RELEVANT CONTAINMENT LOGICS AND CERTAIN FRAME PROBLEMS OF AI

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Abstract. Relevant containment logics, which combine relevant logics with content containment requirements, are motivated and explained. Semantics for some of these logics are introduced and shown to be adequate. In the light of the semantics the logical theory is improved, and other directions for elaboration are indicated. Finally, the logics are applied to one significant part of the vexatious frame problems of *AI*, and a route to implementation is suggested.

One many occasions there appears to be a real need for a relevant implication relation which does not introduce or carry superfluous or irrelevant information. In an implication connection A → B, B contains superfluous content if B contains content not represented in A. An implication meets a *tight* containment requirement if it does not sanction the introduction of such superfluous content. The standard relevant logics do not meet such a containment requirement (though they can satisfy *other* plausible accounts of containment; see [1], p. 155). They fail it because, in particular, of principles like Addition, A → A ∨ B and B →. A ∨ B, which tack on disjunctively what may be fresh or extra information.

Nor is an obvious alternative to a relevant logic, a Parry logic, or analytic implication system, altogether satisfactory. For while such containment logics, also weakly relevant, do exclude additive expansion, they do not exclude other types of damaging (and implausible) explosion, notably (but unsurprisingly given their modal bases) of necessary and of inconsistent information. ([1]) For instance, they underwrite implicative explosion in

([1]) The trouble with Parry logics by no means end there. For a detailed critique of these logics and their motivation, see [11], esp. p. 96ff., where such logics are called *conceptionist* (and indexed thus).
It needs stressing that standard relevant logics straightforwardly satisfy important containment requirements, such as that A → B holds true iff the content of B is included in that of A; see e.g. [11] pp. 216-7.

such forms as A →. A → A, A →. (A → A) → (A → A), ..., in demodalised systems, or similar chains with antecedents such as A → B or B → A in modalised versions. Nor do they entirely exclude additive effects; for Addition is available in rule form, thus apparently violating adequacy conditions (as Kielkopf has shown; see further [11] p. 101). Indeed the systems are worthless for major paraconsistent purposes, since they spread contradictory information everywhere, by virtue of implications such as (A & ~A) & B →. B & ~B and (A & ~A) & (B ∨ ~ B) →. B & ~ B, direct consequences of the relevant bête noire, Disjunctive Syllogism, A & (~A ∨ B) → B, rightly excluded in main relevant and paraconsistent logics (cf. Deutsch, p. 139). Since one of B or ~B holds true, an isolated contradiction, A & ~A say, gets spread everywhere (at least where B is determinate), to arbitrary contradiction B & ~B.

It looks as if the best of both, relevant and containment logics without the defects, can be had by combining the two, essentially product-wise. So result, in one way or another, relevant containment logics. But, as will emerge, amalgamation is by no means straightforward (especially once the simple deceptions of modal confines are left behind); and combinations are far from uniquely determined.

1. Reasons for the enterprise and for dissatisfaction with modally-based approaches, such as analytical implication.

There are systems which satisfy containment requirements, and which, as a corollary, are relevant; namely, Parry's system of "analytic implication" and systems in its vicinity. But though some of these systems have pleasant properties, such as neat semantics and algebraic analyses, they nonetheless include a certain amount of junk. The reason for this is that they amount to restrictions of normal modal logics (or even their degenerate limit, classical logic), and carry over the junk from these systems, some of it already exhibited. Other undesirable junk includes "relevantised" versions of all the noxious paradoxes of implication. By contrast, the aim of the present enterprise is to start from systems from which modal and classical junk has *already* been appropriately removed, such as certain (deeper) relevant logics, and to add containment requirements in one way or another to such systems.

Put differently, one main raison for proceeding beyond logics like Parry's is that such logics offer inadequate *control*. For example, Parry's logic spreads identity about in a quite profligate way, similarly inconsistency. A mere *filter* on strict implication is far from satisfactory; the rot has already set in extensively with strict implication. As a result, systems like Parry's are not particularly useful for several of the main applications envisaged; and some requisite work has already in effect gone into modifying Parry logics with a view to reducing some of these problems (e.g. Deutsch, Daniels).

