21′) and (22′), in the way that 'P(ϵxPx)' rules out '-P(ϵxAx)' in connection with (28′) and (29′). One must remember that Leibniz' Law merely requires that 'Fs' and '-Ft' are inconsistent if s = t, and 'Ba-St' does not have the form '-BaSt', hence, even if ϵxSx = ϵxTx (21′) is consistent with

BbJ(ϵxJx) . Ba-S(ϵxJx).

Now we have shown this without introducing Edelberg's dubious ontology of 'thought objects' which (p. 18) 'exist in beliefs': 'ϵxSx' refers to the man who shot Smith and that is an extensional object, no matter what is thought about it (or whether anyone shot Smith). Equally we have an extensional object with the ϵ-term defined in terms of Hob's beliefs in (7′). So we have dispelled that 'opacity' which was the historic source of difficulties with intentional identity, and done so simply by getting hold of some good names for particulars, i.e. ϵ-terms. The temptation was, in Quine's classic case (see 'Quantifiers and Propositional Attitudes', *The Ways of Paradox*, New York 1966) to say that Ralph did not have beliefs about particulars if he believed the man in the brown hat was a spy and tha man at the beach was not a spy, when the man in the brown hat was the man at the beach. But resistance to such a reading, we can now see, came from an inability to formulate the right expressions. And once those expressions are formulated, there is no difficulty about 'quantifying in': for we have

$$(Ex)BrSx\,.\,(Ey)Br\text{-}Sy\,.\,\epsilon xBrSx = \epsilon yBr\text{-}Sy$$

and the two objects are identified outside a belief context, and so undoubtedly are extensional.

Now the possible inconsistency of thinkers, which this case also reveals, looks like it shows that we cannot take 'possible worlds' to be conceivable ones defined in terms of the contents of men's minds, i.e. in terms of Psychic Logic. So it looks like it rules out not just a certain Logicism there might be in Psychology, but also the well known Psychologism there has been in Logic. However there are some detailed features of Modal Logic which we must have settled before we can be entirely confident about this. We must produce a clear, and positive, alternative account of the basis of Logic before Psychologism can be completely routed, and it is towards that ultimate objective we must now turn.

2

Now, as was mentioned before, the present theory of descriptions, which Hughes and Cresswell allow as an adequate account of descriptions in non-modal contexts, runs into trouble, in their eyes, when applied unamended to the modal case. The difficulty they have lies in their inability to find an extensional semantics for it, in this wider application. They say (pp. 204-5):

> ...we need a rule for evaluating $(ix)\varphi x$ in a semantic model. In non-modal LPC it is easy to say: if there is exactly one member of D which is φ, let $V((ix)\varphi x)$ be that member of D; and if there is no such member of D let $V((ix)\varphi x)$ be some arbitrary object... And in modal systems which are prepared to admit intensional objects we could, analogously, let $V((ix)\varphi x, w_i)$ be the member of D which is φ in w_i if there is exactly one such and some arbitrary member of D if there is not. But for the other modal systems it is not at all clear

> what the corresponding rule should be. For even if in each world (in a given model) there is exactly one thing which is φ, this may not be the same thing in all worlds; and in such a case there will be no member of D which we can fix on as clearly the value to be assigned to $(\iota x)\varphi x$. The one kind of model in which it *is* clear what we should do is one in which one and the same member of D is the only thing which is φ in each of the worlds; i.e. in which there is some $u_k \epsilon D$ such that, for every $w_i \epsilon W$, u_k is φ in w_i and nothing else is; for then it is obvious that this u_k is 'the φ' in that model and should be assigned to $(\iota x)\varphi x$.

As a result of these considerations, Hughes and Cresswell modify their axiom for descriptions to

$$(E!x)L\varphi x \rightarrow \varphi((\iota x)\varphi x)$$

although even this they are dissatisfied with (see p. 206).