The sorts of applications that have been considered, in one way or another, or that should be, include these. They fall into two overlapping groups, roughly:

1. Cases and logics involving circumscribed contexts, where additional (irrelevant) information is, in some way, inappropriate:
• logics of fiction and stories, where additions are in serious doubt. According to the story story, stories don't introduce new content (see e.g. Daniels, pp. 221-2).
• communication networks, and polylogue theory. For communication and dialogue, like stories upon which they generalise, are characteristically information circumscribed. (For the many places in polylogue theory where relevant containment logics tend to enter, see Sylvan.)
• meaning, inclusion of meaning synonymy, analysis analysis. For example, even if inclusion-of-meaning theory and Tarski-Davidson meaning-through-truth theory are respectively rectified by setting the theories on relevant logic foundations (see [11] for the first and, for the second, Priest and Crosthwaite), odd puzzles at least remain. For, primarily by virtue of Addition, relevant logics permit the introduction of superfluous meaning, through such biconditionals as $p \leftrightarrow. p \& (p \vee q)$, $p \leftrightarrow p \& (p \vee q) \& (p \vee r)$..., etc., where q, r, ..., may have nothing to do with p. But these relevantly equivalent expressions don't have the same meaning. So in particular a "meaning-giving" T-schema of the form, s is true $\leftrightarrow$ p, should not deliver, s is true $\leftrightarrow. p \& (p \vee q) \& (p \vee r)$. By switching to relevant containment logics (which are derivationally adequate for the purposes at hand), *these* difficulties are avoided.
• frame problems. In important respects, frame problems (investigated in more detail below) typify this whole class of problems.

2. Cases and logics where tight relevance is required, or at least paradoxes and like puzzles are resolved through such relevance considerations. As

useful surveys of such cases are available elsewhere (see especially Weingartner and Schurz), there is excuse for even briefer listings:

- doxastic logics, assertion logics, epistemic logics and so forth, where again superfluous additions are in much dispute. Belief and assertion functors are normally closed under some relevant operations significantly tighter than implication. A suggestion to toy with, seriously, is that some at least of the main intensional functors involved are closed under relevant containment.
- deontic logics, preference logics, and volitive logics (with operators like "desires that", "wishes that", "feels that"). Again something in contained in the judgements made, from which something relevant emerges logically; some relevant commitments are made.
- legal reasoning where (Ross's "paradox", has some exposure), and relevant reasoning more generally (cf. Bollen for what is to be varied).
- logics of explanation and confirmation, where superfluity again generates several puzzles (see Weingartner and Schurz, whose classical tack-on resolutions are *not* however endorsed, for reasons essentially given in Kielkopf and [11]).

Most of these problems – those that are genuine, that is, for not all are – can be tackled, in a fairly uniform fashion, through (deep) relevant containment logics. Some admittedly can be dealt with in Parry logics; but most cannot, so uniformity of treatment would be lost. To be sure, there are economic advantages that logics in the vicinity of Parry's do have, namely that variable inclusion (*if* that is what is sought) can be represented *within* the logics, e.g. through formulae like B →. A → A and B → B →. A → A. A perhaps surprising feature of relevant logics is that there is (so it is conjectured) no way of representing such variable inclusion within the logic. It is possible to represent *overlap* of variables, e.g. A ⦿B iff A → B v. B → A. But to define inclusion in terms of overlap requires (presumably) what amount to propositional quantifiers, e.g. C ⊇ D iff (B) (if B ⦿D then B ⦿C).

To couple inclusion notions with relevant (first order) logics, requires then further logical apparatus. Specificially, it calls for an appropriate connective ⊇, to symbolise inclusion of content or information. This content-inclusion connective, obtainable perhaps by definition, should – apart from satisfying expected inclusion conditions such as transitivity – hold *where* variable inclusion obtains (*but* not necessarily conversely);

i.e. if the variables of B are included among those of A then $A \supseteq B$. Nor, however, is variable inclusion, though governing $\supseteq$, always enough for containment implication, as implication explosion reveals. A good underlying implication connection, such as deep relevant logics furnish but modal logics do not, is an essential prerequisite.

Relevant containment logics combine then relevant logics with a further inclusion requirement, in product fashion. A relevant containment logic is one which includes an implication $-\gg$ which is both relevant and satisfies tight containment, symbolized through connective $\supseteq$. Relevant containment $-\gg$ accordingly is defined: $A -\gg B$ iff $A \rightarrow B$ & $A \supseteq B$. (2) The inclusion of content concerned would ordinarily be presented more explicitly through a content function, c say, and an inclusion relation on terms, i.e. $A \supseteq B$ would expand to $c(B) \succ c(A)$. But for the present, c functions have conveniently been absorbed. The combined effect of relevant implication *and* containment is to peel off such orthodox principles as $A \rightarrow . A \vee B$, $A \rightarrow . A \rightarrow A$, $A \leftrightarrow . A \vee (A \& B)$, etc., principles which in many circumstances prove an embarrassment, because they can introduce superfluous content. For $A -\gg A \vee B$, $A -\gg . A -\gg A$ and the like do not hold. The first of these fails because content inclusion breaks down, as with analytic implication; the second is invalid for relevant reasons (see the discussion of Mingle in [1] and [11]).

Already a *categorematic* account of content inclusion – where connectives (and wider context) do not contribute essentially to content – has been insinuated. But, despite its historical sponsorship, such an account is in no way dictated, and indeed is decidedly dubious. There are (out there in *aussersein*) other, less restrictive and more satisfactory, theories of content and content inclusion. Content is a determinable with many determinates, which vary considerably in character and quality. Nonetheless, the initial logical theory advanced will begin with categorematic theories of content inclusion; they are places to start from that have much to offer.

(2) In effect this adapts to relevant settings an old idea revived by Gödel (and worked out by Fine) for Parry's system AI. In essence the idea is ancient, reaching back perhaps to Boethius but certainly to Abaelard.

2. *Some details of initial relevant containment logics and their semantics*

To begin with, there is an underlying relevant logic, L say, the *carrier logic*, with conventional connective set $\{\rightarrow, \&, \vee, \sim\}$ for instance (as in [11]). Later, the carrier logic may be removed, with implication $\rightarrow$ supplanted by containment connective $-\gg$. But, for the present, containment logic is superimposed upon the carrier logic. This is done by adding to L, as on the surface in relational logic (³), a relation r of the right type, a two-place connective, with the following features: By virtue of its grammatical categorisation, where A and B are wff, so is r(A, B). The chief logical feature required of r is that it enables the definition of a certain order relation, a (lattice-ordered content) *containment* relation, $\supseteq$, on wff.

Relation $\supseteq$ is required to be transitive (achieved of necessary by taking the transitive closure of r) and is variable governed, and it meets appropriate semi-lattice conditions, e.g. it composes fully. Specifically,

$\supseteq$1. $A \supseteq B \ \& \ B \supseteq C \rightarrow . \ A \supseteq C$.
$\supseteq$2. $A \supseteq B$ where $\vee(A) \supseteq \vee(B)$, with $\vee(A)$ representing the set of variables of A.
$\supseteq$3. $C \supseteq A \ \& \ D \supseteq B \rightarrow . \ C \ \& \ D \supseteq A \ \& \ B$.
$\supseteq$4. $A \supseteq B \leftrightarrow . \ A \ \& \ B \supseteq A \ \& . \ A \supseteq A \ \& \ B$. Scheme $\supseteq$ 4, though dubious in a relevant setting, supplies the standard lattice linkage, of order with identity. For it reduces to $A \supseteq B \leftrightarrow . \ A \ \& \ B \equiv A$, where $C \equiv D$ is defined as $(C \supseteq D) \ \& \ (D \supseteq C)$. Together with the vital connection
$\supseteq$0. $A -\gg B \leftrightarrow . \ A \rightarrow B \ \& . \ A \supseteq B$, this completes axiomatisation of the basic initial logic CLI. The axiomatisation admits of much tampering; for instance (given replacement) the awkward lattice principles of $\supseteq$4 can be broken down into the perhaps more amenable trio $A \supseteq B \rightarrow . \ A \ \& \ B \supseteq A$, $A \supseteq B \rightarrow . \ A \supseteq A$, $A \supseteq A \ \& \ B \rightarrow . \ A \supseteq B$.