We can trace their problem to their conception of 'intensional object' (p. 197):

> Consider, for example, the expression 'the top card in the pack', as it occurs in the rules of a card game. The rules may, without ambiguity, specify that at a certain point in the play the top card is to be dealt to a certain player; yet on one occasion the top card may be the Ace of Spades and on another it may be the Queen of Hearts. Thus the phrase 'the top card in the pack' does not designate any particular card (individual piece of pasteboard), except in the context of a particular state of the pack; yet we can in one sense think of it as standing for a single object, contrasted with *the bottom card in the pack* and so forth. Such 'objects' are often called *intensional objects*, and the rules we have been considering [for contingent identity systems] would seem to provide a semantics for a logic in which ... the individual-variables range over intensional objects. We shall, however, not attempt to construct such a logic here, though, as the above discussion has indicated,
>
> (1) $L(Ex)\varphi x \rightarrow (Ex)L\varphi x$
>
> would be valid in it.

Now principle (1) is clearly valid if we simply apply our ϵ-calculus to the normal modal language, since

$$L(Ex)\varphi x \rightarrow L\varphi(\epsilon x\varphi x)$$

and $$L\varphi(\epsilon x\varphi x) \rightarrow (Ex)L\varphi x.$$

So it might seem as though, if we simply kept to the description axiom in its original form, we would be committed to allowing 'individual' variables to range over non-constants like 'the top card in pack i'. However there is an important distinction which Hughes and Cresswell overlook here, between the top cards in various packs (or the same pack at different times) and the possible top cards in *that* particular pack, *then*. For in the case of multiple packs (or multiple times) we must formalize 'In every pack there is some one top card' as

$$(i)(Ex)\varphi xp_i$$

in which 'φxp_i' is 'x is the top card of pack i', and since this is

$$(i)\varphi(\epsilon x\varphi xp_i)p_i$$

in which '$\epsilon x\varphi xp_i$' is not free for 'y' in '$(i)\varphi yp_i$', we cannot use the Hilbertian axiom to get

$$(Ex)(i)\varphi xp_i$$

i.e. 'There is some card which is the top in every pack'. Hence there is no object *of any kind* here, which is contingently identical, say sometimes with the Ace of Spades and sometimes the Queen of Hearts. If in packs 1, 2, 3, etc. the top cards are a, b, c, etc., there is a common *form of name* '$\epsilon x\varphi xp_i$' (the top card in pack i) which each of the top cards may have; but a, b, c, etc. are not identical in any sense with this variable: they are its *values*, and we have a series of proper identities, '$a=\epsilon x\varphi xp_1$', '$b=\epsilon x\varphi xp_2$', '$c=\epsilon x\varphi xp_3$' etc., to prove this.

In the case of the possible top cards of *that* pack *then*, on the other hand, we must symbolize the matter

$$L(Ex)\varphi' x$$

i.e. $$(i)(V((Ex)\varphi' x,w_i) = 1)$$

or $$(i)(Ex)(V(\varphi' x,w_i) = 1)$$

where '$\varphi' x$' says 'x is the top card of that pack then', and '$V(\varphi' x,w_i)=1$' says not that x *is* the top card of that pack then in some one of several *actual* circumstances w_i, but that it *would be* the top card of that pack then in some possible circumstance w_i. And now we are assured of a single something which in every possible circumstance *is* the top card of the pack, not because it has the required formal name of such things, but simply because the *actual* top card, i.e. $\epsilon x\varphi' x$, fills this bill. So we get, from the above two semantic expressions

$$(i)(V(\varphi'\epsilon x\varphi' x, w_i)=1)$$

and $$(i)(V(\varphi'\epsilon x[V(\varphi' x, w_i)=1],\ w_i)=1)$$

giving, $$(i)V(\epsilon x\varphi' x\ =\ \epsilon x[V(\varphi' x, w_i)=1],\ w_i)=1)$$

because of the uniqueness condition in 'φ'' and the fact that possible worlds, unlike psychological minds, must be logical. And here we do have a contingent identity, in each case, but one between two straight-forward objects, namely the actual top card ($\epsilon x\varphi' x$), and the card which would be top in that case ($\epsilon x[V(\varphi' x, w_i)=1]$). In other words, if the top card actually is the Ace of Spades, and it is supposed the top card is the Queen of Hearts, then what would have to be true for those circumstances to obtain would be for the Ace of Spades to be the Queen of Hearts. Indeed if what would have to be true was not so contrary to fact then those circumstances would not be contrary to fact, either.