(³) The approach differs from the relational logic of the literature (presented in most detail in a special issue of *Philosophical Studies* 36 (1979), which is criticised in [10]). For here (as in effect in Deutsch) variable-sharing is *not* taken as a sufficient condition for relevance, and accordingly transitivity is not abandoned. Furthermore, main paraconsistent purposes are not sacrificed, as in standard relational logics, which like their close relatives Parry logics, simply tack relations onto irrelevant logics, of strict or classical varieties. Thus the main objections lodged in [10], pp. 137-8, against relational logics are avoided on the approach taken here.

The axiom schemes in $\supseteq$ can be argued to independently of their role for the semantical theory (see [12]). But in fact the schemes reflect exactly what is required for a modified equivalence-class completeness proof on inclusion of content, with contents getting modelled canonically through equivalence defined classes of wff. But, just as it proves possible to provide Lindenbaum algebras for logics where appealing equivalence connections are lacking, so it is possible to provide somewhat messier completeness proofs where various underwriting schemes in $\supseteq$ are missing. Certainly, as we shall see, it is not too difficult to peel off the less convincing semi-lattice linkages imposed in $\supseteq$4. However the conditions are not demanding by conventional standards, and transitive closures of more familiar variable-connecting relations r will meet them.

More or less any adequate semantics for the carrier logic L can be used, so long as it affords a way of separating out regular theories or worlds or structures, that is those where all theorems of CLI hold. Here we shall suppose that something like the usual relational frames or model structures, of form $M = \langle T, O, K, R, * \rangle$, for relevant logics are used, with $O \subseteq K$ the class of regular situations, and actual situation T in O.

Interpretations on model structures for the carrier logic are also as for relevant logic (for details see [11]); whether wff A holds at a, symbolised $I(A, a) = 1$, is defined for every situation or world a in K. What further semantical apparatus is required in order to represent content inclusion? The requisite additional apparatus can be reached in a straightforward naive fashion. Since, naively, $A \supseteq B$ holds true just where the content of A includes that of B, i.e. $I(A \supseteq B, T) = 1$ iff $c(B, T) < c(A, T)$, then, by straightforward world relativisation,

$$I(A \supseteq B, a) = 1 \text{ iff } c(B, a) < c(A, a), \text{ for every a in K.}$$

The (categorematic) content of A at a is in turn governed by (the sum of) that of its sentential parameters, as follows, for regular situations a in O:

$$c(A, a) = c(p_1, a) \wedge \ldots \wedge c(p_n, a),$$

where $p_1, \ldots, p_n$ are precisely the sentential parameters of wff A. In regular situations A says what its parameters jointly say. In regular situations, moreover, the operation $\wedge$ amounts simply to a (semi-)lattice operation on contents, given for instance by the least upper bound on the content structure ordered by $\supseteq$ ([4]) (for semi-lattice properties, see Curry, p. 131ff). For each situation a not in O, c(A, a) is assigned content arbitrarily from C(a), the information or content at situation a, subject only to the restriction that c(A & B, a) = C(A, a) $\wedge$ c(B, a). For such nonregular situations more general structures than semi-lattices are required, because at them reflexive and other lattice properties may, like all logical principles, fail. An appropriate structure at each such a is an *order-structure* $<C(a), <, \wedge>$, or $<C(a), \wedge>$ since order can be defined in the usual way: for α, β in C(a), $\alpha < \beta$ iff $\alpha \wedge \beta = \beta$. The relation $<$ is transitive on C(a), composes fully, and satisfies the (definitional) condition $\alpha = \beta$ iff $\alpha < \beta$ & $\beta < \alpha$. It follows immediately that $(\alpha \wedge \beta < \beta)$ & $(\beta < \alpha \wedge \beta)$ iff $\alpha < \beta$. It is an elaborate story for a structure which underwrites no theses!

To pull all this together, the structure required is given by a functor C supplying a set C(a) of contents or bits of information, at each situation a in K, with the set C(a) ordered by $<$ with meet operation $\wedge$. It is enough to add C and $\wedge$ to L model structures (m.s.) to obtain CLI m.s.. Thus a CLI m.s. is a structure $<T, O, K, R, {*}, C, \wedge>$, with $<(C(a), \wedge>$ an order-structure for each a in K.

A CLI (mark 1 containment logic) *model* adds to a CLI m.s. two valuation functions, I(or $\vee$) and c. Valuation I, from sentential parameters and situations to holding values 1(on) and 0(off), subject to a hereditariness requirement, is as in relevant semantics. The hereditariness requirement and several of the rules resemble those for intuitionistic semantics. Hereditariness is extended to include the further connective $\supseteq$ of CLI; specially, wherever a $\leq$ b and I(A $\supseteq$ B, a) = 1 then I(A $\supseteq$ B, b) = 1. Valuation c is also from sentential parameters and situations, but maps to situationally-associated content; i.e. c(p, a), with c representing the information or content of p at a, belongs to C(a). Both I and c are extended from initial sentential wff to all wff inductively. The evaluation clauses for I, as applied to connectives &, $\vee$, $\sim$, $\rightarrow$, are just those of relevant semantics. The further clauses for c are just those already given.

([4]) As it happens, operations can be taken either as a meet (l.u.b.) – more natural for the intended summation of contents story – or as a join (g.l.b.), how it may look. For all that is required in the semantics is that $\wedge$ is commutative, associative and idempotent. Then whether $\wedge$ is a meet or join will depend upon which order is adopted: $\beta \supseteq \gamma$ iff $\gamma \wedge \beta = \beta$, or the converse: $\gamma \subseteq \beta$ iff $\gamma \wedge \beta = \beta$.

3. *An outline of soundness and completeness arguments*

In the *soundness* argument, the following properties drawn from relevant semantics much simplify verification of postulates: $I(C \rightarrow D, T) = 1$ iff for every situation a, where $I(C, a) = 1$ then $I(D, a) = 1$. Similarly $I(C \leftrightarrow D, T) = 1$ iff for every situation a, $I(C, a) = I(D, a)$. These connections enable immediate validation of $\supseteq 0$, given the evaluation rule for $- \gg$ at an arbitrary world simply conjoins those for $\rightarrow$ and $\supseteq$.

ad $\supseteq 1$. By the foregoing it suffices to show that where $c(B, a) < c(A, a)$ and $c(C, a) < c(B, a)$ then $c(C, a) < c(A, a)$ for arbitrary a in K. But this is a result of transitivity of $<$ in C(a).

ad $\supseteq 2$. Suppose $\vee(A) \supseteq \vee(B)$. Then for a in O, $c(B, a) = \Sigma c(p_B, a) < \Sigma c(p_A, a) = c(A, a)$, where p_A ranges over variables of A and p_B over those of B and Σ indicates $\wedge$ summation. Hence, for a in O, $I(A \supseteq B, a) = 1$, whence as T is in O, $A \supseteq B$ is valid.

ad $\supseteq 3$. and $\supseteq 4$. By virtue of the simplifying features and the connection $c(A \& B, a) = c(A, a) \wedge c(B, a)$, these reduce to order-structure properties at arbitrary a in K.

Completeness is established through a canonical modelling. To the canonical m.s. for the carrier logic, defined in a familiar way, are added further details for C and $\wedge$. The requisite canonical order-structure is arrived at by adapting standard methods for Lindenbaum algebras. A quasi-equivalence on wff, $\tilde{a}$ is defined for each situation a thus: $A \tilde{a} B$ iff $A \equiv B \in a$, i.e. iff $A \supseteq B \in a$ & $B \supseteq A \in a$. Then $\tilde{a}$ is symmetric and transitive, and reflexive for $\tilde{a} \in O$. Let $|A|_a$ be the quasi-equivalence class of A under $\tilde{a}$, i.e. $\{B : B \tilde{a} A\}$. Then C(a) is the class of all these classes, i.e. $C(a) = \{|A|_a : A \text{ is a wff}\}$; and correspondingly $|A|_a \wedge |B|_a = |A \& B|_a$. By virtue of $\supseteq 3$, $|A \& B|_a$ is suitably independent of the choice of wff A and B.