Now, from

$$(i)(V(\varphi'\epsilon x[V(\varphi' x, w_i)=1], w_i)=1)$$

we certainly cannot get

$$(Ex)(i)(V(\varphi' x, w_i)=1)$$

since '$\epsilon x[V(\varphi' x, w_i)=1]$' is not free for 'y' in '$(i)(V(\varphi' y, w_i)=1)$', as before. But from

$$(i)(V(\varphi'\epsilon x\varphi' x, w_i)=1)$$

we can get

$$(Ex)(i)(V(\varphi' x, w_i)=1)$$

since '$\epsilon x\varphi' x$' denotes a constant, the actual top card in the pack. Hence from '$L(Ex)\varphi' x$' we have got '$(Ex)L\varphi' x$', again, showing (1) is even more secure. But notice that from

$$(i)(V(\epsilon x\varphi' x=\epsilon x[V(\varphi' x, w_i)=1], w_i)=1)$$

while we can get

$$(Ey)(i)(V(y=\epsilon x[V(\varphi' x,w_i)=1],w_i)=1),$$

we cannot get

$$(Ey)(i)(y=\epsilon x[V(\varphi' x,w_i)=1])$$

since all the contingent identities between the actual top card and the various cards which would be top in other circumstances are each only true in those other circumstances, which means they do not lead to any grand identity of all the cards which would be top, and make the actual top card the only possible one there could have been.

To recapitulate the essentials of all this with a different example: if Tom, Dick and Harry are the contestants in some race then the winner of that race will be a certain one of them, say Tom. But while the winner of the race might have been Dick, in certain circumstances, that does not mean (because Tom *is* not Dick) that it is not Tom which would have been Dick, in those circumstances. If Dick would not be Tom, *then*, then supposing Dick was the winner of the race would not be contrary to fact, since the 'winner of the race' in this case would just be the winner of the race *then*, i.e. Dick, and not the winner in fact, i.e. Tom. So we do not have to invent a nebulous 'intensional' entity, a variable 'the winner', which *is* not any of the contestants, but might have been each one, to account for the possibilities in the case. We have an entity which *is* one of the contestants, but might have been any one of the others, and it's not being one of the others is what settles the facts of the matter. Nor does Tom have to have been the only possible winner, i.e. the only contestant, before we can identify a (constant) winner here.

It was this latter sort of case which Hughes and Cresswell took to alone determine a clear reference for 'the φ' (see the end of the first quotation from them, above), and it was thus what guided their change of the axiom for descriptions into one with '$(E!x)L\varphi x$' in its antecedent. But the case does not have this expression, as we have seen; and indeed it has no common expression outside the standard semantical language (augmented with ϵ-constructions) we have been using: it determines not 'the φ' but 'the necessary φ', and would be symbolized using '$(L\varphi)x$' rather than '$L(\varphi x)$' in the more subtle object language Hughes and Cresswell sketch in their note on pp. 199-200. But we do not need a separate object language, i.e. mode of expression, and we certainly do not need a more subtle object or meta-language than that used in standard modal semantics, since *it* (augmented with ϵ-constructions) can express all we currently need: though if we are to use it more commonly, its complication suggests we could do with a simpler version. Also, of course, it is now clear that we do not need the complicated description axiom which Hughes and Cresswell construct, since the original Hilbertian one, properly understood, is serving us even better. In settling on a formalism, therefore, I shall stay with the basic ϵ-calculus defined in section 1, but write '$V(\varphi x, w_i)=1$' simply as 'Wiφx', which is to be read 'It would be the case in circumstance i that φx'. Then '$V(\varphi x, w_i)=0$' is, indifferently, '-Wiφx' and 'Wi-φx', so we have a quasi-metric-tense expression for the fundamental form of statement in Modal Logic. This form of expression was once hidden away in the semantics, but is now overt; all other needed statements can be defined in terms of it, with 'L(φx)' being '(i)Wiφx' and '(Lφ)x' being '(i)(x = ϵyWiφy)' etc. The operator, 'It would be the case in circumstance i that', removes the old difficulty about 'Modal Realism': 'Wiφx' is not a relational expression, like, say, '$\varphi x p_i$', stating what is in fact the case, but a subjunctive expression supposing some case is a fact and saying what *would be* in that case. Cases and circumstances are described by means of stories, so unearthing the basic mode of expression 'Wiφx' shows us that stories are the reality which Modal Logic is about. These stories are maximally consistent, but, of course, that doesn't make them true: 'Wiφx' is what may be true though 'φx' is false, so it is not the story, but what would be true in the case which we can know to be true at first hand. A maximally consistent story obeys the following laws.