• $<C(a), \wedge>$ is an order-structure, for each a in K. Requisite properties follow from the CLI axiom schemes using connections established below. Canonical valuations are defined as expected: $I(p, a) = 1$ iff $p \in a$ and $c(p, a) = |p|_a$, for every sentential parameter p and every situation a. These interconnections are extended inductively or definitionally to every wff A involved. The definitional element is the general stipulation, for a not in O, $c(A, a) = |A|_a$. Then requisite details, beyond those for the carrier logic (which are as in [11]), are these:

• $c(A, a) = |A|_a$, for every wff A and situation a.
The generalising step, for a in O, is as follows, where $p_1, ..., p_n$ are all the sentential parameters of A:

$$\begin{aligned} c(A, a) &= \Sigma c(p, a) && \text{by its evaluation rule} \\ &= \Sigma\ |p_j|_a && \text{, by the given basis} \\ &= |p_1 \& \ldots \& p_n|_a, && \text{by iteration of } |p \& q|_a = |p|_a \wedge |q|_a \\ &= |A|_a \end{aligned}$$

It is in this final step that restriction to situations in O is crucial. By ⊒2, both the following results are derivable: p_1 & ... & p_n ⊒ A and A ⊒ p_1 & ... & p_n. Since a is regular, both those results belong to a. Hence p_1 & ... & p_n $\tilde{a}$ A, vindicating the final step.

• c(A & B, a) = c(A, a) ∧ c(B, a). By the preceding argument and definition of ∧.

• I(A ⊒ B, a) = 1 iff A ⊒ B ϵ a.
Because of the evaluation rule for ⊒, it suffices to show c(A, a) < c(B, a) iff A ⊒ B ϵ a. Now

c(A, a) < c(B, a) iff $|A|_a < |B|_a$, as above
iff $|B|_a \wedge |A|_a = |A|_a$, by order-structure definition
iff $|A|_a = |A \& B|_a$, by definition of ∧
iff A $\tilde{a}$ A & B, by canonical definitions, symmetry and transitivity
iff A ⊒ A & B ϵ a &. A & B ⊒ A ϵ a, by canonical definitions
iff A ⊒ B ϵ a, by ⊒4.

The connection also enables validation of hereditariness as regards ⊒. For I(A ⊒ B, a) = 1 and a ≤ b supply A ⊒ B ϵ a and a ⊆ b, which guarantee A ⊒ B ϵ b, whence I(A ⊒ B, b) = 1. And that finishes the outline of the completeness argument.

As usual, the logical theory can be strengthened by further postulates and corresponding modelling conditions. Let us consider just one important case, that of ↔ replacement, a case which reveals as well just how precious the basic logical theory CLI is algebraically (e.g. it doesn't permit replacement of C & C by C everywhere). The replacement rule, SE. A ↔ B / D(A) → D(B), reduces in the setting of relevant affixing logics to the cases for ⊒, i.e. to

$\supseteq$SE. A $\leftrightarrow$ B / A $\supseteq$ C $\rightarrow$. B $\supseteq$ C
/ C $\supseteq$ A $\rightarrow$. C $\supseteq$ B

The corresponding modelling condition, which interconnects valuations I and c, is a trifle curious. It is in effect this: If I(A, d) = I(B, d) for every d in K, then c(A, a) = c(B, a).

4. *Upgrading the logical theory*

The logical system CLI presented, while decidedly appealing as regards the $\rightarrow$ and $-\gg$ theorems it delivers and even more for what is fails, still has noticeable shortcomings. For it still yields such dubia as A $\supseteq$ B $\rightarrow$. A $\supseteq$ A. And though it properly avoids such atrocities as A $\rightarrow$. A $\supseteq$ A, and worse B $\rightarrow$. A $\supseteq$ A, it does so at a cost in complexity of specially contrived semantical apparatus. Without that, without substitution of order-structures for semi-lattices at nonregular worlds, it would validate those and other noxious results, by virtue of the evaluation rule I(A $\supseteq$ B, a) = 1 iff c(A, a) < c(B, a), whence A $\supseteq$ A and other lattice themes would hold at *all* situations supplied. To make requisite room for impossible situations where lattice linkages fail, a better procedure is to modify the evaluation rule, for instance as follows:

for a in O, I(A $\supseteq$ B, a) = 1 iff c(A, a) < c(B, a); and otherwise, i.e. for a in K-O, I(A $\supseteq$B, a) is assigned arbitrarily, except so far as constraints like hereditariness are to be met. In short, for a $\notin$ O, I(A $\supseteq$ B, a) is treated like I(p, a), as an initial assignment.

Such an adjustment also enables dubious lattice connections to be removed, in particular the ungainly $\supseteq$4. However an obvious way in which to accomplish this feat – by weakening $\supseteq$1 and $\supseteq$3 to rule forms – does not prove robust enough to carry the style of completeness argument so far used. Fortunately an alternative is ready to hand, which averts difficulties; namely, make the semi-lattice conditions, what they seem to be, enthymematic. For this purpose, appeal is made to the standard constant t of relevant logic, with t construed contentwise as a zero-place connective. Constant t, which serves to represent the conjunction of theorems, satisfies the two way rule: A // t $\rightarrow$ A. Given t the new semi-lattice conditions are the following enthymematic forms:

t$\supseteq$1. t & A $\supseteq$ B & B $\supseteq$ C $\rightarrow$. A $\supseteq$ C, i.e. A $\supseteq$ B & B $\supseteq$ C $\supset$. A $\supseteq$C, where $\supset$ is an intuitionistic-style connective, defined $D \supset E =_{Df} t \,\&\, D \rightarrow E$.

$t{\supseteq}3.\ t \,\&\, C \supseteq A \,\&\, C \supseteq B \rightarrow .\ C \supseteq A \,\&\, B.$

Note that since t is a theorem, rule analogues are immediately forthcoming. Postulates $\supseteq 2$ and $\supseteq 0$ of this revised, mark II, containment logic CLII are as before. An analogue of $\supseteq 4$ is no longer required; so far as such lattice connections are required they can be derived.

Some theorems applied in showing completeness are next recorded.

T1. $A \supseteq A$, by $\supseteq 2$; whence $A - \gg A$.
T2. $A \,\&\, B \supseteq A$; $A \,\&\, B \supseteq B$, by $\supseteq 2$.
T3. $A_1 \supseteq A_2 \,\&\, t \rightarrow .\ A_1 \,\&\, B \supseteq A_2 \,\&\, B$.
For, as $t \rightarrow .\ A_1 \,\&\, B \supseteq A_1$ from T2, $A_1 \supseteq A_2 \,\&\, t \rightarrow .\ A_1 \,\&\, B \supseteq A_2$. But, by T2, $t \rightarrow .\ A_1 \,\&\, B \supseteq B$ also; whence by $\supseteq 3$, $A_1 \supseteq A_2 \,\&\, t \rightarrow .\ A_1 \,\&\, B \supseteq A_2 \,\&\, B$.
T4. $B_1 \supseteq B_2 \,\&\, t \rightarrow .\ A \,\&\, B_1 \supseteq A \,\&\, B_2$. Similar to T3.
T5. $A_1 \supseteq A_2 \,\&\, B_1 \supseteq B_2 \,\&\, t \rightarrow .\ A_1 \,\&\, B_1 \supseteq A_1 \,\&\, B_2$. By T3 and T4.

The model structure is like that for CLI *except* that $<C(a), \wedge>$ can now simply revert everywhere regular to more amenable semi-lattice form. That is, operation $\wedge$, defined (if you prefer) in C(a) for every a in K, is *within* O a semi-lattice operation: commutative, associative *and* idempotent. Soundness and completeness arguments for CLII vary those given already for CLI. Soundness is much as before, and verification of $t{\supseteq}1$ will indicate all main points. The previous argument as to $\supseteq 1$ will work when duly restricted to arbitrary a in O. But that a is in O is guaranteed by the adjunction of t to the antecedents. For by the relevant rule for t, $I(t, a) = 1$ iff $a \in O$. Furthermore, it is evident why $\supseteq 1$ itself, by contrast with $t{\supseteq}1$, breaks down. For $a \notin O$, $I(A \supseteq A, a)$ can be assigned value 0 when the antecedents are all assigned 1.