$$(Wip\,.\,Wiq) \leftrightarrow Wi(p\,.\,q) \qquad (A)$$

$$Wip \leftrightarrow -Wi\text{-}p \qquad (B)$$

to which we may add $\quad Wop \leftrightarrow p \qquad (C)$

though (C) must not be taken to imply we can, by inspection, determine which story should be indexed 'o', i.e. which story is true. This gives us a propositional system, W, something like the common system T: 'Lp → p' follows from the quantificational principle '(i)Wip → Wop'; 'L(p → q) → (Lp → Lq)' follows from '(Wip . Wi-q) → Wi(p.-q)' since then 'Wi(p → q) → (Wip → Wiq)'; also the rule of necessitation is obtainable, since, for instance, we cannot have 'Wi(p.-p)', by (A) and (B), and so, by (B), we must have 'Wi(pv-p)', and in a like manner we can derive 'Wit' for any tautology 't'.

There is no way that any reduction law for iterated modalities works, in this system, since, now that the fictional basis for modal logic has been exposed, any 'seeing' or 'accessibility' relation clearly could not be generally necessary, since consistent stories have been stipulated so that what would be true in them was otherwise. But the unavailability of any general thesis about iterated modalities means in particular that we do not have '$p \rightarrow LMp$', and so the remaining major theoretical problem with identity in modal (and psychic) contexts is over: for then, while we can derive from Leibniz' Law

$$x=y \rightarrow L(x=y)$$

we cannot derive

$$x\neq y \rightarrow L(x\neq y)$$

(see Hughes and Cresswell, *op. cit.*, p. 190). Absence of the latter allows things non-identical in real life (like Tom and Dick, an Ace and a Queen) to be identified in some consistent story; but presence of the former means that things identical in real life (like the murderer of Smith and the murderer of Jones) may be thought to be non-identical only at the risk of confusion. It also means that identicals stay identical in all stories, and all thoughts: a false tale does not prevent a true allusion, indeed it can be false only if it does this.

3

Apart from the detailed points I have made throughout this paper the overall tentative conclusion I draw from it is the one foreshadowed at the end of section 1, on the relation between Modal and Psychic Logic. It used to be thought that Modal Logic was about the imaginable and conceivable, and this psychologizing of the topic not only subjectivized the *a priori*, but also, in reverse, made it seem men's minds were consistent, and automatically drew conclusions, i.e. were logical. We must, I think, sever both these associations: men's minds can be non-logical and thoroughly inconsistent, and what is logical is something objective, namely a read story or a heard speech. The 'imaginable', 'conceivable', 'supposable' thus give way to the *consistent*, in an area where '$Wi\text{-}\varphi x$' contradicts '$Wi\varphi x$'; contrarywise the subjective mind is given free reign in an unbounded fantasy land where 'Bn-Sx' is quite disconnected from 'BnSx'. In neither area is there 'opacity', or the traditional difficulty with 'quantifying in', for the theoretical problems with Intensional Identity are resolved simply by using a good theory of (i.e. expression for) descriptions, and an extended predicate calculus which supports this.

If there is any particular example which can summarize the essence of the matter it is of the sort given by Hughes and Cresswell, when attacking 'L(Ex)φx → (Ex)Lφx'. They said (*op. cit.*, p. 144):

> Let φx be 'x is the number of the planets'. Then the antecedent is true, for there must be some number which is the number of the planets (even if there were no planets at all there would still be such a number, viz. 0): but the consequent is false, for since it is a contingent matter how many planets there are, there is no number which *must* be the number of the planets.

But 'L(Ex)φx' i.e. '(i)Wi(Ex)φx', while it does not entail '(Ex)(i)(x = ϵyWiφy)', i.e. '(Ex)(Lφ)x', does entail '(Ex)L(φx)' i.e. '(Ex)(i)Wiφx'; for there is a number, namely 9, which though it needn't have been the number of the planets is, as a matter of fact, the number of the planets, and so is still that in each consistent story, i.e. each contingency.

The University of Western B.H. SLATER
Dept. of Philosophy
Australia