For completeness, the definition of $|A|_a$ is upgraded, inasmuch as that, for $a \in O$, standard equivalence-class requirements are now met. For instance, as $t \rightarrow .\ A \supseteq A$, $\tilde{a}$ is reflexive at each a in O. For $a \notin O$, $|A|_a$ is virtually a Don't Care: any passable definition will do, as nothing extradefinitional has to be established. For definiteness let $|A|_a$ be $<A, a>$ for $a \notin O$. Other crucial new details concern the canonical modelling of $\supseteq$, i.e. proof that $I(A \supseteq B, a) = 1$ iff $A \supseteq B \in a$. For a not in O, this is a matter of stipulation for initial wff, and hereditariness follows in the usual way. Suppose then a is in O. The argument is as before except for the final step,

$$A \supseteq A \,\&\, B \in a \,\&.\, A \,\&\, B \supseteq A \in a \text{ iff } A \supseteq B \in a,$$

which previously applied $\supseteq 4$. Suppose now $A \supseteq A \,\&\, B \in a$. As $a \in O$, $t \in a$ and $A \,\&\, B \supseteq B \in a$ because all theorems do. But a is closed under adjunction and provable implication, so by $t{\supseteq}1$, $A \supseteq B \in a$. Suppose conversely, $A \supseteq B \in a$. As $a \in O$, $t \in a$ and $A \supseteq A \in a$, so by $t{\supseteq}3$, $A \supseteq A \,\&\, B \in a$. Also as $a \in O$, $A \,\&\, B \supseteq A \in a$, completing the argument.

With a batch of logics in hand, which yield through one of their primary implication connections, namely $-\gg$, only tight (nonsuperfluous) relevant consequences, we are better prepared for applications to typical problem areas. Applications to issues which call for removal of $A -\gg. A \vee B$ (such as Ross's "paradox" in deontic logic) are immediate. Let us consider then a less immediate and much more ambitious application.

5. *An application to certain frame problems*

The main problem, as often portrayed, consists in suitably confining the informational and inferential frame of an automaton or suchlike artefact – in particular, so it can act in requisite time on *relevant* information and not be sidetracked on *irrelevant* inference-making. The very general setting of the problem is this: In the course of planning and acting, an automaton needs both a representation – model or some such – of its environment and also to be able to update that representation as circumstances change, for instance because of its own changing position and impact on the local environment. Several of the problems in the bundle of problems that go under the head of "the frame problem" are then but pointed instances of much more general problems. Such are the problems of excess information and of requisite pruning of information bases and restriction of search spaces, and also those problems bound up with suitable formalisation of nondemonstrative inference. (5)

(5) The many and various problems that now go under the designation "the frame problem" are distentangled in another paper, "Frame problems of artifical intelligence, relevance and alternative structures", typescript, Canberra, 1987.

But a major problem, sometimes accounted a "control problem", just is that of relevance of what is inferred. The logical apparatus of an automaton typically enables it to make (potentially) infinitely many inferences from the information it has stored. But much of that information and many of the inferences it might make will, even if modally valid, be irrelevant to solving specific issues confronting it. A significant part of the problem reduces to that of limiting inferences to those relevant to the issues at hand.

A relevant containment implication which does not allow the iterated introduction of extraneous material can accomplish much of what is sought. For such problem-solving situations the automaton's logic is a relevant containment one. Thus it does not derive extraneous material, or waste important time piling up irrelevant consequences of the problem data. This *internal* resolution of the logical problem can be contrasted with Dennett's external tagging suggestion. Dennett's designer tried to teach their newest and best model "*the difference between relevant implications and irrelevant implications* ... and to ignore the irrelevant ones. So they developed a method of tagging implications as either relevant or irrelevant to the project at hand." But this robot-relevant-deducer spent its time listing implications and tagging as irrelevant those it should ignore, when urgent evasive action was required. An automaton programmed with a relevant containment logic would spend no time on such irrelevant details. Given relevant inputs (a significant control matter), it would arrive, and arrive directly, only at tightly relevant conclusions.

It is not pretended that such relevant programming would solve "the" frame problem; of course it does not. But it could chip a worthwhile piece off one of the conglomeration of problems involved. How then is tight relevance programming to be implemented; how, in particular, is relevant theorem proving to be duly mechanised?

A promising approach reformulates relevant containment logics as subscripted tableaux or natural deduction systems. These systems work with expressions of the form A_c, where A is a wff and index c is a situational subscript. The tableaux formulation in fact corresponds directly to an operational reformulation of the semantics, and the natural deduction and tableaux formulations are effectively transformations of one another (on both points cf. [11] chapter 11). The idea is that *subscripted* tableaux or natural deduction theorem proving procedures can take advantage of very many of the features already worked out for corresponding *un*subscripted procedures. But that idea has yet to be followed through; the hard work at work-stations lies ahead. Any relevant workers about?

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